STUDIES IN RADAR CROSS SECTIONS XXVIII

THE PHYSICS OF RADIO COMMUNICATION VIA THE MOON

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PREFACE

This paper is the twenty-eighth in a series growing out of studies of radar cross sections at the Engineering Research Institute of The University of Michigan. The primary aims of this program are:

1. To show that radar cross sections can be determined analytically.

2. A. To obtain means for computing the radiation patterns from antennas by approximate techniques which determine the pattern to the accuracy required in military problems but which do not require the exact solutions.

   B. To obtain means for computing the radar cross sections of various objects of military interest.

(Since 2A and 2B are interrelated by the reciprocity theorem it is necessary to solve only one of these problems.)

3. To demonstrate that these theoretical cross sections and theoretically determined radiation patterns are in agreement with experimentally determined ones.

Intermediate objectives are:

1. A. To compute the exact theoretical cross sections of various simple bodies by solution of the appropriate boundary-value problems arising from electromagnetic theory.

   B. Compute the exact radiation patterns from infinitesimal sources on the surfaces of simple shapes by the solution of appropriate boundary-value problems arising from electromagnetic theory.

(Since 1A and 1B are interrelated by the reciprocity theorem it is necessary to solve only one of these problems.)

2. To examine the various approximations possible in this problem and to determine the limits of their validity and utility.

3. To find means of combining the simple-body solutions in order to determine the cross sections of composite bodies.

4. To tabulate various formulas and functions necessary to enable such computations to be done quickly for arbitrary objects.

5. To collect, summarize, and evaluate existing experimental data.

K. M. Siegel

SECTION I

INTRODUCTION

In order to determine the optimum frequency range for communication by reflection from the moon, a thorough understanding has to be obtained of the effect on transmission of the media between the earth and the moon, as well as the effect on reflection of the surface characteristics of the moon. A frequency has to be chosen which will be high enough to get through the ionosphere without significant attenuation or scattering. A thorough analysis is devoted to meteor scattering, to establish the extent of its effect upon a moon communication system. Even though point to point communication may require that the points be far enough separated that the transmitter's main beam cannot be considered coincident with the receiver's main beam (since we limit this discussion to the surface of the earth and do not investigate communication with vehicles in outer space), only a small angular difference exists between the axes of symmetry of these main beams. Nevertheless, since it is not much harder to obtain the bistatic answer than the monostatic answer, we have determined the Faraday effect, i.e., the effect of the rotation of the plane of polarization during propagation through the ionosphere, as a function of the locations of transmitter and receiver. Practically speaking, for purposes of communication via the moon it seems wiser to use circular polarization or to choose a high enough frequency so that this physical effect is not measurably significant.

A great mystery to all who have studied the moon is the nature of the surface of the moon. A major requirement on any antenna system is to understand the properties of the antennas themselves. The moon in this case acts as a parasitic antenna, and it is very important to understand the effect of its surface on reflection of electromagnetic waves before one can design optimally a communication system which has this type of reflection inherent in its operation.

In Section II, experimental and theoretical support is given for the hypothesis that at radar frequencies the moon can be considered to be a smooth reflector. The shape of the echo pulses under this hypothesis is found not to contradict the available experimental evidence.

By routine application of the usual antenna far-zone criterion, the earth would be considered to be in the near zone of the moon for all frequencies which would go through the ionosphere. However, for the special case (not previously specifically analyzed) in which the dimensions of the scatterer are extremely large compared with the wavelength, while still very small compared with the ranges to the transmitter and receiver, a more refined criterion is possible. In Section III, this analysis is carried out and it is demonstrated that far zone information is available for at least part of the pulse.

In Section IV, we determine the dielectric constant and conductivity of the surface of the moon, by using the physical picture given in Section II together with the experimental results of several experimenters (Yaplee, Trexler, Leadabrand, Blevis, Evans), which cover a wide range of frequencies.

The effect of the Faraday rotation on transmission with linear polarization is derived in Section V.

The theory of meteor scattering by under-dense trails is developed in Section VI, with emphasis on its complex character at the higher frequencies; for the latter, the results differ substantially from previous less sophisticated analyses. While the over-dense trails (which are not treated) produce a stronger and longer lasting return than the under-dense trails, their occurrence is sufficiently rare not to affect significantly communication considerations. This Section will of course be of interest to people concerned with meteor scatter communication, and serves to delimit meteor considerations for moon communication.

