The Nature of Space and Spacetime

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Chapter 1: History of the Debate

In our lives, everything, if happens physically, takes place in space. We are so used to space as the background of everything that we often neglect the peculiarity of its nature. Is it a substantial entity that fills the universe? Or is it pure emptiness or nothingness, i.e., an absence of any substantial entity? And what is the nature of the spatial position of an object in space? Is there an absolute space, in which we could locate an absolute coordinate, or is the spatial position of an object merely relational, defined by its relation to other objects in the universe? In the past thousands of years, philosophers endeavored to resolve these questions. With the development of modern natural science, the answers to these questions gradually revealed themselves to us. In this thesis, I will discuss the development of the philosophical debate over the nature of space, the development of the relevant physical theory, and examine closely two modern views under the general theory of relativity. In the this very first chapter of the thesis, I will introduce the history of the discussion that predates the development of classical physics.

Philosophers debated the nature of space since ancient Greece. From the modern point of view, we can divide all these philosophers vaguely into two broad categories, namely the relationalists and the substantivalists. In short, the substantivalists believe in an absolute space, and the relationalists believe that absolute space does not exist, and that the essence of space in the relations of material objects. Even though the distinction between the two schools became clear after Newton and Leibniz, some earlier philosophers such as Aristotle are also considered relationalist. Aristotle's discussion of space can be found in his *Physics*. In Book IV chapter 4 of *Physics*, Aristotle talks about the "place", which is a concept close to what we call "space" now. Aristotle defines it as "the first thing surrounding [the object]; and [is] not anything pertaining to the object; [is] neither less nor greater than the object. That every place should have 'above' or 'below', that each body should naturally move to remain in its proper places, and this it must do either above or below."[1] This sounds misleading, because it seems that "not pertaining to object" and other descriptions of Aristotle refer to an absolute space. This, however, is not quite what Aristotle means.

In simpler terms, in Aristotelian cosmology, matter is a plenum, i.e., the world is filled with matter.[2] Every body is surrounded by other bodies. And "place", under this cosmology, is thus the boundary between the surrounding bodies and surrounded body. Aristotle's line of reasoning is as follows. Space is one of the following four options: form, matter, some natural extension, or the extension from the magnitude of the body. Aristotelian form is the quiddity of the object. In this discussion, it can be roughly understood as the abstract shape of the object, actualize by "matter", i.e., physical material arranged in the shape outlined by the form. Aristotle rules out the possibility of form as place by arguing that "the form is a limit of the object, and the place of the surrounding body"[3]. In other words, the form is an intrinsic attribute of the object, while place is not. Aristotle rules out matter, because he thinks that matter is neither "separable from the object nor does it surround it, while place has both properties"[4]. In other words, like form, Aristotelian matter is also an intrinsic attribute that forms objects and cannot be separated from the objects. Aristotle also rules out some natural extension, which is a concept similar to an absolute space in modern discussions. Aristotle describes a natural extension as "static" and "always present and different from the extension of the object that changes position."[5] Aristotle thinks that it cannot be space because if it is, then when the world as a entirety moves, "place will have another place and there

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[1] Aristotle, Physics, Book IV Chapter 4, 211a 1-12, in *A New Aristotle Reader* trans. by Ackrill.
[2] Belot, 2011, Appendix C, pp. 158.
[3] Aristotle, 211b 10-14.
[4] Aristotle, 212a 1-3.
[5] Aristotle, 211b 18-21.

will be many places together", "but there is not a different place of the part, in which it is moved, when the whole vessel changes its place: it is always the same: for it is in the (proximate) place where they are that the air and the water (or the parts of the water) succeed each other, not in that place in which they come to be, which is part of the place which is the place of the whole world."[6] In other words. Aristotle thinks that the existence of a natural, static extension would create a pointless symmetry when the world moves as a unity. When the world in its entirety moves relativity to this natural static extension, it generates many different "places" for each object for each different position relative to the natural extension. But since all the objects in the world move altogether, each individual object in the world does not move relative to each other. The positional relations between objects remain the same. Their place relative to their surroundings is identical, but their place relative to the natural extension is different. Aristotle clearly thinks that the prior is what really matters, as he considers place to be something that surrounds the object. Thus, he says that a natural extension cannot be place because it pointlessly generates many symmetries that are essentially the same.

Ruling out the three other options, Aristotle naturally argues that space must be the only option left, namely the "limit of the surrounding body". Although Aristotle does not think space is an intrinsic property of the surrounded object, he does not think it can exist independent of any object. It is the boundary of the surroundings of an object. It relies on the relations between one object and all of its surrounding objects, which can thus be understood as a relational view of space.

The Aristotelian view does not exactly agree with some of the early modern views of relationalism that appeared later in history, such as those of Descartes and Leibniz. However, some

[6] Aristotle, 211b 21-29.

of his points remain central to later relationalism. Three of them remain particularly crucial as far as this thesis is concerned. The first is the to view the world as filled by a plenum. Essentially, this means that the world is entirely filled by something. Most variations of relationalism and substantivalism later follow this tradition of a plenum-view of the world, though exceptions do exist, as will be discussed in later sections. Different philosophers have different opinions about the nature of this plenum. For Aristotle, this plenum is matter. For some later substantivalists, this plenum might be the smallest unit of an absolute space or spacetime. For some later relationalists, this plenum might the smallest unity of an entity that has a more complicated definition. Nevertheless, among most substantivalists and relationalists, there is a general consensus about the existence of such a plenum, whatever it exactly is.

The second is the rejection of symmetry. As discussed earlier, Aristotle argues against absolute space (or in his terms a natural extension) using the argument of symmetries. He argues that if in several worlds the only difference between them is their positional relation to the absolute space, and in all the worlds every object has the same relations to everything else, there is no substantial difference among these worlds. This continues to be a central difference between relationalism and substantivalism. The relationalists continue to insist that such symmetries are impossible: they are essentially the same world, while the substantivalists continue to look for arguments in response to this worry. In the early modern discussion, Leibniz proposed an almost same worry about symmetry using Leibniz shifts and boosts, which are essentially a variation of Aristotle's example in Newtonian physics. This problem continues to exist in the general theory of relativity. In general relativity, it takes the form of "the hole argument", questioning the symmetry of different metric manifolds that are associated by a hole transformation.

The third point that is worth mentioning is the relationship between space and motion. Aristotle argues that "place would not be a subject of inquiry" if there were no motion, i.e., the change in place.[7] There is no point to talk about the abstract concepts of position and space in addition to the material world unless motion or the change in place is relative to it. This does not mean that Aristotle thinks entire world is in motion. Rather, he thinks that everything in the world has a potence for motion, and thus everything has a place and place is a topic worth discussing. Therefore, we see that many arguments he uses to rule out other options of space, for example the arguments he uses to rule out a natural static extension, concerns motion. This close association between space and motion continues to exist in the history of the discussion about space. For example, Newton defines the absolute motion as being relative to an absolute space. The possibility of the measurement of the absolute motion continues to be a debated subject even after the development of the theories of relativity. These will be discussed in later sections.

As we can see from above, Aristotle's view is a good place to start our inquiry about the history of the relational-substantival debate about space. It is one of the earliest formulations of relationalism, and we can trace many traditions in the modern discussion to it. Yet, it was not unchallenged. Some contemporaries of Aristotle disagreed with him, proposing their own interpretation of the nature of space.[8] I will not go into too much detail about these variations, not only because I am limited by the length of this thesis, but also because the ancient philosophers, however well-organized their theories are, based their theories mainly on their own contemplations and limited observations. At that time, it was beyond their capability to test whether these theories are accurate in describing our actual world (and most of the theories are not). The accuracy of

[7] Aristotle, 211a 11-24.
[8] Belot, 2011, pp. 160-172.

natural philosophy improved to a degree that can be called a natural science today only much later in the 16th to 17th century. With the improvement of natural science, philosophers developed new views about the nature of space. The early modern natural scientists and natural philosophers such as Newton and Leibniz applied both mathematical and experimental methods in founding the basis of classical physics. In their discussion of the nature of space, they argue both from logical reasoning and from the physical phenomena they observe and calculate, and offer some insights into the nature of space.

Chapter 2: Accounts from Classical Physics

Newton was perhaps the most significant substantivalist in the history of our debate. With the development of his classical physics theories (replacing the inaccurate Aristotelian physics), Newton argues for the existence of absolute space. In this chapter, I will introduce the basics of Newtonian or classical physics, and discuss the views of substantivalism and relationalism under classical context.

To understand Newton's substantivalism, we must first understand his physics. In Aristotelian physics, some motions could happen spontaneously. For example, heavy objects such as stone are considered to have a nature to fall.[9] This is impossible in Newtonian physics. The three fundamental laws of mechanics in Newtonian physics are as follows: 1. Objects remain at rest or at a constant velocity unless a force is exerted upon it to change this inertial state. By a constant velocity we mean a vector, i.e., that both the magnitude and the direction of its speed are constant. This tendency of objects to remain unchanged in their state of motion is called inertia. 2. The net force exerted on an object is equal to the product of its mass and its acceleration. 3. Every force

[9] Bodnar, *Stanford Encyclopedia of Philosophy*; and Augustine, *On Free Choice of Will*, 3.1.2.6-3.1.2.11.

exerted by one object on another object always has a reaction that is the same in magnitude and opposite in direction. Apart from these three laws of motion, Newton also derived a law of universal gravitation between any two masses m_1 and m_2, $F = G\frac{m_1 m_2}{r^2}$, which states that the gravitational force is proportional to the product of the two masses divided by the square of their distance by a factor of G which is a gravitational constant whose value is empirically found.

Now, having defined these physical laws of motion, Newton now attempts a philosophical explication of the concept of motion itself. Like Aristotle, Newton defines motion as a change of space, but he has a different opinion about what this change is relative to. Earlier philosophers such as Aristotle and Descartes hold a view closer to relationalism: they think motion is relative to its surroundings or some other objects.[10] Newton argues against this. He thinks that it is impossible to define some object being at motion or at absolute rest by examining its motion relative to its immediate surroundings, because according to this definition, it might be possible that some near object and some far object could both be at absolute rest relative to their immediate surroundings, but at motion relative to each other. Instead, Newton proposes the notion of absolute motion of the entire substantial universe. More specifically, Newton considers all the substances in the universe altogether as a single entity, and proposes that this entity is in a motion. This motion is what Newton calls an absolute motion. All the motions of specific objects in the world, the motion we see in every life, such as the falling of a stone or the flying of a bird, are merely internal motion of within the universal entity. They are thus merely parts and components of the absolute motion.[11]

To put it in a simpler way, the world is analogous to an elevator (imagine there is just the elevator and no gravity here). These motions are analogous to throwing a ball horizontally in a

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[10] Newton, *De Gravitatione*, 1.-4..
[11] Newton, *De Gravitatione*, 6.-8., 10.; and Scholium to the Definition in *Principia*, definitions II.-IV.

moving elevator. The motion of the ball is an internal motion within the elevator, and is a part of the motion of the elevator as a whole. The internal motions only matter if we are in the system, i.e., the elevator. They do not affect the overall motion of the system from an outside point of view, just like we cannot tell if the person is throwing a ball or not in the elevator if we stand outside of the elevator (assuming it is not transparent). Nor can we tell, if we are inside the elevator, whether the elevator and thus ourselves inside the elevator are at rest or moving at a constant velocity. Locally, inside the elevator, we might feel we are at rest relative to our immediate surroundings, i.e., the elevator walls, but the elevator as whole might be moving relative to an elevator shaft at a distance from us. Newton's absolute space is analogous to the elevator shaft here. If this absolute motion exists, it must be relative to something. The absolute motion of the entire universe is relative to some background at absolute rest. Otherwise, this motion cannot be identified as a motion. There must thus be something at rest aside from this universe in motion. And this is what Newton defines to be the absolute space.

