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Electromagnetic Field Penetration into a Cylindrical Cavity

THOMAS B. A. SENIOR

Abstract—For an E-polarized plane wave incident on a perfectly conducting cylindrical shell having a longitudinal slit aperture, the fields inside the cavity are determined by a numerical solution of the E field integral equation. Selected data are presented and the first few complex frequency (SEM) singularities are determined for a variety of aperture sizes.

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The author is with the Radiation Laboratory, Department of Electrical and Computer Engineering, The University of Michigan, Ann Arbor, Michigan 48104.

As a contribution to the general study of field penetration through an aperture into a cavity beyond, we recently investigated [1] the excitation of a spherical cavity by a plane electromagnetic wave incident on a circular aperture in the shell. To profit from the spherical symmetry of the problem, the interior and exterior fields were expanded in spherical modes, but because of the slow convergence of the expansions in the vicinity of the boundary, it was necessary to resort to a least squares matching of the fields in the aperture and to adopt other measures as well. The resulting method of solution was neither elegant nor efficient, though still adequate for the task.

An alternative is to use an integral equation approach and this has the advantage of being convenient for the determination of the complex frequency singularities of the singularity expansion method (see, for example, [2]). If the fields are again expanded in spherical modes, the integration can be confined to the aperture with the aperture fields as the unknowns. Unfortunately, the convergence difficulties are now worse than before, but if the integrals are taken over the shell instead, the introduction of the spherical mode expansions can be avoided. The unknowns are then the components of the currents induced in the shell, from which the aperture and interior fields can be obtained by a further integration.

To test this approach, we consider here the simpler problem of a cylindrical shell having a slit aperture of angular width $2\phi_0$ parallel to the z axis of the cylinder. The shell is assumed infinitesimally thin and perfectly conducting and is defined in terms of the cylindrical polar coordinates ρ, ϕ, z by the equation $\rho = a$, $\phi_0 \leq \phi \leq 2\pi - \phi_0$ (see Fig. 1). A plane wave is incident in a plane perpendicular to the z axis with its electric vector parallel to the axis (E-polarization). The problem is now a two dimensional one and the incident electric vector is taken as

$$\underline{E}^i = \hat{z}\, e^{-ik\rho\cos(\phi-\alpha)}$$

where a time factor $e^{-i\omega t}$ has been assumed and suppressed.

Since the shell is equivalent to an electric current sheet of strength $J_z(\phi')$ where $\underline{J} = \hat{z} J_z$ is the total current it supports, the scattered electric field is

$$\underline{E}^s = -\hat{z}\frac{kaZ_0}{4}\int_{\phi_0}^{2\pi-\phi_0} J_z(\phi')\, H_0^{(1)}(kR)\, d\phi' \tag{2}$$

where Z_0 is the intrinsic impedance of free space, $H_0^{(1)}$ is the Hankel function of the first kind, and R is the distance between the integration and observation points. The total field is obtained by adding (1) and (2), and if we now allow the observation point to lie on the shell and use the boundary condition at a perfect conductor, the following integral equation results:

$$e^{-ika\cos(\phi-\alpha)} = \frac{kaZ_0}{4}\int_{\phi_0}^{2\pi-\phi_0} J_z(\phi')H_0^{(1)}(kR)\, d\phi' . \tag{3}$$

This is only a particular case of the E field integral equation for two-dimensional resistive sheets and can be solved numerically by the moment method using the computer program developed [3] for the more general situation. We note in passing that the program will also solve the integral equation for an H-polarized incident plane wave. Having found the current J_z, the field at any point can be calculated from eqs. (1) and (2), and the program was specialized to compute the total field in the aperture and at a number of selected points within the cavity.

To illustrate the results obtained, Fig. 2 shows the aperture field as a function of ϕ for $\phi_0 = 30^o$, $\alpha = 0$ and a variety of ka. The behavior is quite similar to that for a sphere. The amplitude increases with ka up to the first resonant frequency of the cavity and then falls rapidly before increasing again to a maximum occurring just below the next resonance. This variation is brought out in Fig. 3 where the field at the center of the aperture is plotted as a function of ka for $\phi_0 = 10^o$ and 30^o. When $\phi = 10^o$, the maximum at ka = 2.40 exceeds 6.5 times the (unit) magnitude of the incident electric field. For this same aperture the fields within the cavity are illustrated in Fig. 4, showing the amplitude as a function of the normalized distance ρ/a from the z axis out to the center of the aperture, and Fig. 5 gives the frequency variation of the axial field for $\phi_0 = 10^o$ and 30^o. With the smaller aperture, the field has a strong resonance at ka = 2.40 and here $|E_z(0)| > 40$, but there is almost no evidence of the (anti-)resonance at ka = 3.83. Increasing the aperture angle to 30^o markedly detunes the cavity, shifting the first resonance to ka = 2.30 (where $|E_z(0)| = 7.26$), and revealing the second. Additional data can be found in [4], where the program used is also described.

