HIGH-FREQUENCY ELECTROMAGNETIC SCATTERING
FROM A FINITE CIRCULAR CYLINDER

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The problem of the diffraction of a plane wave by a solid, right circular metallic cylinder is almost a classical one. The physical optics expression for the scattered field has been available for many years and for many practical purposes this constitutes an adequate approximation at high frequencies. More refined approximations taking into account single (or first order) diffraction at the edges have been obtained by Ufimtsev using his physical (or fringe wave) theory of diffraction, and by Bechtel (and others) using the ray techniques of the geometrical theory of diffraction. The results are in rather good agreement with CW measurements of, for example, the backscattered field, but nevertheless there are circumstances when some of the second order effects become important, and these present a number of interesting practical and theoretical problems.

For simplicity, I will consider only backscattering, and will use primarily GTD, through I will make some use of the derived method of equivalent currents as well.

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The geometry is shown here:

- cylinder of radius a and length ℓ, both large compared with the wavelength.
- plane wave incident at angle θ to axis. Sufficient to assume θ between 0 and $\pi/2$ inclusive.
- will consider the two principal polarizations: that in which the electric vector is in the y direction, and which I will call E-polarization; and that in which the magnetic vector is in the y direction, called H-polarization.

At wide angles — θ bounded away from 0 and $\pi/2$ — get first order diffraction at the flash points P_1, P_2 and P_3 on the illuminated portions of the edges. In addition, we may have a creeping wave which is diffracted at the rear edge, and which is a pseudo first order term in the sense that it undergoes a single edge diffraction. And in the directions $\theta = 0$ and $\pi/2$ we also have the contributions from the rays specularly reflected from the front face and side of the cylinder.

The second order ray paths at wide angles are as shown in this lower diagram:

- along the generator $P_2 \; P_3$ of the illuminated side;
- across the diameter $P_1 \; P_2$ of the front face;
- and along a perpendicular line $P_5 \; P_6$ whose location depends on the angle of incidence — leading to the term 'migratory path'.

Each path is, of course, described in both directions. We could also have creeping waves diffracted at both front and rear edges, but their contribution is insignificant.

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2 Let us look now at the first order contribution from P_1, P_2 and P_3. At wide angles, it is a trivial matter to write this down:

- $\frac{1}{2}(X \mp Y)$ are local wedge diffraction coefficients as defined by Keller. Upper and lower sign for E or H polarization respectively; suffix indicates point from which contribution arises.

- X, Y are just ratios of trig functions, X being constant and Y depending on θ. You should note that Y_1 and Y_2 are infinite when $\theta = 0$, because this is a caustic of the geometrical optics rays and implies diffraction along the reflected wave boundary. Similarly, Y_2 and Y_3 are infinite when $\theta = \pi/2$.

- phase factors are explicitly displayed, and here you see the divergence factor which also becomes infinite when $\theta = 0$. Since the whole edge now contributes, the axis is a caustic of the diffracted rays.

- the quantity S is a complex far field amplitude, i.e., the coefficient of $\frac{e^{ikR}}{kR}$ in the far field, the time factor being $e^{-i\omega t}$. In terms of S, the scattering cross section is as shown.

To obtain a uniform expression valid for all θ from 0 to $\pi/2$ it is necessary to match this into the finite values at the caustics. There are several techniques for doing so, but the one that we have found most convenient is the method of equivalent currents in which the wide angle contributions are used to define equivalent filamentary electric and magnetic currents coincident with the edge. In the case of the front edge, the known behavior of Y_1 and Y_2 as functions of θ as well as of angular position around the edge leads to a specific integral expression for the field which can be evaluated analytically to produce an expression involving the Bessel functions J_0, J_1 and J_2 of argument ξ.

The matching of the rear edge contribution into the axial caustic is just a little bit more tricky, but if we follow Ufimtsev's argument, we are led to introduce the matching functions J_1 and J_2, which are zero on axis.

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3 The result is this, finite for all θ:

- in the axial direction, the result is independent of ℓ and just the physical optics contribution from the front face;
- at broadside we have the physical optics contribution from the side, plus diffraction contributions from P_2 and P_3 and (as indicated by the phase factor) from P_1.