The parts and components of the absolute motion are called relative motions by Newton. Newton distinguishes relative motions from the absolute motion by defining that the absolute motion can be generated or altered only when forces are exerted on the body, while relative motion can be generated or altered without the presence of force. This is derived from his definition of absolute and relative motion: since relative motions are merely parts of the absolute motion, what we observe as objects in (relative) motion could merely be an apparent effect of the absolute motion which is itself not apparent. Therefore, the absolute motion, if altered, must require an external force on the entire universe, whereas a change in a relative motion might just be reflection of effects of the absolute motion without experiencing a real external force.[12] Consider, again, our

[12] Newton, Scholium, definition IV, VIII-XI; and Sklar 182-191

elevator analogy. In an elevator accelerating in the horizontal direction, a person inside the elevator experiences an inertial effect. Relative to the reference frame of the elevator, the person seems to be moving left or right. But this change in the state of motion of this person is not caused by any force exerted on this person. There is not external force exerted on this person. This change in relative motion has to be understood as a part of the absolute motion of the elevator. By Newton's laws of motion, when the elevator is experiencing a net acceleration, there must be a net external force. Therefore, a relative motion can be generated or altered without the presence of an external force, while a change in the absolute motion requires an external force.

After defining the absolute motion as relative to absolute space, Newton looks for physical evidence for such an absolute space. Newton supports the existence of the absolute space he defined with several experiments in physics. The first one of them is the example of a rotating water bucket. In this experiment, Newton imagines a bucket containing some water being hung by a rope to the ceiling. The rope is twisted as much as possible. Then, once the rope is released, the rope itself and the bucket both will start to rotate about the vertical axis of the rope. The rotation will have the most potential energy at first, for at this point the rope is twisted the tightest. Therefore, the force of tension and thus the acceleration will be the strongest at the beginning and, the acceleration will gradually decrease. The speed of rotation will increase until the potential energy from the rope is completely converted to kinetic energy (and converted to heat through friction), and then the rotation will start to recede (skipping over the steps of fluctuations) until the energy is completely exhausted through friction and the bucket becomes at rest again. The rotation of water, however, will never be synchronized with the rotation of the bucket due to inertia. The water in the bucket will not rotate with the bucket at first, but will gradually rotate faster and faster until it catches up with the rotation of the bucket. But then, as the rotation of the bucket recedes, the rotation of the water continues to exist, and when the bucket is at rest again, the water alone will still be in rotation. During this process, the surface of the water in the bucket will be flat at first, but will gradually become concave. Water will leave the center of the bucket and aggregate near the walls of the bucket, causing the surface to be concave. This does not happen at the beginning, when the rotation is the strongest (i.e., when it has the most acceleration). It only gradually happens when the motion of the bucket becomes weaker and weaker, and after the bucket is at rest again, the surface of water will remain concave as the water will continue to rotate. It will become flat when its energy is finally exhausted and when the water becomes completely at rest again, but this happens only much later than the cessation of the rotation of the bucket.[13]

Based on these physical phenomena, Newton gives his own philosophical interpretation of space. He argues that the force that makes water ascend up the walls of the bucket and descend in the center of the bucket cannot be due to the rotation of water relative to the bucket, as its strength does not correspond to the strength of this relative rotation. Likewise, we cannot attribute such an effect to any other ordinary motion relative to the walls or the laboratory etc.. Therefore, if the effect is not a result of any relative motion, it must be because of some true, absolute rotation beyond the apparent, relative rotation between the water and the bucket. This supports the existence of the absolute motion of the entire universe, as we defined earlier.

In addition to the water bucket experiment, another thought experiment seems to hint at the existence of absolute space as well. Newton uses this example to demonstrate that inertial effects alone are indications of motion. Considers a world with only a system of two globes and one rope connecting them. The system of the two globes could either be rotating about its center

[13] Newton, Scholium, XII.

of mass (the central position between the two globes), or it could be at rest. If it is at rest, nothing happens. If they are rotating about their center of mass, then the two globes both experience a centrifugal force, an effect that gives them a tendency to move away from the center. This is an inertial effect similar to what the water in the bucket experiences when rotating. In such a rotation, there is nothing else in the world the motion is relative to. In addition, the distance between the two globes is not changed. The position of the globes relative to the rope does not change either. No positional relation has changed for the system of two globes. Thus, for the relationalists, they cannot define the system to be in motion despite that it clearly is rotating. For them, the globes are at rest even though the two globes are rotating about each other. However, even if the relationlists define the system as stationary, the inertial effects and the tension still exist in the rope connecting the two globes as a result of the centrifugal force of the two globes. These effects (the tension and centrifugal forces) exist without the presence of a relative motion. In addition to this, it is clear that by our intuition it does not make sense to say that such a pair of rotating globes are not in motion. Newton thus considers these inertial effects precisely the indication of a motion. Since these effects cannot come from any relative motions as they do not exist in this world, there must be an absolute motion and thus an absolute space. Therefore, a Newtonian substantivalist would consider this to be a serious issue for the relationalists, as they cannot attribute the inertial effects to any motion, while they are clearly indications of some motion.[14]

These ingenious examples about rotation seem to imply the existence of an absolute space. Like Einstein suggests, rotation of system is something that does not belong to the system itself. It has to be understood from a viewpoint outside the system.[15] Therefore, this seems to imply that

[14] Newton, Scholium, XIV.
[15] Einstein, 1920, *Ether and Relativity*.

the existence of material world, is insufficient to explain rotation. There needs to be something irreducible to the material world to serve as a background for rotation. And substantivalists like Newton argues that this has to be an absolute space. The relationalists, however, takes an alternative approach to the problem of rotation, attempting to explicate the problem without resorting to an absolute space. One attempt is to dismiss the problem of rotation by defining rotation as a compound of rectilinear motions. Leibniz, for example, suggests that all motions, including rotations, are essentially rectilinear motions. Leibniz argues that the existence of centrifugal force precisely proves this, because the centrifugal forces are in the tangential direction, i.e., the particles (the smallest parts) of a rotating object in fact experience linear motion in the tangential direction, but they are continually redirected at every instant by their immediate surroundings through impact. In other words, they instantaneously move in a straight line, but at every moment, they bump into another particle surrounding them, which redirect their trajectory into a polygon with infinitely many sides and angles, which eventually becomes a circle. Therefore, all circular motions and rotations can be broken down into segments of rectilinear motions.[16] Thus, it seems that we can resolves the problem of rotating water bucket by arguing that the concavity of the surface of water is a result of collisions caused by rectilinear motion, not by effects of absolute motion relativity to absolute space.

Leibniz's account is quite obscure. Another relationist, Huygens, has a better and clearer explanation of the inertial effects that agrees with Newtonian physics. Consider the rotating globes again. As Leibniz said, if the rope were cut, then the globes would fly away in a linear motion in the direction of the tangential centrifugal force. Huygens argues that the rope is essentially exerting an acceleration against this motion. This way, he breaks down the rotation into two linear motions:

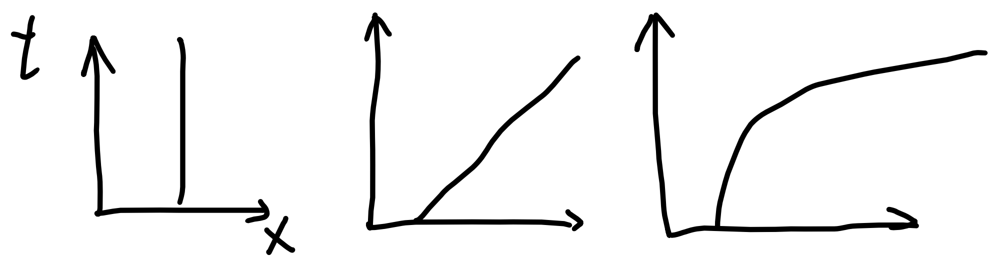
[16] Stein, pp. 3-7.

one uniform linear motion in the tangential direction and one acceleration in the direction towards the center of mass. Huygens suggests a new definition of motion: not by the change in the relative position of objects, but rather by a difference in velocity. A circular motion involves a constant change not in the magnitude of the velocity but in the direction of it, and thus should be considered as a motion even though there is no change in the relative position within the system. Therefore, in rotation, although the relative position between the two globes remains the same, there is a difference in velocities. According to Stein, the insight of this seemingly lame argument is to argue from here that the absolute space is not required to explain the inertial effects. The centrifugal force is indeed an indication. But it is not an indication of an absolute motion but rather that of a circular motion. There is no direct evidence that requires an absolute space as long as we find a sufficient way to explain the motion. Stein suggests that Huygens's definition sufficiently explains motion using an observable concept, velocity-difference, without having to resort to an unobservable absolute space.[17]

What Stein means here is not a "velocity difference" in the sense of a car accelerating when we step on the gas. Rather, he is talking about a "velocity-difference" in a much more mathematical way, introducing Gallilean spacetime, the geometric representation of Newtonian space and time. Space in Newtonian physics can be represented mathematically by a Euclidean space.[18] In popular language, a one-dimensional Euclidean space is simply a collection of points, and each can be assigned a parameter from negative infinity to infinity. In Newtonian physics, space is represented by a three-dimensional Euclidean space. At every moment, we can construct a three-dimensional Euclidean space with a coordinate system to represent objects and their spatial position with three

[17] Stein, pp. 7-10.
[18] Maudlin, 2012, Chapter 1, pp. 5-8.

mathematical parameters. Whether the coordinate system itself and the choice of origin is absolute or not is a topic of debate for the substantivalists and relationalists, but nevertheless they all agree that space can be represented in such a mathematical manner. If we put together all the Euclidean spaces from all instants by their order in time, we get a series of Euclidean space in the evolution of time. We can thus see the trajectory of an object in space over time. Although Newtonian physics, space and time are considered to be two completely different things, we can still make such a space-time (not spacetime) plot just for the clarity of mathematical presentation. Of course, a real space-time diagram in this form is four-dimensional with three dimensions of space and one dimension of time. For the sake simplicity, the example here uses just one dimension of space and one dimension of time, but the gist behind the diagrams should be clear. The position of an object at one instant, considering only its position in the x-direction, can be represented by a point on a number line. If we stack many points on many number lines from many instants, we can make space-time diagram for the object just like the following:



The above three diagrams represent, from left to right, the space-time trajectory of an object at rest, at constant velocity, and in acceleration in the x-direction respectively. The velocity is defined by $\frac{dx}{dt}$, or the change of position at ever instant. Algebraically, it is the slope of the tangent line of the trajectory of the object at every instant. The velocity or the slope for an object at rest is 0. The velocity or the slope of an object in uniform motion is a constant number. The velocity of an object in acceleration or even more complicated motion is a polynomial dependent on time. The velocity-difference of Huygens, according to Stein, should be understood in this sense. To say that objects are in real motion in this definition, the velocity of it, or the slope of it in such diagram, must be different from the velocity or the slope of another object. Therefore, under such a definition, motion has to be relative, because his difference in velocity has to be defined with at least two objects. The reasoning for this is intuitive: if we place ourselves in the reference frame of any object in constant velocity or at rest (so that Newtons laws preserve), the space-time diagram of it will always just be a vertical line. The object, from its own frame of reference, always assume itself to be at rest and other things to be moving relative to it. When we look out through the window of a moving train, we intuitively think that the station and the views are themselves moving backwards away from us. When we look through the glass of an elevator moving upwards, the floors seem to be descending away from us. But no matter which reference frame we choose, as long as there are two objects moving at different velocities, this difference must be reflected in the slopes in the space-time diagrams. If we are in the train frame, the station must have a different velocity different from the train that seems to be at rest, and if we are in the station frame, the train must have a different velocity from the station that seems to be at rest. And this difference in velocity is precisely what Stein suggests that Huygens defines as the criterion for motion. Even if this is not what Huygens was thinking about, at least, this is an alternative offered by some later, modern successors of relationalism. With this new definition, relationalists can have an absolute criterion for motion without resorting to unobservable absolute space.[19]

