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REDUCTION OF RADAR CROSS SECTION OF DUCTS

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By
John J. Bowman and Vaughan H. Weston

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FOREWORD

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ABSTRACT

As a preliminary to an investigation concerning radar scattering by a rectangular duct lined with absorbing materials, the simplified problem of plane-wave scattering by a semi-infinite parallel plane waveguide is considered. The surfaces of the guide are assumed to obey impedance boundary conditions, where the impedance on the interior surface may be different from the impedance on the exterior surface. A case of particular interest would be that in which the exterior surface is perfectly conducting. Ray-optical techniques based upon known results for a semi-infinite screen with two face impedances are employed to calculate both the field scattered into the far zone and the field generated at the mouth of the guide. The ray optical procedure is also applied to the case of a perfectly conducting, open-ended parallel plane waveguide for which an exact solution is available, and the ray optical result is compared with the asymptotic expansion of the exact solution.

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I

INTRODUCTION

The purpose of this contract is to investigate the radar cross section of an air intake duct similar to that found on jet aircraft. In order to reduce the radar cross section, the interior of the duct is supposed to be lined with a layer of radar absorbing material, thereby providing an absorption of the energy launched into waveguide modes, which may be reflected by the termination of the duct and reradiated back toward the radar transmitter. The opening of the duct is taken to be rectangular, although the cross sectional shape gradually changes to circular at the rear of the duct. The tasks of the contract were therefore, divided into two parts; the first being to calculate the energy launched into the duct and its consequent reradiation based upon an unknown reflection coefficient at the termination of the duct, and the second being to calculate the effect of the gradual cross sectional change of shape of the duct on the waveguide modes carrying the energy. From this knowledge, it is hoped to produce a good estimate for the radar cross section due to the interior of the duct.

The theoretical approach we shall employ is to use ray optical techniques to calculate the field generated at the mouth of the duct and from this information to determine the energy launched into the waveguide modes. As a first approximation to this very complicated problem we have studied the two-dimensional problem of a plane wave incident upon a semi-infinite parallel plane waveguide wherein the outside surfaces of the guide are perfectly conducting and the interior surfaces of the guide obey an impedance boundary condition. Ray optical techniques are employed to obtain both the field generated at the mouth of the guide and the field scattered into the far zone by the edges forming the waveguide opening. The ray treatment is intimately dependent on the known solution for plane wave diffraction by a half plane with arbitrary face impedances. At the present time, it is no extra complication to consider the outside surfaces of the guide as being governed by a separate constant surface impedance. The ray optical methods are also applied to the case of a perfectly conducting parallel plane waveguide for which an exact solution is available, the motivation being to compare the ray optical result with the asymptotic expansion of the exact solution.

For application to a rectangular waveguide, these results must be extended to include the case of oblique incidence on the edges forming the waveguide opening. This aspect of the problem is presently under investigation. At the same time, the waveguide modes existing in a rectangular waveguide with an impedance boundary condition are being investigated. The orthogonality properties of these modes must be determined in order to match the field in the guide to the field generated at the mouth.

II

THE HALF PLANE

Before we consider the problem of diffraction by a semi-infinite parallel plane waveguide we must first examine the pertinent half plane results. To fix our notation, we shall employ natural units with free-space constants ϵ_0, μ_0 set equal to unit and suppress the harmonic time dependence $\exp(-i\omega t)$ throughout. A plane electromagnetic wave of unit amplitude is assumed incident at an angle θ_0 to the semi-infinite screen as shown in Fig. 1 . The screen is assumed to be comprised of material of such a kind as to make the total tangential field components satisfy the following impedance boundary condition on the surface ($\hat{n}$ is the unit outward normal to the surface)

$$\underline{E} - (\hat{n} \cdot \underline{E})\,\hat{n} = \eta\, \hat{\underline{n}} \wedge \underline{H} \ , \tag{1}$$

where $\eta = \eta_1$ on the upper surface and $\eta = \eta_2$ on the lower surface. The face impedances η_1 and η_2 are complex constants whose real parts, because of energy considerations, must be non-negative. Further, the surface impedances are assumed to account for the presence of thin layers of highly refractive absorbing materials applied as a coating on a perfectly conducting half-plane. Although the validity of the impedance boundary condition near an edge of the diffracting structure is then open to question (Weston, 1963), one nevertheless expects some general features of the scattering process to emerge when the overall effect of an absorber coating is treated in terms of a constant surface impedance (see, e.g., Bowman, 1967). It may be noted that if the surface is to be considered as perfectly conducting, then the corresponding impedance must be set identically equal to zero. In particular, a special case of interest would be that of a perfectly conducting half plane coated on one side with radar absorbing material.

