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# A GENERAL POLARIMETRIC CALIBRATION TECHNIQUE

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## Abstract

A polarimetric calibration procedure useful for both laboratory and field measurements is introduced. The procedure requires measurements of three known targets in order to determine the distortion matrices that characterize the effect of the measurement system on the transmitted and received waves. The form of the scattering matrices for the known targets is arbitrary, with the restriction that at least one scattering matrix be invertible. The measured scattering matrix associated with this target must also be invertible. The approach involves forming matrix products from the measured scattering matrices to obtain a similarity transformation where the transforming matrix is the unknown distortion matrix. The relationships between the eigenvalues and eigenvectors of the similar matrices are then used to solve for the distortion matrices to within an unknown absolute phase. A special case, wherein the transmit and receive distortion matrices are the transpose of one another, is considered also. This form can be used with some single antenna systems, and it has the advantage that only two known targets need to be measured. In this case, the measured and theoretical scattering matrices of both targets must be invertible. Finally, application of the technique to measuring the propagation characteristics of random media is briefly discussed.



# 1 Introduction

A Traditional radar system transmits and receives a single polarization. Hence, the scattering characteristics of the illuminated scene are obtained for only one transmit and receive polarization combination. Because the radar measures only the amplitude of the scattered wave, any information contained in the phase or polarization of the wave is lost. However, a polarimetric radar system measures the complete scattering matrix (amplitude and relative phase) of the illuminated scene using an orthogonal set of polarization configurations, and this information can be used to synthesize the scattering characteristics of the scene at any arbitrary transmit and receive polarization combination [1,2]. Much of the early research in polarimetry concentrated on point targets, and an extensive review of this history is given by Guili [3]. Polarimetric radars and techniques have received increased attention in the last few years, following the development of several polarimetric imaging radar systems [1,4,5]. Laboratory and truck-based polarimetric radars have also been developed with the advent of the HP 8510 vector network analyzer [6,7,8,11]. With an increasing number of operating polarimetric radars, it has become important to develop effective techniques for accurate polarimetric calibration.

Calibration of polarimetric radar systems has been considered by several investigators in recent years. A technique proposed by Barnes [9] characterizes the errors introduced by the transmitter and receiver in terms of distortion matrices that alter the measured scattering matrix of the target. This calibration tech-

nique requires the measurement of three known targets, some of which have zero elements in their scattering matrices. Two algorithms are proposed by Barnes, differing only in the type of known targets to be measured. A technique similar to the one by Barnes was used by Freeman, et al. [10] where the known target scattering matrices were realized by Polarimetric Active Radar Calibrators (PARC's). A technique introduced by Riegger, et al. [11] characterizes the system errors in terms of coupling coefficients between elements of the theoretical scattering matrix and elements of the measured scattering matrix. It is in essence the same model as used by Barnes, except that Riegger has expanded the matrix product which resulted in twice the number of unknowns. In its most general form, this technique would require the measurement of four known targets to solve for sixteen unknown coupling coefficients. However, Riegger neglects four of the coupling coefficients and reduces the required number of known targets to three.

The calibration techniques described above have a number of disadvantages. The algorithms used by Barnes and Freeman depend upon the actual known targets that are measured. If a new set of targets is used, the derivation must be repeated. In addition, if the scattering matrices of at least some of the targets do not contain zeros, it becomes difficult (if not impossible) to solve for the elements of the distortion matrices. In many cases, the scattering matrix elements are known to be non-zero, but one must assume they are zero for derivation of the calibration algorithms. Relying on certain elements of the scattering matrix to be zero places a considerable restriction on the targets that can be used. This

is seen as a major disadvantage, because one would like to examine a variety of known targets and choose the best possible set. The technique used by Riegger, as already mentioned, introduces more unknowns than required and is therefore inefficient.

Two additional polarimetric calibration techniques are of interest because they require only a single non-depolarizing target (such as a sphere or trihedral) to correct for co-polarized channel imbalance and absolute magnitude errors. The cross-polarization coupling (or cross-talk) errors are corrected using unknown targets. The first technique, proposed by Sarabandi, et al. [12] achieves calibration of the cross-talk errors by measuring any arbitrary depolarizing target. Knowledge of the scattering matrix of the arbitrary target is not required. A similar technique by van Zyl [13] uses measurements of distributed natural targets to determine the cross-talk errors. The advantage of these techniques is their insensitivity to target positioning, which makes them particularly useful in field calibration. The disadvantages, however, include (1) the radar systems are assumed to be reciprocal (i.e., the distortion matrix for reception is the transpose of that for transmission), (2) the cross-talk errors are assumed to be small (i.e., cross-pol isolation is good), and (3) in the case of the technique by van Zyl, the co-polarized and cross-polarized scattering matrix elements from natural distributed targets are assumed to be uncorrelated.

