SCATTERING BY A CONDUCTING SINUSOIDAL SURFACE

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We consider here the problem of a plane electromagnetic wave at oblique incidence on a perfectly conducting sinusoidal sheet, and seek to estimate the scattered (or diffracted) field.

During the last few years, several numerical procedures have been developed (e.g. Neureuther and Zaki [1]) for finding the scattered field exactly. Most of these are essentially point matching schemes whereby the integral equation for the surface field is converted to a set of simultaneous linear equations whose solutions are then found by matrix inversion. Since the matrix size is governed by the number of sampling points, the methods are most appropriate for surfaces whose period is less than or comparable to the wavelength. For this reason, there is still some interest in exploring other approaches that may be effective when the period is larger, even though such methods may be only approximate.

Although no exact analytical methods are known, a variety of approximate techniques have been developed, many of them stemming from the pioneering work of Lord Rayleigh [2]. One physically-based approach is the physical optics method in which an explicit form for the surface field is postulated, thereby reducing the determination of the scattered field to quadratures. In many instances, however, and the sinusoidal sheet is one, the evaluation of the physical optics integral is a difficult task, particularly in those cases where a portion of the surface is shadowed, and some of the shortcomings of physical optics results are often attributable to approximations made in the evaluation of the integral. In still other cases, the accuracy of the physical optics result can be improved by ignoring all shadowing,

i.e. by using the so-called 'extended' physical optics method, and it is of interest to observe that for a sinusoidal sheet it is then possible to provide an exact evaluation of the integral. Needless to say, the result is still subject to the unknown approximation introduced by the physical optics assumption.

In order to indicate the derivation, consider a perfectly conducting sheet having the equation

$$y = a \cos Kx \tag{1}$$

where a is the amplitude of the corrugations and $2\pi/K$ is the period. A plane wave is incident in a direction making an angle θ with the negative y axis and if, for convenience, we take it to have its magnetic vector parallel to the corrugations, i.e. to be H polarized, the incident field can be written as

$$\underline{E}^i = (\cos\theta\, \hat{x} - \sin\theta\, \hat{y})\, e^{-ik(x\sin\theta + y\cos\theta)},$$

$$\underline{H}^i = Y\, \hat{z}\, e^{-ik(x\sin\theta + y\cos\theta)}, \tag{2}$$

where Y is the intrinsic admittance of free space. MKS units are employed and a time factor $e^{-i\omega t}$ suppressed throughout.

Because of the two dimensional nature of the problem, the scattered magnetic field can have only a z component, and it is sufficient to work in terms of this. In a half space above the sheet, and certainly for $y \geq a$, the scattered field can be represented as an angular spectrum of plane waves which are outgoing as regards the sheet. By virtue of the periodicity of the surface it now follows that the spectrum is discrete and hence we can write

$$H_z^s = Y \sum_{m=-\infty}^{\infty} A_m\, e^{ik(x\sin\theta_m + y\cos\theta_m)} \tag{3}$$

where

$$k \sin\theta_m = mK - k\sin\theta$$

$$k \cos\theta_m = \sqrt{k^2 - (mK - k\sin\theta)^2}. \tag{4}$$

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Only those waves having $\left|\sin\theta_m\right| \leq 1$ are propagating, and for the remaining (evanescent) waves, the branch of the square root that must be chosen is that having positive imaginary part (to satisfy the radiation condition).

According to the physical optics approximation, the current induced in the surface is

$$\underline{J} = 2\,\hat{n}_\wedge \underline{H}^i \tag{5}$$

($\hat{n}$ is the outward normal) in the lit region, with $\underline{J} = 0$ in the geometric shadow. If we use the extended version of physical optics,(5) is postulated over the entire surface, implying

$$\underline{J} = (\hat{x} - a\,K\sin Kx\,\hat{y})\,\frac{2\,Y}{\sqrt{1 + (aK\sin Kx)^2}}\,e^{-ik(x\sin\theta + a\cos\theta\cos Kx)} \quad ,$$

from which we have

$$H_z^s = \frac{Y}{2\pi}\iint\limits_{-\infty}^{\infty}\left\{(a\cos Kx' - y) + (x' - x)aK\sin Kx'\right\}e^{-ik(x'\sin\theta + a\cos\theta\cos Kx')}$$
$$\cdot\frac{1}{R}\frac{\partial}{\partial R}\left(\frac{e^{ikR}}{R}\right)dx'\,dz' \tag{6}$$

