Recovery of Fine Resolution Information in Multispectral Processing[1]

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ABSTRACT

In this paper, we consider multiple-sensor processing and develop a unified method for representing multiple-sensor data. When resolution varies between sensors, such a multiple-sensor system can be viewed as samples of a scale-space signal representation. We show that if the spatial transfer function of the sensors are Gaussian, then scale-space filtering can be used to recover small scale (fine resolution) information through extrapolation in scale. As an example of multiple-sensor processing, we consider multispectral processing of remote sensing in which images of surface scenes are simultaneously generated at different (center) frequencies.

INTRODUCTION

Scale-space filtering was introduced in the early 1980's as a technique for signal analysis over multiple scales [1,2]. The origins of scale-space filtering lie in the edge detection concerns of computer vision but have since been applied to other problems in computer vision [3,4] as well as a model for multiresolution systems [5]. In this latter context, scale-space analysis provides a mathematical framework for data integration in multiresolution systems that are characterized by classes of sensors varying along a dimension called *scale*. For example, multispectral analysis in remote sensing requires integration of data from constant-Q bandlimited sensors that vary in center frequency [6]. A simple approximation of such a multiresolution system is that of a multiscale system in which a single parameter, e.g., bandwidth, characterizes the primary differences among each of the bandlimited sensors. Under this approximation, the data from each sensor represents *samples* of the scale-space representation of the imaged object.

Formally, scale-space filter theory describes the effects of filter scale on functions e(x,s) of the form [1,2,7],

$$e(x,s) \equiv O\{r(x,s)\},$$

where O{.} is a linear operator and r(x,s) is a filter output signal given by,

$$r(x,s) \equiv h(x,s)*i(x).$$

The function i(x) is the input signal and h(x,s) is a family of filters which is parameterized by a continuous variable s. This variable s is inversely proportional to filter bandwidth and denotes the *scale* of the filter.

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Scale-space filtering represents the signal i(x) by the two-dimensional function r(x,s) and draws inferences about variations of i(x) from the threshold crossing contours of e(x,s). The location of these threshold crossings, x(s), is dependent on scale and calculated from

$$e(x(s),s) = \gamma(x(s)), \tag{1}$$

for some specified *threshold function* $\gamma(.)$. The threshold crossings form contours in the x-s plane. In the nomenclature of scale-space theory, the x-s plane is the *scale-space*, the function e(.,.) is the *scale-space image*, and the threshold crossing contours, x(s), are the *fingerprints*.

Although Zuerndorfer and Wakefield [7] have shown that the requirements can be relaxed while still preserving major points of the theory, the strongest theorems of scale-space signal representation [2,8] assume Gaussian kernels of the form

$$h(x,s) = \frac{1}{s\sqrt{2\pi k}}\exp\left[-\frac{1}{2}\left(\frac{x^2}{s^2k}\right)\right], \tag{2}$$

and a Laplacian operator for O{.}. In this case, the fingerprints present a continuous track of the inflection points of the signal i(.) as it is filtered over scale. The inflection points can be used for locating "edges" in the signal [9].

Given the use of Gaussian kernels, Gaussian filtering can be used to degrade the resolution (broaden the PSF) of fine-resolution sensors in multiresolution systems to match that of a coarse-resolution sensor. Alternatively, given the features of a signal measured by a coarse-resolution sensor, it is useful to register their locations with signal features measured by fine-resolution sensors. Modelled as a multiscale system, these two problems represent *interpolation* and *extrapolation*, respectively, of the sampled fingerprints. In the following, we present a formal development of extrapolation in scale-space and then apply extrapolation to multispectral processing of remote sensing [6].

EXTRAPOLATION IN SCALE-SPACE

Extrapolation concerns determining the threshold crossing contours, x(s), given by,

$$h(x(s),s)*i_I(x(s)) \equiv r(x(s),s) = \alpha(x(s)), \tag{3}$$

where $\alpha(.)$ is a threshold function. The function $i_I(.)$ is an *indicator* composed of a linear combination of single sensor data,

$$i_I(.) \equiv \sum_{j=1}^{N} a_j i_j(.),$$

where $i_j(.)$ is the ideal output signal from the j^{th} sensor, and the a_j's are coefficients. The function $i_j(.)$ is the image output from the j^{th} sensor for a device having infinitesimal spatial resolution. Without loss of generality, the sensor data are ordered by increasing scale of the sensor. Comparing (3) with (1), the operator O{.} in (3) is the identity.

The fingerprints x(s) in (3) are estimates of boundary locations in $i_I(.)$. True boundaries occur between surface regions having different $i_I(.)$ values, so that boundaries occur at x values where $i_I(.)$ crosses the threshold function,

$$i_I(x) = \alpha(x). \qquad (4)$$

The threshold function $\alpha(.)$ is a linear function of x.