The purpose of this paper is to develop a general polarimetric calibration technique that is independent of the scattering matrices of the known targets to be

measured. The errors are modeled as distortion matrices (see Section 2) in the same way as was done by Barnes [9]. No assumptions are made about the magnitude (or form) of the distortion matrices. Instead of solving a set of possibly nonlinear equations for the elements of the distortion matrices, an eigenvalue approach is employed. Two types of polarimetric radar systems will be considered: (1) dual antenna systems for which the distortion matrices for transmit and receive are unrelated and (2) specialized single antenna systems for which the distortion is reciprocal. The first type is considered in Section 3, and the second type is considered in Section 4. Finally, we consider application of the technique to measuring the propagation characteristics of random media in Section 5.

## 2 Distortion Model

When using an ideal polarimetric radar, the measured scattering matrix  $\mathbf{M}$  of a point target would be equal to its theoretical (or actual) scattering matrix  $\mathbf{P}$ . Because this is rarely the case, the errors introduced by the radar system must be determined and then the process must be inverted in order to obtain an estimate of the actual scattering matrix. In the present work, we consider two types of errors: additive errors due to the presence of some unknown background and multiplicative errors which modify the polarization, amplitude, and phase of the transmitted and received waves. The multiplicative errors occur because of unknown gain and phase differences between the vertical and horizontal channels of the system. In order to account for these errors, we write the measured scattering matrix  $\mathbf{M}$  for some



point target  $\mathbf{P}$  as

$$\mathbf{M} = \mathbf{B} + e^{j\phi} r_{vv} t_{vv} \mathbf{R} \mathbf{P} \mathbf{T}, \quad (1)$$

where the matrix  $\mathbf{B}$  represents the effect of the background, and the distortion matrices  $\mathbf{T}$  and  $\mathbf{R}$  represent the effect of the antenna system (or the multiplicative errors) for transmit and receive, respectively. Throughout the paper, all matrices will be considered in the linear (vertical and horizontal) polarization basis; therefore the matrices of equation (1) are represented by

$$\mathbf{M} = \begin{bmatrix} m_{vv} & m_{vh} \\ m_{hv} & m_{hh} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} b_{vv} & b_{vh} \\ b_{hv} & b_{hh} \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} p_{vv} & p_{vh} \\ p_{hv} & p_{hh} \end{bmatrix}, \quad (2)$$

$$\mathbf{R} = \begin{bmatrix} 1 & r_{vh} \\ r_{hv} & r_{hh} \end{bmatrix}, \quad \text{and} \quad \mathbf{T} = \begin{bmatrix} 1 & t_{vh} \\ t_{hv} & t_{hh} \end{bmatrix}. \quad (3)$$

Notice that  $\mathbf{R}$  and  $\mathbf{T}$  are relative matrices, meaning that the entire matrix has been divided by the first element which is then used as a common scalar constant. The phase factor  $e^{j\phi}$  accounts for propagation to the target and back, and it depends on the exact position of the target phase center.

With some single antenna systems, the transmit and receive distortion matrices are simply the transpose of one another (i.e., the system is reciprocal) resulting in the equation

$$\mathbf{M} = \mathbf{B} + e^{j\phi} a_{vv}^2 \mathbf{A}^T \mathbf{P} \mathbf{A}, \quad (4)$$

where the distortion matrix  $\mathbf{A}$  is given by

$$\mathbf{A} = \begin{bmatrix} 1 & a_{vh} \\ a_{hv} & a_{hh} \end{bmatrix}. \quad (5)$$

If the matrices  $\mathbf{B}$ ,  $\mathbf{T}$ , and  $\mathbf{R}$  (or  $\mathbf{A}$  in the reciprocal case) can be determined, then the actual scattering matrix  $\mathbf{P}$  can be obtained from  $\mathbf{M}$  through one of the expressions

$$\mathbf{P} = e^{-j\phi} \frac{1}{r_{vv}t_{vv}} \mathbf{R}^{-1}(\mathbf{M} - \mathbf{B})\mathbf{T}^{-1} \quad (6)$$

$$\mathbf{P} = e^{-j\phi} \frac{1}{a_{vv}^2} (\mathbf{A}^T)^{-1}(\mathbf{M} - \mathbf{B})\mathbf{A}^{-1}, \quad (7)$$

which are obtained by rearranging equations (1) and (4). The propagation phase factor  $e^{-j\phi}$  in equations (6) and (7) is difficult to measure, since the phase center of the target and the target position must be known exactly. However, in most cases only the relative phase of  $\mathbf{P}$  is desired.