In some of the experimental data, double fades have been observed. That is, if one transmits at one polarization and receives at two orthogonal polarizations, a fade sometimes occurs simultaneously in both receiving antennas. A theory for this type of return has been formulated, but we have left it out of this report because enough time has not elapsed to show that the double fade phenomenon is truly reproducible.

In Section VII, we discuss the factors which determine a suitable choice of frequency and antenna size, and point out that at high frequencies the phase shift as a function of modulation spectrum can be ignored. The optimum pulse width for pulse communication is also considered.

In Section VIII we present some conclusions of this investigation, and recommendations concerning communication via the moon.

SECTION II

REFLECTIONS FROM THE MOON

The moon has been characterized by astronomers as a "rough" (meaning non-uniformly rough) surface, and this has led many scientists to assume that the laws of scattering from a rough surface are applicable to the moon at all frequencies - radar as well as optical. Because of the assumed roughness, some experimentalists have rejected the idea of using the moon as a means of communication between points on the earth.

It is our opinion that the moon is not rough in the radar sense of the word, and that a more accurate description is "quasi-smooth". The meaning to be attached to this terminology will become clear later.

One of the reasons for abandoning the roughness concept is provided by the experimental results obtained by Blevis (Ref. 1). With circular polarization, at 488 Mc., Blevis compared the signals scattered from the moon using two orthogonally polarized receivers and found that a maximum signal in one receiver was generally accompanied by a minimum in the other. The existence of maxima and minima is no surprise, and is quite in accord with both rough and smooth scattering theories. Some of the minima, however, are below the noise level, and this fact strongly suggests a smooth scattering surface.

To see this, let us consider the radar return from a corner reflector whose sides are very large in respect to a wavelength and upon which there are protuberances whose magnitudes are of order $\lambda/2$. When orthogonally polarized receivers are used it is found that the minimum signals always contain at least thirty per cent of the total received energy. On the other hand, when a smooth corner reflector is rotated and one of the receivers has the transmitted polarization, the energy contained in a signal at minimum is between 0 and 1 per cent. Which receiver has the minimum depends upon the aspect angle; if the transmitted signal is circularly polarized, the dominant return is in the receiver with the opposite (same) polarization when the aspect is such as to demand an odd (even) number of reflections. The reason for 99 per cent instead of 100 per cent is because of edge effects. Thus the fact that the minima in the moon's return can be below the noise level shows that the moon has a reflection property in common with a smooth body.

In view of the preceding results, it is evident that the deep nulls are best explained by assuming only even or only odd numbers of bounces in the predominant return. Since there must obviously be a large single bounce return it follows that it is the odd number of bounces that are contributing. Since the contributions from double bounces are negligible, and since the above-mentioned results hold for a wide range of aspects, it is probable that single bounces provide the bulk of the received energy. Such a statement does not rule out the existence of double bounce reflections, but it does imply that the corresponding energy is not usually returned to a receiver on the earth.

All this is not in disagreement with what the astronomers have determined. By visual observation it is known that the moon is liberally strewn with mountains and craters, and in this sense the moon is rough. With this definition of roughness, a 3-sided corner reflector one of whose sides was measured in millions of wavelengths, while the other two sides were only thousands of wavelengths in dimension, would be considered a rough surface, since the smaller sides could be likened to the mountains on the moon. From the radar point of view, however, the surface would behave as a smooth one.

Optical measurements have shown that the light reflectivity of the moon can be accounted for by the assumption that the moon is a rough surface (rough in the sense of diffusely reflecting) and a directivity factor of 5.7 has been ascribed to the moon. But a roughness at optical wavelengths implies only the existence of irregularities whose dimensions are measured in microns, and obviously a fine sandy surface could provide an appropriate type of roughness. At radar frequencies, however, this surface may well behave as a smooth reflector. In short, we admit the fact that the moon has irregularities whose dimensions are of the order of many microns; we also admit the presence of the large natural features such as mountains and craters. But these facts do not preclude the behavior of the moon as a substantially smooth reflector at radar frequencies. Obviously there will be some return from the mountains and craters at these frequencies superimposed on the smooth sphere return and, in general, rather small compared to it, and it is for this reason that we have used the word "quasi-smooth" (rather than smooth) to describe the moon's surface for wavelengths in the meter and centimeter bands.