The introduction of these space-time diagrams allows relationalists not only to be able to define motion without resorting absolute space, but it also helps them to explicate the Newton's rotating globes in a relationalist way. By introducing a frame of reference centered around the

[19] Stein, pp. 9-10; and Maudlin, 1993, section 4, pp. 192-196.

center of mass of the system and fixing the frame of spacetime at one moment t_0, (i.e., this frame of reference does not rotate with the system), the trajectory of the rope and the globes can be easily represented in the spacetime diagrams as motion. This is because the reference frame, or the coordinate system, is fixed at one instant. The system in motion always has a different position in relation to its position at the previous moment, if looking from the initial fixed reference frame.[20]

This might sound like cheating. It seems that relationalists are regulating an absolute origin, which sounds very much like substantivalism. For the relationalists, arguably, this is allowed because it is compatible with the relational definition of space. The reference frame (or, the coordinate system) is fixed not absolutely, but rather fixed relative to the objects at one instance. Therefore, whenever a relationalist determines the spatial position of an object at a different time relative to this coordinate system, he is not making a reference to a coordinate system based on an absolute space that exists across all time frames even if there is no object in it. Instead, the relationalists are making a reference to objects at a different time frame, which means the spatial position is still relational in terms of objects, not absolute. The relationalists are merely overlapping the positions of a later time frame on those of the initial time frame. This response, satisfying or not, at least opens up a possibility for a cosmology without an unobservable absolute space. Perhaps the substantivalist could argue that fixing the reference frame at one instant is not really relationalist. Nevertheless, other parts of the argument allow us to define motion without using anything absolute, or at least not anything absolute in the Newtonian sense of absolute sense. The relationalists, for the sake of parsimony, seems to be favored now, as they do not include in their theories a possibly redundant concept of absolute space.

[20] Maudlin, 1993, section 3, pp. 188-192.

Of course, the relationalists are not merely responding to attacks from substantivalists. They also actively make some attacks on substantivalism themselves. One argument the make is the argument from symmetry, similar to the problem discussed by Aristotle. Leibniz, for example, argues that an absolute space violates certain principles that he holds to be true. Leibniz, in his correspondence with Clarke, suggests that from his principle of sufficient reason, that "there ought to be a sufficient reason why things should be so, and not otherwise", one could derive all propositions in metaphysics as well as all principles in natural science. Leibniz argues that the existence of an absolute space is incompatible with this principle of sufficient reason. This is because Leibniz thinks that the absolute space has no inherent regional differences: apart from the things located there, every point in the absolute space is essentially the same. No point possesses any haecceitist quality, i.e., qualities that uniquely belong to each individual point. Then, when God created the world, there cannot be any sufficient reason for why he placed something here rather than there.[21] Certainly, we do not have to accept Leibniz's theology to use his argument against substantivalism. But the argument could also be phrased as "there cannot be sufficient reason for something to be here rather than there" without mentioning anything about God. In other words, to accept the existence of an absolute space, one must also accept many more possible worlds that are exactly the same as ours in every respect except that the entire world is shifted by a distance in the coordinate system of the absolute space. The existence of these possible worlds is problematic, because there cannot be any sufficient reason why the world is the world we are in right now, but not the world that is five meters to the right.

Leibniz also suggests another principle, the identity of indiscernibles, which is common understood as something like "one would exactly be the same thing as the other" if the two have

[21] Leibniz, second paper reply, paragraph 1, pp. 15-16, from Alexander, *The Leibniz-Clarke Correspondence*.

no discernible difference.[22] This principle does not directly attack substantivalism like the principle of sufficient reason does. Rather, it undermines substantivalism indirectly, hinting at relationalism as a better alternative. For, if we accept the principle of sufficient reason, and if we accept that spatial points have no haecceitist property except their coordinate in the absolute space, then substanvalism would lead us into a dilemma, for we can find no sufficient reason for the world to be the way it is in our current world. But the principle of identity of indiscernibles provides us an easy way out. We can claim that all these possible worlds are essentially identical as long as we give up the only haecceitist property each spatial point has, namely their coordinates in the absolute space. In other words, as long as we discard substantivalism, the principle of identity of indiscernibles allows us to claim that there is no symmetry, but instead really just only one world (with perhaps different mathematical representations), and thus easily avoid the problem from the principle of sufficient reason. And what choice do we have left other defining space as absolute? It seems the best and the only option is relationalism, namely by defining space, not prior to objects in it like substantivalism does, but rather posterior to objects by the relations between the objects.

Some later substantivalists also seek help from the Galilean space-time diagrams just like the relationalists. Likewise, they attempt to get rid of the concept of absolute space and position and even velocity, but still maintain certain degree of absoluteness. Their solution, instead of using a velocity(slope) difference like Huygens, is to use slope itself. The slope of an accelerating object has to be curved, and that of an object at rest or in constant motion has to be a straight line, regardless of what reference frame we choose. Therefore, they use this property of curvedness of space-time trajectory that carries over all the possible worlds to define a real motion, namely acceleration. For, if the velocity and position of an object can possibly be different in different

[22] Leibniz, fifth paper, paragraph 53, pp. 74, from Alexander, *The Leibniz-Clarke Correspondence*.

worlds with possible shifts or boosts, then, arguably, these properties might not be real. This is because many properties are invariant across all these different worlds, such as the laws of physics. These invariant properties clearly matter, and particularly, the laws of physics, which is invariant, seem to be an essentially property for our world to be the world that it is. Naturally, one would doubt that invariance is a criterion for real significant properties, and suspect position and velocity to be unreal, perhaps merely gauge, i.e., redundant mathematical formalism. The substantivalists do not intend to give a verdict about whether this is true, but if they have an alternative definition that can avoid such a doubt from its roots, then why not adopt it? This new definition, unlike that Huygens, does not require a relation between objects. The curvedness lies in the trajectory of only one object and does not depend on the relations between objects. Although this sounds very far from the substantivalism Newton and his contemporaries' conjecture, but it nevertheless maintains a criterion for absolute acceleration and stands against the relationalist view of motion.[23]

Even if we neglect this possible response, Leibniz's argument for relationalism still does not seem to be very strong. It requires several premises, namely the acceptance of the two principles of Leibniz and the rejection of haecceitism for individual spatial points. And none of these are really that self-evident and immune from any challenge. Nevertheless, although the consequence of indiscernibility is challenged, the indiscernibility itself is quite commonly accepted. There is no way that we can find out whether our world is here or there (if there really is a difference between here and there), i.e., whether there is a Leibnizian shift, nor is it possible for us to find whether the world altogether is at rest or moving at some constant velocity, i.e., whether there is an Leibnizian boost. Some modern philosophers argue against this[24], but their

[23] Maudlin, 1993, section 4, pp. 192-196.
[24] Middleton and Ramirez.

argument seems to me not convincing enough to be commonly accepted, and I will go no further into this. I will proceed with a temporary conclusion that if we reject the Leibnizian view, we are in some position of inaccessibility to the physical truth about the world.

Of course, this worry is not necessarily problematic. Although limiting things to the observables by the law of parsimony seems to be a good thing, but, as some argue, physics is not a science merely about the observables. It usually makes predictions about the unobservables based on observable evidence.[25] It seems, then, that in classical Newtonian physics, there are arguments on both sides, and neither substantivalism nor relationalism can claim for itself a substantial victory. How would the philosophical discussion evolve, as we move into the next stage in the development of physics, when the revolutionary theories of modern physics, especially the theories of relativity, come into play and provide a totally different description of space and time?

Chapter 3: Physical Theories of Relativity

In the 20th century, Einstein, one of the most prominent and well-known physicists in human history, developed the special and general theories of relativity. Relativity is now commonly accepted as the most accurate theory we have to describe the macroscopic mechanical phenomena in life, and greatly influence our understanding of the nature of space. This chapter presents an introduction to the physical theory of the special and general theories of relativity.

The special theory of relativity started from the particle-wave debate of light. Physicists once regarded light as a wave. It was thought that a medium is necessary for the propagation of any wave. Ether was thought to be the medium for light and other electromagnetic waves. The Michelson-Moley experiment conducted in the 19th century, however, shows that ether, in the

[25] Maudlin, 2012, Chapter 2, pp. 46.

traditional sense as matter, does not exist. Based on this, Einstein makes the revolutionary postulate that the speed of light is invariant in any reference frame. Using this postulate and the postulate that the same laws of physics hold in all inertial frames, Einstein derives a systematic physical theory about spacetime from them. This theory is the famous special theory of relativity.

The second postulate about the invariance of physical laws seems intuitive, as it exists in classical physics as well. But the first postulate leads to many curious consequences. Consider, for example, a moving train. Its speed relative to a stationary person standing near the railway is v. Suppose another dog is running on the ground, relative to this stationary person, at a speed u_0. What would the speed of the dog relative to the person be, if it is to run at the speed u' with the same magnitude as u_0, relative to the train on the train? (For mathematical presentation, the primed values are in the moving frame and the unprimed values are in the rest frame). Intuitively, this new speed u is just the sum of u' and v, and the perception of time for a person in the frame of the stationary person, the train, and the dog should all be the same. To put it mathematically, $u = u' + v$ and $t = t'$. The position of a point x in the stationary frame, would be $x = x' + vt$ in the moving frame, as the point would be moving away from the proceeding train at the speed of v. With some mathematical manipulation, we can also represent the values in the moving frame using the values in the stationary frame: $x' = x + vt$, $t' = t$, and $u' = \frac{dx'}{dt'} = u + v$. This is the Galilean transformation between different frames in classical physics. But what happens if add to this imaginary moving train Einstein's postulate about the invariance of the speed of light? What happens if we replace the running dog with a beam of light moving at speed c, the speed of light? Using the Galilean transformation, it should be $c' = c + v$. But the postulate does not allow this. Instead, it claims that $c' = c$. Something must have changed for the light beam on a moving train. The velocity of light can be expressed as the time derivative of its position. If the velocity of light is invariant, then the distance that it travels and the time it takes to travel must have changed in order to keep the velocity invariant. Mathematically, this is made possible through Lorentz transformation. Lorentz transformation states that $x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}$, $t' = \frac{t-\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$, and , $u' = \frac{dx'}{dt'} = \frac{u-v}{1-\frac{uv}{c^2}}$. When $u = c$, $u' = \frac{c-v}{1-\frac{cv}{c^2}} = \frac{c-v}{\frac{c-v}{c}} = c$. The factor $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is denoted as γ. It is dependent on the ratio between the speed of the frame and the speed of light. The higher the speed of the frame v is, the larger this factor is.

