Waveguide: A Program for Equivalent Circuit Representation of Two-Dimensional Rectangular Waveguide Discontinutities

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Introduction

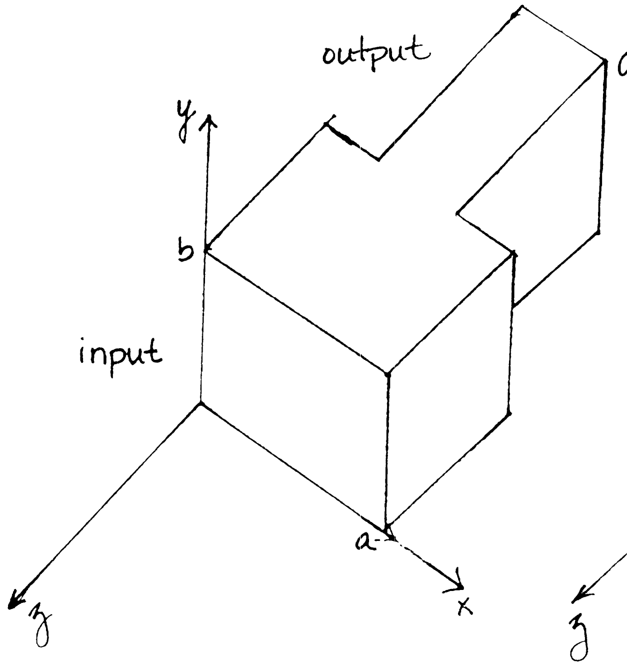
Waveguide is a Fortran program which will characterize simple two-dimensional rectangular waveguide discontinuities, using quasi-static equivalent circuit methods. Specifically, it calculates the normalized reactance for a change in width or the normalized susceptance for a change in height at a waveguide junction. The circuit models examined are found in Marcuvitz's *Waveguide Handbook.*[1] The equations used to derive these models may be found in Appendix A.

Description of Equivalent Circuits

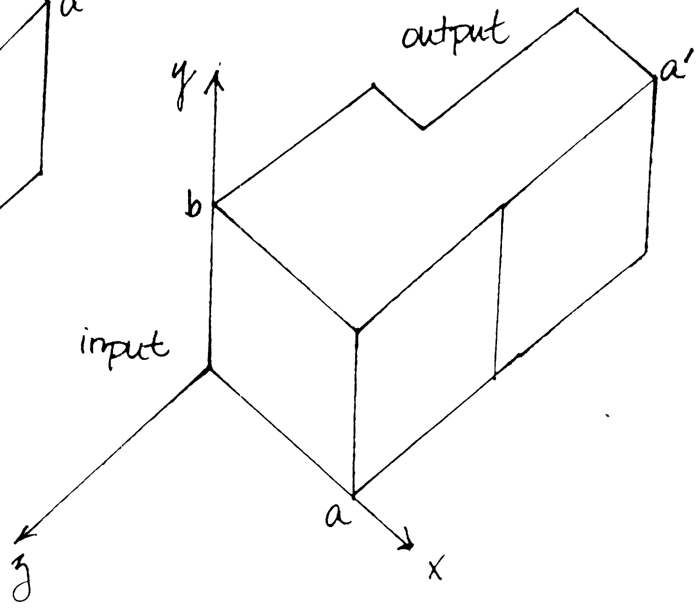
Waveguide is largely based on equations for two-terminal structures (equivalent to one-port circuits) found in Marcuvitz's *Waveguide Handbook.* In general, a rectangular waveguide has a width, x, and a height, y, with propagation in the negative z direction (Fig 1 & 2). This program characterizes a waveguide discontinuity. The input section preceding the discontinuity is a uniform rectangular waveguide of width a and height b. Dominant mode propagation is assumed in this section. The output region is a section of rectangular waveguide of width a′ and height b′, whose dimensions are such that the operating frequency is above the cut-off frequency of the dominant mode. B/Yo, calculated for a change in height at the waveguide junction, represents the ratio between the susceptance and the characteristic admittance of the guide. Likewise, X/Zo, calculated for a change in width, represents the ratio between the reactance and the characteristic impedance of the guide. Four types of equivalent circuit models are examined.

The **first case** is that of *an axially symmetrical junction of two rectangular guides of unequal widths but equal heights* (Fig. 1a). This case has dominant mode propagation in the large rectangular guide and no propagation in the small guide. By terminating the circuit with the inductive characteristic impedance of the smaller guide, an equivalent circuit is obtained (Fig. 1c). It is valid within the following parameters. First, the ratio of the width of the first waveguide to the wavelength must lie between 0.5

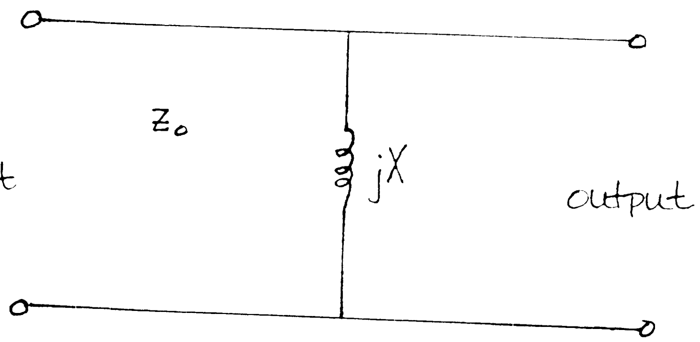
[1] Marcuvitz, N., *Waveguide Handbook.* McGraw-Hill Book Company, Inc., New York, 1951.



