THE BACK SCATTERING CROSS SECTION OF A CONE–SPHERE

by

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SUMMARY

The metallic cone–sphere is the prototype for many low cross section shapes and has received an increasing amount of attention in recent years. In spite of the simplicity of its mathematical form, it is still one for which no exact solution of the boundary–value problem is available, and to calculate the cross section we must rely on approximate methods with such refinements and extensions as experimental data demands. We here present some new data for both the back scattered and surface fields at nose–on incidence and deduce therefrom a modification to the accepted theory sufficient to explain the results.

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INTRODUCTION

The design of shapes to have a low back scattering cross section for some range of aspect angles is a problem of continued interest in scattering theory. To remove the possibility of any specular reflection at these aspects, the chosen shape is often a pointed body of revolution, smoothly terminated at the rear to minimize the contribution from any ring singularity there, and a typical example of this configuration is the cone-sphere (or 'carrot') in which the first derivatives of the surface profile are matched at the join. Although it has not been demonstrated that this is in any sense an optimum as regards its cross sectional behavior at nose-on aspects, it does at least have the advantage of a relatively simple mathematical form, but even so it is still a shape for which no exact solution of the boundary-value problem is available. In consequence, to calculate the scattering cross section it is necessary to rely on approximate methods such as physical optics, creeping wave theory, etc., with the accuracy of the predictions judged by comparison with experimental data.

The first reported measurement of a cone-sphere was by Sletten who employed this shape to simulate a semi-infinite cone, but it was not until 1960 that a reasonably complete set of data was published. Using a Doppler system operating at a wavelength of 3.22 cm, Gent et al. measured the nose-on cross section of 50 cone-spheres with vertex angle 30° and base radii spanning the range \( 1.00 \leq a/\lambda \leq 2.125 \), and from an analysis of the results the magnitudes of the contributing components were deduced. Additional data for cone-spheres of the same angle was later obtained by Kennaugh and Moffatt and Moffatt who
measured six models of different sizes with a variable frequency X-band system to give 39 cross section values in the range 1 \leq ka < 7, where k = 2\pi/\lambda.

When these were compared with the predictions of physical optics, the geometrical theory of diffraction\(^5\) and the 'impulse approximation' method\(^6\), the superiority of this last approach was clearly apparent, but a systematic discrepancy between theory and experiment still remained. This was pointed out by Blore\(^7\), who measured cone-spheres of angles 15, 30, 40, 60 and 75\(^\circ\) to produce the most comprehensive body of data yet available for his shape. For each cone angle the cross section was determined for as many as 100 values of ka ranging from (about) 0.3 up to 7.5 using five large sets of models and a combination of X and K\(_a\)-band frequencies. At the lower end of the interval the results are in good agreement with the modified Rayleigh formula proposed by Siegel\(^8\), and this in turn matches in remarkably well with the impulse approximation curve. For ka greater than (about) 2, however, the curve lies two or more db below the measured peak values of the cross section and as much as 7db below the minima, with some evidence of a displacement in the position of the latter. Though the overall agreement is, nevertheless, quite gratifying, an understanding of the nature of the discrepancy is important for its own sake as well as for the effect that it may have on estimates of the scattering cross sections of other and more general shapes, and a study was therefore undertaken to discover its origin. It is the purpose of this paper to detail the investigation.
2. Basic Theory

It is appropriate to begin with a review of the basic theory as it applies to the calculation of the nose-on cross section for values of the base radius greater than a wavelength or so.

Consider a perfectly conducting cone–sphere of semi-vertex angle \( \alpha \) and base radius \( a \) illuminated by a plane wave at nose-on incidence. In terms of a Cartesian coordinate system \((x, y, z)\) whose origin is at the center of the spherical cap, the axis of the cone is taken as the \( z \) axis, and since there is no loss of generality in choosing the electric vector to lie in the \( x \) direction, we write

\[
E = A e^{-ikz}
\]

where a time factor \( e^{-i\omega t} \) has been assumed and suppressed. From symmetry the back scattered field has the same polarization as the incident field and hence, in the far zone, we can define a scattering amplitude \( S \) by the equation

\[
E^S = \frac{A e^{-ikr}}{kr} S
\]

from which it follows that

\[
\sigma = \frac{\lambda^2}{\pi} \left| S \right|^2.
\]  

(1)

If the physical optics approximation is applied, the determination of the scattered field is reduced to quadratures, and the resulting expression for \( S \) is

\[
S = -\frac{i}{4} \left( \tan^2 \alpha e^{-2ika \csc \alpha} - \sec^2 \alpha e^{-2ika \sin \alpha + 1} \right).
\]  

(2)
The sources of the individual terms can be identified by the phase factors.

