EQUIVALENT CURRENTS FOR A RING DISCONTINUITY [*]

by

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Abstract:

Based on the geometrical theory of diffraction, equivalent filamentary currents are derived for a linear element of a surface singularity for general incidence and polarization. The currents are used to obtain an analytical expression for the bistatic field diffracted by a ring singularity, valid within the caustic region as well as at wide angles.

The geometrical theory of diffraction (GTD) is a valuable method for estimating high frequency scattering from non-trivial objects, but like all ray techniques it does suffer the disadvantage of predicting infinite field strengths at caustics. This difficulty with geometrical optics can be overcome by using ray techniques to specify the surface field and an integral representation to obtain the scattered field. The resulting method is physical optics and if the surface is not itself a caustic, the scattered field obtained thereby is finite everywhere. Moreover, if the observation point is far from the surface and not at a caustic, the integral can be evaluated by the stationary phase method. The stationary phase point is the specular point of geometric optics, and a first order evaluation of the integral yields precisely the geometric optics field.

Although it is natural to attempt a similar procedure with GTD, the surface is now a caustic and GTD does not provide a valid description of the surface field. In practice, however, our main interest is in the far field, which could be obtained were an equivalent surface field or current known. The essential requirement for equivalence is that a first order stationary phase evaluation of the integral expression for the diffracted field must yield precisely the GTD result at points away from

caustics. This is analogous to the geometrical-physical optics connection. The integral must also produce finite fields at a caustic and thereby provide a uniform transition between the "wide angle" and caustic fields.

The use of integral representations as a means of caustic matching is not new (see [1], for example), but the explicit concept of equivalent currents was introduced by Ryan and Peters [2] in studying the ring singularity at the base of a cone. However, the near axial (caustic) result they obtained does not have the expected form [3,4] and is not supported by experimental data. It was this discrepancy that prompted us to undertake the present study of the GTD equivalent currents for a ring discontinuity in slope for general conditions of incidence, scattering and polarization.

We choose the axis of the ring discontinuity to coincide with the z-axis of the Cartesian coordinate sytem (x,y,z) shown in Figure 1. The entire ring is illuminated by a plane wave incident in the direction $\hat{i}$ having polar angles γ_i and ϕ_i, and we seek the scattered field in the direction $\hat{s}$ having polar angles γ_s and ϕ_s. According to the prescription of first order GTD, the diffracted field arises from a pair of diametrically opposed flash points and our first task is to find the field due to each. The relationship between the incident and diffracted field can be written in terms of a diffraction tensor, and at a remote point the field $\underline{E}^d$ is

$$\underline{E}^d = -\frac{\Gamma e^{iks}}{\sin^3\beta}\left\{(\underline{E}^i\cdot\hat{t})(X-Y)\hat{s}\times(\hat{s}\times\hat{t})+Z_0(\underline{H}^i\cdot\hat{t})(X+Y)\hat{s}\times\hat{t}\right\} \qquad (1)$$

where $X \stackrel{+}{-} Y$ are the scalar diffraction coefficients for a wedge, $\hat{t}$ is a unit tangent vector at a flash point, s is the distance between a flash point and the remote point of observation, Γ is a divergence factor accounting for the spreading of the rays away from a flash point, Z_0 is the impedance of free space, and $\underline{E}^i,\underline{H}^i$ are the incident field vectors.

The diffraction coefficients are

$$X = \frac{1}{n}\sin\frac{\pi}{n}\left\{\cos\frac{\pi}{n}-\cos\frac{\alpha-\theta}{n}\right\}^{-1}$$

$$Y = \frac{1}{n}\sin\frac{\pi}{n}\left\{\cos\frac{\pi}{n}+\cos\left(\frac{\pi}{n}-\frac{\alpha+\theta}{n}\right)\right\}^{-1} \qquad ,$$

where $n\pi$ is the exterior wedge angle and α and θ are the angles of incidence and scattering in terms of the local wedge coordinates at a flash point.