What does this mean physically? Consider, for example, a length between two points, x_1 and x_2, in a stationary frame, $L = x_1 - x_2$. If the two points are moving at velocity v, by Lorentz transformation, their positions are x'_1 and x'_2, and their length is $L' = x'_1 - x'_2 = \frac{x_1-vt}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{x_2-vt}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{x_1-x_2}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma L$. The higher the velocity v, the larger the factor γ, the greater the longer the stationary length L appears to be in the moving frame. Relatively, as the length moves faster, it appears to be shorter in the stationary frame. This phenomenon is called length contraction. Likewise, a time interval $T = t_1 - t_2$ will become $T' = t'_1 - t'_2 = \frac{t_1-\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{t_2-\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{t_1-t_2}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma T$. The time interval experienced in the stationary frame becomes longer in the moving frame, and relatively the time interval in a moving frame appears shorter in the stationary frame. In other words, everything else moving at a much lower velocity would look like slow motion to a person moving at high velocity. This phenomenon is called time dilation.

Length contraction and time dilation are not merely some unverified mathematical fabrications. Increasing physical evidence confirm the accuracy of special relativity. One of the earliest and the most famous examples is the muon experiment. Muons are small particles that have a very short lifetime. They disappear after only a few microseconds. They exist in cosmic radiation. They can pass through the atmosphere and are detected at the ground level of earth. From the classical perspective, this is impossible: it takes much longer than its life for a muon to penetrate the atmosphere and reach the surface of earth. This is thus considered evidence for special relativity: special relativity provides a good explanation of this. Muons are moving at high velocities. Therefore, although their lifetimes are still only a few microseconds in their own moving frame, in our comparatively stationary (or low-speed) frame of earth, the lifetimes of muons are time-dilated, and thus can exist much longer than they should. As for the muons, they experience length contraction: the supposedly long distance from the atmosphere to the ground becomes length-contracted, allowing the muons to travel through the distance within its short lifetime of only a few microseconds.

These phenomena, together with the Lorentz transformation, imply that the classical view of space and time is wrong. The length of objects and the lapse of time are relatively to reference frames, and there is not an absolute measure of them. Further, time and space are not two separate and independent entities. Rather, as we can see from the Lorentz transformation, the transformation of time and space are dependent on each other. Therefore, in special relativity, the two are considered together as a unity, namely spacetime. The discussion of the nature of space, described in the previous sections of this thesis, is really a discussion of the nature of spacetime (although the time-like features beyond the scope of this thesis).

The geometry of spacetime in special relativity is similar to the geometry of Galilean space-time. Likewise, the four-dimensional spacetime in special relativity is represented by a four-dimensional Euclidean space with three dimensions of space and one dimension of time. The difference is that the Galilean space-time is merely placing many consecutive three-dimensional spatial structures by the order of time, while the spacetime in special relativity is a four-dimensional unity. This spacetime is called the Minkowski spacetime.

The fusion of space and time in special relativity leads to the disappearance of absolute spatial length or simultaneity, but it also leads to a new invariant concept, namely the invariance of spacetime intervals. Spacetime interval is defined the spacetime interval ds is given by $ds^2 = dt^2 - dx^2 - dy^2 - dz^2$, where dt, dx, dy , and dz represent the change in time and in the three dimensions of space respectively. The spacetime interval of two points are invariant across all possible transformations in special relativity, i.e., $ds^2 = dt^2 - dx^2 - dy^2 - dz^2 = dt'^2 - dx'^2 - dy'^2 - dz'^2$ for any transformed coordinates dt', dx', dy', and dz'.

These are what the physical theory of special relativity tells us. They are already point towards some philosophical implications already. But there is no need to hurry: the role of special relativity in the debate between substantivalism and relationalism will be further discussed in later chapters. Like Einstein, at this point, we should still not be satisfied with merely the special theory of relativity. The special theory of relativity leaves many things unexplained, for example how gravity acts on massive objects, or why light bends around the sun. These have to be answer by a physical theory that takes a step further than the special theory of relativity, namely Einstein's the general theory of relativity.

There is very little controversy over the formulation of the special theory of relativity. Physicists have reached a consensus about the guiding principles for it, namely the two postulates. For general relativity, things are quite different. Although general relativity is a physical project about gravity, Einstein developed it following some principles that appear to be more philosophical than physical. Modern textbooks do not have a consensus about the principles count as the foundation of general relativity. In this thesis, I consult mainly the Ray d'Inverno textbook and the Sean M. Carroll textbook.[26]

The first key principle in general relativity is the equivalence principle. Its weak form of it states that the gravitational mass of an object is the same as its inertial mass. This comes the fact that the mass in the formulation of the force need to cause an acceleration $F = ma$ and that of gravity $F = G\frac{m_1 m_2}{r^2}$ are the same. (Here I use the classical formulae, but in special relativity they are not essentially different. They are formulated a little more complicated, and some use the concept of relativistic mass; but generally, it is agreed that rest mass, i.e., the mass that is similar to mass in classical physics, instead of the relativistic mass, should be used in discussion of matter). In other words, locally, the freefall caused by a gravitational field is the same as any motion caused by a uniform acceleration. This can be easily understood, if we think about the experience of taking an accelerating elevator or roller coaster: we cannot distinguish the effects of their acceleration from the effects of gravity or loss of gravity. Frames in acceleration or in a gravitational are not inertial frames. In classical physics and special relativity, everything is considered in inertial frames, as equivalence was believed to hold only in those frame. The equivalence principle, as we now see now, suggests that some degree of equivalence holds not only in inertial frames, but also in non-inertial frames. In the weak formulation, it is limited to the equivalence of effects of uniform acceleration and gravity. With the development of special relativity, which relates mass to a different physical property, energy (the famous formula $E = mc^2$ is a simplified demonstration of this), Einstein believed that such an equivalence should be limited to inertial mass and gravitational mass. In stronger formulations, this equivalence is extended to all laws of physics. This form states that in all frames, inertial or affected by a gravitational field, all laws of physics are the equivalent,

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[26] d'Inverno, pp. 120-132; and Carroll pp. 48-90, 151-152, 177-181,

including the laws about gravity, in the sense that they should all reduce to the form of the physical laws in special relativity.

Another principle that is very similar and closely related to the principle of equivalence is the principle of general covariance. It states that all observers are equivalent regardless of their reference frames. It appears to say the same thing as the strong equivalence principle, but the principle of general covariance focuses on the observers, while the equivalence principle focuses on the physical laws (and makes a specific constraint limiting the laws to only those in special relativity). In addition, to enable the equivalence by covariance, this principle also implies a constraint for the mathematical formalism of general relativity, namely that equations of physics should have tensorial form. Tensors can be understood as a sort of complicated vectors with more "dimensions". In order to determine a component of tensor, vector which is a rank-one tensor requires only one index, either x, y, or z. For a matrix which is a rank two tensor, we need two indices to determine one component of it. Since I would like to limit the discussion of mathematics in this thesis to a very basic and understandable level, I will not go further into the mathematical nature of tensors. For simplicity, we can understand it as an improved version of parameters about the world, improved particularly for general relativity to enables transformations between reference frames. The mathematical transformation between different frames is called a diffeomorphism, and in short, it is a process of mapping the spacetime points in one frame to another according to some general rules, keeping the metric relations the same. The transformation from a spacetime curved in one way to a spacetime curved in another way is an example of diffeomorphism.

The third principle, Mach's principle, is a rather philosophical principle than physical principle. It comes from physicist Mach's thoughts in response to Newton's water bucket experiment. Mach believes that all motions are relative, and there is no point to conceive an absolute motion relative to an absolute space. The inertial effects are caused by the interaction among the totality of all masses in the universe. In the context of Einsteinian general relativity, Mach's principle implies that matter determines the geometry of spacetime, and there cannot any geometry without matter. The latter half of this statement turns out to be contradictory to some solutions of the Einstein's equation, as will be discussed later.

In addition to these three, two other principles set some constraints about the mathematical foundation of general relativity. The first is the principle of minimal gravitational coupling. In, short, this is a law of parsimony. It states that from the transition from special to general relativity should take the simples way. No unnecessary mathematical terms should be added. The second is the principle of correspondence, which is an intuitive principle that requires general relativity to agree with Newtonian physics on classical limits.

With these principles, we can now construct the geometry of spacetime in general relativity. The spacetime structure in general relativity is called a manifold. It resembles a four-dimensional Euclidean space in many ways, except that it is not necessarily Euclidean, or flat. It might appear to be Euclidean locally, but its overall structure can take a non-Euclidean, i.e., curved, shape, caused by the presence massive objects in the structure. A main difference is that the geodesic, or the shortest distance between two points is a straight line in a flat geometry, while the geodesic in a curved spacetime is curved as well. A simple example is the distance on a sphere. If we intend to find the shortest distance on the surface of sphere, we cannot pass through the inside of the sphere. Therefore, shortest distance is the curve on the surface of the sphere along the arc between the two points. The overall geometry of the spacetime curvature is described by Einstein's field equations $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}$. The left-hand side is the Einstein's tensor $G_{\mu\nu}$ defined as $R_{\mu\nu} -$

$\frac{1}{2}Rg_{\mu\nu}$, using the Ricci tensor $R_{\mu\nu}$, the Ricci tensor R, and the metric tensor $g_{\mu\nu}$. The Ricci tensor and the Ricci scalar are both obtained from the Riemann tensor, which is in turn derived from the metric tensor $g_{\mu\nu}$. The metric tensor itself and the other tensors derived from it describe the geometry of the spacetime. In other words, the lefthand side of the equation is a description about the geometry of spacetime. The right-hand side is the stress-energy tensor $T_{\mu\nu}$, which is a tensor describing the density and flux of energy and matter. By equating the two side, Einstein's equations establish a mathematical representation of how the distribution of matter shapes the spacetime structure, which is the key innovation in general relativity. In short, spacetime in general relativity is no longer an independent background of matter. Instead, it interacts with matter and its geometry is shaped by matter in it. Apart from these, Einstein added a term involving the cosmological constant, $\Lambda g_{\mu\nu}$, to the lefthand side, in order to balance out the influence from matter and energy density on the expansion of the universe to create a static universe. However, physical observations shows that the expansion of the universe is indeed happening. This term is now used to account for the influence of dark energy, i.e., the energy that is not directly observable and fully explainable right now. It is also noteworthy that there also exists dark matter, the matter that is observable and explainable right now. Unlike dark energy, dark matter is described by $T_{\mu\nu}$, the tensor that describes regular matter and energy.

Einstein's equations do not have a single unique solution. There are already many possible solutions known to physicists, and there are very like many more. Each solution is also called a metric, as they describe a unique spacetime geometry. Among them, there are many non-vacuum solutions describing the gravitational field of some kind of massive object, but there also exists vacuum solutions. The stress-energy tensor represents the density of matter. When it is set to zero, it means that the universe is empty. The solutions to Einstein's equation in this case are vacuum solutions. Vacuum solutions are also used to approximate the gravitational field in our world. The Minkowski spacetime from special relativity, is in fact a simple vacuum solution. In a sense, special relativity is a special case of general relativity. Other vacuum metrics that are often used in describing our universe are the Schwarzschild metric and the de Sitter metric. The Schwarzschild metric is used to approximate the field around a spherical mass. The de Sitter metric has a zero $T_{\mu\nu}$, but a positive Λ, suggesting it is a vacuum in the sense that it has no matter and no "traditional" energy, but it might not be vacuum in a broader sense since it has dark energy. This metric is used to approximate the universe in the very early stage of the big bang.