By using the above theory it is possible to predict the form of a radar pulse returned from the moon's surface. The energy due to single reflections would come primarily from the specular point and from the mountains or craters. If the mountains are near to the center of the moon, they would be the first features of the moon to give rise to a scattered signal and their contributions would represent the initial portion of the returned pulse. If the mountains (or surface indentations) are on the limbs of the moon, their contributions would appear as fluctuations superimposed on that portion of the pulse representing the specular return. In addition, they would give rise to a tail to the pulse, the relative length of the tail depending upon the pulse width. These fluctuations may also be a function of aspect, and hence of time. If the surface of the moon has large areas with comparatively small curvature (which areas could be likened to parts of an oblate spheroid), the specular region of reflection could move, relative to a fixed point on the surface, as the aspect changes. The phase relations between the individual contributions to the returned pulse would then be a function of time, leading to continuous changes in the fluctuations.

Still other types of fluctuations can be expected to appear. Both the Canadians and the British have observed a fluctuation rate of 9 cycles per second in their moon data at 120 Mc. and 488 Mc. which has been ascribed to the libration of the moon. The Faraday effect in the ionosphere will also produce fluctuations in the received signal, and fades occuring about every two minutes at 488 Mc. have been attributed to this effect. To prove conclusively that the fades had this origin it would be necessary to measure the whole scattering matrix, rather than merely the signals at two orthogonally polarized receivers from one transmitted signal. And finally there are the fluctuations due to the diurnal variations in ionization and to the presence of stray clouds of high ionization density.

Although these fluctuations are important in confirming our views concerning the medium between the earth and the moon, they do not represent all the information which can be obtained from the moon data. Even if we exclude such astronomical considerations as better determinations of the distance from the earth to the moon, additional information is available if it is realized that, for most of the time, the fluctuations are superimposed on the specular return. Consequently by selecting the high-pulse returns and averaging out the fluctuations, it is possible to obtain a measure of the specular return. The theory outlined in Section IV can then be used to determine the permittivity, permeability and conductivity of the relevant portion of the moon's surface.

A knowledge of the specular return at two different frequencies is theoretically sufficient to specify the constitutive parameters of the moon's surface, on the assumption that they are frequency independent in the frequency range considered, and by using the results at more than two frequencies it is possible to test the consistency of the theory.

An objection to the above work might be based upon a feature of the data obtained using very short pulses. Thus, Trexler found that with a 12 microsecond pulse only fifty per cent of the received energy was returned in the first 50 microseconds (Ref. 2). Still shorter pulses of 5 microseconds were used by the Royal Radar Establishment (Ref. 3), and a typical return obtained by them is reproduced in Figure 2-1. In spite of

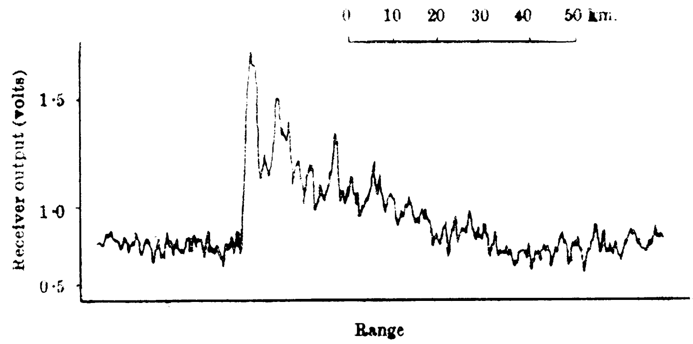

FIG. 2-1: AMPLITUDE OF THE MOON ECHO (Ref. 3)

the appreciable lengthening of the received pulse, we do not feel that this violates our theory. The theory merely states that, in the strong portion of the return, the signal is due to specular reflection upon which fluctuations are superimposed: and the time at which the strong portion is received is, in general, consistent with specular reflection. But the theory does not assume that energy cannot be reflected from points other than the specular point; if, for example, there are mountains further back, they will contribute to the received signal and their contribution will appear at some time after the specular signal has been received. As an extreme example of this behavior, consider a spherically-tipped cone having a disc at its rear extending beyond the cone's surface. When viewed at, or near, head on, the initial return would be from a specular reflection at the spherical tip and by concentrating upon this portion of the return it would be possible to determine the electromagnetic properties of the sphere by using data obtained at several frequencies. This becomes even easier if the radius of the sphere is known. At some time after the specular return a strong signal would be received from the disc at the rear of the cone and thus the complete received signal would be spread out in time by an amount depending upon the length of the cone and the transmitted pulse width. Thus, the fact that significant returns can be obtained from parts of the moon other than the specular point does not prevent us from determining the electromagnetic properties of the moon near its specular point. Naturally the limb of the moon would not contribute as strongly as the disc in the above example, but it could be important enough, at some aspects, to spread out the received energy over a time much greater than that of the transmitted pulse. In practice, the time is limited by the radius of the moon, since it is hard to believe that, at these frequencies, any significant amount of energy could travel sufficiently far around the moon to get back to earth.

