THE POLARISATION CHARACTERISTICS OF SCATTERED FIELDS

E. F. Knott and T. B. A. Senior*

Waves scattered by all obstacles are depolarised to some degree except in certain symmetrical situations. Expressions of some generality are derived that demonstrate this fact and, furthermore, it is shown that the method of physical optics predicts no depolarisation, even in the absence of symmetry.

The polarisation characteristics of the electromagnetic field scattered by an obstacle are strongly dependent on the obstacle geometry. These characteristics are potential sources of information about the body and, in particular, the cross polarised component (here defined as the component perpendicular to that of the incident wave) can be used as a measure of the 'edginess' of the obstacle. There are many instances, however, in which there is no depolarisation of the scattered wave, even if there are edges or other 'electrical' sources of depolarisation present.

By way of illustration, consider a perfectly conducting object illuminated by an electromagnetic wave whose field intensities are $\underline{E}^i$ and $\underline{H}^i$. The far zone scattered fields $\underline{E}^s$ and $\underline{H}^s$ are given by

$$\underline{E}^s \sim -ikZ \frac{e^{ikr}}{4\pi r} \hat{r} \times \hat{r} \times \int_A e^{-ik\hat{r}\cdot\underline{r}'} \underline{J}(\underline{r}')\, dA \ , \qquad (1)$$

*The University of Michigan Radiation Laboratory, Department of Electrical Engineering, Ann Arbor, Michigan, 48105, USA

$$\underline{H}^s \sim ik \frac{e^{ikr}}{4\pi r} \hat{r} \times \int_A e^{-ik\hat{r}\cdot\underline{r}'} \underline{J}(\underline{r}') \, dA \quad , \tag{2}$$

where $\underline{J} = \hat{n} \times (\underline{H}^i + \underline{H}^s)$ is the surface current density, $\hat{n}$ is a unit outward normal of the surface A, $\underline{r}$ and $\underline{r}'$ are the position vectors of the points of observation and integration, k is the propagation constant, $Z = 1/Y$ is the intrinsic impedance of free space, and a time factor $e^{-i\omega t}$ has been suppressed. These formulae follow directly from the Stratton-Chu relations[1] and show that $\underline{E}^s$, $\underline{H}^s$, and $\underline{r}$ are mutually perpendicular in the far zone.

If the body possesses a plane of symmetry, say the plane y = 0 of a Cartesian coordinate system (x, y, z), two cases may be considered depending on whether $\underline{E}^i$ or $\underline{H}^i$ is perpendicular to that plane. Let vertical polarisation be the case in which a plane wave incident in the plane of symmetry has its electric vector oriented along the y direction:

$$\underline{E}^i = \hat{y}\, e^{ik\hat{k}\cdot\underline{r}} \quad , \qquad \underline{H}^i = Y(\hat{k} \times \hat{y})\, e^{ik\hat{k}\cdot\underline{r}} \quad . \tag{3}$$

Symmetry shows that $\hat{y} \cdot \underline{J}(\underline{r}')$ is an even function about y = 0 whereas $\hat{y} \times \underline{J}(\underline{r}')$ is an odd function, and, if the point of observation lies in the plane y = 0 ,

$$\underline{E}^s = \hat{y} \frac{e^{ikr}}{kr} S_V \, , \qquad \underline{H}^s = Y(\hat{r} \times \hat{y}) \frac{e^{ikr}}{kr} S_V \, , \tag{4}$$

where

$$S_V = \frac{ik^2 Z}{4\pi} \int e^{-ik\hat{k}\cdot\underline{r}'} \hat{y} \cdot \underline{J}(\underline{r}') \, dA \quad . \tag{5}$$

Thus $\underline{E}^s$ is parallel to $\underline{E}^i$, implying that no depolarisation occurs.

---

* Stratton, J. A., Electromagnetic Theory, McGraw-Hill, 1941, p. 466.

