

## LOW FREQUENCY SCATTERING BY SPACE OBJECTS\*

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RL-611 = RL-611

## Abstract

The relevance of the polarizability tensors in low frequency scattering is discussed. Data are presented for a variety of perfectly conducting, rotationally symmetric bodies simulating simple aerospace objects, and it is shown how the presence of any coating influences the results.

## 1. Introduction

The development of low frequency radars has increased the importance of understanding the scattering characteristics of objects of linear dimensions much smaller than the operating wavelength. This low frequency regime is designated the Rayleigh region and it is well known that the scattered field can then be attributed to equivalent radiating electric and magnetic dipoles. The far field pattern has the characteristic dipole shape and is completely determined by the induced dipole moments, which are functions of the incident field. For plane wave incidence, Keller et al. [1] have shown that the induced electric dipole moment can be expressed as the dot product of an electric polarization vector with an electric polarizability tensor whose elements are functions only of the target geometry. They also derive a similar expression for the induced magnetic dipole moment. The results are particularly simple in the case of bodies of revolution where only three independent tensor elements suffice to specify the low frequency scattering behavior in its entirety. The elements can be computed by solution of the appropriate potential problems and data are presented here for a variety of space objects.

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This work was supported by the U.S. Air Force Office of Scientific Research under Grants 74-2634 and 72-2262.

These results are also applicable to a body surrounded by an idealized form of plasma sheath or lossy dielectric coating. Since the sheath effects only the electric dipole moment whose value is the same as for a perfectly conducting body whose surface configuration is that of the sheath, this provides a basis for using low frequency scattering data to obtain geometric information about the body and its sheath separately.

## 2. Metallic Bodies

Consider a finite, closed, perfectly conducting rotationally symmetric body illuminated by a linearly polarized, plane electromagnetic wave. With the notation of Keller et al. [1] in which the z-axis coincides with the axis of symmetry, the Rayleigh term of the scattered far field is [2]

$$\begin{aligned} \underline{E}^s(\underline{r}) \sim - \frac{e^{ikr}}{4\pi r} k^2 \left\{ P_{11} \hat{r} \times (\hat{r} \times \hat{a}) + (P_{33} - P_{11})(\hat{a} \cdot \hat{z}) \hat{r} \times (\hat{r} \times \hat{z}) \right. \\ \left. - M_{11} \hat{r} \times \hat{b} - (M_{33} - M_{11})(\hat{b} \cdot \hat{z}) \hat{r} \times \hat{z} \right\} \end{aligned} \quad (1)$$

where the unit vectors  $\hat{a}$ ,  $\hat{b}$ , and  $\hat{r}$  denote the incident electric polarization, incident magnetic polarization, and receiver direction respectively, and  $P_{11}$ ,  $P_{33}$ ,  $M_{11}$ ,  $M_{33}$  are the nonvanishing elements of the polarizability tensors. The scattered far field determines the radar cross section  $\sigma$  through

$$\sigma = 4\pi r^2 \left| \underline{E}^s \right|^2 \quad (2)$$

In the particular case of backscattering ( $\hat{r} = \hat{b} \times \hat{a}$ )

$$\begin{aligned} \sigma = \frac{k^4}{4\pi} \left\{ \left[ P_{11} + M_{11} + (P_{33} - P_{11})(\hat{a} \cdot \hat{z})^2 + (M_{33} - M_{11})(\hat{b} \cdot \hat{z})^2 \right]^2 \right. \\ \left. + (P_{33} - P_{11} + M_{11} - M_{33})^2 (\hat{a} \cdot \hat{z})^2 (\hat{b} \cdot \hat{z})^2 \right\} \quad (3) \end{aligned}$$

The tensor elements  $P_{11}$ , etc. are functions of the target geometry alone and are subject to a number of relations [1, 2]. We cite only one,

$$M_{33} = \frac{1}{2} P_{11} \quad (4)$$

provided the body is not ring shaped (toroidal). This reduces to three the number of quantities needed in (1) and (3), and these can be expressed as weighted integrals of the solutions of certain potential problems for the geometry in question [1].