SECTION III

FAR FIELD CRITERIA APPLIED TO THE MOON

On the assumption that the dimensions of the transmitting/receiving antenna are small compared to the scattering object, the criterion for the application of far-zone approximations is

$$r \gg \frac{4a^2}{\lambda} \qquad (1)$$

where r is the range and 2a is the maximum dimension of the scatterer in the plane perpendicular to the direction of propagation. Equation (1) is based on a maximum spread in phase angle of $\pi/4$. In the case of the moon we identify a with its radius (1080 miles). We choose the largest value of λ (which therefore corresponds to the minimum range) to be 5×10^{-3} miles and thus obtain

$$r \gg 8 \times 10^{8} \text{ miles.}$$

Since the range of the moon is about 2×10^{5} miles, we see that, by ordinary antenna theory, the moon must be treated as being in the near zone.

Let us now consider a pulse scattered by a sphere of radius a at a distance r from the radar. If Equation (1) is violated, then at least part of the received pulse will be determined by near-zone scattering laws. However, an initial part of the pulse of time length t_1 can only correspond to reflection from the small initially-irradiated portion of the sphere. For a sufficiently small t_1, this part of the pulse will correspond to a far-zone return, although this is certainly not true for later parts of the pulse.

We shall now determine an upper bound for the time t_1 during which far zone considerations can be applied to the moon. If Equation (1) is replaced by the less stringent condition

$$r \gg \frac{4a^2}{\lambda} , \qquad (2)$$

the critical radius for the illuminated zone is $\frac{1}{2}\sqrt{r\lambda}$. The depth of this zone on the moon can easily be shown to be

$$d = \frac{2a - \sqrt{4a^2 - r\lambda}}{2}$$

and since $\sqrt{r\lambda}$ must be much less than 2a,

$$d \simeq \frac{r\lambda}{8a} .$$

The corresponding value of t_1 is

$$t_1 = \frac{r\lambda}{4\,ac} \qquad (3)$$

where c is the velocity of light. Extreme values for t_1 are obtained by taking $\lambda = 10$ m., giving $t_1 \approx 2\,\mu$sec, and $\lambda = 3$ cm, giving $t_1 = 0.006\,\mu$sec.

If the scatterer is a smooth sphere with voltage reflection coefficient R, there are two cases which can be considered. If the range r satisfies Equation (1), the optics scattering cross section is

$$S = |R|^2 \, \pi a^2 . \qquad (4)$$

This is the leading term in an asymptotic expansion proceeding in inverse powers of $(ka)^{2/3}$, where $k = 2\pi/\lambda$, and,when ka is much greater than unity, Equation (4) gives the cross section to a high degree of accuracy. If r violates Equation (1), let us confine our attention to the "far-zone" portion of the returned pulse. If the dimensions of the corresponding area on the sphere are large compared with λ, geometrical optics can again be used to give the scattering cross section of Equation (4).

So much for the initial portion of the return. For application to the problem of reflections from the moon, the transmitted pulse length τ can be assumed to exceed t_1. Let us measure time t from the instant at which a return is first received. During the interval $0 < t < t_1$, Equation (4) is applicable. Thereafter, a near-zone contribution will be superimposed upon the far-zone return, but since the former is of a lower order of magnitude, the overall level of the signal may be expected to remain unchanged. This will continue until time $t = \tau$. When $t > \tau$, no far-zone contribution is present, and the level of the return will be appreciably reduced, ultimately falling to zero.

All this applies to a smooth moon. It will be observed that in this case the magnitude of the reflection coefficient can be deduced from the initial part of the pulse, notwithstanding the fact that the moon as a whole is in the near zone. Moreover, if the moon is "quasi-smooth" in the sense described in Section II, the essential shape of the returned pulse will be unchanged except for the presence of spikes corresponding to specular reflections from other (discrete) areas of secondary importance.