It is obvious that the fields in the cavity and the aperture are strongly influenced by the interior resonances and these are in turn related to the complex frequency (SEM singularities from which the transient response can be computed. If the aperture were closed, the singularities would be the real resonant frequencies corresponding to the zeros of $J_n(ka)$, i.e.,

$$\omega = u_{mn}\, c/a$$

where u_{mn} is the mth zero of $J_n(u)$. When ordered in increasing magnitude, the first three are

$$u_{10} = 2.405, \qquad u_{11} = 3.832, \qquad u_{12} = 5.136.$$

Each has its counterpart in the case of a sphere and the lowest ones are even similar in magnitude for the two cavities. If the aperture is now opened, the complex frequencies must take on a negative imaginary part associated with the radiation damping, and in addition it is expected that the real part will decrease with increasing aperture size. This has been verified by computing the zeros of the determinant generated in the solution of the integral equation (3) by the moment method, and data for the first two complex frequencies as a function of the aperture size are listed in Table 1. As regards the determinants considered (30 by 30 for the first resonance and 48 by 48 for the second) the data are accurate to the third decimal place, but since there was no detailed exploration of the influence of matrix size, no statement of the absolute accuracy is possible. As seen from the Table, the aperture's effect increases rapidly with increasing ϕ_0. With the first resonance, however, the effect is small for $\phi_0 = 10^\circ$, and with the second resonance even $\phi_0 = 30^\circ$ produces very little change.

There are also the singularities associated with the exterior region $\rho \geq a$. When the aperture is closed, the complex "resonant" frequencies are

$$\omega = v_{mn}\, c/a$$

where v_{mn} is the mth zero of $H_n^{(1)}(v)$, and in addition there is the branch point at $\omega = 0$ which has no analogue when the body is finite in all dimensions. The zero with the smallest imaginary part is

$$v_{10} = -2.404 - i0.3405 \quad ,$$

and since its imaginary part exceeds that of the first interior resonance for all $\phi_0 \lesssim 55^\circ$, the exterior singularities will be relatively unimportant in any SEM calculation of the transient excitation of a cavity whose aperture is small.

References

[1] T.B.A. Senior and G. A. Desjardins, "Electromagnetic Penetration into a Spherical Cavity," IEEE Trans. Electromagn. Compat., vol. EMC-16, pp. 205-208, Nov. 1974.

[2] C. E. Baum, "Singularity Expansion of Electromagnetic Fields and Potentials Radiated from Antennas and Scattered from Objects in Free Space," U.S. Air Force Weapons Lab. Sensor and Simulation Note 179, May 1973.

[3] V. V. Liepa, E.F. Knott and T. B. A. Senior, "Scattering from Two-Dimensional Bodies with Absorber Sheets," The University of Michigan Radiation Lab., Ann Arbor, Michigan, Report 011764-2-T, May 1974.

[4] T. B. A. Senior, "Field Penetration into a Cylindrical Cavity," The University of Michigan Radiation Lab., Ann Arbor, Michigan, Report 012643-2-T, Jan. 1975.

Figure Captions

Fig. 1: The geometry.

Fig. 2: Aperture field amplitude for $\phi_0 = 30^{\circ}$, $\alpha = 0$ and various ka.

Fig. 3: Field amplitude at the center of the aperture for $\phi_0 = 10^{\circ}$ and 30°, and $\alpha = 0$.