By making a single measurement with no point target present, we can directly determine the matrix  $\mathbf{B}$  when the background is stationary ( $\mathbf{M} = \mathbf{B}$  when  $\mathbf{P} = \mathbf{0}$ ). Considering only a stationary background, the dual antenna and specialized single antenna problems in (1) and (4) are thus reduced to

$$\mathbf{N} = e^{j\phi} r_{vv}t_{vv} \mathbf{R}\mathbf{P}\mathbf{T} \quad (8)$$

$$\mathbf{N} = e^{j\phi} a_{vv}^2 \mathbf{A}^T \mathbf{P} \mathbf{A}, \quad (9)$$

where  $\mathbf{N} = \mathbf{M} - \mathbf{B}$  is now a known matrix. If  $\mathbf{T}$  and  $\mathbf{R}$  (or  $\mathbf{A}$ ) are known, the actual scattering matrix  $\mathbf{P}$  can be obtained from one of the expressions

$$\mathbf{P} = e^{-j\phi} \frac{1}{r_{vv}t_{vv}} \mathbf{R}^{-1} \mathbf{N} \mathbf{T}^{-1} \quad (10)$$

$$\mathbf{P} = e^{-j\phi} \frac{1}{a_{vv}^2} (\mathbf{A}^T)^{-1} \mathbf{N} \mathbf{A}^{-1}, \quad (11)$$

where we have simply replaced  $\mathbf{M} - \mathbf{B}$  with  $\mathbf{N}$  in (6) and (7).

In the next two sections, techniques for determining the distortion matrices for the general dual antenna problem and the specialized single antenna problem are discussed. We will assume that the background scattering matrix has been removed as in equations (8)-(11) above.

### 3 General Dual Antenna System

Consider measurements of the form in (8) on three different targets with known scattering matrices. Using subscripts to denote the corresponding target and measurement scattering matrices, we obtain three matrix equations;

$$\mathbf{N}_i = e^{j\phi_i} r_{vv} t_{vv} \mathbf{R} \mathbf{P}_i \mathbf{T}, \quad \text{with } i = 1, 2, 3 \quad (12)$$

where  $\mathbf{N}_i$  and  $\mathbf{P}_i$  are known matrices, but  $\mathbf{R}$  and  $\mathbf{T}$  are unknown. Notice that a subscript is also used on the absolute phase to account for the positioning of the targets. The phase centers of the three targets will generally be located at different distances from the radar.

The following derivation requires that both the measured scattering matrix  $\mathbf{N}_i$  and the target scattering matrix  $\mathbf{P}_i$  be invertible for at least one of the targets. Without loss of generality, we assume that this requirement is satisfied with the first target. Premultiplying both  $\mathbf{N}_2$  and  $\mathbf{N}_3$  by  $\mathbf{N}_1^{-1}$  and denoting the products

as  $\mathbf{N}_T$  and  $\overline{\mathbf{N}}_T$ , we obtain the similarity relations

$$\mathbf{N}_T = e^{j(\phi_2 - \phi_1)} \mathbf{T}^{-1} \mathbf{P}_T \mathbf{T} \quad (13)$$

$$\overline{\mathbf{N}}_T = e^{j(\phi_3 - \phi_1)} \mathbf{T}^{-1} \overline{\mathbf{P}}_T \mathbf{T}, \quad (14)$$

where  $\mathbf{N}_T = \mathbf{N}_1^{-1} \mathbf{N}_2$ ,  $\overline{\mathbf{N}}_T = \mathbf{N}_1^{-1} \mathbf{N}_3$ ,  $\mathbf{P}_T = \mathbf{P}_1^{-1} \mathbf{P}_2$ , and  $\overline{\mathbf{P}}_T = \mathbf{P}_1^{-1} \mathbf{P}_3$ .

We now consider an important property relating the eigenvalues and eigenvectors of similar matrices [14, pp. 165-166]. The eigenvalues and eigenvectors of the matrices  $\mathbf{N}_T$  and  $\mathbf{P}_T$  in equation (13) satisfy the relations

$$\mathbf{P}_T \mathbf{X}_T = \mathbf{X}_T \Lambda'_T \quad (15)$$

$$\mathbf{N}_T \mathbf{Y}_T = \mathbf{Y}_T \Lambda_T, \quad (16)$$

where  $\Lambda'_T$  and  $\Lambda_T$  are the diagonal matrices composed of the eigenvalues of  $\mathbf{P}_T$  and  $\mathbf{N}_T$ , respectively. The eigenvalue matrices are related by the expression

$$\Lambda'_T = \Lambda_T e^{j(\phi_1 - \phi_2)}. \quad (17)$$

Notice that the propagation phase difference,  $\phi_1 - \phi_2$ , between the phase centers of any two known targets can be determined using equation (17), even though this fact is not used in the present development. The corresponding eigenvectors of  $\mathbf{P}_T$  and  $\mathbf{N}_T$  form the columns of  $\mathbf{X}_T$  and  $\mathbf{Y}_T$ , respectively. Equations (15)-(17) state that the eigenvalues of similar matrices are equal. Furthermore, the eigenvectors of  $\mathbf{P}_T$  and  $\mathbf{N}_T$  are related by the expression