Thus, the first term is associated with the tip, the second with the cone-sphere join and the third with the shadow boundary. The cross section attributable to the tip is therefore

$$\sigma_{\text{tip}} = \frac{\lambda^2}{16\pi} \tan^4 \alpha$$  \hspace{1cm} (3)

and this is in general agreement with the result obtained from the exact solution for scattering by a semi-infinite cone. The cross section of the join is similarly

$$\sigma_{\text{join}} = \frac{\lambda^2}{16\pi} \sec^4 \alpha,$$  \hspace{1cm} (4)

which has the same wavelength dependence as the tip contribution, but a much greater magnitude. Thus, for example, when $\alpha = 15^0$

$$\sigma_{\text{tip}} = 1.026 \times 10^{-4} \lambda^2$$

whereas

$$\sigma_{\text{join}} = 2.286 \times 10^{-2} \lambda^2,$$

and though there is as yet no exact analysis to support the formula, it is believed to give a valid estimate of the return from the join when the first derivatives of the surface profile are matched. Since the surface in the neighborhood of the join is entirely in the illuminated region, the current distribution on which (4) is based should not be seriously in error for $ka \cos \alpha >> 1$. 

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For the shadow boundary contribution the physical optics answer is spurious and arises because of the current discontinuity introduced by this approximation independently of the presence of the cone. From an analysis of the field scattered by a sphere it is found that the return from the boundary is produced by creeping waves, which are launched in the vicinity of this point and arrive at the receiver only after having traversed the rear of the body. Since the separation (in wavelengths) between the cone-sphere join and the shadow boundary increases with increasing frequency, it seems reasonable that this contribution should be determinable by reference to the sphere alone, and in place of the third term in equation (2) we now have

\[
S_c = \left( \frac{ka}{2} \right)^{4/3} e^{i\pi(ka-2\beta)} \frac{1}{\left( \beta \text{Ai}(-\beta) \right)^2} \left\{ 1 + \frac{8\beta}{15}(1 + \frac{9}{32\beta}) \left( \frac{2}{\text{ka}} \right)^{2/3} \frac{i\pi}{3} \right\} e^{\exp \left\{ -\beta \left( \frac{ka}{2} \right)^{1/3} - \frac{i\pi}{6} - \frac{\beta^2\pi}{60} \left( 1 - \frac{9}{\beta^2} \right) \left( \frac{2}{\text{ka}} \right)^{1/3} \frac{i\pi}{6} \right\}}
\]

(5)

where

\[
\beta = 1.01879 \ldots \text{ and } \text{Ai}(-\beta) = 0.53565 \ldots
\]

This is a more complete expression for the major creeping wave component than is usually employed, and is accurate to about one percent for \( ka \gg 5 \).

At still lower frequencies, however, even (5) becomes inadequate owing to the increased importance of the higher order waves, and no simple formula is then available. Nevertheless, numerical values can be obtained by subtracting the specular return for a sphere (Logan) from the complete scattering amplitude computed from the Mie series. This has been done
by Gent et al.\(^2\) for selected values of \(ka\) and, implicitly, by Kennaugh and Moffatt\(^6\).

If this new prescription for cone-sphere scattering is basically correct, certain conclusions follow immediately. Since the creeping wave is exponentially attenuated with increasing \(ka\), it contributes nothing to the high frequency limit, and hence

\[
S = \frac{i}{4} \left\{ \tan^2 \alpha \ e^{-2ika \csc \alpha} - \sec^2 \alpha \ e^{-2ika \sin \alpha} \right\}
\]

for sufficiently large \(ka\). The first term is also negligible in comparison with the second, so that finally

\[
S \sim \frac{i}{4} \sec^2 \alpha \ e^{-2ika \sin \alpha}
\]

and this is reasonably consistent with data for the largest \(ka\) at which measurements have been made.