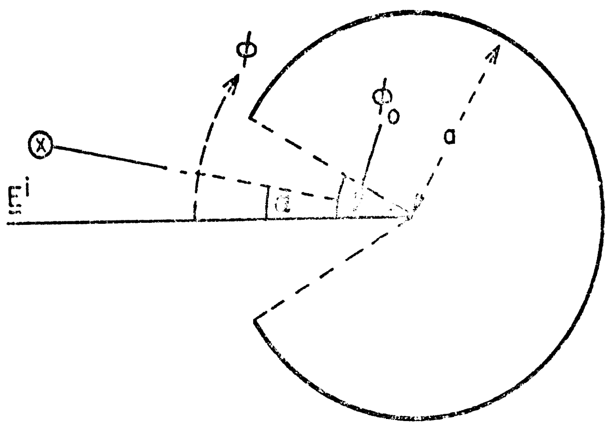
Fig. 4: Interior field amplitude for $\phi_o = 10^{\circ}$, $\alpha = 0$ and various ka.

Fig. 5: Field amplitude at the center of the cavity for $\phi_0 = 10^{\circ}$ and 30°, and $\alpha = 0$.

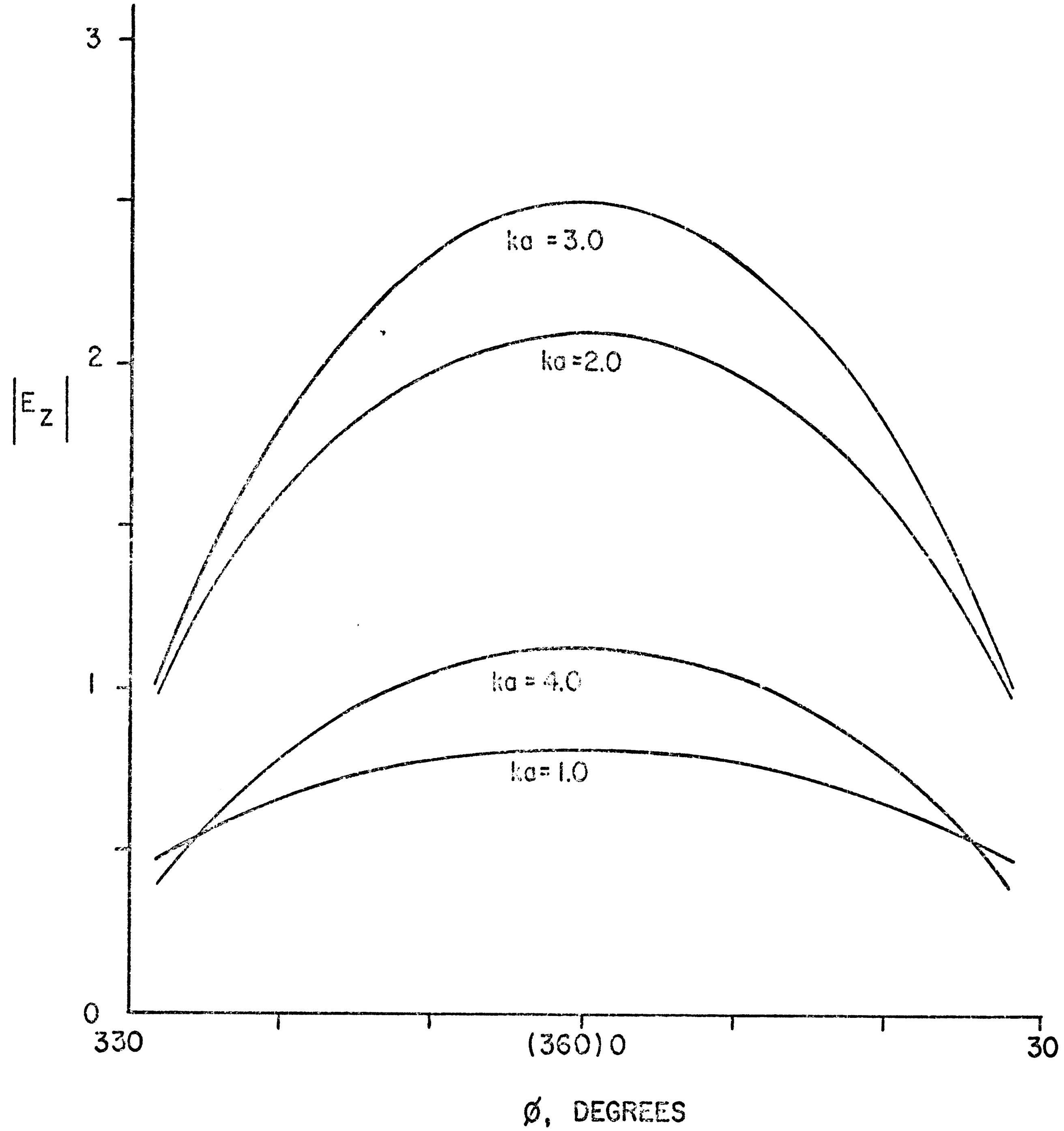
Table 1