The exact solution for plane-wave scattering by an absorbing half plane with two face impedances is available from Maliuzhinets (1958, 1960), who treated the more general problem of diffraction by a wedge with arbitrary face impedances. For the

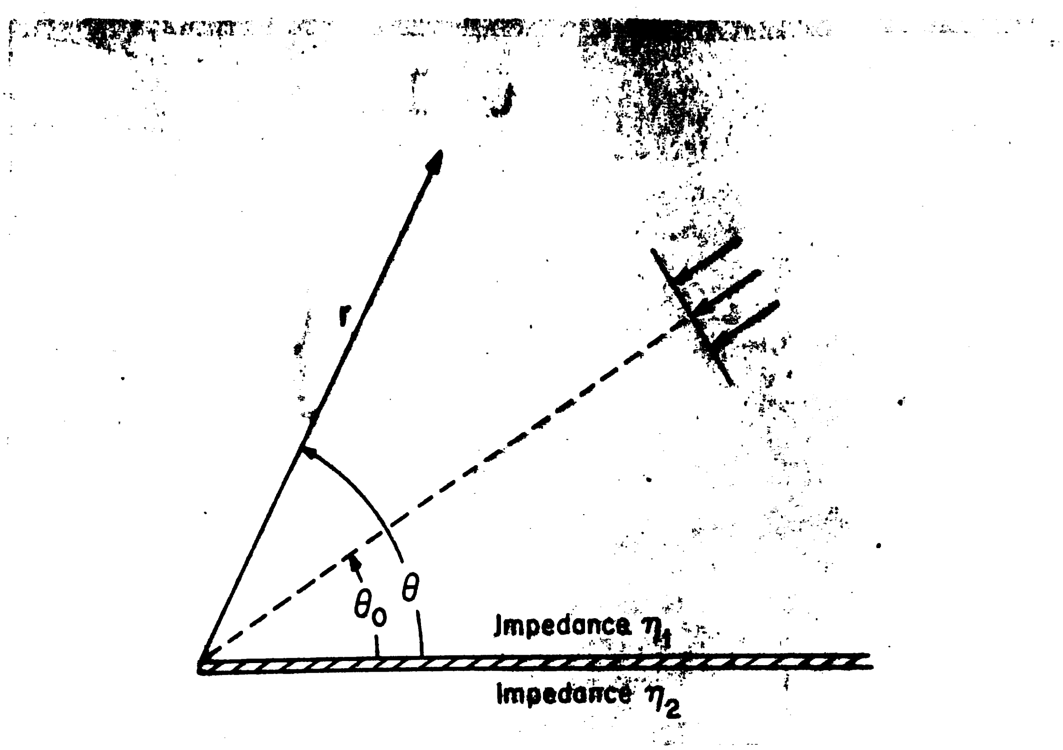

FIG. 1: PLANE WAVE INCIDENCE ON ABSORBING HALF PLANE WITH TWO FACE IMPEDANCES

application of ray-optics techniques, we shall require the asymptotic far-field form of the solution. This result, obtained by means of a steepest descent approximation to the exact contour integral solution, may be written in the form

$$u^d \sim \frac{1}{4\pi i} \sqrt{\frac{2\pi}{kr}} \exp\left[i(kr - \frac{1}{4}\pi)\right] U(\theta, \theta_0) \tag{2}$$

representing a cylindrical wave emanating from the edge of the semi-infinite screen and produced by a plane wave incident at an angle θ_0 to the screen (see Fig. 1, where the geometrical configuration is different from that used by Maliuzhinets). The total exact solution, u, which is a function of two complex quantities α_1 and α_2, is derived by imposing boundary conditions of the third kind,

$$\frac{1}{ikr} \frac{\partial u}{\partial \theta} \pm u \cos \alpha_{1,2} = 0 \qquad (\theta = 0, 2\pi) \,, \tag{3}$$

on the faces of the semi-infinite screen. The quantities α_1 and α_2 are constants whose real parts lie in the closed interval $[0, \theta/2]$. For observation angles θ bounded away from the geometric optics boundaries (i.e. for $\theta \neq \pi \pm \theta_0$), the amplitude factor $U(\theta, \theta_0)$ appearing in (2) is given by

$$U(\theta, \theta_0) = \frac{\sin(\theta_0/2)}{\psi(\pi-\theta_0)} \left\{ \frac{\psi(-\theta)}{\sin(\theta/2)+\cos(\theta_0/2)} + \frac{\psi(2\pi-\theta)}{\sin(\theta/2)-\cos(\theta_0/2)} \right\}, \quad (4)$$

where the auxiliary function $\psi(\beta)$ is expressed in terms of a special meromorphic function $\psi_\pi(\beta)$ by the product

$$\psi(\beta) = \psi_\pi(\beta+\pi+\alpha_1)\psi_\pi(\beta+\pi-\alpha_1)\psi_\pi(\beta-\pi-\alpha_2)\psi_\pi(\beta-\pi+\alpha_2) \quad (5)$$

and $\psi_\pi(\beta)$ has the representation

$$\psi_\pi(\beta) = \exp\left\{ -\frac{1}{8\pi} \int_0^\beta \frac{\pi \sin\nu - 2\sqrt{2}\pi \sin(\nu/2) + 2\nu}{\cos\nu} d\nu \right\}. \quad (6)$$

Maliuzhinets (1960) mentions that the special function $\psi_\pi(\beta)$, along with its generalization for the wedge problem, has been tabulated by M.P.Sacharowa, although no reference to the literature is given. The important analytical properties of the functions $\psi_\pi(\beta)$ and $\psi(\beta)$ are given in Appendix A.

An examination of the boundary conditions (1) and (3) indicates that the scalar function u may be employed to represent the half-plane solution for either of the two fundamental electromagnetic polarizations. Thus, in the case of H polarization, the function u represents the z component of the magnetic field H_z , provided we make the identifications

$$\eta_1 \equiv \cos\alpha_1, \qquad \eta_2 \equiv \cos\alpha_2 , \quad (7)$$

while in the case of E polarization, it represents E_z under the identifications

$$\eta_1 \equiv 1/\cos\alpha_1, \qquad \eta_2 \equiv 1/\cos\alpha_2 . \quad (8)$$

This property is a result of the natural duality of Maxwell's equations and the impedance boundary condition (1) under the transformation $E \rightarrow H$, $H \rightarrow -E$, $\eta \rightarrow 1/\eta$.