In most cases, however, the intermediate values of \(ka\) are of more practical interest. The creeping wave now gives a significant contribution and if we again neglect the tip return, the scattering amplitude can be written as

\[
S = \frac{i}{4} \sec^2 \alpha \ e^{-2ika \sin \alpha} + S_c
\]

The cross section which then results is identical to that obtained by Kennaugh and Moffatt\(^3\) using the impulse approximation technique, but differs from the formula originally proposed by Keller\(^5\) through the inclusion of a return from the cone-sphere join, the neglect of the tip contribution, and the use of a more complete expression for the creeping wave component.
3. **Experiment**

From a consideration of equation (5) it is seen that the phase difference between the two terms on the right hand side of (7) is essentially a linear function of $ka$ for $ka \gg 1$, and by comparison the amplitude of $S_c$ is slowly varying. The nose-on cross section will therefore oscillate as a function of $ka$ in a manner which is almost sinusoidal in any small interval, and in practice the oscillations are significant from the edge of the Rayleigh region up to $ka = 100$ or more. Under these circumstances it is obvious that for a valid check on equation (7) the cross section should be measured at a series of closely spaced values of $ka$ sufficient to embrace one or more periods of the oscillations, and by comparison isolated determinations are of relatively little worth.

To get such data, either $a$ or $\lambda$ (or both) must be varied. The first method automatically implies a large group of models each slightly different in size from the others, and this is the technique by which most of the more comprehensive sets of data have been obtained so far. There is, however, a disadvantage associated with it. If the cone and sphere portions of the model are incorrectly mated, the scattering from the join may be profoundly disturbed. Thus, for example, a ridge at this point equivalent to an angular difference $\delta$ between the tangents to the cone and the sphere would increase the first term of (7) by a factor $\phi$

$$1 + 2ika \cos \alpha \tan \delta$$  \hspace{1cm} (8)

approximately, as follows by a straightforward application of physical
optics, and the second term of (3) may easily dominate through its 
ka factor. The n đề h -l of the join is certainly one of the most critical 
regions for a cone-sphere, and care is necessary to ensure accurate 
modeling here. Not unnaturally, the probability of imperfections increases 
with the number of models employed, and many of the irregularities apparent 
in the data of Gent et al^2 may be due to such imperfections amongst their 
50 models.

The difficulty can be overcome, at least in part, by using one carefully 
constructed model and achieving the variation in ka by shifting frequencies, 
and this is the procedure that we adopted. The first model had $a = 12.4^\circ$ 
and $a = 4.519\,\text{cm}$, and was made from rolled aluminum stock machined to a 
high degree of surface finish. For convenience, attention was directed at 
X-band frequencies and within the range 7.98 to 10.97 Gc a total of 17 individual 
frequencies were employed. All were generated from a stabilized oscillator of 
continuously varying frequency or from phase-locked oscillators. At each one 
a pattern was recorded showing the back scattered return out to the specular 
flash or beyond, followed by a minimum of 10 (and on average 25) separate 
measurements of the nose-on cross section $\sigma / \lambda^2$. Every effort was made to 
get the highest possible accuracy and to have the measurements truly independent. 
The means and standard deviations were then calculated, and on the assumption 
that the errors are random, the means should be accurate to about 0.3 db. The 
data is plotted as a function of ka in Fig. 1. Similar results have also been 
obtained for a cone-sphere of semi vertex angle $71^\circ$ and base radius 5.613 cm.
The expected sinusoidal oscillation is clearly in evidence, but to determine the full extent of the agreement between the theoretical values and the measured data, it is necessary to compute $S$ using equation (7). This in turn requires the calculation of $S_c$ from equation (5), and for the required range of $ka$ it is found that $S_c$ can be written as

$$S_c = (0.5026 - 0.01467 ka) \exp \left\{ i\pi (1.0256 ka - 0.95410) \right\}.$$

Note that this is purely a numerical representation based on the values computed from (5), and though accurate over the above range, it has no physical significance per se. Nevertheless, it is convenient inasmuch as it allows us to derive an explicit theoretical expression for $\sigma/\lambda^2$ and from equations (1), (7) and (9) with $\alpha = 12.4^{0}$ we have

$$\frac{\sigma}{\lambda^2} = 0.02190 \left| (1.916 - 0.05593 ka) + \exp \left\{ i\pi(1.45410 - 1.16335 ka) \right\} \right|^2.$$  

The corresponding curve is shown dashed in Fig. 1.