INTERIOR SEM SINGULARITIES

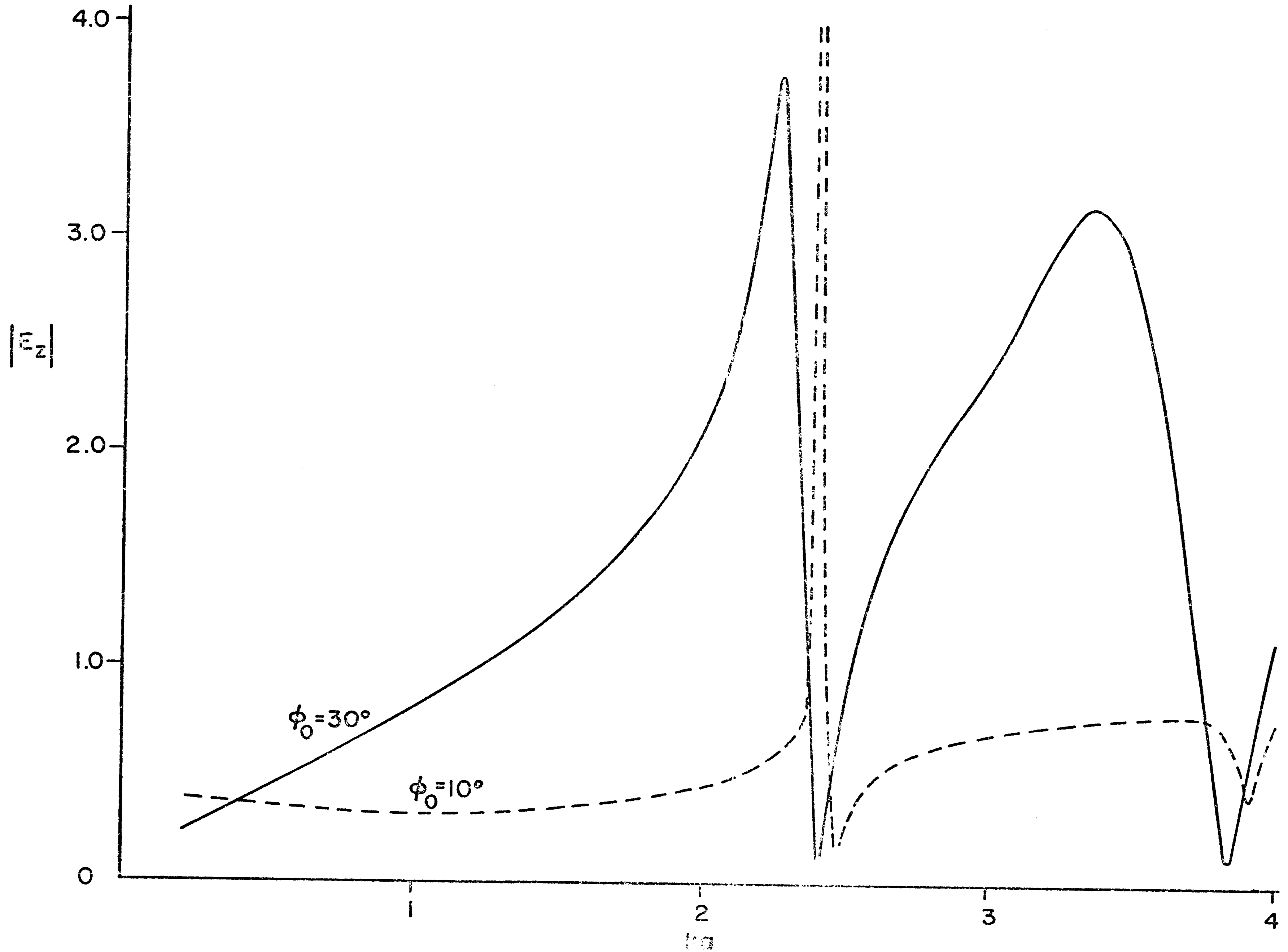
ϕ_0, deg. \ $\omega c/a$	first	second
0	2.405	3.832
10	2.400 - i0.001	3.832 - i0.000
20	2.359 - i0.015	3.831 - i0.000
30	2.320 - i0.067	3.827 - i0.003
40	2.255 - i0.160	3.795 - i0.024
50	2.227 - i0.281	
60	2.220 - i0.424	