Similarly, when the incident electric vector is taken in the plane y = 0, corresponding to horizontal incident polarization,

$$\underline{E}^i = (\hat{k} \times \hat{y})\, e^{ik\hat{k}\cdot\underline{r}}, \qquad \underline{H}^i = -Y\,\hat{y}\, e^{ik\hat{k}\cdot\underline{r}}; \tag{6}$$

the symmetry now shows that $\hat{y} \cdot \underline{J}(\underline{r}')$ is an odd function of y and $\hat{y} \times \underline{J}(\underline{r}')$ is even. The scattered fields in this case are

$$\underline{E}^s = -(\hat{r} \times \hat{y})\,\frac{e^{ikr}}{kr}\,S_H, \qquad \underline{H}^s = Y\,\hat{y}\,\frac{e^{ikr}}{kr}\,S_H, \tag{7}$$

where

$$S_H = -\frac{ik^2 Z}{4\pi}\int e^{-ik\hat{r}\cdot\underline{r}'}\,\hat{r}\cdot\hat{y}\times\underline{J}(\underline{r}')\,dA. \tag{8}$$

Again there is no depolarisation, this time because $\underline{H}^s$ is parallel to $\underline{H}^i$.

Passing on to a more general situation in which the electric vector is inclined at some angle β to the y axis, the polarisation can be expressed in terms of $\hat{\xi}$ and $\hat{\eta}$,

$$\hat{\xi} = \hat{y}\cos\beta + (\hat{k} \times \hat{y})\sin\beta, \qquad \hat{\eta} = (\hat{k} \times \hat{y})\cos\beta - \hat{y}\sin\beta.$$

Then

$$\underline{E}^i = \hat{\xi}\, e^{ik\hat{k}\cdot\underline{r}}, \qquad \underline{H}^i = Y\,\hat{\eta}\, e^{ik\hat{k}\cdot\underline{r}} \tag{9}$$

and by superposition of the previous two cases,

$$\underline{E}^s = \frac{e^{ikr}}{kr}\left\{\hat{y}\,S_V\cos\beta - (\hat{r} \times \hat{y})\,S_H\sin\beta\right\}. \tag{10}$$

For observation in the backscattering directions, $\hat{r} = -\hat{k}$ and the far field amplitude of the direct, or parallel, polarized component $\underline{E}^s \cdot \hat{\xi}$ becomes

$$S_{\parallel} = \frac{1}{2}(S_V + S_H) + \frac{1}{2}(S_V - S_H)\cos 2\beta , \qquad (11)$$

and that for the cross, or perpendicular, polarised component is

$$S_{\perp} = -\frac{1}{2}(S_V - S_H)\sin 2\beta . \qquad (12)$$

It is customary to refer to plots of $|S_V|^2$ and $|S_H|^2$ versus angle as principle plane bistatic patterns, but in the present case they become the principle plane backscattering patterns. In general S_V and S_H differ, implying that some depolarisation occurs for all $\beta \neq 0, \pi/2$; when incidence is out of the plane of symmetry, the backscattered field will almost certainly suffer depolarisation, but no simple expression for the cross polarised component is then obtainable.

A common means of estimating scattering by obstacles is physical optics, a method which approximates the surface current density by

$$\underline{J}^{p.o.}(\underline{r}) = 2\Gamma(\hat{n} \times \underline{H}^i)$$

where $\Gamma = 1$ over the lit surfaces of the body and $\Gamma = 0$ in the geometric shadow. For backscattering from a perfectly conducting obstacle of entirely arbitrary shape

$$\hat{r} \times (\hat{r} \times \underline{J}^{p.o.}) = 2\Gamma(\hat{k} \cdot \hat{n})\,\underline{E}^i$$

and Eq. (1) now shows that the scattered electric vector is <u>always</u> parallel to that of the incident plane wave. Thus no depolarisation is predicted by physical optics and, in fact, for the cases given above it is trivial to verify from Eqs. (5) and (8) that physical optics implies $S_V = S_H$.

The method of physical optics has served as a valuable tool in electromagnetic scattering but its failure to predict any depolarisation in the scattering by a metallic object is significant. If, for example, the method is used to estimate any one of several contributions to the backscattering from a complex object, the deduced polarisation characteristics will almost always be in error in spite of (possibly) more precise procedures used to obtain the remaining contributions. In the light of this shortcoming, one also questions the validity of present attempts to combine numerical methods (based on a direct digital solution of the integral equation) with physical optics estimates of the currents over selected regions of the surface.

We have found it useful to visualize depolarisation as the consequence of geometrical constraints on the current paths. A sphere offers no such constraints to the currents, those in the equatorial direction having the same opportunities as those running along polar directions, hence one is willing to accept that $S_H = S_V$ But a thin wire by contrast permits the currents to flow in only one direction, whence $S_V = 0$ and $S_H \neq 0$. The wire offers the clearest example of the source of depolarisation being a result of geometrical constraints on the surface current.