a. symmetrical
step change in width

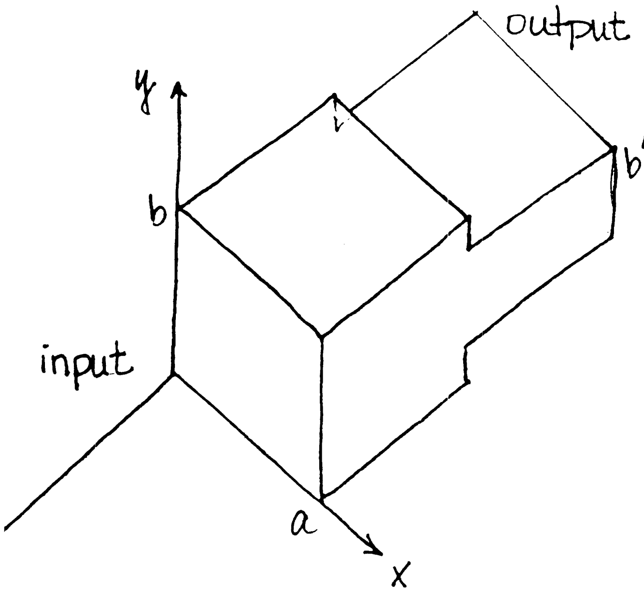


b. asymmetrical
step change in width

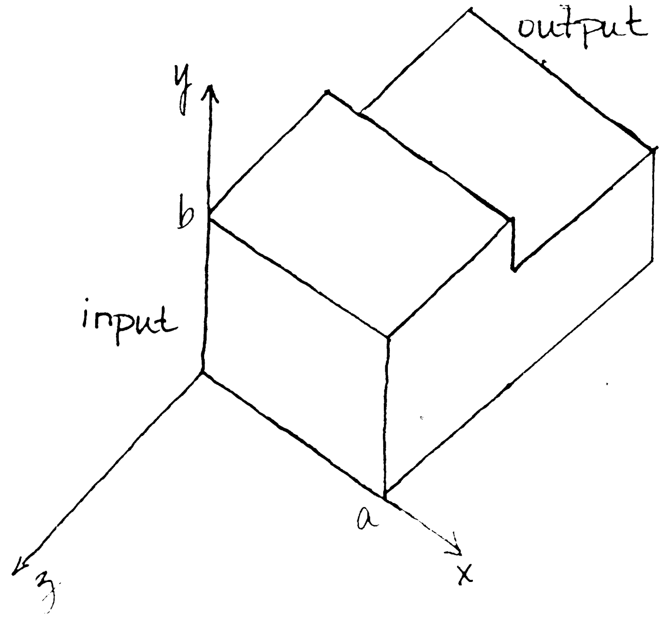


c. equivalent circuit
step change in width

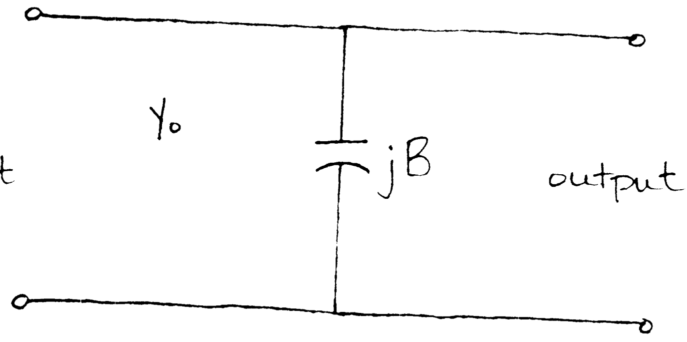
Figure 1.



a. symmetrical
step change in height



b. asymmetrical
step change in height



c. equivalent circuit
step change in height

Figure 2.

and 1.5 ($0.5 < a/\lambda < 1.5$). Second, the wavelength must be larger than twice the width of the second guide ($\lambda > 2a'$), such that the second guide is at cut-off.

The **second case** is that of *an axially asymmetrical junction of two rectangular guides of unequal widths but equal heights* (Fig. 1b). This, again, has the H_{10}-mode in the large rectangular guide and no propagation in the small guide. The equivalent circuit (Fig. 1c) is valid within the same parameters with the following exception. The ratio of the input width to the wavelength must lie between 0.5 and 1.0 ($0.5 < a/\lambda < 1.0$).

The **third case** is that of *an axially symmetrical junction of two rectangular guides of equal widths but unequal heights* (Fig. 2a). This is equivalent to the H_{10}-modes in both rectangular guides. The equivalent capacitive circuit (Fig. 2c) is valid while the ratio of the height of the first guide to the wavelength of the guide is less than 1 ($b/\lambda_g < 1.$). The equations used were obtained by the equivalent static method employing a static aperture field due to the incidence of the two lowest modes and is correct to within 1% in the range previously specified.

The **fourth case** is that of *an asymmetrical junction of two rectangular guides of equal widths but unequal heights* (Fig. 2b). This is simply the asymmetric version of the third case and is also correct to within 1% in the range $b/\lambda_g < 1$. Its equivalent circuit representation is shown in Figure 2c.

Review: Rectangular Waveguides

Waveguide characterizes a hollow rectangular waveguide. The dominant mode is the TE_{10} mode, having the lowest cutoff frequency. Using x as the width and y as the height of the guide, the general equation for cutoff frequency is :

$$f_c = 1/(2\sqrt{\mu\varepsilon})\sqrt{(n/y)^2+(m/x)^2}$$

Since the cutoff frequency is a function of the modes and guide dimensions, the physical size of the waveguide will determine the propagation of the modes. Assuming $x>y$, we take m = 1 and n = 0, and $f_c = c/2x$.

Likewise, the cutoff wavelength ,

$$\lambda_{oc} = 2/\sqrt{(n/y)^2 + (m/x)^2} = 2x$$

The wavelength of the guide is given by:

$$\lambda_g = \lambda_o/\sqrt{1 - (f_c/f)^2}\ , \lambda_o = v_p/f$$

Theory: Waveguide Disconituities as Equivalent Circuits

The waveguide structures dicusses in this report consist of two uniform sections of different dimensions separated by a discontinuity. Generally, the fields within a uniform section of waveguide may be described by the dominant mode. At a discontinuity, however, a complete description of the fields within that region requires an infinite number of nonpropagating modes in addition to the dominant propagating mode. These additional modes are evanescent and decay exponentially in both directions away from the discontinuity, resulting in a localized storage of reactive energy. Thus, the fields at the discontinuity may be approximated as lumped parameter circuit models.

Getting Started with *Waveguide*

To run *Waveguide,* follow these instructions:

1. Type the executable to initiate the program and a <CR>.
   On the apollos, using the Unix operating system, this would be:

   % **waveguide <CR>**

2. *Waveguide* asks you to choose one of the four types of junctions. You may simply respond by typing the number of your choice (1,2,3, or 4) and <CR>. For example, if you were interested in a symmetrical junction, inductive equivalent circuit you would type 1 and a <CR>:

   %PLEASE SELECT WAVEGUIDE JUNCTION.