An alternative representation for $U(\theta, \theta_0)$ may be obtained by successive use of the identity given in (A.10); in particular, after some trigonometric reduction, we obtain the following expression:

$$U(\theta,\theta_0)=\frac{\left[\psi_\pi(\frac{\pi}{2})\right]^8\sin\frac{\theta}{2}\sin\frac{\theta_0}{2}}{\psi(\pi-\theta)\psi(\pi-\theta_0)}\;\frac{\left(\frac{1}{2}-\cos\frac{\alpha_1}{2}\cos\frac{\alpha_2}{2}\right)+\frac{1}{\sqrt{2}}\left(\cos\frac{\theta}{2}+\cos\frac{\theta_0}{2}\right)\left(\cos\frac{\alpha_2}{2}-\cos\frac{\alpha_1}{2}\right)+\cos\frac{\theta}{2}\cos\frac{\theta_0}{2}}{\cos\theta+\cos\theta_0} \qquad (9)$$

which is valid so long as $\theta \neq \pi \pm \theta_0$. The quantity $\psi_\pi(\pi/2)$ is given in (A.9). The expression in Eq. (9) is manifestly symmetrical in the two variables θ, θ_0, thereby confirming the principle of reciprocity: $U(\theta, \theta_0)=U(\theta_0, \theta)$. For $\alpha_1=\alpha_2=(\pi/2)$, corresponding to a perfect conductor with H polarization, we have upon employing (A.10) and (A.15)

$$\psi(\pi-\theta)=\frac{1}{2}\left[\psi_\pi(\frac{\pi}{2})\right]^4\sin\frac{\theta}{2} \quad ,$$

and therefore (9) reduces to

$$U(\theta,\theta_0)=\frac{4\cos(\theta/2)\cos(\theta_0/2)}{\cos\theta+\cos\theta_0} \quad . \qquad (10)$$

On the other hand, for $\alpha_1=\alpha_2 \rightarrow \pm i\infty$, corresponding to a perfect conductor with E polarization, one finds asymptotically

$$\psi(\pi-\theta)\sim\frac{1}{4}\left[\psi_\pi\;(\frac{\pi}{2})\right]^4 e^{|\alpha_1|/2} \quad ,$$

so that (9) now reduces to

$$U(\theta,\theta_0)=-\frac{4\sin(\theta/2)\sin(\theta_0/2)}{\cos\theta+\cos\theta_0} \quad . \qquad (11)$$

These limiting expressions are in accord with known results for a perfectly conducting half plane.

In order to explore the effects of multiple interaction between the two edges forming the mouth of a semi-infinite parallel plane waveguide, we shall also require an asymptotic expression for the half-plane solution along the ray-optics reflection boundary $\theta=\pi-\theta_0$. It may be shown from the Maliuzhinets (1958, 1960) contour integral solution that for $\theta=\pi-\theta_0$ the total field behaves as

$$u \sim e^{ikr\cos 2\theta_0} - \frac{1}{2}\,\frac{\cos\alpha_1-\sin\theta_0}{\cos\alpha_1+\sin\theta_0}\,e^{ikr} + O\left(\frac{1}{\sqrt{kr}}\right) \tag{12}$$

provided $0<\theta_0<\pi$. For incidence from the lower half space $\pi<\theta_0<2\pi$, the same expression obtains except that α_1 is replaced by α_2. The result in (12) is hardly surprising physically and shows that far along the reflection boundary the scattered field is given by the perfectly conducting result multiplied by the infinite flat plane reflection coefficient. However, to apply the ray-optical procedure to the semi-infinite parallel plane waveguide, we need to know the field generated along the reflection boundary by a line source located at a finite distance from the semi-infinite screen. Rather than solve the line source problem for an absorbing half plane, we shall draw upon an analogy (in view of Eq. 12) with the known result for an isotropic line source over a perfectly conducting half plane. Thus, for an incident field given by

$$u^i = H_0^{(1)}(k|\underline{r}-\underline{r}_0|) \sim \sqrt{\frac{2}{\pi k|\underline{r}-\underline{r}_0|}}\; e^{ik|\underline{r}+\underline{r}_0|-i\frac{\pi}{4}} \tag{13}$$

it is physically reasonable to take

$$u^s \sim -\frac{1}{2}\,\frac{\cos\alpha_1-\sin\theta_0}{\cos\alpha_1+\sin\theta_0}\,\sqrt{\frac{2}{\pi k(r+r_0)}}\; e^{ik(r+r_0)-i\frac{\pi}{4}} \tag{14}$$

as the scattered field far along the reflection boundary $\theta=\pi-\theta_0$, where $0<\theta_0<\pi$. For $\pi<\theta_0<2\pi$, replace α_1 by α_2 in (14).

III

THE SEMI-INFINITE PARALLEL PLANE WAVEGUIDE

The semi-infinite waveguide to be considered here consists of two parallel half planes specified by $y = \pm a$, $x > 0$, as shown in Fig. 2, where the polar coordinates r, θ measured from the center of the guide are also illustrated. The upper half plane is referred to as ① and the lower as ② , and the distances from edges ① and ② to the field point (r, θ) are respectively denoted as r_1 and r_2 . The interior surfaces of the waveguide are governed by a constant surface impedance η_2, while the exterior surfaces are governed by an impedance η_1 which in future applications will generally be taken as zero. Finally, a plane wave is assumed incident upon the structure from the direction θ_0, where $(\pi/2) < \theta_0 \leq \pi$.