It differs only insignificantly from that obtained by Ufimtsev and the agreement with CW measured data is illustrated here

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using Bechtel's data for a cylinder about 2 λ in radius and 9 λ in length. The only discrepancies are of order 2 or 3 dB at the intermediate aspects.

This agreement is mainly due to the dominating influence of the physical optics returns from the front and side. If, in the experiment, short pulses are used instead, short enough to resolve individual contributors to the scattering, the limitations of the first order theory are more apparent. For example, the rear edge is actually quite a strong source of scattering even at axial incidence, and, in addition, a number of apparent sources are seen whose position in space varies with θ. Because of the growing use of short pulse techniques in inverse scattering, it is important to be able to understand and interpret such results.

So let's look at some of the other types of contribution.

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Taking first the edge-diffracted creeping wave, there are two points of theoretical interest. As you can see from the geometry, and I show here the unrolled cylinder on which the creeping wave paths are straight lines:

- for all θ for which these paths exist, the backscattering direction is a caustic for these rays,

- and if creeping waves are diffracted, it would appear to be the surface field which is required and not the field in some boundary layer as is usual in GTD.

Fortunately, for a cylinder it is easy enough to work with the surface fields. For typical ray shown

- creeping wave excitation of the edge;

- specification of the local strength of equivalent (magnetic) edge current;

- using known expression for the field of an azimuthal magnetic dipole on the cylinder, get the far field contribution;

- integrate over the angular extent of the excited edge

- factor 2 for 2-way path.

The result is as shown, where $\phi_2 - \phi$ is zero for small θ, then rises linearly to π, and is constant thereafter.

This contribution has apparently been detected by Liang at General Dynamics using short pulses, but its magnitude is rather small.

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For the second order contributions, the wide angle expressions can be obtained by repeated application of first order edge diffraction. For the migratory path, the result is this top one where $G_{5,6}$ is a product of diffraction coefficients. Observe that the axis is a caustic, but the required caustic matching function can be obtained using the equivalent current method and is a finite-range Fresnel integral. The result is then finite for all θ.

For the path across the diameter we again have an axial caustic and the required matching function is just the complex conjugate of the previous one. But there is an additional complication: one of the diffraction coefficients embodied in $G_{1,2}$ becomes infinite at $\theta = \pi/2$, since we then have forward diffraction from (say) P_2 to P_1, and within a transition region about $\pi = \pi/2$ a modified diffraction coefficient must be employed.

As θ approaches zero, the sum of these two contributions for E and H polarization become equal in magnitude and opposite in sign, as required. On axis the result is $O(\overline{ka}^{1/2})$, which is $(ka)^{3/2}$ smaller than the physical optics value. It is, moreover, the third — not the second — order term in the axial expansion.

The remaining second order path is along a generator and its contribution is of particular interest. One of the diffraction coefficients in $G_{2,3}$ is now infinite at $\theta = 0$ — again, because of forward diffraction — and since we also have an axial caustic, it is by no means evident how to derive the caustic matching functions, though the $(\sin\theta)^{-1/2}$ behavior suggests some form of circular function.

To see how to proceed, let's look at the magnitudes of the various contributions compared with the first order ones

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These are for the same size cylinder as before (here is the creeping wave contribution, for example), but I want to concentrate on the rear edge contributions for H polarization. For small θ , the first and second order returns are of similar magnitude, suggesting that they must be treated jointly to produce the matching into the axial caustic.

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This you can do by considering the field close to the surface of the local wedge at P_2. The net field incident at P_3 is then as shown, embodying a uniform diffraction coefficient Y^1 , and remaining valid as $\theta \rightarrow 0$. You can then multiply by the diffraction coefficient at P_3 and repeat this combined treatment to find the backscattered field. What we are <u>really</u> doing is including terms through third order in the wide angle expression, but as $\theta \rightarrow 0$ each second order term becomes $-\frac{1}{2}$ x first, cancelling it, and leaving the third order (say) term, which is $(-\frac{1}{2})^2$ x first. When this net contribution is matched into the axial caustic using, for example, the equivalent current method, the result is as shown. The second part is what we previously had in our first order uniform expression, and the first part, remaining non-zero on axis, is the result of including second and third order terms at wide angles. This is actually the <u>second</u> order term in the axial expansion, which points up the re-ordering of terms that can occur between wide angle and caustic.