$$\mathbf{Y}_T = \mathbf{T}^{-1} \mathbf{X}_T. \quad (18)$$

However, equation (18) does not uniquely specify  $\mathbf{T}$  since the eigenvectors comprising  $\mathbf{X}_T$  and  $\mathbf{Y}_T$  have arbitrary scale factors. Upon independently solving the eigenvalue problems in (15) and (16) for the matrices  $\mathbf{X}_T$  and  $\mathbf{Y}_T$  (arbitrarily choosing the scale factors), the matrix  $\mathbf{T}$  is uniquely specified by

$$\mathbf{Y}_T = \mathbf{T}^{-1}\mathbf{X}_T\mathbf{C} \quad \text{or} \quad (19)$$

$$\mathbf{T} = \mathbf{X}_T\mathbf{C}\mathbf{Y}_T^{-1}, \quad (20)$$

where  $\mathbf{C}$  is the diagonal matrix with elements  $c_1 \neq 0$  and  $c_2 \neq 0$  on the diagonal.

In the same way, the eigenvalues and eigenvectors of the matrices  $\bar{\mathbf{N}}_T$  and  $\bar{\mathbf{P}}_T$  in equation (14) satisfy the relations

$$\bar{\mathbf{P}}_T\bar{\mathbf{X}}_T = \bar{\mathbf{X}}_T\bar{\Lambda}'_T \quad (21)$$

$$\bar{\mathbf{N}}_T\bar{\mathbf{Y}}_T = \bar{\mathbf{Y}}_T\bar{\Lambda}_T, \quad (22)$$

where again  $\bar{\Lambda}'_T$  and  $\bar{\Lambda}_T$  are composed of the eigenvalues, and  $\bar{\mathbf{X}}_T$  and  $\bar{\mathbf{Y}}_T$  are the matrices whose columns are given by the corresponding eigenvectors. The eigenvalues of  $\bar{\mathbf{P}}_T$  and  $\bar{\mathbf{N}}_T$  are related by the expression

$$\bar{\Lambda}'_T = \bar{\Lambda}_T e^{j(\phi_1 - \phi_3)}. \quad (23)$$

From these results, we obtain another matrix equation for  $\mathbf{T}$ ;

$$\mathbf{T} = \bar{\mathbf{X}}_T\bar{\mathbf{C}}\bar{\mathbf{Y}}_T^{-1}, \quad (24)$$

where  $\bar{\mathbf{C}}$  is the diagonal matrix with elements  $\bar{c}_1 \neq 0$  and  $\bar{c}_2 \neq 0$  on the diagonal.

Equating (20) and (24), we obtain

$$\mathbf{X}_T\mathbf{C}\mathbf{Y}_T^{-1} = \bar{\mathbf{X}}_T\bar{\mathbf{C}}\bar{\mathbf{Y}}_T^{-1}, \quad (25)$$

and after rearranging, (25) becomes

$$\mathbf{C}\mathbf{Y}_T^{-1}\bar{\mathbf{Y}}_T = \mathbf{X}_T^{-1}\bar{\mathbf{X}}_T\bar{\mathbf{C}}. \quad (26)$$

If the eigenvalues of  $\mathbf{P}_T$ ,  $\mathbf{N}_T$ ,  $\bar{\mathbf{P}}_T$ , and  $\bar{\mathbf{N}}_T$  are distinct, then the corresponding eigenvectors are linearly independent. Therefore, the matrices  $\mathbf{X}_T$ ,  $\mathbf{Y}_T$ ,  $\bar{\mathbf{X}}_T$ , and  $\bar{\mathbf{Y}}_T$  have rank two, which means that they are nonsingular and invertible [14, p. 149].

Expanding equation (26) and writing it in terms of four scalar equations, we have

$$c_1\Delta(\mathbf{X}_T)(y_{22}\bar{y}_{11} - y_{12}\bar{y}_{21}) = \bar{c}_1\Delta(\mathbf{Y}_T)(x_{22}\bar{x}_{11} - x_{12}\bar{x}_{21}) \quad (27)$$

$$c_1\Delta(\mathbf{X}_T)(y_{22}\bar{y}_{12} - y_{12}\bar{y}_{22}) = \bar{c}_2\Delta(\mathbf{Y}_T)(x_{22}\bar{x}_{12} - x_{12}\bar{x}_{22}) \quad (28)$$

$$c_2\Delta(\mathbf{X}_T)(y_{11}\bar{y}_{21} - y_{21}\bar{y}_{11}) = \bar{c}_1\Delta(\mathbf{Y}_T)(x_{11}\bar{x}_{21} - x_{21}\bar{x}_{11}) \quad (29)$$

$$c_2\Delta(\mathbf{X}_T)(y_{11}\bar{y}_{22} - y_{21}\bar{y}_{12}) = \bar{c}_2\Delta(\mathbf{Y}_T)(x_{11}\bar{x}_{22} - x_{21}\bar{x}_{12}), \quad (30)$$