Two rectangular guides of:
1) Unequal widths but equal heights at an axially symmetrical junction (inductive equiv. ckt)
2) Unequal widths but equal heights at an axially asymmetrical junction (inductive equiv. ckt)
3) Equal widths but unequal heights at an axially symmetrical junction (capacitive equiv. ckt)
4) Equal widths but unequal heights at an axially asymmetrical junction (capacitive equiv. ckt)

% **1 <CR>**

3. You will first be prompted for the dimensions of the first guide. Enter all guide dimensions in meters. If either of the **inductive** equivalent circuit cases is chosen (1 or 2), the restrictions on the dimensions of the second guide will be briefly stated and you must then enter an **incremental change** in the **width** of the second guide. For example, if you wanted to test a junction in which guide #1 was 19.55 cm X 15.3 cm with increments of .01:

%Enter values for width1 and height1(in meters):
%**.1955,.153 <CR>**

%Height2 will equal height1.
Width2 will range from 0.0 to width1 so 0.0 < alpha < 1.0.
By what incremental change shall width2 increase?
% **.01 <CR>**

Alpha represents the ratio between width2 (a') and width1 (a). Alpha's limiting value is one to ensure the second waveguide's dimensions do not exceed the first.
Conversely, if either of the **capacitive** equivalent circuit cases is chosen (3 or 4), the restrictions on the dimensions of the second guide will be briefly stated and you must enter an **incremental change** in **height** of the second guide.

%Width2 will equal width1.
Height2 will range from 0.0 to height1

so 0.0 < alpha < 1.0.
By what incremental change shall height2 increase?
% **.01 <CR>**

In the capacitive case, alpha represents the ratio between height2 (b′) and height1 (b). Again, alpha's limiting value is one to ensure the second waveguide's dimensions do not exceed the first.

4. Next, select frequency range (in GHz). For example, if your desired range was from .96 GHz to .97 GHz and you wanted to know values every .01 GHz:

%Enter beginning freq.,ending freq., increments(GHz):
% **.96, .97, .01**

5. For an **inductive case**, *Waveguide* treats input data in the following manner. At the initial frequency value, the ratio **X/Zo** is computed for each incremental change in width2. This incremental change is represented by alpha (a′/a), which ranges from 0 to 1. When alpha reaches its limit of 1 , the frequency is incremented and the calculations are repeated. The program continues to loop until the limiting frequency is reached.

6. For a capacitive **case**, *Waveguide* treats input data in the following manner. At the initial frequency value, the ratio **B/Yo** is computed for each incremental change in height2. This incremental change is represented by alpha (b′/b), which ranges from 0 to 1. When alpha reaches its limit of 1 , the frequency is incremented and the calculations are repeated. The program continues to loop until the limiting frequency is reached.

7. **Results** are stored in two files: **info.dat** and **plot.dat**. **Info.dat** contains **general information** such as the junction chosen, the guide dimensions, the frequency, alpha, and either X/Zo or B/Yo. **Plot.dat** is specifically for **plotting**. Column 1 contains values of **alpha** while

column 2 contains values of **X/Zo** if a inductive case has been run, **or B/Yo** if an capacitive case has been run. Figures 6a., 6b., and 6c. are examples of info.dat, plot.dat, and an actual plot for case 1. Likewise, Figures 7, 8, and 9 represent files and plots for cases 2, 3, and 4 respectively. The plots presented in this report were generated using //hybrid/users/norman/plotting/nplot.

Results for symmetric inductive case:
Dimensions: .1955m by .1530m , W2 by , .1530m

f	W2	alpha	X/Zo
0.96000	0.01000	0.05115	0.00198
0.96000	0.02000	0.10230	0.00799
0.96000	0.03000	0.15345	0.01828
0.96000	0.04000	0.20460	0.03319
0.96000	0.05000	0.25575	0.05320
0.96000	0.06000	0.30691	0.07900
0.96000	0.07000	0.35806	0.11151
0.96000	0.08000	0.40921	0.15202
0.96000	0.09000	0.46036	0.20247
0.96000	0.10000	0.51151	0.26583
0.96000	0.11000	0.56266	0.34709
0.96000	0.12000	0.61381	0.45548
0.96000	0.13000	0.66496	0.61072
0.96000	0.14000	0.71611	0.86700
0.96000	0.15000	0.76726	1.48512
w <OR= 2*W2,EQUIV. CKT. NOT VALID			
0.97000	0.01000	0.05115	0.00203
0.97000	0.02000	0.10230	0.00822
0.97000	0.03000	0.15345	0.01881
0.97000	0.04000	0.20460	0.03416
0.97000	0.05000	0.25575	0.05479
0.97000	0.06000	0.30691	0.08141
0.97000	0.07000	0.35806	0.11498
0.97000	0.08000	0.40921	0.15689
0.97000	0.09000	0.46036	0.20919
0.97000	0.10000	0.51151	0.27509
0.97000	0.11000	0.56266	0.36005
0.97000	0.12000	0.61381	0.47429
0.97000	0.13000	0.66496	0.64037
0.97000	0.14000	0.71611	0.92368
0.97000	0.15000	0.76726	1.69747
w <OR= 2*W2,EQUIV. CKT. NOT VALID			

Figure 6a. info.dat

```
5.1150892E-02 1.9765685E-03
0.1023018 7.9938713E-03
0.1534527 1.8279992E-02
0.2046036 3.3186160E-02
0.2557545 5.3203840E-02
0.3069053 7.9003461E-02
0.3580562 0.1115088
0.4092071 0.1520242
0.4603580 0.2024691
0.5115089 0.2658284
0.5626597 0.3470916
0.6138107 0.4554784
0.6649615 0.6107232
0.7161124 0.8670046
0.7672634 1.485118
5.1150892E-02 2.0332176E-03
0.1023018 8.2244696E-03
0.1534527 1.8812237E-02
0.2046036 3.4163967E-02
0.2557545 5.4794662E-02
0.3069053 8.1408598E-02
0.3580562 0.1149799
0.4092071 0.1568911
0.4603580 0.2091908
0.5115089 0.2750949
0.5626597 0.3600459
0.6138107 0.4742852
0.6649615 0.6403701
0.7161124 0.9236836
0.7672634 1.697470
```