The metric tensor, as it governs the arrangement of spacetime manifold, is not merely a one-line solution to a set of equations. It is represented by the metric field that spans across the entire spacetime manifold. The metric field assigns to every point in spacetime a metric tensor. Specifically, every point in spacetime with some coordinate (t, x, y, z), or, in spherical coordinates, (t, r, θ, φ), has a specific metric tensor describing the geometry at that point according to the overall metric tensor. For example, in the Schwarzschild spacetime, the overall metric tensor represented in spherical coordinates is $\begin{bmatrix} 1-\frac{2GM}{r} & 0 & 0 & 0 \\ 0 & -(1-\frac{2GM}{r})^2 & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 sin^2\theta \end{bmatrix}$. As we can see, it is dependent on the specific coordinates of r and θ. And each specific point will have a specific metric tensor calculated using its specific coordinates with different r and θ.

The general relativistic explanation of spacetime resolves many problems bothering the physicists. For example, when spacetime was thought be flat, the bending of light around the sun was not explicable. Light should always travel in the shortest distance. The outer space is near a vacuum and thus should have very little change in the density of the medium to bend the propagation of light. General relativity provides a perfect explanation: the spacetime itself is curved due to the mass of the sun, and the shortest distance is a curve, not a straight line. There is thus no surprise that we should see the bending of light. However, despite resolving many physical problems like this, general relativity also provokes a tremendous number of philosophical problems and implications. The new physical theory about spacetime seems to point at a completely new understanding of the nature of spacetime. In the next chapter, I will discuss one of the most important philosophical issues of general relativity, namely the hole argument.

Chapter 4: The Philosophical Implications of Relativity

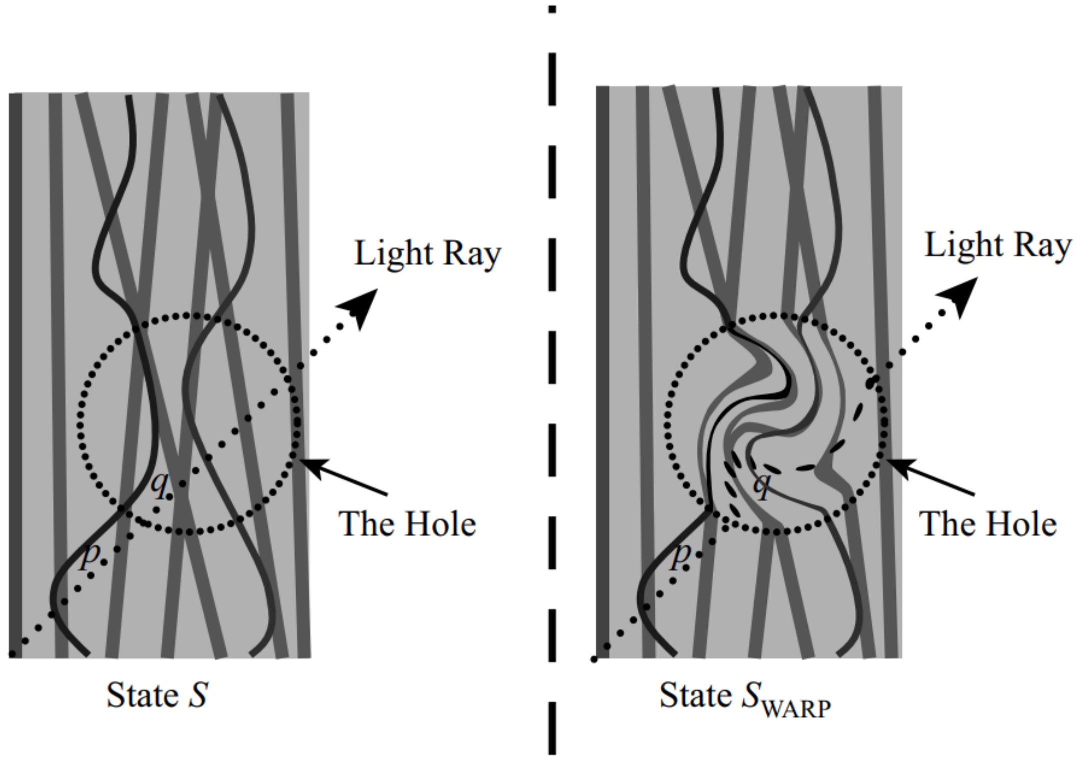
The hole argument was first conceived by Einstein himself. Modern discussions of it were reopened by Norton and Earman, who present the hole argument in a form different from Einstein's original conjecture. Today, the discussion of the hole argument is generally around Norton and Earman's version of it.[27] In this chapter, I will briefly discuss the hole argument itself, some debates over its mathematical foundation, and briefly discuss some sophisticated substantivalist response to it.

The hole argument is, again, a thought experiment, similar to Leibnizian shift in classical physics. As discussed earlier, spacetime geometry is described by metric tensor that are solutions to Einstein's equation. Therefore, it is mathematically possible that in region, there are multiple metrics that solves the Einstein's equation, each representing a spacetime geometry in that region. These spacetime geometries are related by some diffeomorphism. In other words, every point or event in one spacetime can, after some transformation, can be mapped to one specific point or event in another spacetime. Mathematically, it is also possible to connect smoothly such a region

[27] Norton, et al., *The Hole Argument*, from *Stanford Encyclopedia of Philosophy*, sections 5-8; and Earman and Norton, 1987.

to other regions. Therefore, it is possible to conceive many worlds with diffeomorphisms of a region, i.e., the "hole", smoothly connected with an otherwise identical world. A good graphic demonstration of this is the following graph from Maudlin. In the graph, p and q represents the same events in different diffeomorphisms. As shown in the graph, the identity of p in the two graphs is obvious, whereas that of q seems to require some further contemplation. Are these worlds with different diffeomorphisms of the hole different worlds? Or, are they equivalent forms of the same world? It looks like, then, that we are facing a dilemma similar to that of Leibniz shifts. However, there are some vital differences. The first is that there is no overall shifts or changes to the world. Everywhere else outside the hole is exactly the same. Therefore, the issue at heart in the hole argument is not inaccessibility to the truth about the entire universe, but rather indeterminism about only a small region of the universe despite a definite knowledge about everything in the universe. And the hole could be anywhere, of any size and shape. This suggests a very radical indeterminism: while we know definitely the geometry of the region we live in, with our physical laws we cannot extend this certainty any futher. They physical laws cannot pick between the many possibilities of equivalent physical realities in the holes that possibly all over the universe.[28]

[28] Maudlin, 2012, Chapter 6, pp. 146-152, graph on pp.148.



Indeterminism itself is nothing problematic. The modern theory of quantum mechanics, for example, is a successful indeterministic theory of physics. It discusses everything by probability and does not settle to a definite description about the world. Rather, as Norton suggests, the issue comes from the fact that only substantivalism leads to such a problem of indeterminism. If we accept the equivalence of diffeomorphism, the diffeomorphisms represent the very same physical reality, and there is no indeterminism about the world: no matter which specific diffeomorphism we choose to use to describe the region in the hole, we are describing the same content, with a trivial difference in the mathematical formalism. And indeterminism, in addition to inaccessibility, is thus a direct consequence of substantivalism, which together implies that there might be some problems about substantivalism itself. The hole argument is thus considered an argument against substantivalism.[29]

[29] Norton, et al., *The Hole Argument*, section 6.

Some might insist that even so, indeterminism is acceptable and substantivalism is not a bad option. But more attempt to defend substantivalism by trying to dismiss or argue against the hole argument. There are several ways that substantivalists respond to the hole argument. One of them is to dissolve the hole argument by attacking its mathematical foundation. Some philosophers argue that the interpretation of the physics behind general relativity cannot exceed our understanding of its basic mathematical formalism. They request some formal logic or mathematical representation of each physical interpretation. In particular, some philosophers reject the equivalence of diffeomorphism in the hole argument. They argue that the identity of the manifold before and after a coordinate transformation is not a mathematical truth that the formalism of general relativity is able to grant us. Fletcher, for example, argues that Leibniz's equivalence is in conflict with Lorentz transformation, the foundation of special relativity. His reasoning, according to Pooley, is that the Lorentz transformation does not fully account for the haecceitist, i.e., individual, properties of spacetime points. If such haecceitism is indeed correct, then the indeterminism of general relativity is inevitable: general relativity treats all spacetime points equally, and does not admit any haecceitist properties to any individual spacetime points. Therefore, the hole argument raises a problem of general relativity itself, rather than that of substantivalism. This way, Fletcher intends to defend substantivalism.[30]

Some other formalists, however, hold a different opinion. For example, Weatherall recently argues that Lorentz transformation rightfully treats structures before and after the transformation as the same structure. This is enough for us to make the interpretation that these are physically the same structure.[31] If the Weatherall's view is correct, then based on the interpretation of the

[30] Norton, et al., *The Hole Argument*, section 9.1-2, 9.5; and Pooley, pp. 14-16.
[31] Norton, et al., *The Hole Argument*, section 9.5

mathematical formalism, virtually all forms of structures related by coordinate transformations are identical and equally physically possible.

The specific argument of Weatherall is the following. Mathematics, at least right now, defines sameness with some sort of isomorphism. Isomorphism, literally, means having the same structure. This definition does not really help: what is "structure", and what is "the same structure"? What Weatherall suggests here is roughly this: there is a certain mathematical model that restricts us when we deal with certain mathematical objects. Whatever that model is, if, by the definition of that particular mathematical model, two objects have certain vital identical features, then, the two mathematical objects are the same. Which features matter is usually defined by which specific model is used. Once the standards, i.e., the mathematical model, is decided and fixed, it can take in any input, being capable of judging whether any two arbitrary objects are the same.[32]

Now, since all physical theories have some mathematical foundations, the sameness in physics in its very essence is a mathematical isomorphism. In reality, physics means something more than just mathematics. The sameness in physical situations is not reducible to mathematical identities. However, Weatherall thinks that mathematical models in physics should have the same capabilities as a pure mathematical model. If a mathematical model can be used to represent the isomorphism of two specific objects in physics, then it should also be able to be used to represent any two other arbitrary objects in physics. In his words, "if two isomorphic models may be used to represent two distinct physical situations, then each of those models individually may be used to represent both situations."[33]

[32] Weatherall, section 1, pp. 331
[33] Weatherall, pp. 331-332

Now, it may seem that I used a lengthy paragraph of obscure terms to explain something that sounds really simple. However, this is the point of formalism. There is something very basic, but not clearly defined in physics. Weatherall is thinking about something similar to Kant's legitimate domain of pure reason: are we justified to use this concept in our interpretations? To answer this, he needs to first define this clearly. This concept that Weatherall is concerned with the concept of sameness in physics, and he defined it through mathematics.

Then Weatherall demonstrates a problem with this definition of sameness with a simple example in math. Consider the basic mathematical rules that we are familiar with within a set of integers $\mathbb{Z}$. This is the underlying set, i.e., the range that we could take values from, for two groups we are going to define. Adding 0 to this set creates our first set. This set just represents the integers that even elementary students are familiar with. Within the group, there is an element x defined as "identity" (not the "sameness" we were talking about earlier). Within a group, for every element n in the group, equations $n * x$ and $x * n$ both return n. In this group, identity is 1. The symbol "$*$" does not necessarily mean multiplication. It does in this group, but what it really means is just an operation between two elements. It takes a more general name of "dot". The inverse of an element "dot" by that element returns the identity. Another operation, addition, is defined by additive identity. For every element n in the group, adding the additive identity to it returns itself. For this group, the additive identity is 0 (since the group is created by adding 0 to the underlying set), so $n + 0 = n$. In plain English, everything times one returns itself, and everything plus 0 also returns itself. This is just simple math everyone knows. This group has underlying set $\mathbb{Z}$, addition $+$, and additive identity 0. Let's call this group $(\mathbb{Z}, +, 0)$.