In general, $i_I(.)$ cannot be processed directly due to the finite PSF of the sensor. However, in the absence of noise, the threshold crossings of r(.,s) approach those of $i_I(.)$ as $s \to 0$, since the kernel h(.,s) approximates a delta function as $s \to 0$. To better approximate boundary locations, we seek x(s) for as small a scale as possible. If s_j is the finest scale at which data from the j^{th} channel is available, and $s_1 < \ldots < s_N$, then x(s) can only be determined for $s \geq s_N$. However $s_1 < s_N$, and the boundary estimate is improved if there exists a threshold function $\beta(.)$ such that,

$$h(u(s),s) * i_1(u(s)) \equiv p(u(s),s) = \beta(u(s)), \qquad (5)$$

where u(s) are the fingerprints derived from (5), and

$$u(s) \approx x(s) \quad \text{for} \quad s_N \geq s \geq s_1. \qquad (6)$$

Note that u(s) for a particular threshold function $\beta(.)$ need only approximate x(s) over part of a single contour, and that different threshold functions are used to approximate x(s) for different contours.

To demonstrate the signal and threshold requirements to achieve (6), consider the Taylor expansion of x(s) about s_0,

$$x(s) = x(s_0) + (s - s_0)x'(s_0) + \frac{(s - s_0)^2}{2}x''(s_0) + \ldots . \qquad (7)$$

By the implicit function theorem [10],

$$x'(s_0) = \frac{r_s(x_0, s_0)}{\alpha_x(x_0) - r_x(x_0, s_0)} \qquad (8a)$$

$$x''(s_0) = \qquad (8b)$$

$$\frac{(r_{xx}(x_0, s_0) - \alpha_{xx}(x_0))\,(x'(s_0))^2 + 2r_{xs}(x_0, s_0)x'(s_0) + r_{ss}(x_0, s_0)}{\alpha_x(x_0) - r_x(x_0, s_0)},$$

where $x_0 \equiv x(s_0)$; subscripted variables indicate partial differentiation. In (3) the threshold function $\alpha(.)$ is a linear, so that $\alpha_x(.) = A$ for some constant A, and $\alpha_{xx}(.) = 0$. By solution to the heat equation, the use of a Gaussian kernel yields, $r_s(x_0, s_0) = s_0 r_{xx}(x_0, s_0)$. As a result, (8a) can be written as,

$$x'(s_0) = -\frac{s_0 r_{xx}(x_0, s_0)}{r_x(x_0, s_0)}, \qquad (9a)$$

and (8b) can be written as,

$$x''(s_0) = \qquad (9b)$$

$$-\frac{r_{xx}(x_0, s_0)\,(x'(s_0))^2 + 2s_0 r_{xxx}(x_0, s_0)x'(s_0) + s_0^2 r_{xxxx}(x_0, s_0)}{A - r_x(x_0, s_0)}.$$

Repeating the steps above for u(s) yields,

$$u(s) = u(s_0) + (s - s_0)u'(s_0) + \frac{(s - s_0)^2}{2}u''(s_0) + \ldots, \qquad (10)$$

where,

$$u'(s_0) = -\frac{s_0 p_{xx}(u_0, s_0)}{p_x(u_0, s_0)}, \qquad (11a)$$

$$u''(s_0) = \qquad (11b)$$

$$\frac{p_{xx}(u_0, s_0)\,(u'(s_0))^2 + 2s_0 p_{xxx}(u_0, s_0)u'(s_0) + s_0^2 p_{xxxx}(u_0, s_0)}{B - p_x(u_0, s_0)},$$

and $u_0 \equiv u(s_0)$. The threshold function $\beta(.)$ is linear, so that $\beta_x(.) = B$ for some constant B. Thus, for u(s) to approximate x(s), it is necessary that,

$$x'(s_0) = u'(s_0) \text{ and } x''(s_0) = u''(s_0)$$

or,

$$\frac{h(x,s) * \frac{d^n}{dx^n} i_I(x)}{A - h(x,s) * \frac{d}{dx} i_I(x)} = \qquad (12)$$

$$\frac{h(x,s) * \frac{d^n}{dx^n} i_1(x)}{B - h(x,s) * \frac{d}{dx} i_1(x)} \quad \text{for} \quad n=1,2,3.$$

EXAMPLES

We apply the results of the previous section to a multi-spectral system that integrates data from N sensors, where each sensor operates at a different (center) frequency. In this system, each sensor receives energy using an aperture, e.g., devices such as lenses in optical applications, antennas in microwave application, and arrays in sonar applications that collect radiated energy that is emitted or reflected from a subject of interest. In this system, as the j^{th} sensor is scanned over a subject, the output signal is the convolution of the spatially varying radiation intensity of the subject with the radiation pattern (antenna pattern) of the aperture. The radiation intensity of the subject is equivalent to the scale-space input for the j^{th} sensor, $i_j(.)$. The radiation pattern of the aperture is equivalent to a scale-space filter impulse response of the j^{th} sensor $h(.,s_j)$, where the scale of the impulse response, s_j, is the width (beamwidth) of the radiation pattern; s_j is proportional to the wavelength of the sensor. In this system, true regional boundaries occur at level crossings of $i_I(.)$, and are approximated by the fingerprints x(s).