The one possibility that has not been covered is a moon all of which is rough at the wavelength under discussion. There will then be no specular reflection and the level of the return during the time $0 < t < t_1$ will not be significantly larger than that of the tail. However, there is no moon data at radar frequencies which agrees with rough surface theory.

SECTION IV

THE ELECTROMAGNETIC CONSTANTS OF THE MOON'S SURFACE

In this section we shall attempt to determine the electromagnetic parameters of the moon's surface from measurements of the returned power at different radar frequencies.

Since the radius of the moon is very much greater than the wavelength at all frequencies under consideration, the voltage reflection coefficient R at the specular point is the same as for an infinite slab of the same material as the moon's surface. If a plane electromagnetic wave is incident normally on such a slab,

$$R = \frac{1 - \sqrt{\frac{\mu_0}{\mu}\left(\frac{\epsilon}{\epsilon_0} + i\frac{S}{\omega\epsilon_0}\right)}}{1 + \sqrt{\frac{\mu_0}{\mu}\left(\frac{\epsilon}{\epsilon_0} + i\frac{S}{\omega\epsilon_0}\right)}} \tag{1}$$

(see for example, Stratton (Ref. 4, p.493)), where ϵ , μ and S are the permittivity, permeability, and conductivity respectively of the material. The suffix "o" denotes the same quantities for free space, and ω is the frequency. M.K.S. units are employed and a time dependence $e^{-i\omega t}$ has been assumed. The radar cross section of the moon is

$$\sigma = |R|^2 \pi a^2 \tag{2}$$

where a is the mean radius of the moon.

$|R|^2$ may be written in the form

$$|R|^2 = \frac{X - 1 + 2\left(Y - \sqrt{(X+1)/2}\right)^2}{X - 1 + 2\left(Y + \sqrt{(X+1)/2}\right)^2} \qquad *\,(3)$$

where

$$X = \sqrt{1 + Z\lambda^2}\,, \quad Y = \sqrt{\frac{\mu\epsilon_0}{\mu_0\epsilon}} \qquad (4)$$

$$Z = \frac{S^2\mu_0\epsilon_0}{4\pi^2\ \epsilon^2}$$

$$= \frac{S^2}{\mu^2}\ \frac{\mu_0^3\ Y^4}{4\pi^2\epsilon_0}\ .$$

At this stage it is well to review the assumptions which have been made in carrying out the above analysis. Given the basic assumption that specular reflection takes place (some of the evidence for this has been discussed in Section II) it follows that the radius of curvature in a region surrounding the specular point must be large compared with the wavelength. This in turn justifies the use of a plane slab** to determine an expression for R, and since the distance of the moon from the earth is very much larger than the maximum separation of a transmitter and receiver on the surface of the earth, the transmitted field can be treated as a plane wave incident normally on the specular area. In writing down Equation (2) however, it has been assumed that the

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* For λ very small,Equation (3) takes on the form

$$|R|^2 = \frac{(Y-1)^2}{(Y+1)^2} + \frac{Z}{4}\ \frac{2-Y}{(1+Y)^2}\ \lambda^2$$

** An alternative, but entirely equivalent, method for obtaining Equation (2) would have been to start with the optics expression for the scattering cross section of a dielectric sphere.

radius of curvature of this area is equal to the mean radius of the moon. It is obvious that this will not be true if, for example, the main reflection takes place from a crater, a situation which could arise at particular aspects. Although it seems reasonable to suppose that this will not occur in general, the use of the mean radius a in Equation (2) could be a source of error in comparing different sets of moon data.

In seeking to determine Y and Z by fitting Equation (3) to measured values of $|R|^2$ at different frequencies, it will be assumed that constant values of Y and Z can be used representing averages of Y and Z over the frequency range. This hypothesis is probably a reasonable approximation at the frequencies under consideration and will be put on a firmer basis a posteriori.

As the wavelength increases, so does the depth to which the incident field penetrates below the surface of the moon. The effective reflection takes place at a lower level as λ increases,and if the surface layers of the moon are not homogeneous, or are stratified, the assumption of a uniform slab in the derivation of R will not be valid. Indeed, the available data does provide some slight evidence of non-uniformity as a function of λ (and hence probably of depth), though the accuracy of the experimental values for S is not sufficient for any definite statement on this point.

Let us now return to Equation (2) in which the scattering cross section σ is given as a function of λ. If it is assumed that Y and Z are independent of λ, a knowledge of σ for different values of λ is sufficient to determine Y and Z (and hence ϵ/μ and S/μ). For this purpose the following published data is available.