The predicted locations of the appropriate maxima and minimum are $ka = 8.127$, $9.846$ and $ka = 8.986$. These are in excellent agreement with the data and certainly there is no indication of the displacement reported by Blore$^7$. There is, however, a notable amplitude discrepancy, and the theoretical curve lies almost uniformly below the measured points. Since the maxima and minima are both too low, it would appear that the larger of the two terms in (7) should be increased, implying$^{12}$ an enhancement of the creeping wave contribution, and to see whether this alone would be sufficient, an expression of the form
\[ \frac{\sigma}{\lambda^2} = 0.02190 \left| A(1.916 - 0.05593\,\text{ka}) + B \exp\left\{ i\pi(1.45410 - 1.16335\,\text{ka}) \right\} \right|^2 \] (11)

was postulated. The factors \( A \) and \( B \) apply to the first and second terms respectively in (7), and when (11) was fitted to the data using the method of least squares, the best fit was obtained with \( A = 1.3 \) and \( B = 0.9 \). Such a modification to (10) is sufficient to reproduce even the small decrease in the amplitudes of the successive measured maxima, and the closeness of the fit strongly suggests that there is some mechanism operating whereby the amplitude of the creeping wave is increased over and above the value that it would have for a sphere of the same size. An increase of between 2 and 3 dB would remove most of the discrepancy evident in Fig. 1, and by comparison the slight reduction in the joint contribution produced by the least squares analysis is not felt to be significant. The value of \( B \) is strongly influenced by the measured data near to the minimum in the cross section curve, and this is where the experimental points are most likely to be in error.

4. Creeping Wave Enhancement

An increase in the far field contribution of a creeping wave almost necessarily implies an enhancement of its value on the surface, and since the measured creeping wave amplitude on the surface of a sphere is in excellent agreement with the theoretical expression, it follows that the addition of a conical nose must in some way increase the excitation of the wave.

In an attempt to explain this phenomenon, two possible mechanisms have been investigated. The first of these is associated with the cone–sphere
join alone and is independent of the rest of the cone except insofar as it
would not exist were the cone not there. At the join the first derivative of
the surface profile is continuous, but it still constitutes a singularity (albeit
a weak one) by virtue of the discontinuity in the second and higher derivatives.
It is therefore possible that it could increase the field intensity in the vicinity
of the shadow boundary either through its radiated field or, equivalently, by
the excitation of a creeping wave which is in phase with that generated by the
sphere itself. Although there is no canonical problem whose solution is known
and bears on the matter, it would appear that the situation can be modeled by
a sphere with a semi-active ring slot at a position corresponding to the cone-
sphere join. In the slot either or both of the tangential components of the
electric field are specified as constant multiples of the incident field values,
with the constant factors determined by comparison of the radiated field of the
slot with the back scattered field of the cone-sphere join. A consideration of
the surface field within the shadow then leads to an estimate of the extent to
which the creeping waves are increased by the presence of the singularity.
The increase, however, is far below the level deduced from the far field data
for the cone-sphere, and it must therefore be concluded that the join is not
primarily responsible for the observed enhancement.
From the failure of this first mechanism it would appear that the cone sides are not a negligible factor in the creeping wave enhancement, but to discover the precise role that they do play it is necessary to examine the currents on the surface of a sphere. These can be obtained directly from the standard Mie solution, and in terms of spherical polar angles $\theta$ and $\phi$, the components of the current vector $\mathbf{J}$ are

$$J_{\theta} = -Y \cos \phi \, T_2(\theta)$$

$$J_{\phi} = Y \sin \phi \, T_1(\theta)$$

where $Y$ is the intrinsic admittance of free space and

$$T_1(\theta) = \frac{1}{ka} \sum_{n=1}^{\infty} (-i)^{n+1} \frac{2n+1}{n(n+1)} \left\{ \frac{i}{\xi_n'(ka)} \frac{\partial}{\partial \theta} P_n^1(\cos \theta) - \frac{i}{\xi_n(ka)} \frac{1}{\sin \theta} P_n^1(\cos \theta) \right\}$$