Now, we define another group. We define another group based on the mapping of $\mathrm{n} \rightarrow \mathrm{n} + 1$. In plain words, group is based on adding 1 to the underlying set. The additive identity here is 1.

To align with our style of notion, let's define this additive identity as $\tilde{0}$ which is equal to 1. Due to this mapping, we also define plus, dot, identity, and inverse differently. Plus is now defined as $n \tilde{+} 1 = n$. The symbol $\tilde{+}$ in this group can be expressed with $+$ from the previous group as $n \tilde{+} m = n + m - 1$. Likewise, dot can also be redefined in the new group. I will not go into the mathematical details of calculating the definition of other features of the group, but as we can infer from the definition of addition, they are very different from the features of the previous group. This group $(\mathbb{Z}, \tilde{+}, \tilde{0})$ has underlying set $\mathbb{Z}$, addition $\tilde{+}$, and additive identity $\tilde{0}$.

Here comes the problem. The two groups have the same underlying set, namely all integers, $\mathbb{Z}$. This means they have all the same elements. The two group also have the identical structure. They are defined by the same model: the identity "dot" any element returns the element itself; any element "dot" its inverse returns the identity; any element "add" the additive identity returns itself. The specific value we assign to the additive identity and thus the resulting specific definitions of the operations should not matter. Every element in the two groups is identical. The structures of the two groups are identical. We can conclude, based on the definition Weatherall gave earlier, that the two groups are isomorphic, which by our definition defines sameness. This is problematic, because we showed mathematically that the two groups differ in many ways, but by our definition they are the same. In addition, the fact that we choose to use $(\mathbb{Z}, +, 0)$ instead of $(\mathbb{Z}, \tilde{+}, \tilde{0})$ is purely arbitrary. They are equally possible under the same mathematical model.[34]

Wetherall argues that this example is analogous to the hole argument. Let's start with a simple analogy: Leibniz's shift. Leibniz asks, if the world is shifted to the right by 5 meters, is it still the same world? This can be represented mathematically as adding 5 to the coordinate of each

[34] Weatherall, section 2, pp. 332-334.

spacetime points in the universe, i.e., $x' = x - a$ for $a = 5$. It is not hard to notice that this sounds just like the two mathematical group we just talked about, with one of them had an initial addition of one to the underlying set. Now, the mathematical foundation of relativity, the Lorentz transformation, is nothing more than a complicated version of this. Instead of $x' = x - a$, it is $x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}$. There is not an essential difference between these two. The Lorentz transformation is thus an isomorphic transformation, which by definition means the worlds before and after the transformation are the same. And from the mathematical definition, Lorentz transformation does not need to account for haecceitist properties of spacetime points, because the isomorphism comes from structural properties, not individual properties of every element in the group.[35]

Despite defending the mathematical foundation of the hole argument, Weatherall himself is more inclining towards substantivalism. Weatherall argues that the equivalence before and after the transformtion is compatible with substantivalism. According to Weatherall, Norton and Earman think that substantivalists believe that spacetime points themselves alone represent spacetime. This results in the incompatibility of substantivalism with Leibniz's equivalence. Wetherall suggests that using spacetime points alone to represent spacetime is incoherent with the mathematic foundation he showed. This view suggests that the identity between spacetime points is established prior to the transformation. This is in conflict with mathematical formalism that suggest the transformation in structure is prior to the selection of the specific values of spacetime points. The gist of Weatherall's view is that it should be the opposite: the substantivalists could support that spacetime represented not merely by spacetime points but also by some other related metric. The details of Weatherall's argument are more complicated, involving more discussion of mathematics,

[35] Weatherall, section 3, pp. 334-340.

which is beyond the scope of this thesis (and probably beyond my capacity of mathematical understanding as well).[36] The key role that Weatherall's view plays is to serve as a transition from the mathematical foundation of the hole argument to the discussion of a significant category of responses to the hole argument, namely sophisticated substantivalism.

Weatherall's substantivalism in response to the hole argument belongs to a broad category named sophisticated substantivalism. These views take a more sophisticated stand about the nature of spacetime, suggesting that it is not merely the spacetime points in the manifold that represent spacetime, an attempt to respond to the hole argument. There is a great variety of views that categorized under sophisticated substantivalism, and some might share nothing in common with another except that they are both substantivalism. Some takes an anti-haecceitist view of spacetime points, suggesting that the points have no essential properties that they can carry over transformations. They argue it is these properties, instead of the entirety of substantivalism, generates the many equally possible distinct worlds. Without these properties, the world before and after hole transformation is the same, and thus there is no worry for indeterminism.[37]

Views contrary to this is haecceitist substantivalism. One of the most significant varieties of sophisticated substantivalism, namely Maudlin's metric essentialism, is often categorized as this sort. It suggests that spacetime bears their metric essentially. In the next chapter, I will discuss Maudlin's view in details and compare it to a modern adaption of relationalism.

Chapter 5: Two Specific Views

In this chapter, I will examine two particular modern adaptations of substantivalism and relationalism, namely the substantivalist view of Maudlin and the relationalist view of Rovelli. I

[36] Weatherall, section 3 and 5, pp. 334-340, 342-347.
[37] Norton, et al., *The Hole Argument*, section 9.3-9.4; Pooley, section 4, pp. 9-16.

will discuss their morals in the context of general relativity and the hole argument, and discuss some of their possible insufficiencies.

In section 5 and 6 in his *Buckets of Water and Waves of Space*, Maudlin discusses the adaptations of substantivalism and relationalism with regard to the special and general theory of relativity. In section 5[38], Maudlin considers the special theory of relativity a triumph for the relationalists. The only invariant value in the special theory of relativity for two events is their spacetime interval, which is the essentially the difference between the spacetime coordinates of the two events. As a reminder, the spacetime interval ds is given by $ds^2 = dt^2 - dx^2 - dy^2 - dz^2$, where dt, dx, dy, and dz represent the change in time and in the three dimensions of space respectively. When these spacetime intervals are determined, events can be embedded into the universe to the spacetime points according to their respective spacetime interval. Keeping the spacetime intervals the same, one could transform the placement of the events in a limited way practically with triangulation. For the relationalists, these different arrangements, according to Maudlin, are "consequences of arbitrary convention associated with setting up coordinates systems and reflect no real indeterminacy." Specifically, the Minkowski relationalists, with this sort of technique, establish an arbitrary Minkowski (or 4-dimensional Euclidean) coordinate system with three space axes and one time axis to describe the spacetime points in the universe.

According to Maudlin, the triumph of Minkowski relationslaism in the special theory of relativity comes from several reason. First, the invariance of spacetime intervals, not spacetime coordinates, suggests that spacetime is relational, not substantivalist. A spacetime coordinate system could exist by itself: at least mathematically, there could be an empty coordinate system

[38] Maudlin, 1993, pp. 196-199.

without any event or matter located in any point in the system. Spacetime intervals, unlike the spacetime coordinate system, requires the existence of two events. It cannot exist by itself. Rather, a spacetime interval is always the interval between two points in spacetime, as we can see from its definition $ds^2 = dt^2 - dx^2 - dy^2 - dz^2$. In other words, it is a relational concept between two objects occupying two points in spacetime. And this relation has to be between two objects, or hypothetical objects, because the coordinate system in special relativity is not absolute. It is constructed around an arbitrarily chosen origin relative to some object in spacetime, usually for mathematical convenience to represent the position of the object. Points in spacetime cannot exist by themselves, but rather is a placeholder for an object or a hypothetical object. Therefore, in special relativity, there cannot be an absolute interval between two absolute points independent of objects.

Second, it resolves the problem of rotating bucket. The worldlines of points in a rotating bucket near the axis and the edge are different, travelling a different spacetime interval, whereas their worldlines would be the same in a stationary bucket. The difference in spacetime intervals explains the behaviors of the water, and there is no need for absolute space(time) and acceleration.

Third, the coordinate system resolves the problem of "twice as far" raised by Field and Clarke[39]. Given two material objects at A and B respectively, without the Minkowski coordinate system, when relationalists say that the distance from A to C is "twice as far" as the distance from A to B, it is possible that no material particle exists at C. For relationalists, this is disastrous, because if space is relational, but there is nothing to relate to at that point, this statement of "twice as far" becomes meaningless. In that situation, each ratio must be considered as a new, irreducible

[39] Maudlin, 1993, pp. 197-198; and Clarck, fourth reply, paragraph 41, pp. 52.

predicate. This makes a classical relationalist definition of spacetime infinitely oversized and redundant. With Minkowski coordinate system, however, relationalists find a way around this problem. Given any segment of worldlines with two spacetime points, since the worldline of any object is continuous throughout the history of the universe, a Minkowski relationalist could embed spacetime points on all possible extensions of this segment by triangulation. In other words, these spacetime points are not empty, but rather they represent a position that will be or was occupied by a substantial object. This avoids the situation that when relationalists say "twice as far" without the Minkowski coordinate system, no material particle exists over there. The coordinate system guarantees the existence of possible spacetime points, and thus avoids this problem.

In section 6[40], however, Maudlin drastically turned against relationalism, suggesting its triumph "startlingly ephemeral" and its task "hopeless". Maudlin argues that the construction of a Neo-Newtonian or Minkowski spacetime coordinate systems is independent of the matter in them. Therefore, the embedding of the particles and spacetime events is based on the *a priori* knowledge about the nature of spacetime points. In other words, Maudlin means that in special relativity, spacetime would not change before and after material objects are placed in it. It serves as merely as a background, and does not interact with whatever that is in it. We can know the geometry of spacetime in special relativity without knowing how matter is distributed in it. Therefore, merely knowing the spacetime interval between points would be enough for a Minkowski relationalist to embed all the points, because the background, the spacetime, would not change during the process of embedding. The general theory of relativity, however, does not agree with this. According to general relativity, matter in spacetime shapes the structure of spacetime, and therefore and the structure is not known *a priori* before the embedding. This is because in general relativity, the

[40] Maudlin, 1993, pp. 199-201.

spacetime is described by a metric. And this metric is not fixed *a priori*: it depends on the material objects in the spacetime it describes. Thus, the Minkowski embedding becomes a much harder task: with every point embedded, the spacetime structure changes, and the triangulation about every particle, not only those that are going to be embedded but also those that are already embedded, has to be recalculated. Although there is no direct proof for the impossibility to fix all the points through trial and adjustment, Maudlin clearly thinks that there is very little chance for the success of such a task.

Maudlin thus argues that relationalists in the general theory of relativity have to abandon a particle-like view of the plenum space. Instead, Maudlin suggests that the relationalists are forced to accept a field-like plenum view. Maudlin suggest three possible types of fields to avoid problems caused by vacuum, namely a quantum field, a Higgs field, or a metric field. Based on current physical evidence, Maudlin argues that the metric field is the best choice.[41]

This choice, among the three options, indeed seems the most appropriate. However, it seems that three options Maudlin offers is not quite rigorous. A quantum field is a field that describes the quantum state of a specific type of particle at every point in the universe. In a sense, it can be considered to represent, although not directly, the probability distribution of a type of particle across the universe. A Higgs field is a quantum field for Higgs Bosons, one specific type of elementary particles that gives mass to particles. Therefore, technically, the Higgs field is a specific sub-category under quantum fields. So, essentially, we really only have two options only. We are looking for a field that spans all over the universe. Our first option is the metric field from general relativity that describes gravity, one of the four fundamental forces. Our second option is

[41] Maudlin, 1993, pp. 201-202.

the quantum field from the quantum field theory that can describe the other three of the four fundamental forces (or, everything other than gravity). Together, they describe all four fundamental forces, i.e., they describe everything in the world. However, based on current physical theories, general relativity is not compatible with quantum field theory. Therefore, we have to choose between general relativity and the quantum field theory. Since we are talking about spacetime in the context of general relativity, and since the quantum field theory is not very concerned with spacetime (or perhaps especially time), naturally, our best option for the plenum of spacetime is the metric field.