(i) $\lambda = 0.1$ m $\sigma = 0.0003\ \pi a^2$
(ii) $\lambda = 0.6$ m $\sigma = 0.03\ \pi a^2$
(iii) $\lambda = 1.0$ m $\sigma = (0.05\text{-}0.09)\pi a^2$
(iv) $\lambda = 1.5$ m $\sigma = (0.06\text{-}0.10)\ \pi a^2$
(v) $\lambda = 2.5$ m $\sigma = 0.1\ \pi a^2$

The first of these was obtained by Yaplee et al (Ref. 5) at a frequency of 2860 Mc using 2 microsecond pulses. The relatively low cross section has been attributed by Trexler and Yaplee to modulation loss, and a figure given for this effect is 22 db (private communication from Trexler). The corrected value of σ is then $0.05\ \pi a^2$, a value more in accord with that obtained by Trexler himself at this frequency. It is this result which will be used in our calculations.

The second measurement of σ was made by Blevis (Ref. 1) at a frequency of 488 Mc using two second pulses. For $\lambda = 1.0$ m (300 Mc) the value of σ is due to Trexler (Ref. 2) and has been deduced from his statement that the transmission loss is 258 db when calculated on the basis of isotropic transmitting and receiving antennas. The two values given for σ in (iii) and (iv) are for a moon at perigee and apogee respectively. The fourth result is the average obtained from many measurements made at or near to a frequency of 200 Mc. It has been stated by Trexler (Ref. 2) as a transmission loss of 254 db, although it is pertinent to remark that Trexler himself found an additional loss of 17 db when using 10 μ sec pulses at 198 Mc. This further reduction in power was attributed to modulation loss. For $\lambda = 2.5$ m the value of σ is due to Evans (Ref. 6). The result was obtained using 10 μ sec pulses and is a refinement of that originally measured by Browne et al (Ref. 7). Finally there is a "summary measurement" to which we have not previously referred and which is of considerable assistance. On the basis of a correlation of all the data available to him, Trexler (Ref. 2) states that for frequencies between 20 Mc and 3000 Mc the average transmission loss increases at a rate of 6 db per octave, passing through the value 258 db at 300 Mc; going back to the radar equation, this implies a constant cross section.

The accuracy of the above data is difficult to assess. Evans has given a probable error of $\pm$ 3 db for his result, and an uncertainty of 4 db has been attached to the data quoted by Trexler. It is clear, however, that the magnitude of σ will depend on the manner in which the return is measured, and while most of the results are for the total returned power, it is the initial peak return (corresponding to the main specular reflection) which is required in the present theory.

A good fit of Equation (3) to these experimental results is obtained by choosing Y and Z in Equation (4) to be

$$Y = .645 \qquad Z = 0.1\ m^{-2}. \tag{5}$$

From this result we obtain

$$\frac{\epsilon}{\mu} = 2.4\ \frac{\epsilon_o}{\mu_o} = 1.7 \times 10^{-5}\ (\text{mhos})^2$$

and

$$\frac{S}{\mu} = 1.0 \times 10^{4}\ \text{mhos/henry}\ . \tag{6}$$

If $\mu = \mu_o$

$$\epsilon = 2.1 \times 10^{-11}\ \text{farads/meter}$$

and

$$S = 1.3 \times 10^{-2}\ \text{mhos/meter}\ . \tag{7}$$

These values of ϵ and S are considered to be mean values in the frequency range examined. By comparison of these values to ϵ and S values quoted in von Hippel (Ref. 8) for various soils, it is evident that the present results are not opposed to the hypothesis that the surface layers of the moon are mainly composed of dry soil, though certain varieties of crumbly cheese might be possible materials.

The curve obtained using Equation (5) is superimposed on the experimental points in Figure 4-1.

A further justification for the use of constant values for Y and Z in the curve fitting is now possible. Von Hippel in Ref. 8 reports experimental values of ϵ and S for wavelengths of .03, 0.1, and 1.0 meters, and up. ϵ was found to be quite insensitive to wavelength, while S varied, very roughly speaking, as λ^{-1}. From Equations (3) and (4) it is clear that if a frequency dependence of λ^{-1} for S and λ^{0} for ϵ is assumed, $|R|^2$ would be independent of frequency. If a frequency dependence somewhat closer to that of actual dry soil samples is assumed and the values of Equation (7) assigned to $\lambda = 1$ m, the slope of the resulting curve of $|R|^2$ computed for $.1 < \lambda < 2.5$ m decreases compared to the curve in Figure 4-1, but the $|R|^2$ values remain well within 3 db of those on the curve.