Figure 6b. plot.dat

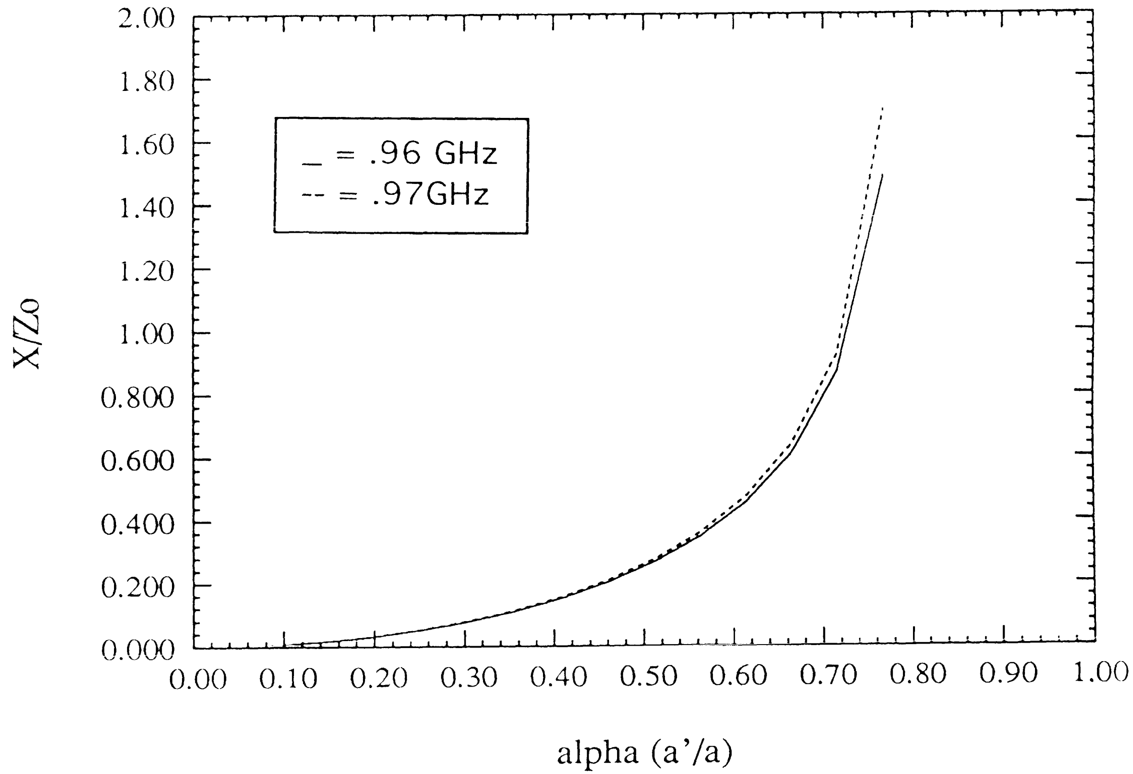


Figure 6c. X/Zo vs alpha

Results for asymmetric inductive case:
Dimensions:

f	W2	alpha	X/Zo
0.96000	0.00000	0.05115	0.00001
0.96000	0.00000	0.10230	0.00022
0.96000	0.00000	0.15345	0.00109
0.96000	0.00000	0.20460	0.00341
0.96000	0.00000	0.25575	0.00820
0.96000	0.00000	0.30691	0.01674
0.96000	0.00000	0.35806	0.03055
0.96000	0.00000	0.40921	0.05142
0.96000	0.00000	0.46036	0.08157
0.96000	0.00000	0.51151	0.12395
0.96000	0.00000	0.56266	0.18285
0.96000	0.00000	0.61381	0.26551
0.96000	0.00000	0.66496	0.38621
0.96000	0.00000	0.71611	0.58123
0.96000	0.00000	0.76726	1.00440
0.97000	0.00000	0.05115	0.00001
0.97000	0.00000	0.10230	0.00022
0.97000	0.00000	0.15345	0.00112
0.97000	0.00000	0.20460	0.00351
0.97000	0.00000	0.25575	0.00844
0.97000	0.00000	0.30691	0.01725
0.97000	0.00000	0.35806	0.03149
0.97000	0.00000	0.40921	0.05305
0.97000	0.00000	0.46036	0.08424
0.97000	0.00000	0.51151	0.12817
0.97000	0.00000	0.56266	0.18945
0.97000	0.00000	0.61381	0.27593
0.97000	0.00000	0.66496	0.40360
0.97000	0.00000	0.71611	0.61505
0.97000	0.00000	0.76726	1.11932

Figure 7a. info.dat

```
5.1150892E-02 1.3668611E-05
0.1023018 2.1785172E-04
0.1534527 1.0921899E-03
0.2046036 3.4085822E-03
0.2557545 8.2008895E-03
0.3069053 1.6742714E-02
0.3580562 3.0548261E-02
0.4092071 5.1418442E-02
0.4603580 8.1572764E-02
0.5115089 0.1239472
0.5626597 0.1828516
0.6138107 0.2655055
0.6649615 0.3862067
0.7161124 0.5812325
0.7672634 1.004403
5.1150892E-02 1.4059674E-05
0.1023018 2.2411690E-04
0.1534527 1.1238480E-03
0.2046036 3.5084779E-03
0.2557545 8.4446706E-03
0.3069053 1.7249178E-02
0.3580562 3.1491984E-02
0.4092071 5.3047806E-02
0.4603580 8.4240593E-02
0.5115089 0.1281711
0.5626597 0.1894479
0.6138107 0.2759297
0.6649615 0.4036019
0.7161124 0.6150534
0.7672634 1.119317
```