The diffracted field of equation (1) can be written in terms of the Hertz vectors

$$\bar{\pi}_e = \hat{t}\,(\underline{E}^i \cdot \hat{t})(X-Y)\,\Gamma e^{iks}/k^2 \sin^3\beta \ ,$$

$$\bar{\pi}_m = \hat{t}\,(\underline{H}^i \cdot \hat{t})(X+Y)\,\Gamma e^{iks}/k^2 \sin^3\beta \ ,$$

(2)

which are locally tangent to the edge of the ring and are determined solely by the corresponding components of the incident field. Since a Hertz vector represents the field of a current element, we postulate that continuous filamentary currents $\underline{I}_e$ and $\underline{I}_m$ exist at all points on the ring C, so that

$$\bar{\pi}_e = \frac{1}{ik} \int_C Z_0 \underline{I}_e \frac{e^{ikr}}{4\pi r} \, d\ell \ ,$$

(3)

$$\bar{\pi}_m = \frac{1}{ik} \int_C Y_0 \underline{I}_m \frac{e^{ikr}}{4\pi r} \, d\ell \ .$$

If we assume that the phase of the currents is that of the incident field, the integrals may be evaluated by the method of stationary phase. Comparison of the results of the integration with equations (2) show that at the flash points the currents are

$$\underline{I}_e = -2\hat{t}\,(\underline{E}^i \cdot \hat{t})(X-Y)/ikZ_0 \sin^2\beta \ ,$$

(4)

$$\underline{I}_m = -2\hat{t}\,(\underline{H}^i \cdot \hat{t})(X+Y)/ikY_0 \sin^2\beta \ .$$

This identification holds regardless of the positions of the flash points, and since the flash points can be rotated around the ring by a displacement of the point of observation, eqs. (4) specify the currents at all points thereon. This implies that quantities X, Y and β — and therefore α and θ — are now functions of the angular

variable ϕ around the ring. Simple substitution then leads to one-dimensional integral expressions for the Hertz vectors and, hence, to the diffracted field at all points in space.

A convenient way to express the results for bistatic scattering is to probe the far field with a linearly polarized receiver whose electric and magnetic polarizations are described by the unit vectors $\hat{e}_r$ and $\hat{h}_r$, and aligned such that $\hat{s} = \hat{e}_r \times \hat{h}_r$. The received signal will therefore be proportional to $\underline{E}^d \cdot \hat{e}_r$, which we relate to a scattering function S via

$$\underline{E}^d \cdot \hat{e}_r = S e^{ikr_0}/kr_0 \ ,$$

where r_0 is the distance of the receiver from the center of the ring. If we define analogous vectors for the incident field so that

$$\underline{E}^i = \hat{e}_i e^{ikr \cdot \hat{i}} \ , \qquad \underline{H}^i = Y_0 \hat{h}_i e^{ikr \cdot \hat{i}} \ ,$$

then

$$S = \frac{ka}{2\pi} \int_0^{2\pi} \frac{e^{ikr \cdot (\hat{i} - \hat{s})}}{\sin^2 \beta} \left\{ e_{it} e_{rt} (X - Y) + h_{it} h_{rt} (X + Y) \right\} d\phi \ , \qquad (5)$$

where the suffix t denotes the tangential component. All quantities in the integrand are functions of position around the ring.

A stationary phase evaluation of the integral can be readily performed and the points of stationary phase are those for which $\hat{t} \cdot (\hat{i} - \hat{s}) = 0$. These are none other than the flash points, where

$$\bar{r} \cdot (\hat{i} - \hat{s}) = \mp iaT$$

with

$$T = \left\{ \sin^2 \gamma_i + 2 \sin \gamma_i \sin \gamma_s \cos(\phi_s - \phi_i) + \sin^2 \gamma_s \right\}^{1/2} \ .$$

If we identify the near (far) flash point associated with the upper (lower) sign by subscript 1 (2), the |result of the stationary phase evaluation is

$$S = \frac{ka}{\sin^2\beta\sqrt{2\pi kaT}} \left\{ \left[e_{it1}e_{rt1}(X_1-Y_1)+h_{it1}h_{rt1}(X_1+Y_1) \right] e^{-i(kaT-\frac{\pi}{4})} \right.$$

$$\left. + \left[e_{it2}e_{rt2}(X_2-Y_2)+h_{it2}h_{rt2}(X_2+Y_2) \right] e^{i(kaT-\frac{\pi}{4})} \right\} . \quad (6)$$

This is precisely the wide angle (kaT $\gg$ 1) bistatic result obtainable via the "standard" GTD method and, for the special case of backscattering, reduces to the expression given by Bechtel [5].