We shall employ the ray optical procedure to calculate first the field scattered into the far zone by the edges forming the mouth of the waveguide, and then the field generated at the mouth of the guide. The latter field will be required in the future when we consider the case of a rectangular waveguide. In particular, an estimate of the field generated at the mouth will play an important role in obtaining the energy launched into the waveguide modes. For waveguides of finite length - such as air intake ducts of aircraft - a portion of the energy carried by the waveguide modes will be reflected back toward the mouth and reradiated into the far zone, thereby making a contribution to the radar cross section of the duct.

The primary diffraction due to the open-ended parallel plane waveguide is obtained by neglecting the mutual interaction process that takes place between the two straight edges forming the waveguide mouth. In this first approximation, then, the two edges behave as independent semi-infinite screens, each in the absence of the other and each excited by the incident field alone. Noting that the distances r_1, r_2 are approximately $r \mp a \sin\theta$ as $r \to \infty$, and taking into account the phase of the incident plane wave at the two edges, we may at once write the scattered field from the parallel plane waveguide as a superposition of the two primary edge waves:

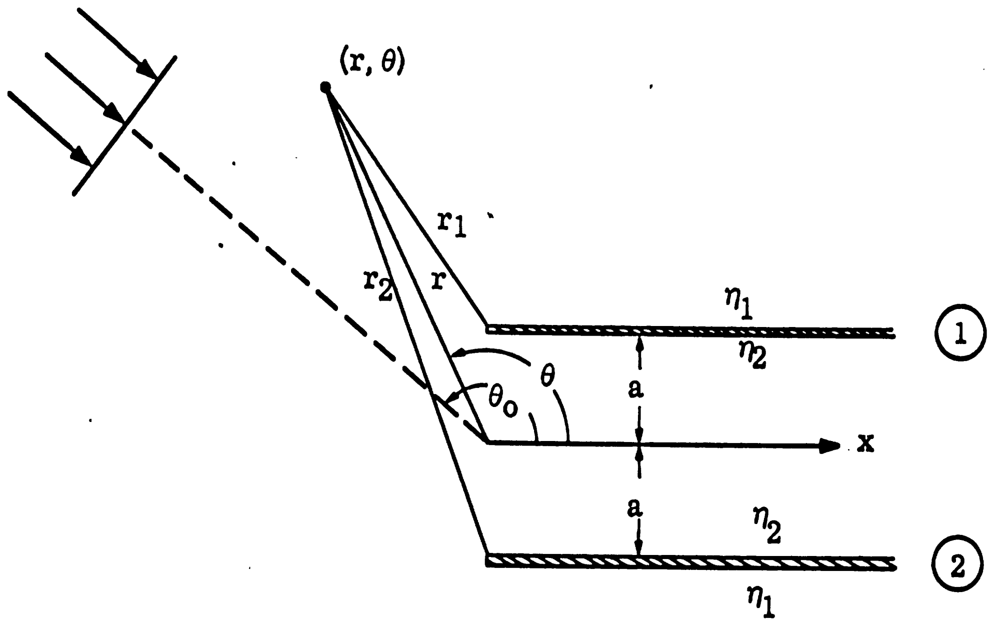

FIG. 2: PLANE WAVE INCIDENT ON A SEMI-INFINITE PARALLEL PLANE WAVEGUIDE WITH TWO SURFACE IMPEDANCES.

$$u^{s} \sim \frac{1}{4\pi i}\sqrt{\frac{2\pi}{kr}}\; e^{i(kr-\frac{\pi}{4})}\left\{ e^{-ika(\sin\theta+\sin\theta_0)}\, U(\theta,\theta_0) + e^{ika(\sin\theta+\sin\theta_0)}\, U(2\pi-\theta, 2\pi-\theta_0) \right\}, \tag{15}$$

where $(\pi/2) < (\theta, \theta_0) \leq \pi$. It may be noted that the function $U(2\pi-\theta, 2\pi-\theta_0)$ properly accounts for the fact that the lower plate ② is illuminated on the η_2 side rather than on the η_1 side as is the case for the upper plate ① . As a special instance of (15), the backscattered field ($\theta=\theta_0$) is

$$u^{BS} \sim \frac{1}{4\pi i}\sqrt{\frac{2\pi}{kr}}\; e^{i(kr-\frac{\pi}{4})}\left\{ e^{-2ika\sin\theta}\, U(\theta,\theta) + e^{2ika\sin\theta}\, U(2\pi-\theta, 2\pi-\theta) \right\}. \tag{16}$$

The immediately preceding approximations are valid so long as $ka \gg 1$, so that the interaction between the edges is weak. However, for more closely spaced waveguides, it may become necessary to include mutual interaction effects in order to improve the accuracy of the ray optical procedure. To calculate the secondary diffraction contribution, for example, each edge is considered to undergo an excitation due to a cylindrical wave emanating from the other edge in addition to the excitation provided by the incident plane-wave field. The cylindrical wave from the edge is assumed to emanate from an equivalent isotropic line source whose strength is chosen, on the basis of Eq. (2), to provide the correct diffraction field in the direction toward the other edge. Knowing how the field of such a line source is diffracted by the half plane representing the other edge, one may write down the first interaction contribution to the far scattered field.