Figure 7b. plot.dat

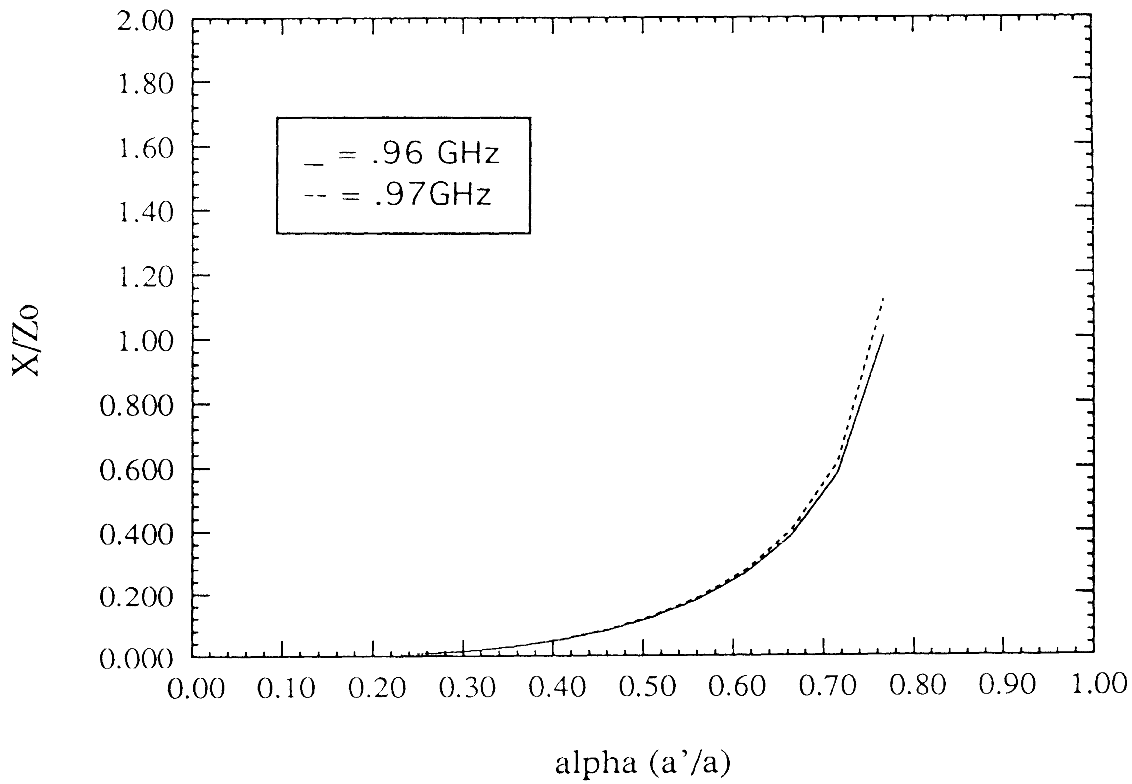


Figure 7c. X/Zo vs alpha

Results for symmetric capacitive case:
Dimensions: .1955m by .1530m , .1955m by H2

f	H2	alpha	B/Yo
0.96000	0.01000	0.06536	1.40867
0.96000	0.02000	0.13072	1.00134
0.96000	0.03000	0.19608	0.76406
0.96000	0.04000	0.26144	0.59722
0.96000	0.05000	0.32680	0.46992
0.96000	0.06000	0.39216	0.36855
0.96000	0.07000	0.45752	0.28590
0.96000	0.08000	0.52288	0.21771
0.96000	0.09000	0.58824	0.16125
0.96000	0.10000	0.65359	0.11467
0.96000	0.11000	0.71895	0.07677
0.96000	0.12000	0.78431	0.04672
0.96000	0.13000	0.84967	0.02403
0.96000	0.14000	0.91503	0.00852
0.96000	0.15000	0.98039	0.00060
alpha (H2/H1) has reached limiting value of 1			
0.97000	0.01000	0.06536	1.45082
0.97000	0.02000	0.13072	1.03174
0.97000	0.03000	0.19608	0.78749
0.97000	0.04000	0.26144	0.61567
0.97000	0.05000	0.32680	0.48450
0.97000	0.06000	0.39216	0.38000
0.97000	0.07000	0.45752	0.29478
0.97000	0.08000	0.52288	0.22446
0.97000	0.09000	0.58824	0.16622
0.97000	0.10000	0.65359	0.11819
0.97000	0.11000	0.71895	0.07911
0.97000	0.12000	0.78431	0.04813
0.97000	0.13000	0.84967	0.02475
0.97000	0.14000	0.91503	0.00877
0.97000	0.15000	0.98039	0.00061
alpha (H2/H1) has reached limiting value of 1			

Figure 8a. info.dat

0.06536	1.40867
0.13072	1.00134
0.19608	0.76406
0.26144	0.59722
0.32680	0.46992
0.39216	0.36855
0.45752	0.28590
0.52288	0.21771
0.58824	0.16125
0.65359	0.11467
0.71895	0.07677
0.78431	0.04672
0.84967	0.02403
0.91503	0.00852
0.98039	0.00060
0.06536	1.45082
0.13072	1.03174
0.19608	0.78749
0.26144	0.61567
0.32680	0.48450
0.39216	0.38000
0.45752	0.29478
0.52288	0.22446
0.58824	0.16622
0.65359	0.11819
0.71895	0.07911
0.78431	0.04813
0.84967	0.02475
0.91503	0.00877
0.98039	0.00061