Although the solutions can be obtained analytically for only a restricted number of bodies, it is a relatively straightforward task to solve the corresponding integral equations numerically and, hence, compute the elements for almost any desired axially-symmetric shape. Such a program has been developed by Senior and Ahlgren [3, 4] and the results for a number of aerospace geometries are given in Table 1.

Table 1

To illustrate the use of this table, consider backscattering from the cone-sphere shown when the direction of illumination and the (electric) polarization are perpendicular to the axis of symmetry. Since  $\hat{z} \cdot \hat{a} \times \hat{b} = \hat{a} \cdot \hat{z} = 0$ , it follows that  $\hat{b} \cdot \hat{z} = \pm 1$ , and eq. (3) now gives

$$\sigma = \frac{k^4}{4\pi} (P_{11} + M_{33})^2 \quad (5)$$

From the table,  $P_{11} = 2.50V$  and (using eq. (6))  $M_{33} = 1.25V$ . Hence

$$\sigma = 0.01693k^4 l^6 \quad (6)$$

### 3. Coated Bodies.

The results of the previous section can be extended to the case when the scattering object consists of a perfectly conducting body surrounded by a homogeneous, isotropic, lossy coating characterized by a complex permittivity. Such coatings include lossy dielectrics, with permittivity

$$\tilde{\epsilon} = \epsilon_1 + \frac{i\tilde{\sigma}}{\omega} \quad , \quad (7)$$

and overdense plasmas, with permittivity

$$\tilde{\epsilon} = \epsilon \left\{ 1 - \frac{\omega_p^2}{\omega(\omega + i\nu_c)} \right\} \quad (8)$$

where  $\epsilon_1$  is the permittivity of the layer,  $\tilde{\sigma}$  is the conductivity,  $\omega (= k/\sqrt{\epsilon\mu})$  is the frequency of the incident electromagnetic field,  $\epsilon$  is the free space permittivity,  $\omega_p$  is the plasma frequency and  $\nu_c$  is the collision frequency. In all cases considered here, the permeability of the layer is assumed to be that of free space.

We denote by C the outside surface of the coating and by B the bare body configuration. The basic assumption is that all electromagnetic fields—the incident and scattered fields, and the field in the layer—may be expanded in powers of  $\omega$ . The dipole moments can then be derived from the leading terms in such expansions, e.g. [5]. The leading terms correspond to  $\omega = 0$  and in that case the electric field in the layer must satisfy\*  $\nabla \times \underline{E} = 0$ ,  $\nabla \cdot \underline{E} = 0$ ,  $\hat{n} \cdot \underline{E} = 0$  on C and  $\hat{n} \times \underline{E} = 0$  on B. This implies [6, 7] that  $\underline{E} = 0$  in the layer. Outside the layer the  $\omega = 0$  term of the total electric field now satisfies  $\nabla \times \underline{E} = 0$ ,  $\nabla \cdot \underline{E} = 0$ ,  $\hat{n} \cdot \underline{E} = 0$  on C and these conditions are exactly the same as the conditions satisfied by the electric field exterior

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\* This is not true if  $\tilde{\sigma} = 0$  unless  $\epsilon = \epsilon_1$  as well.

to a perfectly conducting surface C. Moreover, with  $\underline{E} \equiv 0$  in the layer, it follows that the leading magnetic field term satisfies  $\nabla \times \underline{H} = 0$ ,  $\nabla \cdot \underline{H} = 0$  both inside and outside the layer. Furthermore,  $\hat{n} \cdot \underline{H} = 0$  on B and the normal and tangential components of  $\underline{H}$  are continuous at C. Provided  $\nu_c \neq 0$ , the conditions satisfied by  $\underline{H}$  are now the same as those satisfied by the magnetic field exterior to a perfectly conducting surface B. Thus, a lossy layer is invisible to the leading magnetic field term but opaque to the leading electric field term.

As a consequence of these facts, the dipole moments of the composite body can be described in terms of the moments for perfect conductors:  $\underline{p}$  is the induced electric dipole moment for a perfect conductor whose configuration is C, while  $\underline{m}$  is the induced magnetic dipole moment for the bare body B. If we append the superscripts B and C to indicate the surfaces to which the moments are appropriate, the far zone electric field scattered by the composite body when illuminated by a plane wave is

$$\underline{E}^s \sim -\frac{e^{ikr}}{4\pi r} k^2 \left\{ \frac{1}{\epsilon} \hat{r} \times (\hat{r} \times \underline{p}^C) + Z \hat{r} \times \underline{m}^B \right\} . \quad (9)$$

The dipole moments  $\underline{p}^C$  and  $\underline{m}^B$  can be expressed in terms of the polarizability tensors [1], but it should be noted that the tensors are now appropriate to different configurations.