A more accurate evaluation of the integral can be carried out by analytically accounting for the ϕ-dependence of X and Y around the ring. This can be done by representing X and Y in terms of the first few components of their Fourier expansions and the result is

$$S = -\frac{ka}{2\sin^2\beta} \left\{ (e_{it1}e_{rt1}+h_{it1}h_{rt1})(X_1+X_2)J_2(kaT) \right.$$

$$\left. + (e_{it1}e_{rt1}-h_{it1}h_{rt1}) \left[(Y_1+Y_2)J_0(kaT) - i(Y_1-Y_2)J_1(kaT) \right] \right\} ,$$

$$(7)$$

where the J's are Bessel functions of integer order. This bistatic expression reduces to Ufimtsev's result for the special case of bistatic scattering from a disk in the plane of incidence [3]; it also reduces to the caustically matched first order result obtained by Senior and Uslenghi for backscattering by a finite cone [4].

If each Bessel function in (7) is replaced by the leading term in its asymptotic expansion for large arguments, the wide angle result (6) is recovered; however, if ka is only moderately large, there may be an extended angular region about the axial caustic (where T = 0) for which the substitution is not valid.

A numerical evaluation of the integral (5) provides an interesting assessment of both the wide angle expression (6) and the more exact representation (7). We selected an arbitrary set of parameters involving mixed polarizations and bistatic directions not in the plane of incidence and carried out the integration for a range of γ_s between zero and 30 degrees. The results are shown in Figure 2, and note

that the agreement between numerical integration and the analytical result of equation (7) is very good, even in the vicinity of the nulls. The stationary phase method, by contrast, shows large discrepancies in both amplitude and null position as γ_s swings closer to the ring axis $(\gamma_s = 0)$.

A final test of equation (5) was carried out by means of a laboratory measurement of the bistatic fields scattered by a 40-degree half-angle right circular cone, the large angle being chosen deliberately to provide substantial aspect angle coverage within which the entire rim of the cone was exposed to both the incident and scattering directions. The base diameter was 6.01 inches and the measurement frequency was fixed at 11.50 GHz, implying ka = 18.40. Second order interactions across the base of the cone were suppressed by the use of a pad of absorbent material cemented to the base.

The total bistatic angle was held fixed at 30 degrees, so that as the cone was rotated in aspect, both γ_i and γ_s changed simultaneously. Measurements were made in the plane of incidence for H-polarization (electric field lying in the plane of incidence) and the data are shown as the solid trace in Figure 3. The ordinate is the normalized radar cross section

$$\frac{\sigma}{\lambda^2} = \left| \frac{S}{\sqrt{\pi}} \right|^2$$

in decibels and the abscissa the aspect angle γ_s (with $\gamma_i + \gamma_s = 30$ degrees). The datum points in Figure 3 were computed from equation (5) via numerical integration around the ring and it can be seen that the agreement is very good for aspect angles out to 20 degrees or so.

It is therefore apparent that the equivalent currents (4) provide a basis for the computation of the fields diffracted by any line discontinuity in slope, valid even in caustic regions. In the particular case of a ring discontinuity, the integral is closely approximated by equation (7), in which the scalar diffraction coefficients are displayed explicitly. The result is therefore general and valid for any ring discontinuity, provided the appropriate diffraction coefficients are inserted in place of $X \pm Y$. Although the equivalent currents were determined purely by means of

GTD techniques, the form of the result (7) is analogous to the disk result obtained by Ufimtsev with his fringe wave theory.

References

[1] R.A. Ross, "Investigation of scattering center theory", Cornell Aero. Lab. Tech. Rept. AFAL-TR-67-343, December 1967.

[2] C.E. Ryan, Jr., and L. Peters, Jr., "Evaluation of edge-diffracted fields including equivalent currents for the caustic regions," IEEE Trans. AP-17, pp. 292-299, May 1969. See also correction: AP-18, p. 275, March 1970.

[3] P. Ia. Ufimtsev, "Approximate calculation of the diffraction of plane electro-magnetic waves by certain metal objects," Sov. Phys. - Tech. Phys., 3, pp. 2386-2396, November 1958.

[4] T.B.A. Senior and P.L.E. Uslenghi, "High-frequency backscattering from a finite cone," Radio Sci., 6, pp. 393-406, March 1971.