$$T_2(\theta) = \frac{1}{ka} \sum_{n=1}^{\infty} (-i)^{n+1} \frac{2n+1}{n(n+1)} \left\{ \frac{1}{\xi_n'(ka)} \frac{\partial}{\partial \theta} P_n^1(\cos \theta) - \frac{i}{\xi_n(ka)} \frac{1}{\sin \theta} P_n^1(\cos \theta) \right\}$$

where

$$\xi_n(ka) = ka h_n^{(1)}(ka)$$

and the primes denote differentiation with respect to $ka$

On the shadowed portion ($\theta > \pi/2$) of the sphere, $T_1(\theta)$ and $T_2(\theta)$ can be represented as sums of creeping waves, with $T_1(\theta)$ consisting mainly of 'H waves' whose magnetic vector is normal to the surface and $T_2(\theta)$ composed...
similarly of 'E waves.' Since the latter attenuate less than the H waves, $T_2(\theta)$ is the major current component throughout most of the shadow region.

On the illuminated side, however, an optics contribution is present, and this is dominant at points well forward of the shadow boundary. Thus, for $ka \cos^3 \theta >> 1$

$$T_1(\theta) = -2 \cos \theta e^{-ika \cos \theta} \left(1 + \frac{i \sin^2 \theta}{2 ka \cos^3 \theta} + \ldots\right)$$  \hspace{1cm} (16)$$

$$T_2(\theta) = -2 e^{-ika \cos \theta} \left(1 - \frac{i \sin^2 \theta}{2 ka \cos^3 \theta} + \ldots\right)$$  \hspace{1cm} (17)$$

and by comparison any residual creeping wave effects are negligible. Note that the first term in each of (16) and (17) is identical to the physical optics prediction.

In the region intermediate to that for which (16) and (17) are appropriate and the shadow boundary, no simple expressions for $T_1(\theta)$ and $T_2(\theta)$ are available, and these must now be calculated using either the series expansions shown in (14) and (15) or the integral representations derived by Logan. From such calculations it is found that for ka greater than (about) $3$, $|T_1|$ and $|T_2|$ decrease more or less uniformly as $\theta$ increases from zero to $\pi/2$. The rate for $T_1$ is more rapid than for $T_2$, but even with the latter the quantity

$$\gamma' = \left| \frac{T_2(0)}{T_2(\pi/2 - \alpha)} \right|$$  \hspace{1cm} (18)$$
is appreciably greater than unity for \( \alpha \) small and \( \text{ka} > 3 \) (approx.) but less than some large number dependent on \( \alpha \). To illustrate this behavior, \( |T_2(0)| \) and \( |T_2(77/2^0)| \) have been computed for \( 0 \leq \text{ka} \leq 1 \) using equation (15), and the results are shown in Fig. 2. As \( \text{ka} \) increases from zero, \( |T_2(0)| \) increases from 1.5 to a maximum value 2.42 at \( \text{ka} = 0.94 \), and subsequently oscillates with a rapidly decreasing amplitude about the value 2 indicated by equation (17). It is within two percent of this value for all \( \text{ka} > 4.3 \). \( |T_2(77/2^0)| \) also starts with the value 1.5 at \( \text{ka} = 0 \), but increases more slowly to a maximum of 2.02 at \( \text{ka} = 1.6 \), and thereafter oscillates about a lower level which is almost constant out to the largest \( \text{ka} \) computed. Ultimately, \( |T_2(77/2^0)| \) tends to the same limit as \( |T_2(0)| \); its approach, however, is extremely slow, and even when \( \text{ka} = 100 \) it still differs from 2 by approximately 10 percent. For most cone-sphere applications, therefore, the values of \( \gamma' \) are sufficiently different from unity to be practically significant, in spite of the fact that \( \gamma' \to 1 \) as \( \text{ka} \to \infty \) for all \( \alpha > 0 \).