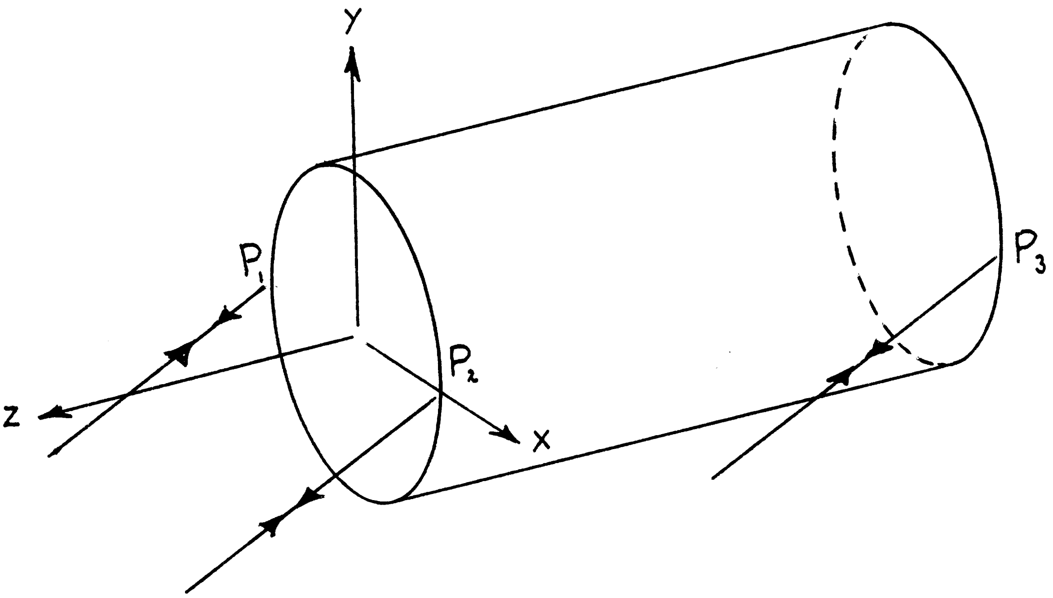
You can, of course, put all these pieces of the theory together, but my interest has been rather to focus on the key features involved:

- caustic matching, particularly of the higher order terms
- creeping wave diffraction at the edges, and
- transitional effects for near-grazing incidence.

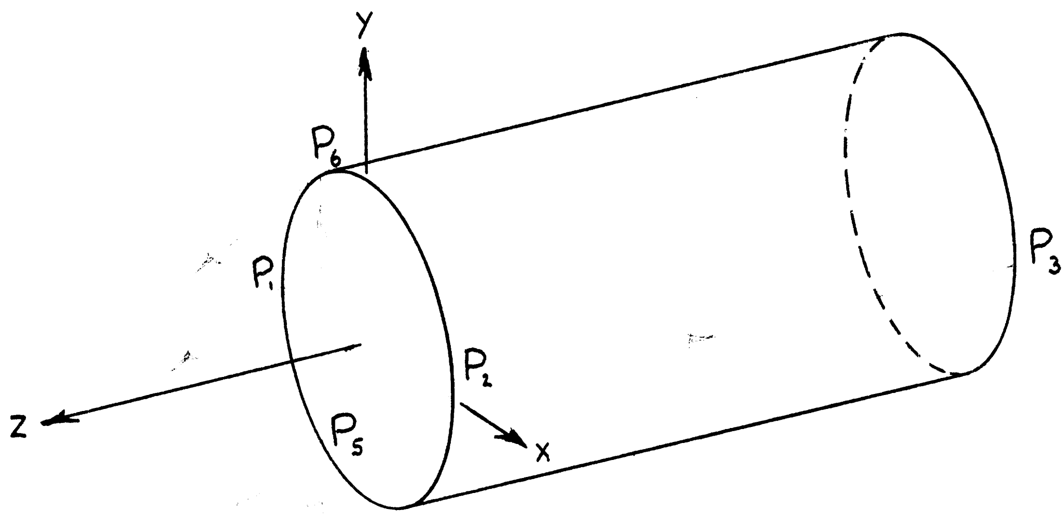
They all have their challenges theoretically and the results are of some practical interest as well inasmuch as most, if not all, of these sources have been detected experimentally.