where  $x_{mn}$ ,  $y_{mn}$ ,  $\bar{x}_{mn}$ , and  $\bar{y}_{mn}$  are the elements of the matrices  $\mathbf{X}_T$ ,  $\mathbf{Y}_T$ ,  $\bar{\mathbf{X}}_T$ , and  $\bar{\mathbf{Y}}_T$ , respectively. The notation  $\Delta(\dots)$  is used to denote the determinant of the argument. Assuming that equations (27)-(30) are all nonzero, we can obtain two expressions for  $c_2/c_1$  and two for  $\bar{c}_2/\bar{c}_1$ ;

$$\frac{c_2}{c_1} = \frac{(x_{11}\bar{x}_{21} - x_{21}\bar{x}_{11})(y_{22}\bar{y}_{11} - y_{12}\bar{y}_{21})}{(x_{22}\bar{x}_{11} - x_{12}\bar{x}_{21})(y_{11}\bar{y}_{21} - y_{21}\bar{y}_{11})} \quad (31)$$

$$\frac{c_2}{c_1} = \frac{(x_{11}\bar{x}_{22} - x_{21}\bar{x}_{12})(y_{22}\bar{y}_{12} - y_{12}\bar{y}_{22})}{(x_{22}\bar{x}_{12} - x_{12}\bar{x}_{22})(y_{11}\bar{y}_{22} - y_{21}\bar{y}_{12})} \quad (32)$$

$$\frac{\bar{c}_2}{\bar{c}_1} = \frac{(x_{22}\bar{x}_{11} - x_{12}\bar{x}_{21})(y_{22}\bar{y}_{12} - y_{12}\bar{y}_{22})}{(x_{22}\bar{x}_{12} - x_{12}\bar{x}_{22})(y_{22}\bar{y}_{11} - y_{12}\bar{y}_{21})} \quad (33)$$

$$\frac{\bar{c}_2}{\bar{c}_1} = \frac{(x_{11}\bar{x}_{21} - x_{21}\bar{x}_{11})(y_{11}\bar{y}_{22} - y_{21}\bar{y}_{12})}{(x_{11}\bar{x}_{22} - x_{21}\bar{x}_{12})(y_{11}\bar{y}_{21} - y_{21}\bar{y}_{11})}. \quad (34)$$

As long as any two of the equations (27)-(30) are nonzero, we can obtain either  $c_2/c_1$  or  $\bar{c}_2/\bar{c}_1$  from at least one of equations (31)-(34). Without loss of generality, we assume for the remainder of the development that  $c_2/c_1$  is known.

The first element of  $\mathbf{T}$  must be unity, and from (20) it is given by

$$\frac{1}{\Delta(\mathbf{Y}_T)} (c_1 x_{11} y_{22} - c_2 x_{12} y_{21}) = 1. \quad (35)$$

This expression can be used to obtain  $c_1$  and  $c_2$  in terms of the ratio  $c_2/c_1$ ;

$$c_1 = \Delta(\mathbf{Y}_T) \left( x_{11} y_{22} - \frac{c_2}{c_1} x_{12} y_{21} \right)^{-1} \quad (36)$$

$$c_2 = \Delta(\mathbf{Y}_T) \left( \frac{c_1}{c_2} x_{11} y_{22} - x_{12} y_{21} \right)^{-1}. \quad (37)$$

The matrix  $\mathbf{R}$  can also be determined using a similar procedure. Postmultiplying both  $\mathbf{N}_2$  and  $\mathbf{N}_3$  by  $\mathbf{N}_1^{-1}$ , we obtain

$$\mathbf{N}_R = e^{j(\phi_2 - \phi_1)} \mathbf{R} \mathbf{P}_R \mathbf{R}^{-1} \quad (38)$$

$$\bar{\mathbf{N}}_R = e^{j(\phi_3 - \phi_1)} \mathbf{R} \bar{\mathbf{P}}_R \mathbf{R}^{-1}, \quad (39)$$

where  $\mathbf{N}_R = \mathbf{N}_2 \mathbf{N}_1^{-1}$ ,  $\bar{\mathbf{N}}_R = \mathbf{N}_3 \mathbf{N}_1^{-1}$ ,  $\mathbf{P}_R = \mathbf{P}_2 \mathbf{P}_1^{-1}$ , and  $\bar{\mathbf{P}}_R = \mathbf{P}_3 \mathbf{P}_1^{-1}$ .