A class of functions for which (12) holds is that where different sensor inputs, $i_j(.)$, are scaled versions of each other. In this case, the indictor is given by,

$$i_I(x) = \left(\sum_j a_j\right)K(x) \qquad (13a)$$

and the finest scale (highest frequency) input signal is given by,

$$i_1(x) = a_1K(x), \qquad (13b)$$

for an arbitrary function K(.). The significance of such functions can be seen in multiplicative models,

$$i_j(x) = I(x)R_j(x),$$

as are often used in image processing and remote sensing. In active remote sensing systems, I(.) and R_j(.) are surface illumination and reflection functions, respectively. In passive systems, I(.) and R_j(.) are surface temperature and surface emissivity, respectively. In both systems, R_j(.) is a function of the surface type and is dependent on frequency. As a result, the indicator is given by,

$$i_I(x) = \sum_j I(x)R_j(x). \qquad (14)$$

An example of a surface satisfying (13a) and (13b) is one consisting of a single surface type, and a spatially varying illumination intensity or surface temperature.

Another class of functions that satisfies (12) is quadratic functions. In remote sensing applications, such signals occur in passive systems where surface temperature and surface emissivity change linearly in the vicinity of a boundary [11]. In this case, I_j(.) and R_j(.) are linear,

$$I(x) = \alpha_I x + \beta_I$$

$$R_j(x) = \alpha_{Rj} x + \beta_{Rj}$$

so that,

$$i_I(x) = \left(\sum_j a_j\right)x^2 + \left(\sum_j b_j\right)x + \left(\sum_j c_j\right) \qquad (16a)$$

and

$$i_1(x) = a_1x^2 + b_1x + c_1 \qquad (16b)$$

where,

$$a_j = \alpha_I \alpha_{Rj}$$

$$b_j = \alpha_I \beta_{Rj} + \beta_I \alpha_{Rj}$$

$$c_j = \beta_I \beta_{Rj}\,.$$

A simplified example of a one-dimensional surfaces model that satisfies (16a) and (16b) is shown in Figures 1-4 (this model is derived from [11]). In the figures, the indicator is composed of data from two sensors,

$$i_I(x) = i_1(x) - i_2(x), \qquad (17)$$

where the scale of sensor 1 is less than the scale of sensor 2. In Figures 1-4, the functions i_1(.) and i_2(.) are quadratic in the vicinity of regional boundaries. The surface type changes at x=125, so that i_1(.) and i_2(.) exhibit different behavior in the vicinity of boundaries for x<125 and x>125 (Figure 1). A threshold level is selected to locate a boundary around the surface region with a low i_I(.) value. The corresponding fingerprint is shown in Figure 2. Figure 3 shows the i_1(.) function and threshold function, and the corresponding fingerprint is shown in Figure 4. Comparing the fingerprints of Figures 2 and 4 shows that the fingerprint of i_1(.) (Figure 4) is a reasonable match to the fingerprint of i_I(.) (Figure 2), particularly at smaller scales. Since the fingerprints of i_I(.) cannot be calculated at small scales, the fingerprints of i_1(.) can be used to approximate the fingerprints of i_I(.) at small scales.

CONCLUSION

For the cases of Gaussian filtering and linear threshold crossings, we've demonstrated that extrapolation of scale can be performed in multispectral processing for signals that satisfy (12). The fingerprints of extrapolated signals approximate the actual multispectral fingerprints at small scales, and can be used when the multispectral fingerprints are not available.

In showing the approximation of extrapolation fingerprints to multispectral fingerprints, only three terms of the Taylor expansion were exploited (i.e., (12) was satisfied for 3 terms). It can be shown that N>3 terms of a Taylor expansion can be used if (12) is satisfied for N terms. As a result, in the absence of noise, extrapolation fingerprints that match the actual multispectral fingerprints at N>3 terms of a Taylor expansion will provide a better approximation at small scales.