The physical situation becomes somewhat more complicated for the successive interactions past the first. This is due essentially to the fact that each edge lies precisely along the ray optical reflection boundary of the half plane corresponding to the other edge. Consider, for example, an isotropic line source of unit strength located at the position of edge ① in the presence of the half plane ② . The field scattered back toward the line source — along the reflection boundary of plate ② — is then (in view of Eq. 14) given by

$$u^{BS} \sim -\frac{1}{2}\,\frac{\cos\alpha_2-1}{\cos\alpha_2+1}\sqrt{\frac{2}{\pi k(r_2+2a)}}\; e^{ik(r_2+2a)-i\frac{\pi}{4}}\,, \tag{17}$$

and this cylindrical wave appears to emanate from an image line source located at a distance 2a behind the half plane ② . From this example, it may be seen that each successive interaction between the edges of the waveguide gives rise to a new image source and that an infinite number of image sources is required to account for the multiple interactions.

Forsaking further details, we shall merely present our final result for the field scattered into the far zone by the parallel plane waveguide:

$$\begin{aligned} u^{S} \sim \frac{1}{4\pi i}\sqrt{\frac{2\pi}{kr}}\, e^{i(kr-\frac{\pi}{4})} \Bigg\{ & e^{-ika(\sin\theta+\sin\theta_0)}\,U(\theta,\theta_0) + e^{ika(\sin\theta+\sin\theta_0)}\,U(2\pi-\theta,2\pi-\theta_0) - \\ & -\frac{e^{i\frac{\pi}{4}}}{\sqrt{4ka}}\sum_{m=0}^{\infty}\Bigg[\frac{R^{2m}\,e^{i(2m+1)2ka}}{2^{2m+1}\,\sqrt{2m+1}}\, e^{-ika(\sin\theta-\sin\theta_0)}\,U(\theta,\tfrac{3\pi}{2})U(\tfrac{3\pi}{2},2\pi-\theta_0) + \\ & + e^{ika(\sin\theta-\sin\theta_0)}\,U(2\pi-\theta,\tfrac{3\pi}{2})U(\tfrac{3\pi}{2},\theta_0)\Bigg] + \\ & + \frac{e^{i\frac{\pi}{4}}}{\sqrt{4ka}}\sum_{m=1}^{\infty}\frac{R^{2m-1}\,e^{i(2m)2ka}}{2^{2m}\,\sqrt{2m}}\Bigg[e^{-ika(\sin\theta+\sin\theta_0)}\,U(\theta,\tfrac{3\pi}{2})U(\tfrac{3\pi}{2},\theta_0) + \\ & + e^{ika(\sin\theta+\sin\theta_0)}\,U(2\pi-\theta,\tfrac{3\pi}{2})U(\tfrac{3\pi}{2},2\pi-\theta_0)\Bigg] + O\left(\frac{1}{ka}\right)\Bigg\} \end{aligned} \tag{18}$$

where

$$R = \frac{\cos\alpha_2-1}{\cos\alpha_2+1}\,. \tag{19}$$

It may be noted that each successive interaction is reduced in magnitude not only by the numerical factors appearing in the denominators of the summands, but also by the reflection coefficient R which for good absorbers is very close to zero.

By a procedure similar to that above, we have also calculated the field generated at the mouth of the waveguide; in particular, for x = 0, $|y| < a$ the diffracted field is

$$u^d(y) \sim \frac{\sqrt{2\pi}}{4\pi i} e^{-ika\sin\theta_o - i\frac{\pi}{4}} U(\frac{3\pi}{2}, \theta_o) \sum_{m=o}^{\infty} \frac{R^{2m} e^{ik(a-y)+2ika(2m)}}{2^{2m}\sqrt{k(a-y)+2ka(2m)}} -$$

$$-\frac{\sqrt{2\pi}}{4\pi i} e^{ika\sin\theta_o - i\frac{\pi}{4}} U(\frac{3\pi}{2}, 2\pi-\theta_o) \sum_{m=1}^{\infty} \frac{R^{2m-1} e^{ik(a-y)+2ika(2m-1)}}{2^{2m-1}\sqrt{k(a-y)+2ka(2m-1)}} +$$

$$+\frac{\sqrt{2\pi}}{4\pi i} e^{ika\sin\theta_o - i\frac{\pi}{4}} U(\frac{3\pi}{2}, 2\pi-\theta_o) \sum_{m=o}^{\infty} \frac{R^{2m} e^{ik(a+y)+2ika(2m)}}{2^{2m}\sqrt{k(a+y)+2ka(2m)}} -$$

$$-\frac{\sqrt{2\pi}}{4\pi i} e^{-ika\sin\theta_o - i\frac{\pi}{4}} U(\frac{3\pi}{2}, \theta_o) \sum_{m=1}^{\infty} \frac{R^{2m-1} e^{ik(a+y)+2ika(2m-1)}}{2^{2m-1}\sqrt{k(a+y)+2ka(2m-1)}} . \qquad (20)$$

Again, the successive interaction contributions become progressively smaller in magnitude, as expected.

It should be emphasized that the ray-optical procedure we have employed above to calculate the mutual interaction effects is based upon the simplifying assumption that the equivalent line sources are isotropic. In order to modify the ray optical method described here, we would require the solution to the complicated boundary value problem of a non-isotropic line source over an absorbing half plane with arbitrary face impedances. Since the higher interaction terms are still expected to be very small, especially in the case of highly efficient absorbers, the slight improvement in accuracy for these terms is most likely not worth the labor involved.