For objects and coatings with axial symmetry, the results of the previous section are applicable. In particular, eqs. (1) and (3) remain valid with the understanding that the electric polarizability tensor elements correspond to C while the magnetic polarizability tensor elements correspond to B. Table 1 can therefore be used to calculate the cross section of a composite shape provided both the layer and bare body geometries are listed. In using the table, it must be kept in mind that (4) holds only between elements for the same shape and not for the  $M_{33}$  and  $P_{11}$  of the composite body. In the present case, the  $M_{33}$  required must be obtained from (4) using the  $P_{11}$  for the bare body.

As an example, consider backscattering from a cone sphere with a prolate spheroidal coating (see figure) when the direction of illumination and the electric polarization are perpendicular to the axis of symmetry. Equation (3) now becomes

$$\sigma = \frac{k^4}{4\pi} (P_{11}^C + M_{33}^B)^2, \quad (10)$$

and from the table

$$P_{11}^C = 2.42 \left[ 0.131(\ell^C)^3 \right], \quad (11)$$

$$M_{33}^B = \frac{P_{11}^B}{2} = 1.25 \left[ 0.123(\ell^B)^3 \right]. \quad (12)$$

In the present example  $\ell^C = 2\ell^B \sec 15^\circ$ . Using this value and eqs. (10) - (12), the cross section is

$$\sigma = 0.701 k^4 (\ell^B)^6, \quad (13)$$

which exceeds the bare body cross section (6) by almost a factor 40. The composite body cross section may also be compared with that of a perfectly conducting prolate spheroid of the same dimensions as the coating for the same incident field. In this case,  $P_{11}^C$  is again given by (11), but  $M_{33}^C$  is obtained from (4) and

$$\sigma = 1.418 k^4 (\ell^B)^6, \quad (14)$$

a value slightly greater than twice that for the composite body.

While the elements of the polarization tensors uniquely specify the radar cross section of the object deep within the Rayleigh region, neither the cross section nor the tensor elements themselves uniquely determine the scatterer shape. Nevertheless, the elements can serve to discriminate among bodies within a class, and scattered field measurements for a sheathed body can be used to specify separately the dipole moments of a body and its coating. For example, back and forward scattering measurements can be added and subtracted to separate these moments. From eq. (1), the forward ( $\hat{r} = \hat{k}$ ) and back ( $\hat{r} = -\hat{k}$ ) scattered fields are

$$\begin{aligned} \underline{E}^S(\pm \hat{k}) \sim & -\frac{e^{ikr}}{4\pi r} k^2 \left\{ P_{11}^C \hat{k} \times (\hat{k} \times \hat{a}) + (P_{33}^C - P_{11}^C)(\hat{a} \cdot \hat{z}) \hat{k} \times (\hat{k} \times \hat{z}) \right. \\ & \left. \mp \left[ M_{11}^B \hat{k} \times \hat{b} - (M_{33}^B - M_{11}^B)(\hat{b} \cdot \hat{z}) \hat{k} \times \hat{z} \right] \right\} \end{aligned} \quad (15)$$

where  $\hat{k} = \hat{a} \times \hat{b}$ . Hence

$$\underline{E}^S(\hat{k}) + \underline{E}^S(-\hat{k}) \sim -\frac{e^{ikr}}{2\pi r} k^2 \left\{ P_{11}^C \hat{k} \times (\hat{k} \times \hat{a}) + (P_{33}^C - P_{11}^C)(\hat{a} \cdot \hat{z}) \hat{k} \times (\hat{k} \times \hat{z}) \right\}, \quad (16)$$

which depends only on the sheath geometry, while

$$\underline{E}^S(\hat{k}) - \underline{E}^S(-\hat{k}) \sim \frac{e^{ikr}}{2\pi r} k^2 \left\{ M_{11}^B \hat{k} \times \hat{b} + (M_{33}^B - M_{11}^B)(\hat{b} \cdot \hat{z}) \hat{k} \times \hat{z} \right\}, \quad (17)$$

which is a function only of the bare body. This separation can also be achieved by combining measurements at other aspects and/or polarizations.