EDGE-DIFFRACTED RAYS

First Order



Second Order



FIRST ORDER

Wide angle

$$S_{E,H}(\theta) = \frac{1}{2}\sqrt{\frac{ka}{\pi\sin\theta}}\left\{ \left(X \mp Y_1\right) e^{-i\pi/4 + i\xi} \right.$$

$$+ \left(X \mp Y_2\right) e^{i\pi/4 - i\xi}$$

$$\left. + \left(X \mp Y_3\right) e^{i\pi/4 - i\xi + 2ikl\cos\theta} \right\}$$

where $\xi = 2ka\sin\theta$.

As $\theta \rightarrow 0$, $Y_1 \sim -Y_2 \rightarrow \infty$, $Y_3 \rightarrow X$

$\theta \rightarrow \pi/2$, $Y_2 \sim -Y_3 \rightarrow \infty$, $Y_1 \rightarrow X$

Note: $\sigma_{E,H}(\theta) = \frac{\lambda^2}{\pi}\left|S_{E,H}(\theta)\right|^2$

Uniform Expression

$$S_{E,H}(\theta) = -\frac{ka}{2}\Big[2X\,J_2(\xi) \pm (Y_1+Y_2)\,J_0(\xi)$$

$$\pm\, i\,(Y_1-Y_2)\,J_1(\xi)$$

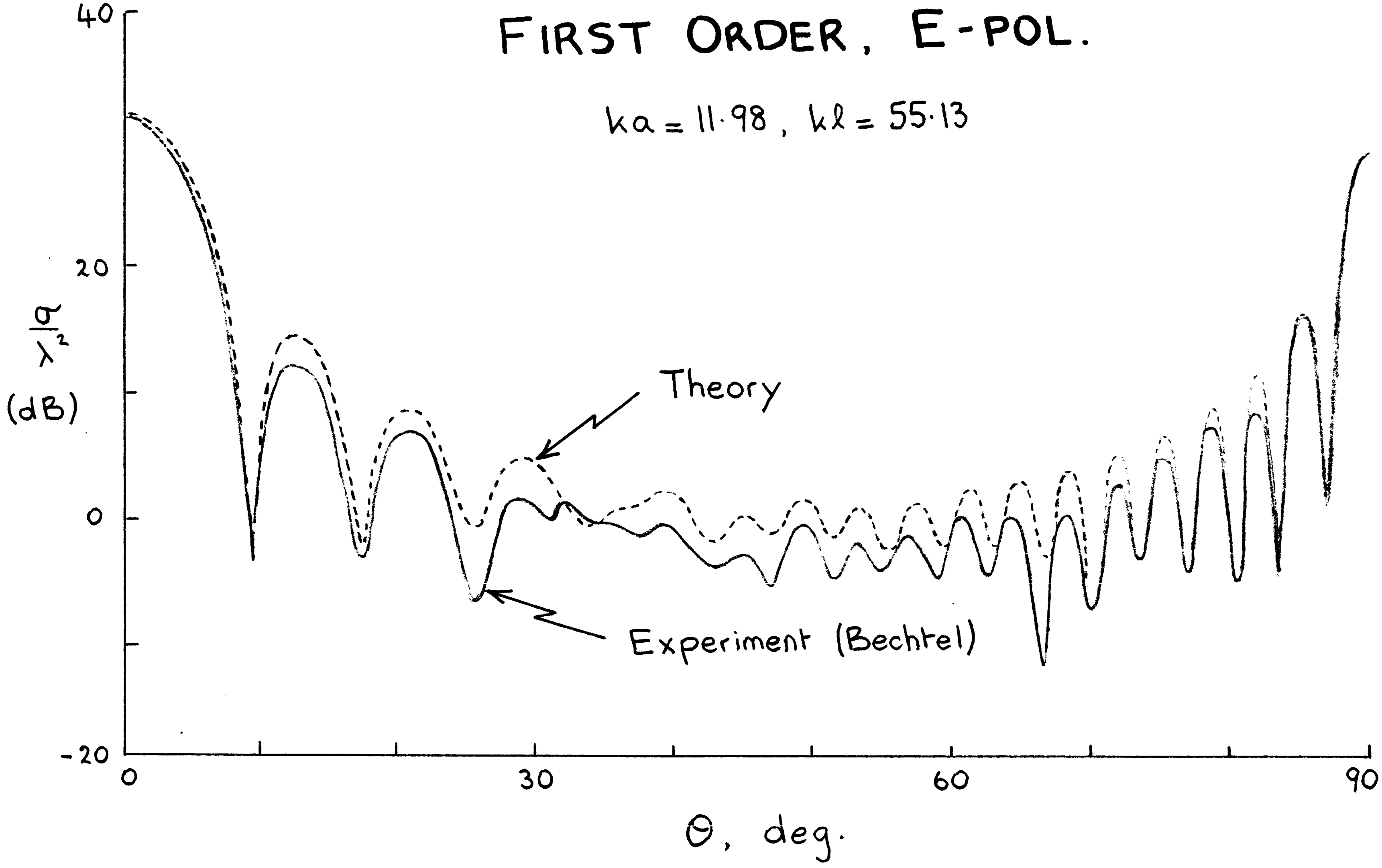
$$+\,(X \mp Y_3)\left\{J_2(\xi) + i\,J_1(\xi)\right\} e^{2ikl\cos\theta}\Big]$$