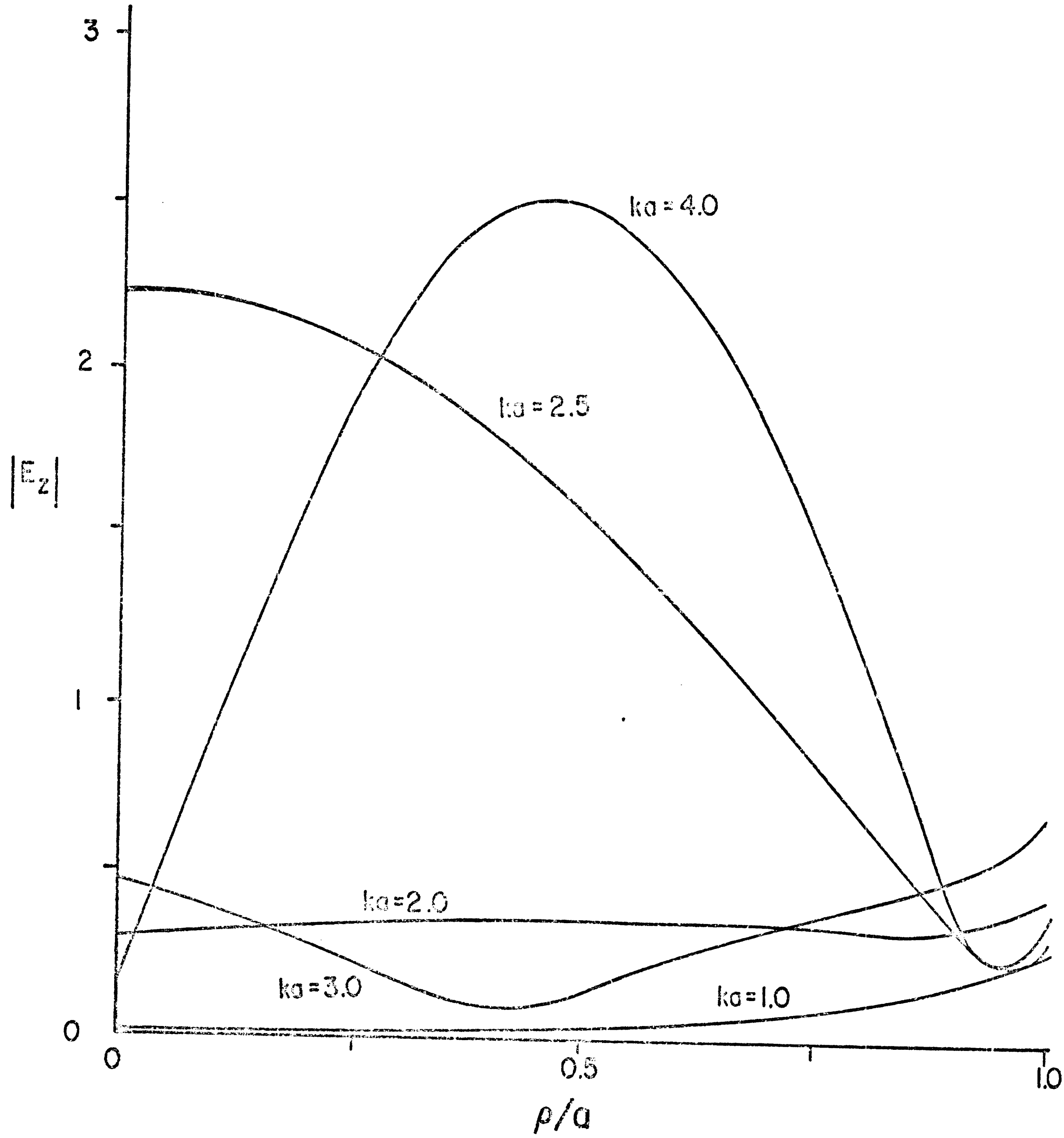
φ
φ₀
a
α
E^i

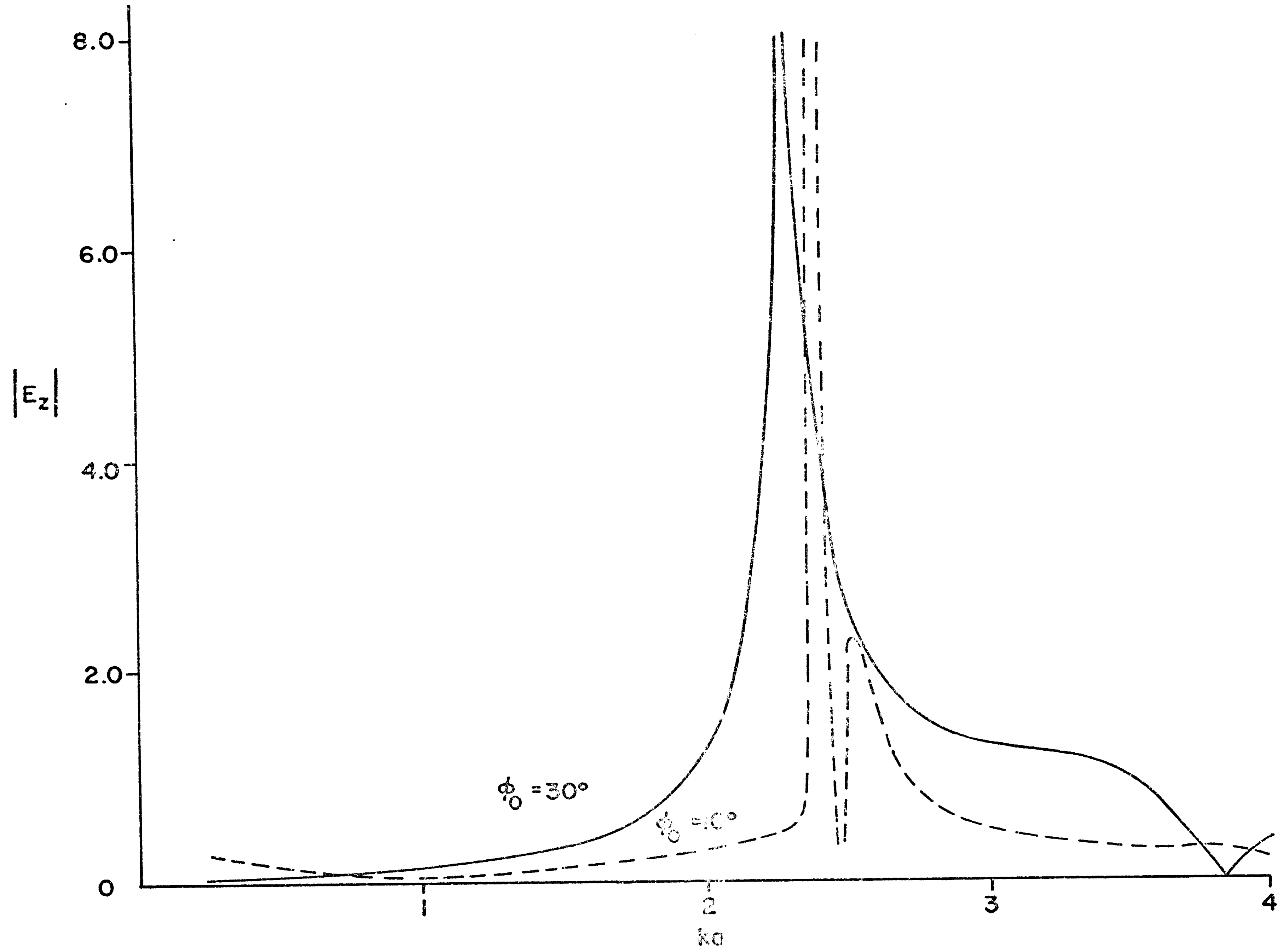


3
2
1
0
|Ez|
ka =3.0
ka=2.0
ka= 4.0
ka= 1.0
330
(360) 0
30
Ø, DEGREES



$|E_z|$
4.0
3.0
2.0
1.0
0
ϕ_0=30°
ϕ_0=10°
1
2
3
4


3
2
1
0
|Ez|
ka = 4.0
ka = 2.5
ka = 2.0
ka = 3.0
ka = 1.0
0
0.5
1.0
ρ/a


|E_z|
8.0
6.0
4.0
2.0
0
ϕ0 =30°
1
2
3
4
ka