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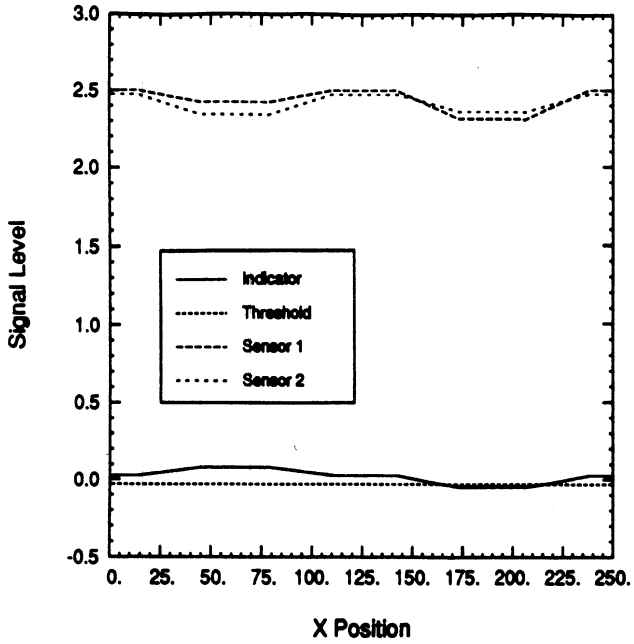

Figure 1. Sensor input signals, indictor, and threshold.

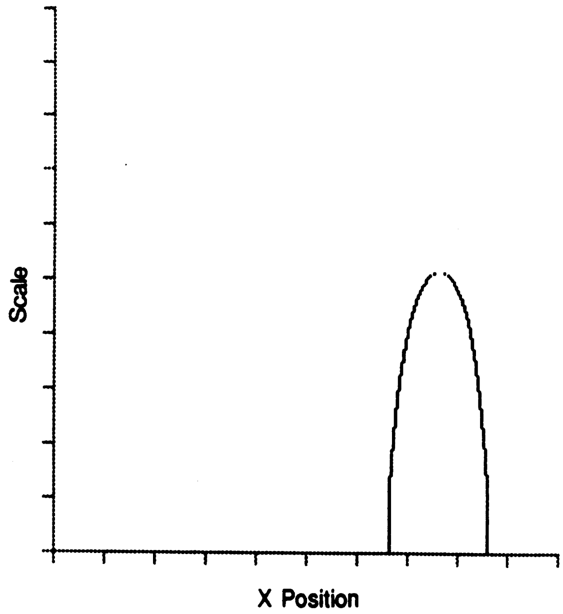

Figure 2. Indicator fingerprints.

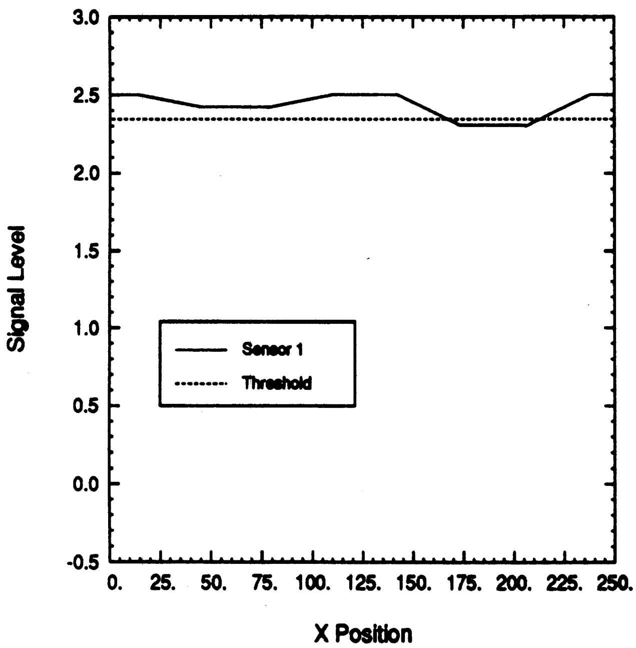

Figure 3. High frequency (fine scale) sensor input signal and threshold.

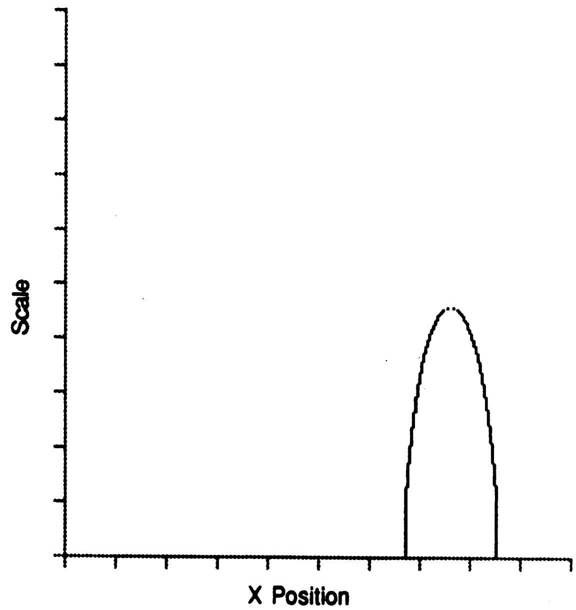

Figure 4. High frequency sensor fingerprints.