In particular

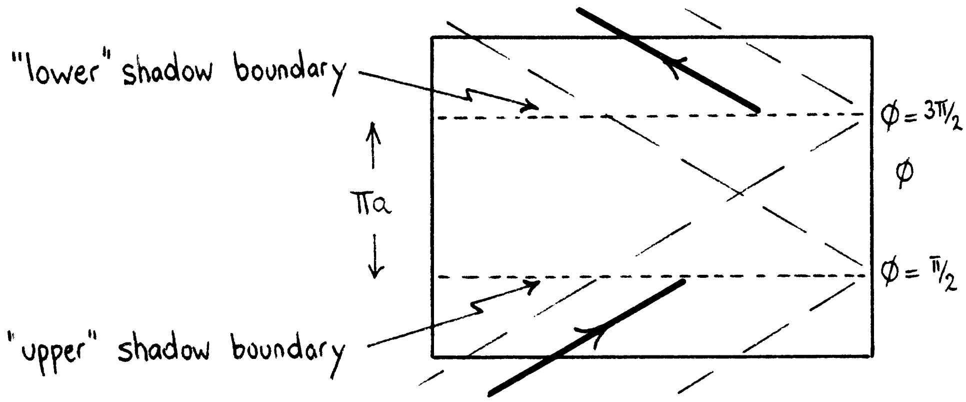
$$S_{E,H}(0) = \pm\frac{i}{2}(ka)^2 \qquad \text{(indep.}^{t} \text{ of } l\text{)}$$

$$S_{E,H}(\pi/2) = \mp\frac{1}{2}kl\sqrt{\frac{ka}{\pi}}\, e^{-2ika - i\pi/4}$$

$$\cdot\left[1 \pm \frac{2X}{ikl}\left\{1 - \frac{i}{2}(1\mp 1)e^{4ika}\right\}\right]$$


FIRST ORDER, E-POL.
ka = 11·98, kℓ = 55·13
Theory
Experiment (Bechtel)
40
20
0
-20
σ/λ²
(dB)
0
30
60
90
Θ, deg.

EDGE-DIFFRACTED CREEPING WAVE



unrolled cylinder

$$S_E(\theta) = 2\int_{\phi_1}^{\phi_2} \frac{e^{ik\ell\cos\theta + i\nu(\phi - \frac{\pi}{2})}}{2\Gamma} \cdot \frac{4X}{ik\cos\theta} \cdot \frac{ik^2 a\cos\theta}{4\pi\Gamma}\, e^{ik\ell\cos\theta + i\nu(\frac{3\pi}{2} - \phi)}\, d\phi$$

$$S_H(\theta) \simeq 0$$

for $0 < \theta < \pi/2$, where

$$\Gamma = \beta \operatorname{Ai}(-\beta), \qquad \nu = ka\sin\theta + \left(\tfrac{1}{2}ka\sin\theta\right)^{1/3} \beta e^{i\pi/3},$$

$$\phi_2 - \phi_1 = \begin{cases} 0, & 0 < \tan\theta < \frac{\pi a}{2\ell} \\ \frac{2\ell}{a}\tan\theta - \pi, & \frac{\pi a}{2\ell} < \tan\theta < \frac{\pi a}{\ell} \\ \pi, & \frac{\pi a}{\ell} < \tan\theta < \infty \end{cases}$$