IV

PERFECTLY CONDUCTING CASE - RAY OPTICS VS EXACT SOLUTION

It would be interesting to compare the ray optical description of scattering by two parallel half planes with the exact solution to the problem; unfortunately, a solution for the case where the parallel plane waveguide is absorbing does not seem to be available. Nevertheless, we can make a comparison in the case where the waveguide is perfectly conducting since in this case exact solutions, based on the Wiener-Hopf technique, are widely known in the literature. Ray optical techniques and their relation to canonical problems with parallel plane geometries involving perfect conductors have been discussed by Yee and Felsen(1967a, b), by Felsen and Yee (1968a, b) and by Yee, Felsen and Keller (1968). Their work has been centered on the study of reflection and radiation of waveguide modes incident on the open end of a waveguide, and they have reported that the ray optical method yields remarkably accurate results even at small ka values. In this section we shall discuss briefly the results that are obtained in the plane-wave scattering case. No numerical calculations have been carried out, so we shall confine our comments to analytical results.

We consider an E-polarized plane wave incident from direction θ_o on a pair of perfectly conducting parallel half planes. By combining the results of Vajnshtejn (1948) and Clemmow (1951), we can write the far field as obtained from the exact solution in the following manner:

$$E^s \sim -\frac{1}{4\pi i}\sqrt{\frac{2\pi}{kr}}\, e^{i(kr-\frac{\pi}{4})}\, \frac{2\sin(\theta/2)\sin(\theta_o/2)}{\cos\theta+\cos\theta_o}\Bigg\{\left(1+e^{2ika\sin\theta}\right)\left(1+e^{2ika\sin\theta_o}\right) e^{V+V_o} + $$

$$+\left(1-e^{2ika\sin\theta}\right)\left(1-e^{2ika\sin\theta_o}\right) e^{U+U_o}\Bigg\} e^{-ika(\sin\theta+\sin\theta_o)} , \qquad (21)$$

where we have assumed $\cos\theta < 0$, $\cos\theta_0 < 0$, and where

$$V = \frac{1}{2\pi i}\int_{\frac{\pi}{2}-i\infty}^{-\frac{\pi}{2}+i\infty} \log\left(1+e^{2ika\cos\tau}\right)\frac{\cos\tau\, d\tau}{\sin\tau-\cos\theta},$$

$$U = \frac{1}{2\pi i}\int_{\frac{\pi}{2}-i\infty}^{-\frac{\pi}{2}+i\infty} \log\left(1-e^{2ika\cos\tau}\right)\frac{\cos\tau\, d\tau}{\sin\tau-\cos\theta} \, . \tag{22}$$

The quantities V_0, U_0 are obtained from (22) upon replacing θ by θ_0. Now for $ka \gg 1$, a steepest descent approximation yields (Vajnshtejn, 1948):

$$V \sim \frac{1}{2\pi i}\int_{-\infty}^{\infty} \log\left(1+e^{2ika-\frac{t^2}{2}}\right)\frac{dt}{t-\sqrt{2ka}\, e^{i\pi/4}\cos\theta} \, . \tag{23}$$

We expand the logarithm under the assumption that k has a vanishingly small positive imaginary part,

$$\log\left(1+e^{2ika-\frac{t^2}{2}}\right) = -\sum_{m=1}^{\infty}\frac{(-)^m e^{2imka}}{m}\, e^{-\frac{mt^2}{2}},$$

and integrate term by term to obtain

$$V \sim -\frac{1}{2}\,\mathrm{sgn}(\cos\theta)\sum_{m=1}^{\infty}\frac{(-)^m}{m}\, e^{2imka}\, G(\sqrt{m}\, w)\, , \tag{24}$$

where $w = \sqrt{ka/2}\, |\cos\theta|$ and

$$G(\xi) = \frac{2}{\sqrt{\pi}}\, e^{-i2\xi^2}\int_{(1-i)\xi}^{\infty} e^{-\mu^2}\, d\mu \, . \tag{25}$$

For values of ka and θ such that $\sqrt{ka/2}\,|\cos\theta| \gg 1$, we have

$$G(w) \sim \frac{e^{i\frac{\pi}{4}}}{\sqrt{2\pi}\,w} \quad , \tag{26}$$

and consequently

$$e^V \sim 1 - \frac{e^{i\frac{\pi}{4}}}{\sqrt{4\pi ka}\cos\theta} \sum_{m=1}^{\infty} \frac{(-)^m}{m^{3/2}} e^{2imka} + O\left(\frac{1}{ka}\right) . \tag{27}$$

Similarly,

$$e^U \sim 1 - \frac{e^{i\frac{\pi}{4}}}{\sqrt{4\pi ka}\cos\theta} \sum_{m=1}^{\infty} \frac{1}{m^{3/2}} e^{2imka} + O\left(\frac{1}{ka}\right) . \tag{28}$$