What are the implications of this choice, according to Mauldin? On the one hand, for relationalists, this seems to be an utter failure. The metric field is the spacetime property itself, which seems to imply the existence of absolute spacetime. On the other hand, for the substantivalists, they only need to make a slight adjustment to their original view, combining spacetime and the metric field which is a property of spacetime. Therefore, Maudlin concludes that the relationalists faces a hopeless task in the general theory of relativity.

Now, according to Maudlin, Leibniz shifts or boosts are not applicable in the general theory of relativity. Therefore, merely looking for haecceities of individual spacetime points seems quite pointless, as there can be no multiplicity generated by Leibniz shifts or boosts. However, there is an analogous but more damaging argument, the hole argument. The hole argument is something that every adaptation of substantivalism needs to face. Therefore, in addition to examining Maudlin's view itself, we will also examine how it responds to the hole argument.[42]

[42] Maudlin, 1993, pp. 199-200.

The hole argument, as we discussed earlier, is about possible problems caused by different possible arrangements of the same events in a region in spacetime called the hole. The response that Maudlin offers in his paper *The Essence of Spacetime* is sometimes referred to as Metric Essentialism. His response to the hole argument offers a more detailed discussion about his view of what spacetime is. Maudlin thinks that the current interpretation of spacetime structure is ambiguous. Scholars tend to mix up two different layers of meaning: the first being the physical and the second being the mathematical. Specifically, in terms of general relativity, there is a purely mathematical definition of everything. A set of equations, such as the Einstein's equation, the geodesic equation, and Lorentz transformation, form the mathematic foundation of general relativity. With these equations alone, we can make a mathematical prediction about the universe without knowing exactly what each term in each equation physical meaning. In other words, we can feed in the parameters of the objects we are concerned with to the mathematical mechanism of general relativity. The mechanism will return us an output, which we can use to predict the interactions of the objects we are concerned with. The mathematical layer of general relativity allows us to make such descriptions or predictions without knowing or explaining what exactly physically happens during the mathematical manipulations. The physical layer of general relativity, distinct from the mathematical equations, does the explaining. It matches the mathematical entities to physical objects or properties. These two layers, though distinct, are often fixed up in the discussion of the hole argument.[43]

Maudlin suggests that we should distinguish the two layers. However, he denies a simple Ramsification of spacetime, i.e., saying that mathematical points are only variables representing the physical spacetime points in really. In a sense, this view deprives the mathematical formalism

[43] Maudlin, 1989, pp. 82-83.

of its meaning. By replacing mathematical concepts with variables, this view denies any difference between different mathematical forms of representations, making this a mere difference in name. What is behind all sorts of different variables is the same physical content. In other words, for different diffeomorphisms, whatever the mathematical formalism says, this Ramsified interpretation considers it to be merely symbols. The mathematical meaning does not extend to any physical meaning. They are different only by name. This avoids the indeterminism caused by the hole argument, because all diffeomorphisms are merely a symbolic difference. They all share the exactly the same physical content. But this is a problematic oversimplification, because it cannot distinguish between Leibniz shifts either. Leibniz shifts are essentially also a mathematical manipulation about the coordinates, moving it by a certain amount. If the mathematical structure is purely by name, then it should have the very same physical content before and after the shift. This is against the very central idea of substantivalism, and therefore a more subtle alternative should be sought.

Instead, Maudlin suggests that spacetime points bear their metric essentially. In plain terms, he adds another "layer" of "metric" between mathematical points and physical points. This layer of metric governs the mathematical structure of the points in spacetime, but also contains some physical meaning. Therefore, altering the arrangement of the points would change the metric of them, which touches on something essential to the points. According to this view, the diffeomorphism in the hole argument is not a symmetry of the same structure, but rather many distinct arrangements. Further, it is not just the world as a whole changes. In addition, each individual point or event becomes a different one from the point or event before the transformation. This is because the essential attribute, the metric, of each point or event is changed. The Leibniz shifts, however, are merely movements along a direction. The overall structure of the points is not changed. Therefore, Leibniz shifts under this view preserves the metric and thus would not create a new world.[44]

More specifically, Maudlin makes an analogy between spacetime points and names as rigid designators. If we consider names as rigid designators, when we swap the names of the person "Tim Maudlin" and the tower "Eiffel Tower", we are not simply matching a reference to an object. Rather, we rigidly designating "Eiffel Tower" and "Tim Maudlin" to something totally different. As a rigid designator, "Tim Maudlin" refer to a human being in all possible worlds, whereas "Eiffel Tower" should refer to a tower in all possible worlds. Certain attributes of the person and the tower might be different in different possible worlds, but the essential ones must be the same. For example, the person could have a different height or weight, or maybe even he could be missing a tooth or having a scar, but he has to a human. The attribute of being a human is an essential attribute to whatever "Tim Maudlin" rigidly designates. Likewise, whatever "Eiffel Tower" rigidly designates, it has to be essentially a tower. Therefore, swapping the names of "Tim Maudlin" and "Eiffel Tower", is not only wrong but also impossible, because in no possible world does "Tim Maulin" designate something whose essence is not a human, and in no possible world does "Eiffel Tower" designate something whose essence is not a tower.[45]

Maulin argues that spacetime time points are rigid designators like names. Each spacetime point rigidly designates a physical particle or event. What, then, is the essential attribute of such a rigid designator, that this rigidly designated event in all possible worlds share? Maudlin's answer is the metric. Maudlin argues against Norton and Earman, who thinks the differentiable manifold, i.e., merely the collection of points regardless of the shape of the points, is the essence of spacetime.

[44] Maudlin, 1989, pp. 83-85.
[45] Maudlin, 1989, pp. 84-85.

Instead, Maudlin thinks that the metric structure, i.e., how all the spacetime points are arranged, is the essence of spacetime. He argues that mathematical solutions to the Einstein equation shows that there are possible spacetimes where other physical fields such as the electromagnetic field do not exist, whereas the metric field necessarily exist in all possible spacetimes in general relativity. A hole transformation alters the metric, and thus represents two distinct situations, while a Leibniz shift preserves the metric and thus represents the same situation. This way, Maudlin resolves the hole problem.

It seems to me that there are several insufficiencies to this view. First, even if we accept all of his previous arguments and adopt a field-like plenum view, it is still too early to give the verdict that it is a complete triumph for the substantivalists over the relationalists. As discussed earlier, Maudlin chooses the metric field as the plenum of spacetime. The metric field describes the overall structure of the universe. It assigns a metric tensor to each point in the spacetime. In solutions such as the Schwarzschild metric or Minkowski metric, symmetry is possible. In other words, spacetime points might have the same metric tensors as other points. This symmetry is not limited merely the one metric tensor at that point, but also to the metric tensors of points in the neighborhood around that point. Therefore, this view does not guarantee haecceitism. We cannot guarantee the absoluteness of each individual point. Nor can we say that one single spacetime point, or one limited group of spacetime points, bears the metric for the entire spacetime. Local distribution of points might have great differences from the overall metric.

To put it more simply, let us consider again Maudlin's response to Norton and Earman. Maudlin argues that Norton and Earman attribute the substantivalist essence of spacetime to the differentiable manifold, or the collection of spacetime points, while he himself thinks it should be the metric. With the former view, we can easily breakdown spacetime into the smallest units. The manifold is just a collection of points, and therefore it makes sense to say that each spacetime points serves as the plenum of spacetime. There is no problem using each point as a rigid designator for an event that takes place at that point. With the latter view, however, it is quite difficult to do so. As discussed earlier, individual points do not have any haecceitist feature, nor can they reflect the metric entirely. Therefore, I think Maudlin's view is insufficient here: unlike what he suggests, the possible symmetry in metric tensors at a specific point suggests that it cannot be a rigid designator. Otherwise, it would rigidly designate several events happening at different points that happen to have the same neighborhood of metric tensors. The only thing that can be called a rigid designator in this view would be the metric for the entire spacetime. It rigidly designates everything altogether in that specific spacetime.

It is very hard to find a non-spacetime analogy for this. Roughly, if such an analogy exists, it should have the following features: it assigns a non-unique value to each of its member, and since it is non-unique, it cannot rigidly designate any single one of its members. Instead, it needs to rigidly designate how all its members are associated together. A concept of family perhaps satisfies some of these criteria, but if family is defined as the descendants of some specific ancestor, or even just a relation to some specific person, then, there is a way to uniquely pick out each member of the family using their relations with that specific ancestor or person. But if family is defined by mere kinship without specifying any relation, then it seems that we cannot assign any position to each member in the family at all. In general, the metric seems to be the only thing that non-rigidly designates its members while rigidly designates the structure of all its members.

This view thus seems to face two problems. First, although this metric-like plenum of spacetime fills the entire universe, it is an inseparable unity as we just discussed. We cannot tag each individual event with a spacetime point as its essential attribute. Such an anti-harcceitist view of spacetime is not inherently wrong, but it is quite against our intuition, and it does not seem to be the picture that Maudlin originally has in mind. Further, it does not have any non-spacetime analogy. This itself seems to imply that such a formulation of spacetime deviates from real life. Second, the metric is not an essential attribute of individual events, but it is an essential attribute of the manifold, i.e., the collection of all the events. How come that the events do not have the attribute individually, but come to have it when they are altogether in a collection? This means that the metric is not an essential attribute to the events, but rather to the mere structure of the events, i.e., *how* the points are collected. The metric is the essence of the manifold, not in the sense that it rigidly designates each individual event, but rather in the sense that it governs how all the events are put together as a whole. In other words, it is the essence of the structure of spacetime, and this does not necessarily mean that it is the essence of the spacetime itself. The metric being the essence of the structure of spacetime, however, seem to be an analytic truth. By definition in general relativity and in Einstein's equation, the metric tensor governs the geometric structure of spacetime. Therefore, Maudlin seems to be saying that what governs the geometric structures spacetime is the essence of the structure of spacetime, which sounds like reading off of its definition misses the real question about the nature of spacetime, not its structure.[46]

Arguably, in response to my objections, one could first question the physical possibility of symmetry in our world. If there are no physically possible symmetries, then metric tensors could be considered a haecceitist feature of each point in spacetime, but as Weatherall suggests, there is no difference in the mathematical possibility of all the solutions. In order to do reject other possible

[46] An immature thought about this is a similar to Baker's global internal symmetry (Baker, 2010, pp.1162-1164), as they are both talking about some sort of structural change, although he specifically defines that to be a change not in spatial or temporal values. The structure of spacetime, the metric relations, is invariant. But the structure of the structure of spacetime might not be. This sounds very sketchy and to be honest I have not yet made a clear explication of it for now.

symmetries, we must reject the physical possibilities of all other solutions to Einstein's equation that allow symmetry such as the Minkowski spacetime or the Schwarzschild spacetime, allowing only a mathematical and maybe a metaphysical possibility. We limit the physical possibility only to the world we are in. As Maudlin suggests in his book *Philosophy of Physics: Space and Time*, "[the world before the hole transformation] S and [the world after the hole transformation] S_{swap} represent distinct situations, but it is not at all clear that S_{swap} represents a physically or metaphysically possible situation".[47] This, however, undermines the foundation of metric essentialism. Recall that, in response to Norton and Earman's view that the metric field is similar to any other physical fields, Maudlin argues that the metric field necessarily exists in all spacetimes, whereas other fields such as the electromagnetic field might not exist in some solution to Einstein's equation. Therefore, limiting the discussion to our spacetime also denies the difference between the metric and other physical field, undermining the underlying reason why we choose the metric field as the essence of spacetime.