where

$$R = \left\{(x - x')^2 + (y - a\cos Kx')^2 + z'^2\right\}^{1/2} \quad .$$

The treatment of the above integral expression is similar to that given by Senior [3], and it is sufficient to describe only the main features. Instead of tackling (6) directly, we consider its Fourier transform

$$\overline{H}_z^s = \int\limits_{-\infty}^{\infty} H_z^s\,e^{i\nu x}\,dx \quad .$$

The z' and x integrations are now carried out asymptotically for $k y \gg 1$ to give

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$$\bar{H}_z^s = Y e^{iy\sqrt{k^2-\nu^2}} \int_{-\infty}^{\infty} F(Kx')e^{ix'(\nu-k\sin\theta)}dx' \tag{7}$$

with

$$F(Kx') = \left\{ 1- \frac{\nu}{\sqrt{k^2-\nu^2}} aK \sin Kx' \right\} e^{-i(k\cos\theta+\sqrt{k^2-\nu^2})a\cos Kx'} \quad . \tag{8}$$

Since $F(Kx')$ is a periodic function of x' of period $2\pi/K$, it can be expanded as

$$F(Kx') = \sum_{m=-\infty}^{\infty} A_m(\nu)e^{imKx'} \tag{9}$$

with amplitudes

$$A_m(\nu) = \frac{1}{2\pi} \int_{-\pi}^{\pi} F(t)e^{-imt} dt \quad . \tag{10}$$

On inserting (9) into (7), the x' integration can be carried out immediately to give

$$\bar{H}_z^s = Y e^{iy\sqrt{k^2-\nu^2}} \sum_{m=-\infty}^{\infty} A_m(\nu) \, \delta(\nu-k\sin\theta+mK)$$

where $\delta(x)$ is the delta function, and hence

$$H_z^s = Y \sum_{m=-\infty}^{\infty} A_m e^{ik(x\sin\theta_m + y\cos\theta_m)} \tag{11}$$

where, from Eq. (8),

$$A_m \equiv A_m(k\sin\theta-mK)$$
$$= (-i)^m J_m\left(ak\left[\cos\theta+\cos\theta_m\right]\right)\sec\theta_m\left(\cos\theta + \frac{mK\sin\theta}{k\left[\cos\theta+\cos\theta_m\right]}\right) \tag{12}$$

and $J_m(x)$ is the Bessel function of order m. Equation (11) has, however, the form

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which an exact evaluation of (6) must possess. Moreover, the A_m are independent of y (as required), and since the only approximation made in the evaluation of (6) is to assume $ky \gg 1$, Eqs. (11) and (12) must represent an exact evaluation of (6) within the region where (3) obtains.

If instead of the incident field (2) we postulate the E polarized field

$$\underline{E}^i = \hat{z} \, e^{-ik(x \sin\theta + y \cos\theta)} \, ,$$

$$\underline{H}^i = -Y (\cos\theta \, \hat{x} - \sin\theta \, \hat{y}) \, e^{-ik(x \sin\theta + y \cos\theta)} \, ,$$

(13)

the scattered electric field in the half space $y \geq a$ can be written as

$$\underline{E}^s = -\hat{z} \sum_{m=-\infty}^{\infty} A'_m \, e^{ik(x \sin\theta_m + y \cos\theta_m)} \, .$$

(14)

The evaluation of the extended physical optics approximation for this case follows closely that which we have already given, with the result that

$$A'_m = A_m \, ,$$

(15)

where A_m is as in Eq. (12). The equality of the coefficients for the two polarizations is in agreement with the known fact that a physical optics estimate of a scattered field is inherently polarization insensitive as long as there are no shadow boundary effects present.

Acknowledgement

References

1. A. R. Neureuther and K. Zaki, Alta Frequenza, 28, p. 282 (1969).

2. Lord Rayleigh, The Theory of Sound, Vol. II, Macmillan and Co., Ltd., London (1878).

3. T. B. A. Senior, Can. J. Phys., 37, pp. 787 and 1572 (1959).