Figure 8b. plot.dat

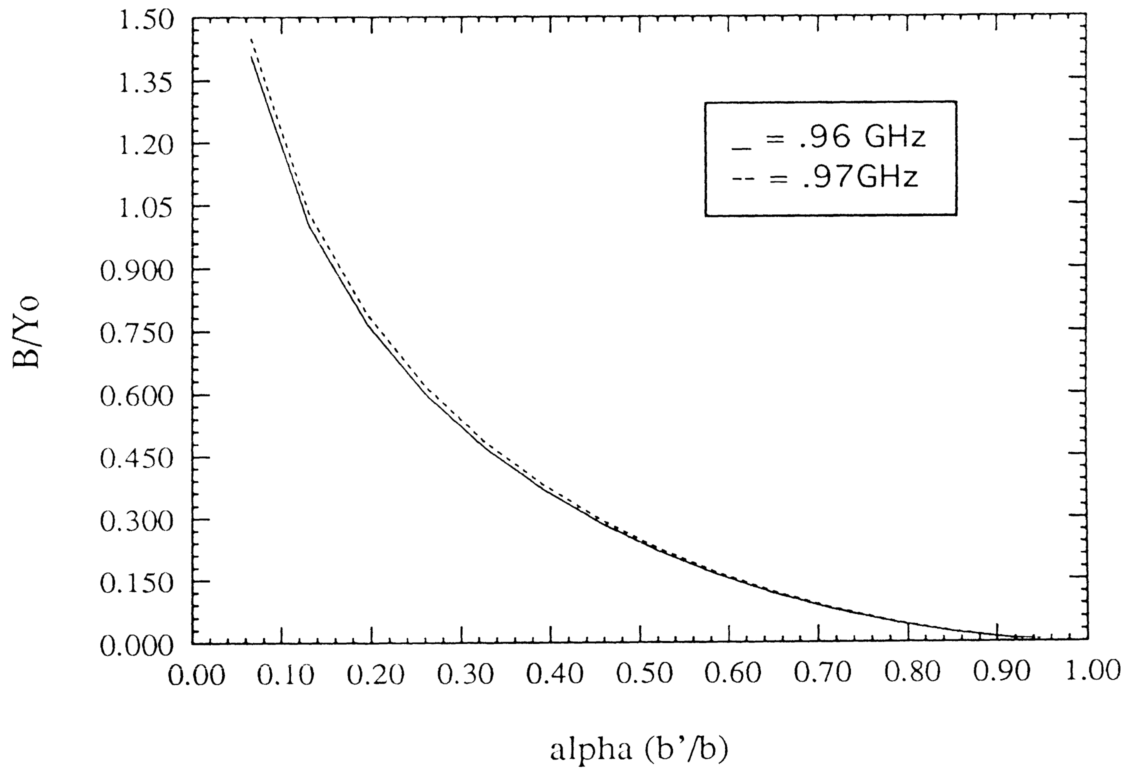


Figure 8c. B/Yo vs alpha

Results for asymmetric capacitive case:
Dimensions: .1955m by .1530m , .1955m by H2

f	H2	alpha	B/Yo
0.96000	0.01000	0.06536	3.05568
0.96000	0.02000	0.13072	2.22648
0.96000	0.03000	0.19608	1.72974
0.96000	0.04000	0.26144	1.36893
0.96000	0.05000	0.32680	1.08505
0.96000	0.06000	0.39216	0.85339
0.96000	0.07000	0.45752	0.66133
0.96000	0.08000	0.52288	0.50135
0.96000	0.09000	0.58824	0.36847
0.96000	0.10000	0.65359	0.25922
0.96000	0.11000	0.71895	0.17116
0.96000	0.12000	0.78431	0.10250
0.96000	0.13000	0.84967	0.05181
0.96000	0.14000	0.91503	0.01805
0.96000	0.15000	0.98039	0.00124
alpha (H2/H1) has reached limiting value of 1			
0.97000	0.01000	0.06536	3.16757
0.97000	0.02000	0.13072	2.31323
0.97000	0.03000	0.19608	1.80007
0.97000	0.04000	0.26144	1.42618
0.97000	0.05000	0.32680	1.13116
0.97000	0.06000	0.39216	0.88985
0.97000	0.07000	0.45752	0.68948
0.97000	0.08000	0.52288	0.52245
0.97000	0.09000	0.58824	0.38367
0.97000	0.10000	0.65359	0.26963
0.97000	0.11000	0.71895	0.17781
0.97000	0.12000	0.78431	0.10633
0.97000	0.13000	0.84967	0.05366
0.97000	0.14000	0.91503	0.01866
0.97000	0.15000	0.98039	0.00128
alpha (H2/H1) has reached limiting value of 1			

Figure 9a. info.dat

0.06536	3.05568
0.13072	2.22648
0.19608	1.72974
0.26144	1.36893
0.32680	1.08505
0.39216	0.85339
0.45752	0.66133
0.52288	0.50135
0.58824	0.36847
0.65359	0.25922
0.71895	0.17116
0.78431	0.10250
0.84967	0.05181
0.91503	0.01805
0.98039	0.00124
0.06536	3.16757
0.13072	2.31323
0.19608	1.80007
0.26144	1.42618
0.32680	1.13116
0.39216	0.88985
0.45752	0.68948
0.52288	0.52245
0.58824	0.38367
0.65359	0.26963
0.71895	0.17781
0.78431	0.10633
0.84967	0.05366
0.91503	0.01866
0.98039	0.00128

Figure 9b. plot.dat

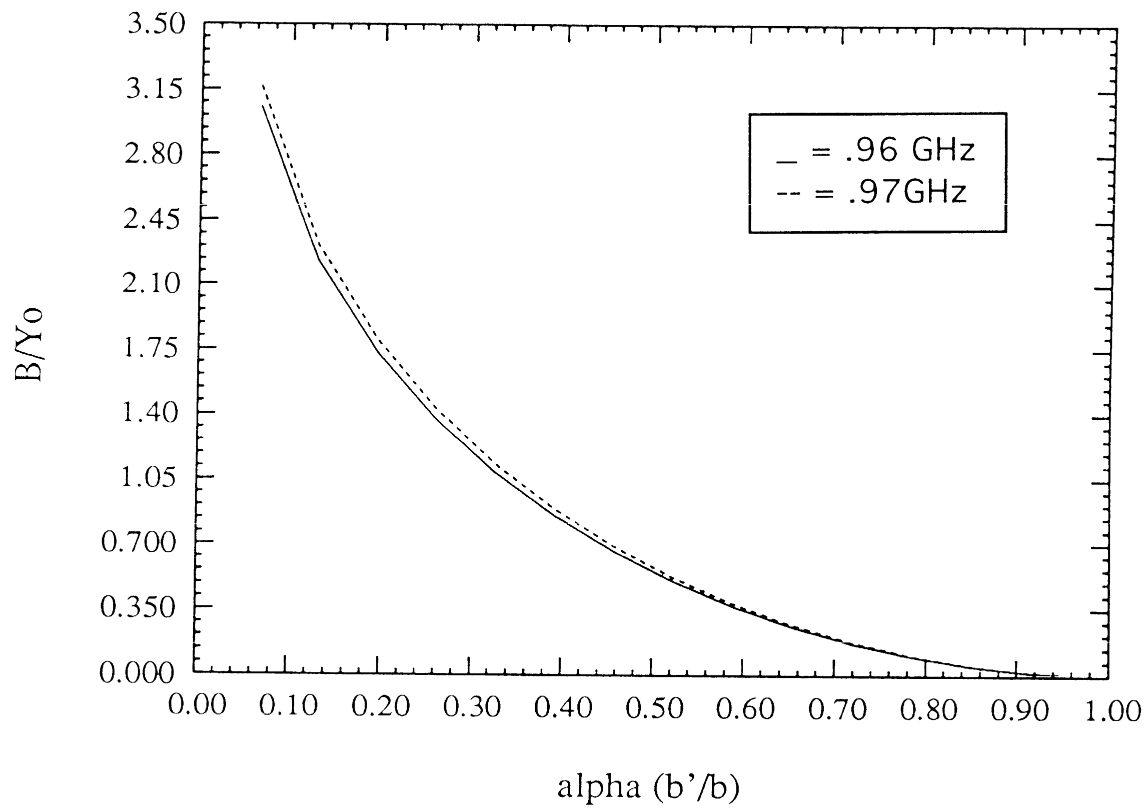


Figure 9c. B/Yo vs alpha

Appendix A:

Equations as Presented by Marcuvitz[1]

CASE 1) SYMMETRICAL INDUCTIVE CASE

$$\frac{X}{Z_0} = \frac{2a}{\lambda_g} \frac{X_{11}\left\{1 - \left[1 - \left(\frac{2a'}{\lambda}\right)^2\right] X_0^2\right\}}{1 - \left[1 - \left(\frac{2a'}{\lambda}\right)^2\right] X_0 X_{22} + \sqrt{1 - \left(\frac{2a'}{\lambda}\right)^2}\,(X_{22} - X_0)},$$

WHERE

$$X_{11} = \frac{A}{1 - \frac{1}{4A}\left[(A+1)^2 N_{11} + 2(A+1)CN_{12} + C^2 N_{22}\right]},$$

$$X_{22} = \frac{A'}{1 - \frac{1}{4A'}\left[(A'+1)^2 N_{22} + 2(A'+1)CN_{12} + C^2 N_{11}\right]},$$

$$X_0 = X_{22} - \frac{X_{12}^2}{X_{11}} \approx A' - \frac{C^2}{A},$$

and

$$A = \frac{(1+R_1)(1-R_2) + T^2}{(1-R_1)(1-R_2) - T^2}, \qquad R_1 = -\left(\frac{1-\alpha}{1+\alpha}\right)^{\alpha},$$

$$A' = \frac{(1-R_1)(1+R_2) + T^2}{(1-R_1)(1-R_2) - T^2}, \qquad R_2 = \left(\frac{1-\alpha}{1+\alpha}\right)^{1/\alpha},$$

$$C = \frac{2T}{(1-R_1)(1-R_2) - T^2}, \qquad T = \frac{4\alpha}{1-\alpha^2}\left(\frac{1-\alpha}{1+\alpha}\right)^{\frac{1}{2}\left(\alpha + \frac{1}{\alpha}\right)}.$$

$$N_{11} = 2\left(\frac{a}{\lambda}\right)^2 \left\{1 + \frac{16R_1}{\pi(1-\alpha^2)}\left[E(\alpha) - \alpha'^2 F(\alpha)\right]\left[E(\alpha') - \alpha^2 F(\alpha')\right] - R_1^2 - \alpha^2 T^2\right\} + \frac{12\alpha^4}{(1-\alpha^2)^2}\left(\frac{1-\alpha}{1+\alpha}\right)^{4\alpha}\left[Q - \frac{1}{2}\left(\frac{2a}{3\lambda}\right)^2\right] + \frac{48\alpha^2}{(1-\alpha^2)^2}\left(\frac{1-\alpha}{1+\alpha}\right)^{\alpha + \frac{3}{\alpha}}\left[Q' - \frac{1}{2}\left(\frac{2}{3}\frac{a'}{\lambda}\right)^2\right],$$

$$N_{22} = 2\left(\frac{a}{\lambda}\right)^2 \left\{\alpha^2 + \frac{16\alpha^2 R_2 E(\alpha)}{\pi(1-\alpha^2)}\left[F(\alpha') - E(\alpha')\right] - \alpha^2 R_2^2 - T^2\right\} + \frac{48\alpha^2}{(1-\alpha^2)^2}\left(\frac{1-\alpha}{1+\alpha}\right)^{3\alpha + \frac{1}{\alpha}}\left[Q - \frac{1}{2}\left(\frac{2}{3}\frac{a}{\lambda}\right)^2\right] + \frac{12}{(1-\alpha^2)^2}\left(\frac{1-\alpha}{1+\alpha}\right)^{4/\alpha}\left[Q' - \frac{1}{2}\left(\frac{2}{3}\frac{a'}{\lambda}\right)^2\right],$$

$$N_{12} = 2\left(\frac{a}{\lambda}\right)^2 \left\{\alpha'^2 - \frac{4E(\alpha)}{\pi}\left[E(\alpha') - \alpha^2 F(\alpha')\right] - R_1 + \alpha^2 R_2\right\} T + \frac{24\alpha^3}{(1-\alpha^2)^2}\left(\frac{1-\alpha}{1+\alpha}\right)^{\frac{1}{2}\left(7\alpha + \frac{1}{\alpha}\right)}\left[Q - \frac{1}{2}\left(\frac{2a}{3\lambda}\right)^2\right] + \frac{24\alpha}{(1-\alpha^2)^2}\left(\frac{1-\alpha}{1+\alpha}\right)^{\frac{1}{2}\left(\alpha + \frac{7}{\alpha}\right)}\left[Q' - \frac{1}{2}\left(\frac{2}{3}\frac{a'}{\lambda}\right)^2\right].$$

$$\alpha = \frac{a'}{a} = 1 - \beta, \qquad \alpha' = \sqrt{1 - \alpha^2}.$$

$$Q = 1 - \sqrt{1 - \left(\frac{2a}{3\lambda}\right)^2}. \qquad Q' = 1 - \sqrt{1 - \left(\frac{2}{3}\frac{a'}{\lambda}\right)^2}.$$

$$\alpha = \frac{a'}{a}, \qquad \lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{2a}\right)^2}}$$

The functions E(α), E(α'), F(α), and F(α') represent the complete Legendre elliptic integrals of the first, F, and second, E, kind.