[5] M.E. Bechtel, "Application of geometric diffraction theory to scattering from cones and disks," Proc. IEEE, 53, pp. 877-882, August 1965.
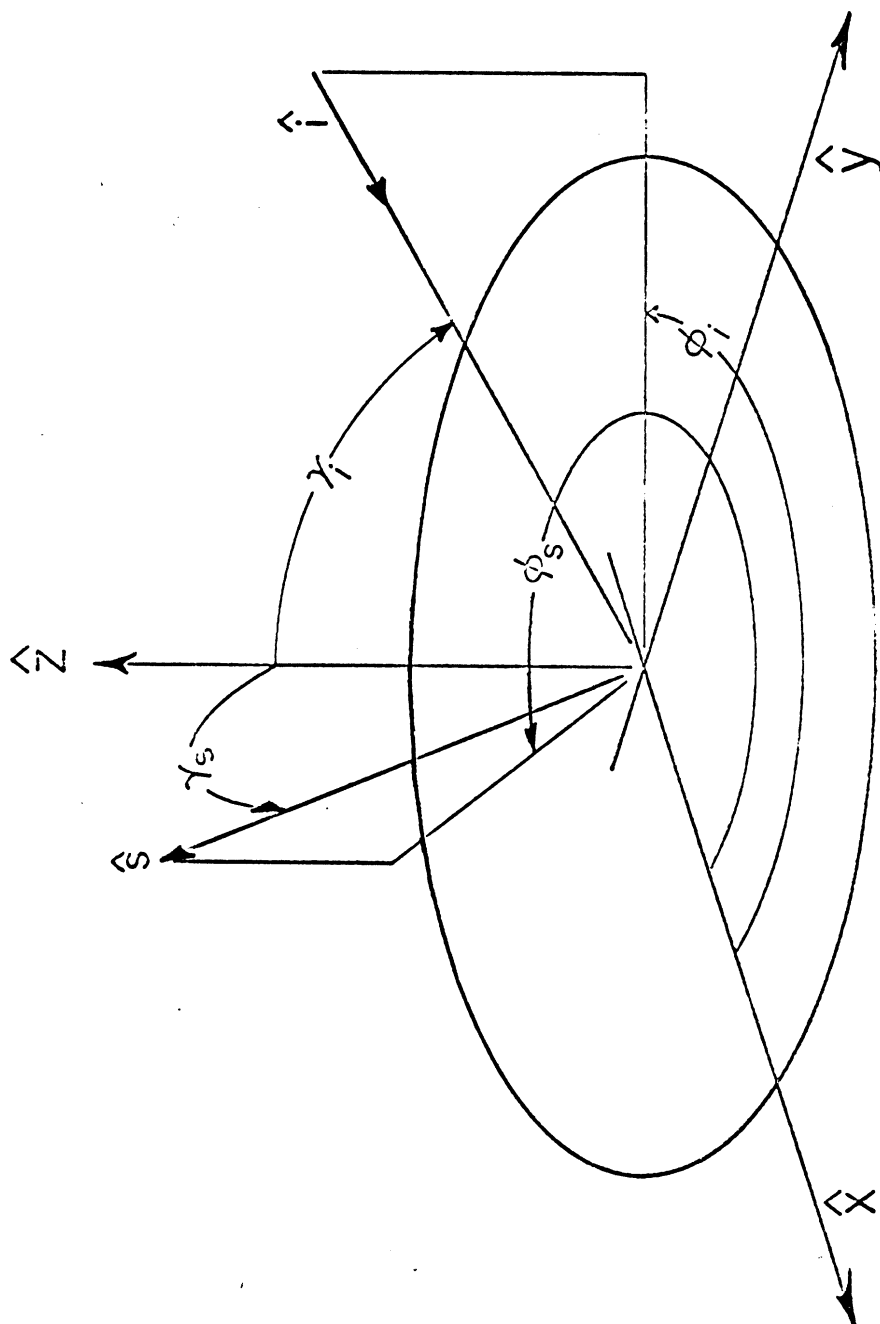
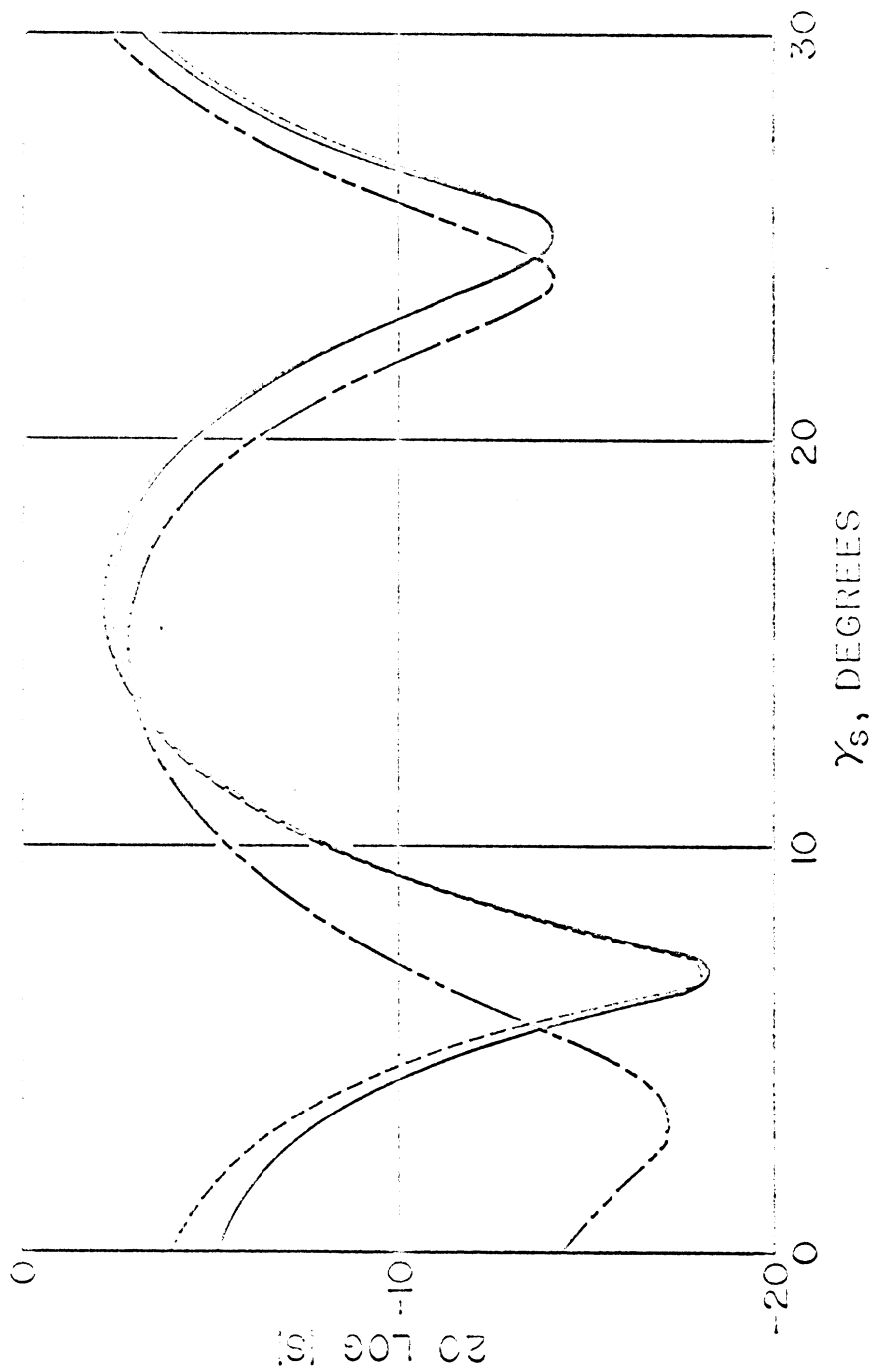Fig. 1. Four angles specify incident and scattered directions.