Since equations (38) and (39) are again similarity transformations, we can uniquely specify  $\mathbf{R}$  by the relations

$$\mathbf{R} = \mathbf{X}_R \mathbf{D} \mathbf{Y}_R^{-1} \quad (40)$$

$$\mathbf{R} = \bar{\mathbf{X}}_R \bar{\mathbf{D}} \bar{\mathbf{Y}}_R^{-1}, \quad (41)$$

where the eigenvector matrices  $\mathbf{X}_R$ ,  $\mathbf{Y}_R$ ,  $\bar{\mathbf{X}}_R$ , and  $\bar{\mathbf{Y}}_R$  satisfy the relations

$$\mathbf{N}_R \mathbf{X}_R = \mathbf{X}_R \Lambda'_R \quad (42)$$

$$\mathbf{P}_R \mathbf{Y}_R = \mathbf{Y}_R \mathbf{\Lambda}_R \quad (43)$$

$$\overline{\mathbf{N}}_R \overline{\mathbf{X}}_R = \overline{\mathbf{X}}_R \overline{\mathbf{\Lambda}}_R' \quad (44)$$

$$\overline{\mathbf{P}}_R \overline{\mathbf{Y}}_R = \overline{\mathbf{Y}}_R \overline{\mathbf{\Lambda}}_R, \quad (45)$$

and the eigenvalue matrices are related by the expressions

$$\mathbf{\Lambda}'_R = \mathbf{\Lambda}_R e^{j(\phi_2 - \phi_1)} \quad (46)$$

$$\overline{\mathbf{\Lambda}}'_R = \overline{\mathbf{\Lambda}}_R e^{j(\phi_3 - \phi_1)}. \quad (47)$$

The matrices  $\mathbf{D}$  and  $\overline{\mathbf{D}}$  are analogous to  $\mathbf{C}$  and  $\overline{\mathbf{C}}$ ; they are diagonal with elements  $(d_1 \neq 0, d_2 \neq 0)$  and  $(\overline{d}_1 \neq 0, \overline{d}_2 \neq 0)$ , respectively. Equations (40) and (41) are of the same form as (20) and (24), therefore equations (31)-(37) can be used to find expressions for  $d_1$  and  $d_2$  by replacing  $c$  with  $d$  and letting  $x_{mn}$ ,  $y_{mn}$ ,  $\overline{x}_{mn}$ , and  $\overline{y}_{mn}$  denote the elements of the matrices  $\mathbf{X}_R$ ,  $\mathbf{Y}_R$ ,  $\overline{\mathbf{X}}_R$ , and  $\overline{\mathbf{Y}}_R$ , respectively.

With the distortion matrices known, the absolute magnitude can be obtained by substituting back into one of the original measurements. Equating the elements of the matrices on both sides of (12) and taking the magnitude, we obtain for the  $mn^{th}$  element

$$|r_{vv} t_{vv}| = \frac{|(\mathbf{N}_i)_{mn}|}{|(\mathbf{R}\mathbf{P}_i\mathbf{T})_{mn}|}. \quad (48)$$

The best estimate of  $|r_{vv} t_{vv}|$  will be obtained by choosing the target for which the theoretical matrix  $\mathbf{P}_i$  is most accurate.

Using (48), the scattering matrix  $\mathbf{P}$  for an unknown point target can be written



in terms of the measured scattering matrix  $\mathbf{N}$ ;

$$\mathbf{P} = e^{-j\phi'} \frac{1}{|r_{vv}t_{vv}|} \mathbf{R}^{-1} \mathbf{N} \mathbf{T}^{-1}, \quad (49)$$

where  $\phi'$  is the unknown absolute phase given by

$$\phi' = \phi + \tan^{-1} \left( \frac{\text{Im}\{r_{vv}t_{vv}\}}{\text{Re}\{r_{vv}t_{vv}\}} \right). \quad (50)$$

## 4 Specialized Single Antenna System

In general, single antenna radar systems, like dual antenna systems, have different distortion matrices for transmit and receive. Even though the single antenna affects the transmitted and received waves in a similar manner, the remaining portions of the transmit and receive paths through the system affect them differently. Therefore, equation (1) should be used to describe the measured scattering matrix for a general single antenna system, and the technique of Section 3 should be used in calibration.

In many cases, the antenna assembly is the major contributor to the distortion errors, and the contributions due to the different transmit and receive paths is negligible. Usually with such systems, care has been taken to make the transmit and receive paths practically identical, and only antenna effects need to be considered. The measured scattering matrix can then be described with equation (4), and a slightly different technique can be used in calibration. The major advantage of the technique is that only two known targets need to be measured to fully calibrate the measurement system. The technique is described in the following development.

Consider measurements of the form in (9) on two different targets with known scattering matrices. Using subscripted notation similar to that of equation (12), the measured scattering matrix of the  $i^{\text{th}}$  target is

$$\mathbf{N}_i = e^{j\phi_i} a_{vv}^2 \mathbf{A}^T \mathbf{P}_i \mathbf{A} \quad \text{with } i = 1, 2 \quad (51)$$

where  $\mathbf{P}_i$  is the known scattering matrix of the target. The unknown distortion matrix is  $\mathbf{A}$ . The calibration technique to be described requires a more restrictive condition on the form of the known scattering matrices than does the general technique of Section 3. With the present technique, both calibration targets must have invertible scattering matrices, whereas only one was required with the general technique.