Hence

$$S_E(\theta) = \frac{kaX}{\pi\Gamma^2}(\phi_2 - \phi_1)\, e^{2ik\ell\cos\theta + i\nu\pi}$$

SECOND ORDER

$P_{5,6}$ — migratory path

$$S_E(\theta) = \frac{i}{\pi}\,\frac{\cos^2\theta}{\sin\theta}\,(1+\sin^2\theta)^{1/2}\,e^{2ika(1+\sin^2\theta)^{1/2}}\,G_{5,6}(\theta)\,f(\theta)$$

$$S_H(\theta) = \tan^2\theta\; S_E(\theta)$$

where $$f(\theta) = \frac{2}{\sqrt{\pi}}\,e^{i\pi/4}\,\mathcal{F}^*\left(\frac{\pi}{4}\sin\theta\sqrt{ka}\right)$$

$P_{1,2}$ — diameter path

$$S_E(\theta) = 0$$

$$S_H(\theta) = \frac{e^{2ika}}{\pi\sin\theta}\,G_{1,2}(\theta)\,f^*(\theta)$$

$P_{2,3}$ — generator path

$$S_E(\theta) = 0$$

$$S_H(\theta) = \frac{2ia}{\pi}\left\{\ell\sin\theta\,(2a+\ell\sin\theta)\right\}^{-1/2} e^{ik\ell(1+\cos\theta)-i\xi}\,G_{2,3}(\theta).$$

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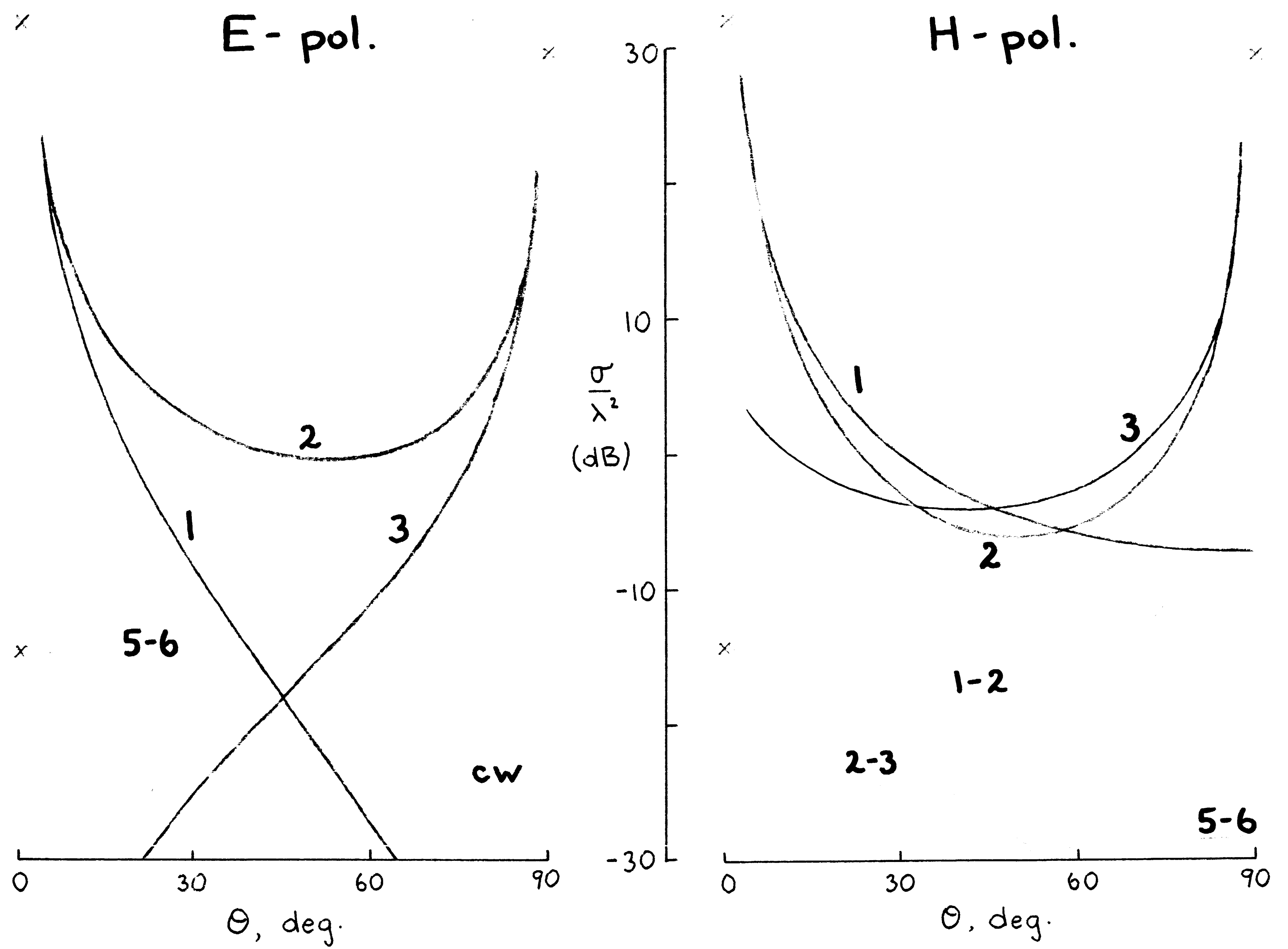
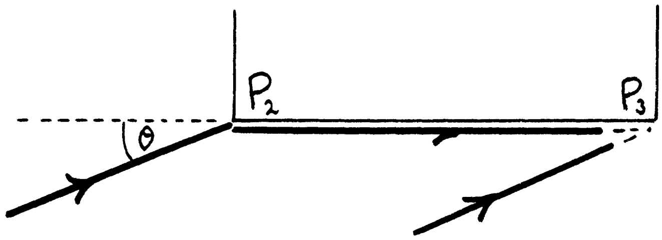