Let us now consider the surface field on the conical portion of the cone-sphere. It would be convenient were we able to estimate this using the rigorous solution for (vector) scattering by a semi-infinite cone, but the complication of the exact expression is such that numerical values are not yet available. It is therefore necessary to resort to approximate methods of which physical optics is most appropriate, and from this we have for the component of current in the direction of the generators

\[
J = 2Y \cos \theta e^{-ikz}
\]
This is basically a travelling wave. Its phase is precisely that of the incident field, and consequently the wave reaches the join with a phase which is almost the same as the sphere current would have had if the sphere had been complete. Since the join is inefficient as a means of launching waves, it seems reasonable to suppose that it will also reflect little of the travelling wave energy and, finding a good match, the wave then flows over the join. Hence, just on the sphere side

\[ J_\theta \approx Y \cos \theta / 2 \ e^{-i k a \sin \alpha}, \]

equivalent to an amplification of the original sphere current by a factor

\[ \gamma = \left| \frac{2}{T_2 \left( \frac{\pi}{2} - \alpha \right)} \right|. \]  

(19)

Beyond \( \theta = \frac{\pi}{2} - \alpha \), of course, the current decreases at a rate typical of the sphere alone, but the amplification through the factor \( \gamma \) leads to a corresponding increase in the creeping wave contributions to both the surface and far fields.

With this modification to the theory the expression for the scattering amplitude of the cone-sphere becomes

\[ S = \frac{i}{4} \sec^2 \alpha \ e^{-2 i k a \sin \alpha} + \gamma S_c \]  

(20)

(cf equation 7), and to show how the scattering cross section is affected, we shall again consider the example of a 25° cone-sphere.

Numerical values for \( \gamma \) can be obtained from equation (19) either by direct computation of \( T_2 \left( \frac{\pi}{2} - \alpha \right) \) using (15) or by representation of \( T_2 \left( \frac{\pi}{2} - \alpha \right) \) in terms of the functions
\[ g^n(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} t^n e^{i\xi t} \frac{dt}{w'(t)} \]  

(21)

tabulated by Logan. In the present case the first approach is more convenient and since \( \gamma \approx \gamma' \) for \( ka > 5.5 \), we have from Fig. 2

\[ \gamma \approx 1.189 \]

for \( 7.5 < ka < 10.5 \), based on a straight line fit to the values plotted there. The new formula for the cross section is therefore

\[ \frac{\sigma}{\lambda^2} = 0.02190 \left[ (2.278 - 0.06649 ka) + \exp\{ir (1.45410 - 1.16335 ka)\} \right]^2 \]

(22)

(cf equation 10) and the corresponding curve is shown in Fig. 1. The raising of the theoretical curve has certainly removed much of the disagreement with the data, but even this enhancement of the creeping wave is not sufficient to remove all of the discrepancy.

5. Discussion

The creeping wave enhancement is a direct consequence of the difference between the true and optics values of the sphere current \( J_\theta \) at the position of the join, and it may therefore seem illogical to apply the physical optics approximation to the cone and not to the sphere. Unfortunately, there is no definite criterion to indicate when physical optics can be used with confidence, and many cases are known in which the approximation gives good results even for small values of the radius of curvature. Nevertheless, the typical requirement is that all radii be large in comparison with the wavelength. Since the radius of curvature of the cone is infinite in the direction of the current flow, the
expectation was that the accuracy of the approximation would be greater for
the cone than the sphere, in spite of the smaller transverse radius of the
former, but to verify this belief it is necessary to measure the surface field
directly.

A number of such measurements have been carried out using a probe
technique, and a description of the experimental equipment has been given
by Senior\(^{16}\). For convenience and accuracy of operation, the frequencies
have been confined to the L and S-band ranges, and even the highest S-band
frequency then requires a model of larger physical size to achieve a \(ka\) value
comparable to those in the far field experiment. Of the models which were
available, the only one for which \(ka\) was in the range 7.5 to 10.5 was a \(30^0\)
cone-sphere with base radius 10.094 cm, and in Fig. 3 the measured amplitudes
of the longitudinal current in the plane of the incident electric vector are
presented for this case. The horizontal scale is the distance along the surface
(in cm) measured from the center of the back of the sphere. The tip of the
cone is to the right, and the locations of the cone-sphere join and the shadow
boundary are indicated. Calibration was with respect to the incident field
intensity in the plane of the support pedestal, and the frequency was 3.714 Gc
corresponding to \(ka = 7.905\). Data similar to the above, but for a frequency
3.066 Gc, is given in Senior\(^{16}\).