Forming the products  $\mathbf{N} = \mathbf{N}_2^{-1} \mathbf{N}_1$  and  $\bar{\mathbf{N}} = (\mathbf{N}_2 \mathbf{N}_1^{-1})^T$ , we obtain the two similarity transformations

$$\mathbf{N} = e^{j(\phi_1 - \phi_2)} \mathbf{A}^{-1} \mathbf{P} \mathbf{A} \quad (52)$$

$$\bar{\mathbf{N}} = e^{j(\phi_2 - \phi_1)} \mathbf{A}^{-1} \bar{\mathbf{P}} \mathbf{A}, \quad (53)$$

where  $\mathbf{P} = \mathbf{P}_2^{-1} \mathbf{P}_1$  and  $\bar{\mathbf{P}} = (\mathbf{P}_2 \mathbf{P}_1^{-1})^T$ . This method can be applied to the specialized single antenna system only because the transmit and receive distortion matrices are related by a transpose. In a manner similar to that in Section 3, we can uniquely specify  $\mathbf{A}$  by the relations

$$\mathbf{A} = \mathbf{X} \mathbf{G} \mathbf{Y}^{-1} \quad (54)$$

$$\mathbf{A} = \bar{\mathbf{X}} \bar{\mathbf{G}} \bar{\mathbf{Y}}^{-1}, \quad (55)$$

where the matrices  $\mathbf{G}$  and  $\overline{\mathbf{G}}$  are diagonal with elements  $(g_1 \neq 0, g_2 \neq 0)$  and  $(\overline{g}_1 \neq 0, \overline{g}_2 \neq 0)$ , respectively. The eigenvector matrices  $\mathbf{X}$ ,  $\mathbf{Y}$ ,  $\overline{\mathbf{X}}$ , and  $\overline{\mathbf{Y}}$  satisfy the expressions

$$\mathbf{P} \mathbf{X} = \mathbf{X} \Lambda' \quad (56)$$

$$\mathbf{N} \mathbf{Y} = \mathbf{Y} \Lambda \quad (57)$$

$$\overline{\mathbf{P}} \overline{\mathbf{X}} = \overline{\mathbf{X}} \overline{\Lambda}' \quad (58)$$

$$\overline{\mathbf{N}} \overline{\mathbf{Y}} = \overline{\mathbf{Y}} \overline{\Lambda}, \quad (59)$$

where the eigenvalue matrices  $\Lambda'$ ,  $\Lambda$ ,  $\overline{\Lambda}'$ , and  $\overline{\Lambda}$  are related by

$$\Lambda' = \Lambda e^{j(\phi_1 - \phi_2)} \quad (60)$$

$$\overline{\Lambda}' = \overline{\Lambda} e^{j(\phi_2 - \phi_1)}. \quad (61)$$

Equating (54) and (55) and then rearranging, we obtain

$$\mathbf{G} \mathbf{Y}^{-1} \overline{\mathbf{Y}} = \mathbf{X}^{-1} \overline{\mathbf{X}} \overline{\mathbf{G}}. \quad (62)$$

Equation (62) is of the same form as (26), so  $g_1$  and  $g_2$  are given by equations (36) and (37) with  $g$  replacing  $c$ . Here, the elements  $x_{mn}$ ,  $y_{mn}$ ,  $\overline{x}_{mn}$ , and  $\overline{y}_{mn}$  denote the elements of the matrices  $\mathbf{X}$ ,  $\mathbf{Y}$ ,  $\overline{\mathbf{X}}$ , and  $\overline{\mathbf{Y}}$ , respectively.

By substituting equation (54) into (51) and equating the elements on both sides, we obtain for the  $mn^{\text{th}}$  element

$$|a_{vv}^2| = \frac{|(\mathbf{N}_i)_{mn}|}{|(\mathbf{A}^T \mathbf{P}_i \mathbf{A})_{mn}|}. \quad (63)$$

As in the general technique, the best estimate of  $|a_{vv}^2|$  will be obtained by using the known target for which the theoretical matrix  $\mathbf{P}_i$  is most accurate.

We now have the effect of the distortion matrices determined to within an unknown phase factor. The scattering matrix for some unknown point target can be written as

$$\mathbf{P} = e^{-j\phi'} \frac{1}{|a_{vv}^2|} (\mathbf{A}^T)^{-1} \mathbf{N} \mathbf{A}^{-1}, \quad (64)$$

where  $\phi'$  is the unknown absolute phase given by

$$\phi' = \phi + \tan^{-1} \left( \frac{\text{Im}\{a_{vv}^2\}}{\text{Re}\{a_{vv}^2\}} \right). \quad (65)$$