When (27) and (28) are employed in (21), we obtain

$$E^s \sim \sqrt{\frac{2}{\pi kr}}\, e^{ikr+i\frac{\pi}{4}} \frac{2\sin(\theta/2)\sin(\theta_o/2)}{\cos\theta+\cos\theta_o} \Big\{ \cos\left[ka(\sin\theta+\sin\theta_o)\right] + $$
$$+ \frac{e^{i\frac{\pi}{4}}}{\sqrt{4\pi ka}} \left(\frac{1}{\cos\theta} + \frac{1}{\cos\theta_o}\right) \sum_{m=0}^{\infty} \frac{e^{i(2m+1)2ka}}{(2m+1)^{3/2}} \cos\left[ka(\sin\theta-\sin\theta_o)\right] -$$
$$- \frac{e^{i\frac{\pi}{4}}}{\sqrt{4\pi ka}} \left(\frac{1}{\cos\theta} + \frac{1}{\cos\theta_o}\right) \sum_{m=1}^{\infty} \frac{e^{i(2m)2ka}}{(2m)^{3/2}} \cos\left[ka(\sin\theta+\sin\theta_o)\right] + O\left(\frac{1}{ka}\right) \Big\} . \tag{29}$$

On the other hand, the ray optical result in (18) reduces to

$$E^s \sim \sqrt{\frac{2}{\pi kr}}\, e^{ikr+i\frac{\pi}{4}} \frac{2\sin(\theta/2)\sin(\theta_o/2)}{\cos\theta+\cos\theta_o} \Big\{ \cos\left[ka(\sin\theta+\sin\theta_o)\right] + $$
$$+ \frac{e^{i\frac{\pi}{4}}}{\sqrt{4\pi ka}} \left(\frac{1}{\cos\theta} + \frac{1}{\cos\theta_o}\right) \sum_{m=0}^{\infty} \frac{e^{i(2m+1)2ka}}{2^{2m}\sqrt{2m+1}} \cos\left[ka(\sin\theta-\sin\theta_o)\right] -$$
$$- \frac{e^{i\frac{\pi}{4}}}{\sqrt{4\pi ka}} \left(\frac{1}{\cos\theta} + \frac{1}{\cos\theta_o}\right) \sum_{m=1}^{\infty} \frac{e^{i(2m)2ka}}{2^{2m-1}\sqrt{2m}} \cos\left[ka(\sin\theta+\sin\theta_o)\right] + O\left(\frac{1}{ka}\right) \Big\} . \tag{30}$$

The two results in (29) and (30) differ because of the numerical factors in the denominators of the summands. It is seen, however, that the primary diffraction, along with the first and second interaction contributions, are in complete agreement. For each successive interaction after the second, the ray optical result (30) underestimates the asymptotic result in (29) obtained from the exact solution. In order to obtain more accurate results the simple ray optical method employed above would have to be modified.

APPENDIX A

PROPERTIES OF THE FUNCTIONS $\psi_\pi(\beta)$ AND $\psi(\beta)$

The meromorphic function $\psi_\pi(\beta)$ is defined in Eq. (6) as

$$\psi_\pi(\beta) = \exp\left\{-\frac{1}{8\pi}\int_0^\beta \frac{\pi\sin v - 2\sqrt{2}\,\pi\sin(v/2) + 2v}{\cos v}dv\right\}, \tag{A.1}$$

from which it will be observed that $\psi_\pi(\beta)$ is an even function of β whose logarithmic derivative is given by

$$\frac{\psi_\pi'(\beta)}{\psi_\pi(\beta)} = -\frac{1}{8}\frac{\sin\beta}{\cos\beta} + \frac{\sqrt{2}}{4}\frac{\sin(\beta/2)}{\cos\beta} - \frac{1}{4\pi}\frac{\beta}{\cos\beta}. \tag{A.2}$$

By means of the elementary integrals

$$\int_0^\beta \frac{\sin v}{\cos v}dv = -\ln(\cos\beta),$$

$$\sqrt{2}\int_0^\beta \frac{\sin(v/2)}{\cos v}dv = -\ln\left[\frac{\sqrt{2}\cos(\beta/2) - 1}{\sqrt{2}\cos(\beta/2) + 1}\cdot\frac{\sqrt{2}+1}{\sqrt{2}-1}\right],$$

we obtain the following alternative representations for $\psi_\pi(\beta)$:

$$\psi_\pi(\beta) = \left[\frac{\sqrt{2}\cos(\beta/2) + 1}{\sqrt{2}+1}\right]^{\frac{1}{2}}\frac{1}{(\cos\beta)^{1/8}}\exp\left\{-\frac{1}{4\pi}\int_0^\beta \frac{v\,dv}{\cos v}\right\}, \tag{A.3}$$

$$\psi_\pi(\beta) = \left[\frac{\sqrt{2}\cos(\beta/2) + 1}{\sqrt{2}+1}\right]^{\frac{1}{2}}\exp\left\{\frac{1}{8\pi}\int_0^\beta \frac{\pi\sin v - 2v}{\cos v}dv\right\}. \tag{A.4}$$