Starting at the tip the current amplitude increases rapidly over a
distance of \(\lambda/2\) or so, and then more slowly for a further 1.5 wavelengths.
The slight oscillation visible there may be due to a small backward-travelling wave, but this damps out in the first $2\lambda$, by which time the amplitude is within 0.2 db of the value predicted by physical optics. It remains relatively constant up to the cone-sphere join with no evidence of any reflection from this point. Immediately beyond, however, the amplitude decreases rapidly following the trend expected of a sphere current, and to confirm this fact the cone-sphere was replaced by a sphere of the same size as the cap, and the measurements repeated for this shape. The results are also shown in Fig. 3, and though the incident field strength was again determined, the relative values of the currents on the two bodies are independent of the calibration. At the front of the sphere the measured amplitude is almost precisely the physical optics value, but in a distance of no more than $\lambda/2$ the amplitude has begun to fall and decreases steadily thereafter. At the position corresponding to the join it is 2.2 db below the value found with the cone-sphere, and averages some 2 db below throughout the shadow region. For the sphere itself the measured values in the illuminated region decrease somewhat more rapidly than the values computed from equation (15), and most of this is believed due to a lack of uniformity in the incident field illumination. In the shadow the agreement with theory is excellent.

Based on the differences between the measured sphere and cone-sphere currents at the join, the shadow boundary and the back of the sphere, the creeping wave enhancement is found to be 2.3 db, compared with the 1.03 db obtained from equation (19). Even when an allowance is made for the irregularity of the illuminating field, the discrepancy is still greater than any intrinsic error.
in the measurements, and suggests an increase in the creeping wave amplitude over and above that provided by the theoretical expression for \( \gamma \). Additional surface field measurements are necessary to confirm this effect, but in the meantime we note that an increase in \( \gamma \) of 1 \( \text{dB} \) is almost precisely that required to achieve complete agreement between the predicted and measured scattering data for a \( 25^\circ \) cone-sphere (see Fig. 1).

On the other hand, it is not obvious in what way the theory should now be refined. The surface field data supports the physical optics approximation to the current on the portions of the cone sides near the join, and this in turn gives added confidence in the theoretical expression for the join contribution shown in (4). The data also confirms the existence of a creeping wave enhancement, and leaves little doubt that the theoretical picture of a travelling wave flowing over the join is basically correct. Indeed, the theoretical values of \( \gamma \) obtained from equation (19) have been used to predict the nose-on cross section of a \( 30^\circ \) cone-sphere for the full range of \( ka \) covered by the experimental data of Kennaugh and Moffatt\(^3\), and to within the accuracy that the measured values can be read from their graph, the agreement with theory is almost perfect. In this respect, the cone-sphere seems akin to a flat-backed cone in that an essentially high-frequency approach is adequate for the calculation of the nose-on cross section down to the edge of the Rayleigh region. The fact that \( \gamma \) is an oscillatory function of \( ka \) then offers an explanation for the markedly different maximum-to-minimum ratios of the measured cross sections (Blore\(^7\)) as the cone angle \( \alpha \) is varied.
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Legends of Figures

Fig. 1. Nose-on backscattering cross section of a 25° cone-sphere with base radius 4.519 cm. The numerals indicate the number of measured values whose means and standard deviations are shown. Original theory—(equation 7); modified theory—(equation 22).

Fig. 2. $T_2(0)$ and $T_2(77\frac{1}{2}^\circ)$ computed from equation (17).

Fig. 3. Measured current amplitudes for a 30° cone-sphere (○) and corresponding sphere (●) at 3.714 Gc.
FIG. 1: Nose-on \( r = 1 \) scattering Cross Section of a 25° Cone-Sphere with Base Radius 4.519 cm. The numerals indicate the number of measured values whose means and standard deviations are shown. Original theory — — (Eq. 7); Modified theory — (eq. 20)
REFERENCES


4. D. L. Moffatt, "Low Radar Cross Section, the Cone-Sphere," The Ohio State University Antenna Laboratory Report No. 1223-5; May, 1962.


12. Experimental confirmation of the fact that the larger contributor is a relatively non-directive one (i.e. a creeping wave return and not a contribution from the join) is obtainable from the back scattering pattern with vertical polarization at aspects away from nose-on.


FIG. 3: Measured Current Amplitudes for a 30° Cone-Sphere (o) and Corresponding Sphere (•) at 3.714 Gc.