Therefore, if we insist to maintain a haecceitist view of spacetime, Maudlin's substantivism runs into problems. However, this does not mean that this view is entirely wrong. Indeed, the metric and the metric field is an important and unique feature of spacetime in general relativity. Is there a way, different from Maudlin's, but still keeps the metric field as an important feature of spacetime? Einstein himself provides an alternative approach. In a speech given in 1920[48], Einstein discusses the possibility to view the metric field as the new "ether" of spacetime. Ether is the matter that was thought to be the medium for the propagation of light. Light was once thought to be a wave, and thus was thought to need a medium for propagation. Later, experimental evidence

[47] Maudlin, 2012, Chapter 6, pp. 151.
[48] Einstein, 1920, *Ether and Relativity*.

shows that such luminous ether does not exist. However, the concept of ether, or a medium that spans across the universe, remains. Material substance does not fill every corner of our universe. Vacuum, or regions with a very low density of matter close to vacuum, such as the outer space, exists. The force of gravity occurs through these regions. The gravity between the sun and the earth is exerted on each other through these almost vacuum regions, without having any direct contact through material objects. This, according to Einstein, is in conflict with our intuition and daily experience that all reciprocal actions must happen through contact. In addition to that, Einstein discusses the problem of rotation that bothered physicists since Newton. The rotation of a system has to be relative to something outside the system. Newton introduces the concept of absolute space, which, according to Einstein, can be considered a sort of ether. As Einstein says, "what is essential is merely that besides observable objects, another thing, which is not perceptible, must be looked upon as real, to enable acceleration or rotation to be looked upon as something real". Mach, in his effort to avoid unobservable things, proposed that instead of Newton's unobservable absolute space, the rotation should be relative to the totality of masses in the universe. But to think about a totality of mass relative to rotation again come back to the issue of interaction at a distance. He again faces the problem of what serves as the medium for the interaction and the effects of inertia.

Following Mach's line of thought, Einstein develops the ether of the general theory of relativity. He suggests that the new ether has no mechanical or kinematical qualities itself, but helps to determine mechanical events. It is "is at every place determined by connections with the matter and the state of the ether in neighboring places, which are amenable to law in the form of differential equations". Although Einstein does not make a direct reference to the metric, it is clear that he is talking about the metric field as the new ether.

Where does this view stand in the debate between substantivalists and relationalists? Since Mach objects to Newton's absolute space (which Einstein describes as Newton's ether), it makes sense to think that this view is closer to relationalism. In fact, one of the modern relationalists, Carlo Rovelli, takes position a very similar to this view. We can take a glimpse at this view from his book chapter *Halfway Through the Woods*.[49] Rovelli, again, emphasizes on the dynamic nature of the metric field in general relativity, i.e., that its geometry is affected by the presence of matter and determined by Einstein's equation. Tracing back in history, Rovelli argues that before general relativity, matter and spacetime were considered differently. This allowed physicists and philosophers to establish object-independent coordinate system i.e., inertial reference frames, from which they could determine absolute position and motion. In these frames, position x and time t could denote absolute location in space and time. But in general relativity, as long as there is matter, there is distortion in spacetime. Everything is affected by a gravitational field (which is the metric field in general relativity). No inertial reference is intact of external force. If constructing an inertial reference frame is impossible, then, the reasoning that leads to it, i.e., that matter and spacetime are distinct, must also be false. In general relativity, position and time t are merely arbitrary labels for computational purposes. Position and motion of objects have to be defined in terms of the relation to each other, not with respect to an external reference frame of absolute space. This leads to Rovelli's tenet c), that location and motion are merely the interaction and the change of the interaction between matter and spacetime, which are of a similar nature. Motion no longer happens in absolute space(time), and thus a substantivalist absolute space(time) does not exist. Instead, the tenets of general relativity suggest a relationalist view of spacetime.

[49] Rovelli, 1998, pp. 183-195.

We see that Rovelli's argument looks very much like a more detailed version of that of Mach and Einstein's. But Rovelli takes a further step to incorporate quantum mechanics, the other important branch of modern physics, into his view of spacetime. As we introduced in the section about the quantum field theory, quantum mechanics usually deals with strange things that happen on a microscopic level, and is usually not concerned with spacetime as much as general relativity is. It discusses the wave nature of all the substances and describes the state of particles based on statistical possibilities derived from mathematical wave equations. The famous example of Schrodinger's cat shows how the measurement made by an observer affects statistical possibilities of the system. For now, physicists consider quantum mechanics not compatible with generality due to their fundamental differences in describing the nature of things. Developing a compatible theory that unifies both general relativity and quantum mechanics is still an ongoing (but seemingly impossible) project.

Despite the difficulties, Rovelli proposes to incorporate quantum mechanics into the discussion of spacetime. He suggests that, analogous to the misunderstanding of simultaneity before we fully understood special relativity, in the current theory of quantum mechanics, there is an incorrect notion of observer-independent state of system. In plain words, he argues that one fundamental principle of physics, that all observers should obverse the same state of the world, is incorrect. He argues that the observer should also be considered an observable part of a system. Every measurement made by each observer would affect the system, causing other observers to make a different observation about the system. This would lead to different measurements on the same system for different observers. In other words, the state of system is relative to different observers, further undermining the possibility of anything absolute and universal. Rovelli argues that with a successful unification (if possible) of general relativity and his version of quantum mechanics, the spacetime curvature would be replaced by a probabilistic contiguity relation between objects. In this sense, then, there cannot be any absolute spacetime, if we still call this thing spacetime. It would be quite relationalist, but whether it is still spacetime is now called into question.[50]

Rovelli's view is very radical and seems to me lacks crucial support from physical theory. It relies on the hypothetical unification of quantum mechanics and general relativity, which right now is nowhere close to realization. If we try to keep the gist of the view, but take a step back from quantum mechanics, we get a somewhat more moderate view focusing on general relativity discussed in another chapter of Rovelli *The Disappearance of Space and Time*.[51] Rovelli argues that if we define electrons and interactions in other physical fields such as the electromagnetic fields, the metric field will resemble matter more than it resembles "spacetime" in the traditional sense. The dynamic interaction between the metric and other material objects seems to be in favor of this. Rovelli further goes on to argue that the mathematical concept of the manifold does not represent spacetime. He considers the diffeomorphisms to be a merely gauge concept, because the invariance of metric relations is observable in these transformations. Due to this measurability of an invariant interval, Rovelli thinks that the diffeomorphisms are redundant mathematical formalism that do not have distinct physical meaning, and thinks that the hole transformation represents physically identical worlds.[52] He therefore further argues that this invariance means that "manifold" is a purely mathematical gauge and has no physical meaning. Therefore, the manifold cannot be used as spacetime points to tag out each event, for the points in the manifold has no physical meaning. Spacetime points in this sense do not exist. Therefore, disqualifying both

[50] Rovelli, 1998, pp. 195-212.
[51] Rovelli, 2006, pp. 28-32.
[52] Rovelli, 2006, pp. 31-32; also discussed in Rovelli, 1991, pp. 300-303; Gaul and Rovelli, 2000, pp. 306-309; Rovelli and Vidotto, 2022 pp. 7-10, with roughly the same explanation.

candidates, the manifold and the metric, Rovelli argues that the concept of spacetime finds no place for it in the general theory of relativity. Therefore, he arrives at the conclusion of the disappearance of spacetime.

This view, still, seems to have some insufficiencies. First, indeed Einstein and Mach describes the metric field as ether, a medium that dynamic interacts with matter and has features that resemble matter. However, they never directly refer to it as matter, and clearly points out the crucial difference that the metric field is completely devoid of any mechanical or kinematical qualities, despite that it helps to determine mechanical and electromagnetic events. This seems to set up a third category besides matter and spacetime for the metric field, rather than looking for a position for the metric field in a dichotomy between matter and spacetime like Rovelli did.

Second, it seems a very risky move to deprive manifold of its physical meaning. Any physical theory is subject to the question that up to what point does its mathematical foundation has a physical meaning. Arguably, mathematical concepts such as tangent space at every point, might not always have a physical meaning. But to deny the physical meaning of the entire manifold seems to undermine the something very basic in the general theory of relativity. Afterall, the TLDR version of general relativity, as according to the current consensus of physicists (not Rovelli's redefined version), is something like "gravity is the curvature of 4-dimensional manifold caused by the presence of mass and energy". To adopt Rovelli's view is to completely rebuild this fundamental definition, and this is a cost that we must deliberate about. Therefore, Rovelli is not a relationalist in the traditional sense that he thinks spacetime is relational. Rather, he is a relationalist in the sense that he thinks all the interactions between matter is relational to each other, and spacetime as concept does not exist at all.

However, I do not think that the gist of Rovelli's view is completely off the track. Einstein in his speech refers to the metric field always as the medium, ether, but never directly as spacetime itself. In fact, Einstein never gives a directly description of spacetime in general relativity. He seems to take an agnostic stance towards spacetime in general relativity, giving no evidence for nor against it. The metric field, or the ether, as a medium, other than spacetime itself, seems adequate for the physical explanation. Spacetime itself might or might not exist, but the answer to this question seems no longer as important as it was in Newtonian physics. This position seems like an even more moderate version of Rovelli, and it does seem to be the reasonable position to take.

Conclusion

So far, we examined the accounts about space and spacetime from Aristotle to modern philosophers and physicists. Every view so far received criticism to some degree and we are far from a settled conclusion. However, we are not completely fruitless in our exploration. The Newtonian absolute space at rest is almost certainly ruled out. Current acceptable views are all variations under the general relativity. Personally, I do find the Einsteinian view most convincing. It is not as damaged by the hole argument as substantivalism. Also, by defining the metric field as an ether, a sort of third category apart from matter and spacetime, it denies the strict dichotomy between spacetime and matter, opening up more possibilities. Such a dichotomy seems unreasonable, especially since we are already aware of dark energy and dark matter, which seems to have a different nature from both spacetime and matter (in the traditional sense) already. At the same time, the concept of ether makes a good description about the interaction between matter and geometry. However, this view is not a perfect answer. It does not give an answer about the nature of spacetime. Although it seems that spacetime is no longer important in this view, we do not know this for certain. And there are far more issues that are not covered in this thesis. In fact, in recent years, some curious findings in cosmology suggest again the possibility for an absolute space. In the analysis of the waves and angles of the cosmic microwave background (CMB), i.e., the background radiation in our universe, astronomers see a possible correlation between the axis of the radiation and the plane of the solar system. This could merely be an observational error, a coincidence, or a psychological effect, but it might also imply an absolute universe with some special position located at the solar system. Afterall, the relationalism under general relativity bears a great deal of resemblance to the Aristotelian view of space. Perhaps there is a chance of a comeback for an absolute space. Even the psychological effect might imply something: together with the hole argument, perhaps we should rethink our position about scientific realism, and consider the possibility of an idealist account of space and spacetime. In conclusion, although the Einsteinian ether view of the metric field seems to me the best argument now, there is still a lot of room for further discussions about the nature of space and spacetime. However, it seems that we are now stalled in our philosophical inquiry, unable to arrive at any consensus. Perhaps, our next progress will only be made when next physical or cosmological breakthrough arrives, in a future near or far.

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