When $|\beta| < (\pi/2)$, the integral in (A.3) can be expanded as

$$\int_0^\beta \frac{v\,dv}{\cos v} = \frac{\beta^2}{2} + \frac{1}{2}\frac{\beta^4}{4} + \frac{5}{24}\frac{\beta^6}{6} + \frac{61}{720}\frac{\beta^8}{8} + \ldots + \frac{(-)^n E_{2n}}{(2n)!}\frac{\beta^{2n+2}}{2n+2} + \ldots, \tag{A.5}$$

where E_{2n} are the Euler numbers. When $\beta = i\infty$, we have (Gröbner and Hofreiter 1949)

$$\frac{1}{\pi}\int_0^{i\infty}\frac{v\,dv}{\cos v} = -\frac{1}{\pi}\int_0^\infty \frac{x\,dx}{\cosh x} = -\frac{2}{\pi}\sum_{n=0}^\infty \frac{(-1)^n}{(2n+1)^2} = -b, \tag{A.6}$$

where b = $(2/\pi)K$ with K = 0.9159656... (Catalan's constant). On the other hand, when $\beta = (\pi/2)$, we employ (A. 4) with a change of integration variable $v = (\pi/2) - u$ to find

$$\psi_\pi(\pi/2) = \left[\frac{2}{\sqrt{2}+1}\right]^{\frac{1}{2}} \exp\left\{\frac{1}{8}\int_0^{\pi/2} \frac{\cos u - 1}{\sin u}\, du + \frac{1}{4\pi}\int_0^{\pi/2} \frac{u\, du}{\sin u}\right\}. \tag{A. 7}$$

The first integral is elementary and the second integral is (Gröbner and Hofreiter 1949)

$$\frac{1}{\pi}\int_0^{\pi/2} \frac{u\, du}{\sin u} = \frac{2}{\pi}\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2} = b. \tag{A. 8}$$

We obtain then

$$\psi_\pi(\pi/2) = \left[\frac{2^{\frac{1}{4}}}{\sqrt{2}+1}\, e^{b/2}\right]^{\frac{1}{2}}. \tag{A. 9}$$

It is easy to verify the following fundamental identity (Maliuzhinets 1958)

$$\psi_\pi(\beta + \tfrac{1}{2}\pi)\psi_\pi(\beta - \tfrac{1}{2}\pi) = [\psi_\pi(\pi/2)]^2 \cos(\beta/4), \tag{A. 10}$$

and by successive application of (A. 10) one obtains

$$\psi_\pi(\beta + \pi)\psi_\pi(\beta - \pi) = \frac{1}{2}\frac{[\psi_\pi(\pi/2)]^4}{[\psi_\pi(\beta)]^2}[\cos(\beta/2) + \cos(\pi/4)], \tag{A. 11}$$

$$\psi_\pi\left(\beta + \frac{3\pi}{2}\right)\psi_\pi\left(\beta - \frac{3\pi}{2}\right) = \tfrac{1}{2}[\psi_\pi(\pi/2)]^2 \frac{\cos(\beta/2)}{\cos(\beta/4)}. \tag{A. 12}$$

From this last equation we observe that the zeros of $\psi_\pi(\beta)$ which are closest to the point $\beta = 0$ and the corresponding poles are the points $\beta = \pm(5\pi/2)$ and $\beta = \pm(7\pi/2)$, respectively. From Eq. (A. 10) one also derives

$$\frac{\psi_\pi(\beta + \pi)}{\psi_\pi(\beta - \pi)} = \frac{\cos(\frac{1}{4}\beta + \frac{1}{8}\pi)}{\cos(\frac{1}{4}\beta - \frac{1}{8}\pi)}, \tag{A. 13}$$

$$\frac{\psi_\pi(\beta + 2\pi)}{\psi_\pi(\beta - 2\pi)} = \cot(\tfrac{1}{2}\beta + \tfrac{1}{4}\pi). \tag{A. 14}$$

The function $\psi(\beta)$ is expressed in terms of the function $\psi_\pi(\beta)$ by the product in Eq. (5):

$$\psi(\beta) = \psi_\pi(\beta + \pi + \alpha_1)\psi_\pi(\beta + \pi - \alpha_1)\psi_\pi(\beta - \pi - \alpha_2)\psi_\pi(\beta - \pi + \alpha_2), \tag{A. 15}$$

from which, by means of (A. 14), we derive

$$\frac{\psi(\pi+\beta)}{\psi(\pi-\beta)} = \frac{\psi_\pi(\beta+\alpha_1+2\pi)\psi_\pi(\beta-\alpha_1+2\pi)}{\psi_\pi(\beta+\alpha_1-2\pi)\psi_\pi(\beta-\alpha_1-2\pi)}$$

$$= \cot(\tfrac{1}{2}\beta+\tfrac{1}{2}\alpha_1+\tfrac{1}{4}\pi)\cot(\tfrac{1}{2}\beta-\tfrac{1}{2}\alpha_1+\tfrac{1}{4}\pi) = \frac{\cos\alpha_1-\sin\beta}{\cos\alpha_1+\sin\beta}, \tag{A.16}$$

and similarly

$$\frac{\psi(-\pi-\beta)}{\psi(-\pi+\beta)} = \frac{\cos\alpha_2-\sin\beta}{\cos\alpha_2+\sin\beta}. \tag{A.17}$$

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13. ABSTRACT

As a preliminary to an investigation concerning radar scattering by a rectangular duct lined with absorbing materials, the simplified problem of plane-wave scattering by a semi-infinite parallel plane waveguide is considered. The surfaces of the guide are assumed to obey impedance boundary conditions, where the impedance on the interior surface may be different from the impedance on the exterior surface.. A case of particular interest would be that in which the exterior surface is perfectly conducting. Ray-optical techniques based upon known results for a semi-infinite screen with two face impedances are employed to calculate both the field scattered into the far zone and the field generated at the mouth of the guide.. The ray-optical procedure is also applied to the case of a perfectly conducting, open-ended parallel plane waveguide for which an exact solution is available, and the ray optical result is compared with the asymptotic expansion of the exact solution. Also briefly discussed are the modes sustained in a circular waveguide whose wall obeys an impedance boundary condition.

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