#### 4. Conclusion

The preceding analysis shows that there is little point in presenting scattering data at low frequencies in any form other than a listing of the elements of the electric and magnetic polarizability tensors. Since the radiation patterns of dipoles are well known, it is sufficient to specify only the induced dipole moments, and as we have seen, these can be written in terms of the polarizability tensors in a way which explicitly exhibits the dependence of the incident plane wave. It is therefore suggested that the elements of these tensors be catalogued for classes of practical aerospace configurations. Such a catalogue would serve as a source of useful parameters in designing configurations to achieve desired low frequency scattering characteristics.

#### References

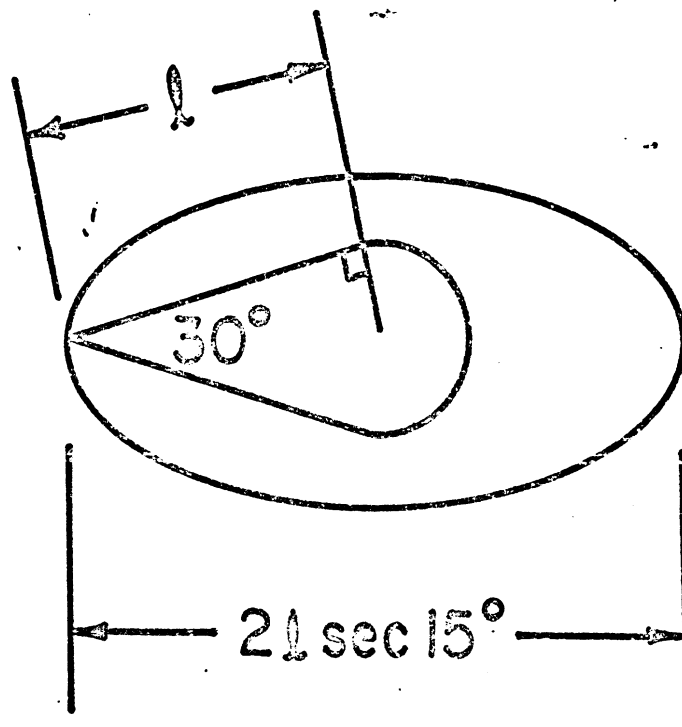
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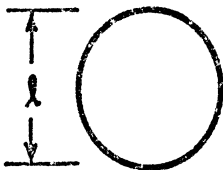
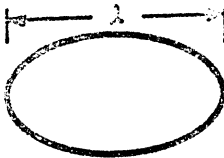
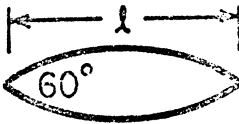


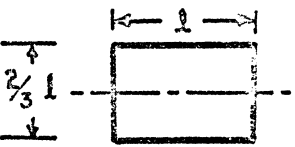
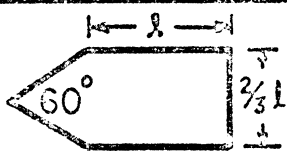
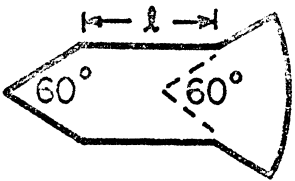
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Figure

DESCRIPTION	V	$P_{11}/V$	$P_{33}/V$	N
SPHERE 	$0.524l^3$	3.00	3.00	1.
2:1 PROLATE SPHEROID 	$0.131l^3$	2.42	5.77	1.
60° OGIVE 	$0.152l^3$	2.57	4.62	1.1
30° ROUNDED CONE 	$0.0714l^3$	2.90	8.21	1.6
30° CONE-SPHERE 	$0.123l^3$	2.50	8.36	1.7
CYLINDER 	$0.349l^3$	2.79	5.37	1.66
60° CONE-CYLINDER 	$0.416l^3$	2.80	9.79	1.81
60° CONE-FLARE 	$0.795l^3$	2.86	9.05	1.71