E - pol.
H - pol.
30
10
-10
-30
$\frac{\sigma}{\lambda^2}$ (dB)
1
2
3
5-6
cw
1-2
2-3
0
30
60
90
θ, deg.
θ, deg.

TRANSITION EFFECT



Using **uniform** diffraction coefficient

$$Y' = \frac{\frac{1}{n}\sin\frac{\pi}{n}}{\cos\frac{\pi}{n} - \cos\frac{\pi-\theta}{n}} + \frac{1}{2}\operatorname{cosec}\frac{\theta}{2} + i\sqrt{2k\ell}\, e^{-ik\ell(1-\cos\theta)}\, F\left[\sqrt{2k\ell}\,\sin\frac{\theta}{2}\right]$$

$$\begin{matrix} E^i(P_3) \\ H^i(P_3) \end{matrix} = e^{-ika\sin\theta + ik\ell\cos\theta}\left\{1 + \frac{1}{\sqrt{\ell\left(1+\frac{\ell}{a}\sin\theta\right)}}\,\frac{e^{i\pi/4}}{\sqrt{2\pi k}}\, e^{ik\ell(1-\cos\theta)}\,\frac{1}{2}\left(Y' \mp Y'\right.\right.$$

$$S^{i3d} + \left(S^{i23d} + S^{i32d}\right) + S^{i232d}$$

$$\underset{\ell\sin\theta \ll 1}{\sim} \frac{1}{4}\sqrt{\frac{ka}{\pi\sin\theta}}\,\frac{1}{2}\left(X \mp Y_2\right) e^{i\pi/4 - i\xi + 2ik\ell\cos\theta}$$

$$S_{E,H}(\theta) = \left[\mp\frac{ka}{4}Y_2\left\{J_0(\xi) + J_2(\xi)\right\} - \frac{ka}{2}\left(X \mp Y_2\right)\left\{J_2(\xi) + iJ_1(\xi)\right\}\right] e^{2ik\ell\alpha}$$

In particular

$$S_{E,H}(0) = \mp\frac{1}{4}kaY_2\, e^{2ik\ell}$$