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THE UNIVERSITY OF MICHIGAN

COLLEGE OF ENGINEERING

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

Radiation Laboratory

ELECTRODYNAMICS OF MOVING MEDIA

Final Report (15 October 1970 - 14 April 1972)

By

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THE UNIVERSITY OF MICHIGAN

FINAL REPORT

Summary

From the period 15 October 1970 to 15 April 1972 most of the research under NSF Grant GK-24102 was devoted to the following problems.

1) The radiation condition for fields in a moving medium,
2) Theory for boundary value problems involving a medium translating parallel to its surface,
3) Radiation from sources in the presence of a moving dielectric column.

Solutions to problems involving radiation from sources in moving media have been discussed by many authors. Such solutions require that the behavior of the fields at large distances be known (the radiation condition). This condition has been assumed in previous work but never explicitly stated. We have determined the mathematical form for this condition.

The general theory for boundary value problems involving a medium moving with a velocity which is parallel to its surface has been developed using the appropriate dyadic Green's function. In particular, symmetry conditions which may be used to find various reciprocity theorems are derived. The general theory for this class of problem has been applied to the particular case of a moving dielectric column for various types of sources.

Discussion of Research

The approach used in these problems is based on the covariance of Maxwell's equations as postulated by the Special Theory of Relativity. These problems were solved by using an auxiliary field as defined by

$$\overline{F} = e^{-i\omega\Omega z}\,\overline{\overline{b}}\cdot\overline{f}$$

where $\overline{F}$ represents either the $\overline{E}$ or $\overline{H}$ field and $\overline{f}$ the corresponding auxiliary field, Ω is a constant proportional to the velocity of the medium and $\bar{\bar{b}}$ is a diagonalized dyad which is a function of the refractive index of the medium and its velocity. The Maxwell-Minkowski equations for the auxiliary field take on a particularly simple form which is very similar to Maxwell's equations for a stationary medium. A dyadic Green's function pertaining to the particular problem is used in the solution of the specific problems.

Problem (1).

The solution for the fields in terms of the sources involves in general both a volume and surface integral involving the dyadic Green's function. We have investigated the condition which is necessary for the surface integral to vanish in the case of an unbounded medium. This leads to a radiation condition in the moving medium which takes the form

$$\lim_{R \to \infty} R \left\{ \bar{\bar{b}} \cdot \nabla \times \left[e^{i\omega\Omega z}\, \overline{E} \right] - ik \left(\frac{f}{a^{1/2}}\, \hat{\phi}\hat{\phi} + \frac{a^{1/2}}{f}\, \hat{\theta}\hat{\theta} \right) \cdot \hat{R} \times \overline{E} \right\}$$

where

$$f = \left[1 + (a-1)\cos^2\theta \right]^{1/2}$$

$$a = \frac{1-\beta^2}{1-n^2\beta^2} \quad .$$

This reduces to the familiar Sommerfeld condition in the zero velocity limit.

Problem (2).

The general solution for problems involving boundaries between stationary and moving dielectric media has been developed in terms of the dyadic Green's function. We have considered problems where the locus of the boundary remains constant with time, i.e., the medium is translating parallel to its boundary.

The properties of the dyadic Green's function for this class of problems have been considered. We have derived the Rayleigh-Carson reciprocity theorem for source and observation points which are located in the same or different media. It is found that the direction of the velocity must be reversed even when both source and observation points are located in the stationary medium. The general form for the Rayleigh-Carson theorem is:

$$\iiint dV_1\, \bar{J}_1(\bar{R}_1) \cdot \bar{E}_2(\bar{R}_1, \bar{v}) = \iiint d\,V_2\, \bar{J}_2(\bar{R}_2) \cdot \bar{E}_1(\bar{R}_2, -\bar{v})$$

Problem (3).

The theory developed in problem (2) was applied to the specific case of a dielectric column moving axially. The dyadic Green's functions for this geometry were found and used to determine expressions for the fields for the following cases.

a) Plane wave incident - $\hat{\theta}$ and $\hat{\phi}$ polarizations,

b) Far field patterns for dipoles oriented axially, radially and azimuthally,

c) Far field pattern for a constant current loop antenna coaxial with the column.

Extension of Time Period

Grant GK 24102 was planned to run for a period of approximately one year ending 15 October 1971. The completion date was extended to 15 April 1972 in accordance with the six-month grace period described in the NSF Brochure on Grants. We are not planning to request a renewal of this research at the present moment. However, we hope to request an extension of an NSF Grant on Transient Radiation (GK 22898) in the near future. Some of the new research problems relating to both moving media and transient radiation will be in that proposal.

Publications Under Grant GK 24102

Stubenrauch, C.F. and C-T Tai., "Dyadic Green's Functions for Cylindrical Waveguides with Moving Media," Applied Scientific Research, Vol. 25, pp. 281-289, December, 1971.

Stubenrauch, C.F., "Radiation from Sources in the Presence of a Moving Dielectric Column", Ph.D. Thesis, The University of Michigan, 1972.

Tai, C-T., and C.F. Stubenrauch, "The Radiation Condition and the Reciprocity Theorem for Fields in Moving Media," presented at USNC/URSI Spring Meeting held April 13-15, 1972, Washington, D.C.

Personnel

NSF Grant 24102 was awarded to The University of Michigan and administered in the Radiation Laboratory, which is under the direction of Professor Ralph E. Hiatt. The Project Director of the Grant has been Professor Chen-To Tai, who was assisted by Mr. Carl F. Stubenrauch, a graduate student in the Department of Electrical and Computer Engineering of the University of Michigan. Mr. Stubenrauch has worked under this Grant during the last two years and has recently completed his Ph.D. thesis. Other students and supporting staff participated from time to time.