CASE 2) ASYMMETRICAL INDUCTIVE CASE

$$\frac{X}{Z_0} = \frac{2a}{\lambda_g} \frac{X_{11}\left\{1 - \left[1 - \left(\frac{2a'}{\lambda}\right)^2\right] X_0^2\right\}}{1 - \left[1 - \left(\frac{2a'}{\lambda}\right)^2\right] X_0 X_{22} + \sqrt{1 - \left(\frac{2a'}{\lambda}\right)^2}\,(X_{22} - X_0)}$$

WHERE THE FOLLOWING EQUATIONS ARE THE SAME AS IN CASE 1:

$$X_{11} = \frac{A}{1 - \frac{1}{4A}\left[(A + 1)^2 N_{11} + 2(A + 1)CN_{12} + C^2 N_{22}\right]},$$

$$X_{22} = \frac{A'}{1 - \frac{1}{4A'}\left[(A' + 1)^2 N_{22} + 2(A' + 1)CN_{12} + C^2 N_{11}\right]}$$

$$X_0 = X_{22} - \frac{X_{12}^2}{X_{11}} \approx A' - \frac{C^2}{A},$$

$$A = \frac{(1 + R_1)(1 - R_2) + T^2}{(1 - R_1)(1 - R_2) - T^2},$$

$$A' = \frac{(1 - R_1)(1 + R_2) + T^2}{(1 - R_1)(1 - R_2) - T^2},$$

$$C = \frac{2T}{(1 - R_1)(1 - R_2) - T^2},$$

$$\alpha' = \sqrt{1 - \alpha^2}.$$

$$Q = 1 - \sqrt{1 - \left(\frac{a}{\lambda}\right)^2}, \qquad \alpha = \frac{a'}{a}, \qquad \lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{2a}\right)^2}}$$

AND R_1, R_2, T_1, N_{11}, N_{22}, N_{12}, AND Q' DIFFER:

$$R_1 = -\left(\frac{1+3\alpha^2}{1-\alpha^2}\right)\left(\frac{1-\alpha}{1+\alpha}\right)^{2\alpha},$$

$$R_2 = \left(\frac{3+\alpha^2}{1-\alpha^2}\right)\left(\frac{1-\alpha}{1+\alpha}\right)^{2/\alpha},$$

$$T = \frac{16\alpha^2}{(1-\alpha^2)^2}\left(\frac{1-\alpha}{1+\alpha}\right)^{\alpha+(1/\alpha)},$$

$$N_{11} = 2\left(\frac{a}{\lambda}\right)^2\left[1+\frac{8\alpha(1-\alpha^2)}{1+3\alpha^2}R_1\ln\frac{1-\alpha}{1+\alpha}+\frac{16\alpha^2}{1+3\alpha^2}R_1-R_1^2-\alpha^2T^2\right]$$
$$+2\left[\frac{32}{3}\frac{\alpha^4}{(1-\alpha^2)^2}\left(\frac{1-\alpha}{1+\alpha}\right)^{3\alpha}\right]^2\left[Q-\frac{1}{2}\left(\frac{a}{\lambda}\right)^2\right]$$
$$+2\left[\frac{32\alpha^2}{(1-\alpha^2)^2}\left(\frac{1-\alpha}{1+\alpha}\right)^{\alpha+(2/\alpha)}\right]^2\left[Q'-\frac{1}{2}\left(\frac{a'}{\lambda}\right)^2\right].$$

$$N_{22} = 2\left(\frac{a}{\lambda}\right)^2\left[\alpha^2-\frac{8\alpha(1-\alpha^2)}{3+\alpha^2}R_2\ln\frac{1-\alpha}{1+\alpha}+\frac{16\alpha^2}{1+3\alpha^2}R_2-\alpha^2R_2^2-T^2\right]$$
$$+2\left[\frac{32\alpha^2}{(1-\alpha^2)^2}\left(\frac{1-\alpha}{1+\alpha}\right)^{2\alpha+(1/\alpha)}\right]^2\left[Q-\frac{1}{2}\left(\frac{a}{\lambda}\right)^2\right]$$
$$+2\left[\frac{32}{3}\frac{1}{(1-\alpha^2)^2}\left(\frac{1-\alpha}{1+\alpha}\right)^{3/\alpha}\right]^2\left[Q'-\frac{1}{2}\left(\frac{a'}{\lambda}\right)^2\right].$$

$$N_{12} = 2\left(\frac{a}{\lambda}\right)^2\left[\frac{(1-\alpha^2)^2}{2\alpha}\ln\frac{1-\alpha}{1+\alpha}-R_1-\alpha^2R_2\right]T$$
$$+\frac{2}{3}\left(\frac{1-\alpha}{1+\alpha}\right)^{5\alpha+(1/\alpha)}\left[\frac{32\alpha^3}{(1-\alpha^2)^2}\right]^2\left[Q-\frac{1}{2}\left(\frac{a}{\lambda}\right)^2\right]$$
$$+\frac{2}{3}\left(\frac{1-\alpha}{1+\alpha}\right)^{\alpha+(5/\alpha)}\left[\frac{32\alpha}{(1-\alpha^2)^2}\right]^2\left[Q'-\frac{1}{2}\left(\frac{a'}{\lambda}\right)^2\right].$$

$$Q' = 1-\sqrt{1-\left(\frac{a'}{\lambda}\right)^2}.$$

CASE 3) SYMMETRICAL CAPACITIVE CASE

$$\frac{B}{Y_0} = \frac{2b}{\lambda_g}\left[\ln\left\{\left(\frac{1-\alpha^2}{4\alpha}\right)\left(\frac{1+\alpha}{1-\alpha}\right)^{\frac{1}{2}\left(\alpha+\frac{1}{\alpha}\right)}\right\} + 2\,\frac{A + A' + 2C}{AA' - C^2} + \left(\frac{b}{4\lambda_g}\right)^2\left(\frac{1-\alpha}{1+\alpha}\right)^{4\alpha}\left(\frac{5\alpha^2-1}{1-\alpha^2} + \frac{4}{3}\,\frac{\alpha^2 C}{A}\right)^2\right],$$

WHERE:

$$\frac{b'}{b} = \alpha$$

$$A = \left(\frac{1+\alpha}{1-\alpha}\right)^{2\alpha}\frac{1+\sqrt{1-\left(\frac{b}{\lambda_g}\right)^2}}{1-\sqrt{1-\left(\frac{b}{\lambda_g}\right)^2}} - \frac{1+3\alpha^2}{1-\alpha^2},$$

$$A' = \left(\frac{1+\alpha}{1-\alpha}\right)^{2/\alpha}\frac{1+\sqrt{1-\left(\frac{b'}{\lambda_g}\right)^2}}{1-\sqrt{1-\left(\frac{b'}{\lambda_g}\right)^2}} + \frac{3+\alpha^2}{1-\alpha^2},$$

$C = \left(\frac{4\alpha}{1-\alpha^2}\right)$ [illegible] 2.718.

CASE 4) ASYMMETRICAL CAPACITIVE CASE

Same equation for B/Y_0 as for CASE 3.

Only difference: $\lambda_g = \lambda_g/2$.