Fig. 2. Evaluation of the integral (5) for $ka = 10$, $\gamma_i = 30^\circ$, $\phi_i = 0^\circ$, $\phi_s = 36^\circ$, $e_{it}e_{rt} = 0.604$, $h_{it}h_{rt} = 0.262$. Numerical integration (———), analytic (———), stationary phase (———).
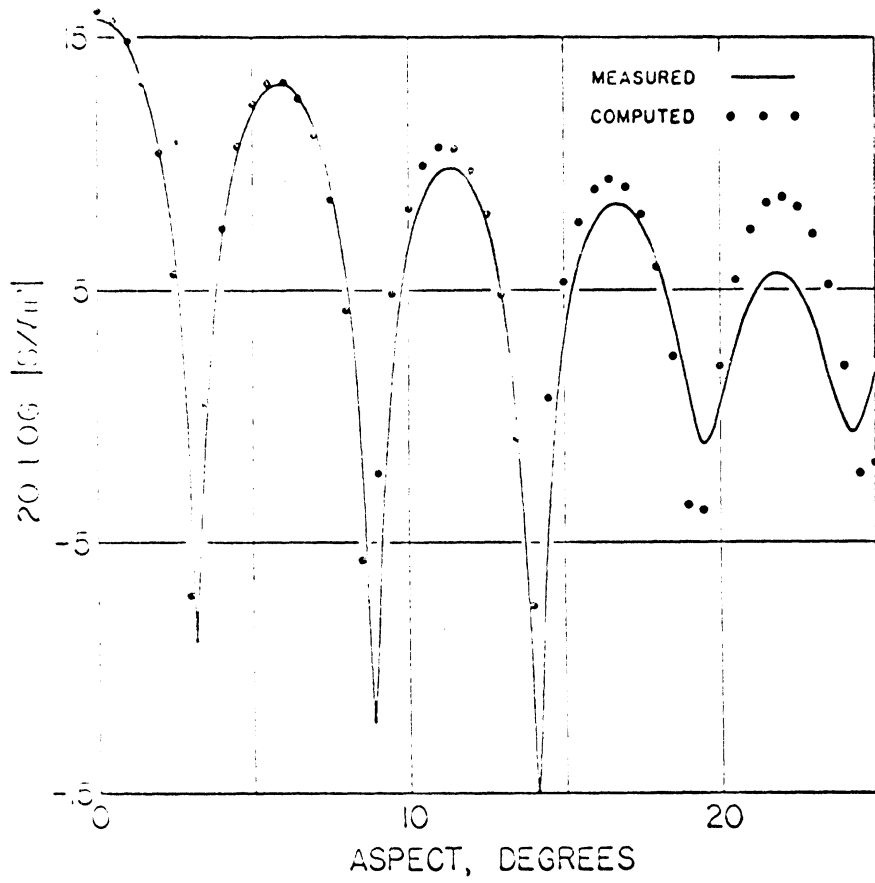
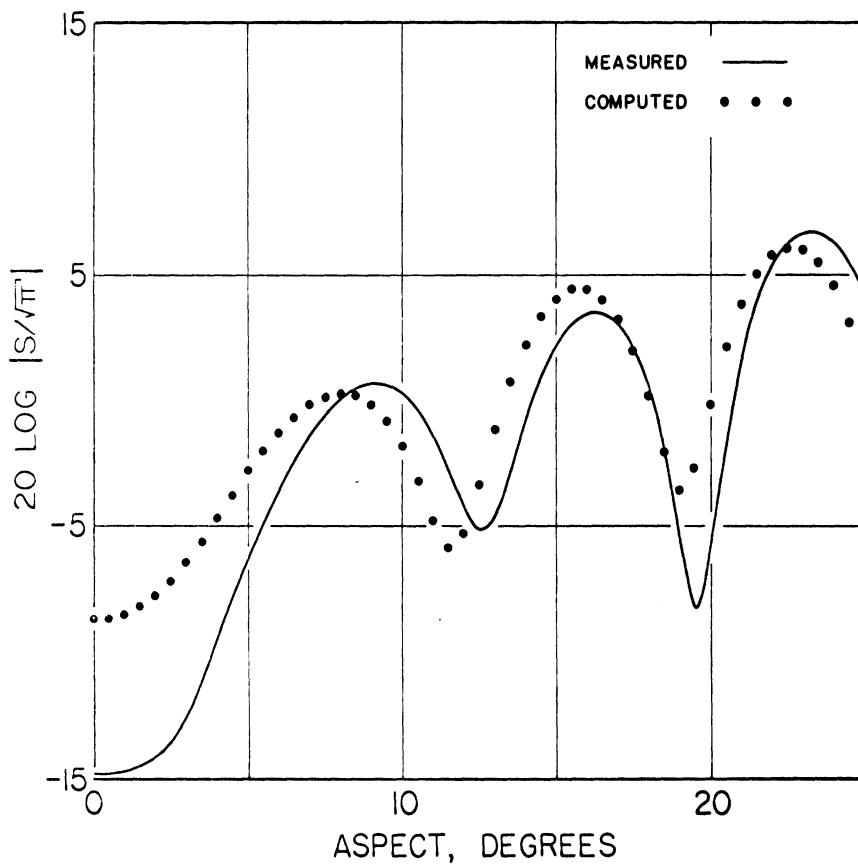Fig. 3.   Results for horizontal polarization and 30-degree total bistatic angle.

Fig. 9. Results for horizontal polarization, 15-degree
tilt angle and 15-degree total bistatic angle.