## 5 Application to Measuring the Propagation Characteristics of Random Media

Experimental investigations into the nature of propagation in random media have traditionally used two measurement configurations. The first type involves the use of a transmitter on one side of the random medium (considered as a layer) and a receiver on the other side. Measurements are made at a number of spatial locations through the layer to determine the statistics associated with the attenuation. In cases where only the extinction in the forward direction is desired, angular resolution is obtained by making the antenna beamwidths small. This technique is cumbersome to use, particularly at oblique incidence to the random layer, due to the difficulty associated with proper pointing of the small beamwidth antennas. A second technique uses a transmitter and receiver placed at the same location (radar mode) on one side of the random layer and a point target on the other side. Because the received signal contains contributions from both the point target attenuated by the medium and the random layer itself, the precision associated with

the measurement of the two-way attenuation depends upon the amplitude ratio of the two contributions [15, pp. 768-770]. Thus, the technique requires a point target with a large scattering cross section to obtain precise attenuation measurements. Furthermore, with this technique only the amplitude of the attenuation is measured; the propagation phase is usually ignored.

Recently, a polarimetric technique for measuring the propagation characteristics of random media was proposed and then demonstrated by measuring the characteristics of a forest canopy [16],[17]. The technique uses the same configuration as the second technique above, with the transmitter and receiver on one side of the random layer and a point target (trihedral) on the other side. The difference is that polarimetric measurements are made of the canopy scattering matrix with and without the presence of the trihedral underneath the canopy. The measurement without the trihedral yields the scattering matrix of the canopy alone, and the scattering matrix of the canopy/target combination is modeled as [16],[17]

$$\mathbf{S} = \mathbf{T} + e^{j\phi} l_{vv}^2 \mathbf{L}^T \mathbf{P} \mathbf{L}, \quad (66)$$

with

$\mathbf{S}$  = scattering matrix of the canopy/target combination

$\mathbf{L}$  = one-way relative propagation (loss) matrix of the canopy

$\mathbf{P}$  = scattering matrix of the point target alone

$\mathbf{T}$  = scattering matrix of the canopy (trees) alone.

The major restriction with the technique as described in [17] is that it assumes  $\mathbf{L}$  to be a diagonal matrix. For the canopy and frequency (L-band) considered, this was a reasonable assumption to make. However, a more general technique without the restriction on the form of  $\mathbf{L}$  would be applicable to a wider range of problems.

The calibration technique described in this paper can be used to extend the method above to cases where  $\mathbf{L}$  is an arbitrary matrix. We notice that equation (66) is in the same form as (4) representing the specialized single antenna system. In essence, the propagation through the random medium is treated as a transformation analogous to that produced by the antenna for the single antenna system. By measuring the canopy alone and two additional known targets underneath the canopy, the method described in Section 4 can be used to determine the two-way extinction and the relative propagation (or loss) matrix  $\mathbf{L}$  of the canopy.

The technique can be further extended by considering the propagation through the canopy as non-reciprocal. Denoting the loss matrices in the upward and downward directions through the canopy as  $\mathbf{U}$  and  $\mathbf{D}$ , respectively, the scattering matrix of the canopy/target combination becomes

$$\mathbf{S} = \mathbf{T} + e^{j\phi} u_{vv} d_{vv} \mathbf{U} \mathbf{P} \mathbf{D}. \quad (67)$$

Since equation (67) is of the same form as (1), the method of Section 3 can be used to determine the two-way extinction and the relative loss matrices  $\mathbf{U}$  and  $\mathbf{D}$ .

The application of the techniques discussed in this paper to measuring the propagation characteristics of random media (specifically vegetation) will be the subject of future investigation by the authors.

## 6 Conclusions

A general polarimetric calibration technique has been developed, requiring the measurement of at most three known targets. The form of the scattering matrices of the known targets is arbitrary (but must be known), with the restriction that at least one target scattering matrix must be invertible. The errors introduced by the transmitter and receiver are modeled by distortion matrices that alter the measured scattering matrix of the illuminated target. A set of eigenvalue problems are then solved on matrix products involving the measured and theoretical scattering matrices to determine the distortion matrices. Calibration of the absolute magnitude is achieved by inserting the measured distortion matrices back into one measurement and solving for the magnitude. The two distinct advantages of the technique are that (1) almost any targets can be used and (2) no assumptions are made about the magnitude of the distortion.

The technique is applicable to both laboratory and field measurements, with the known targets being chosen according to the application. For example, in a laboratory environment the emphasis should be placed on accuracy of the theoretical scattering matrices of the calibration targets. The sensitivity to positioning of the targets is only a secondary consideration, since one would conceivably have very fine control of target orientation. In field calibration, one should choose targets that are generally insensitive to positioning, since this aspect is the most difficult to control. The errors in the theoretical scattering matrices for these calibration targets can be determined with laboratory measurements using a different set of

very accurate calibration targets. Hence, the actual scattering matrices can be determined.

Further research is being conducted to determine the best possible calibration targets to use with the techniques described in this paper. The results of this research and the implementation of the techniques will be considered in an additional paper to follow.

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