For the purposes of Section VII we require also the phase change on reflection. This can be calculated from Equations (1) and (5), and the curve of arg R as a function of λ is given in Figure 4-2.

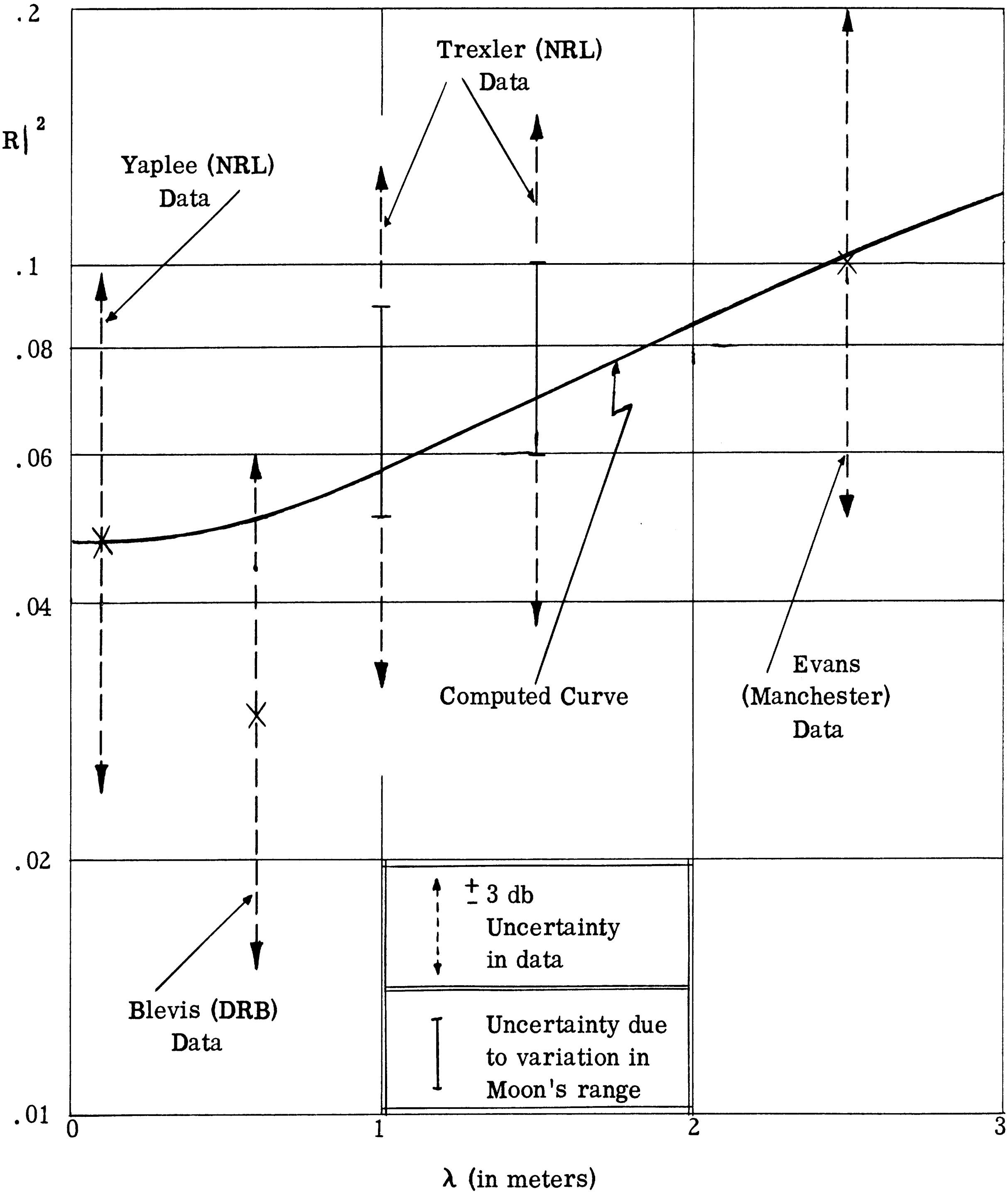

FIG. 4-1: POWER REFLECTION COEFFICIENT $|R|^2$ AS A FUNCTION OF WAVELENGTH

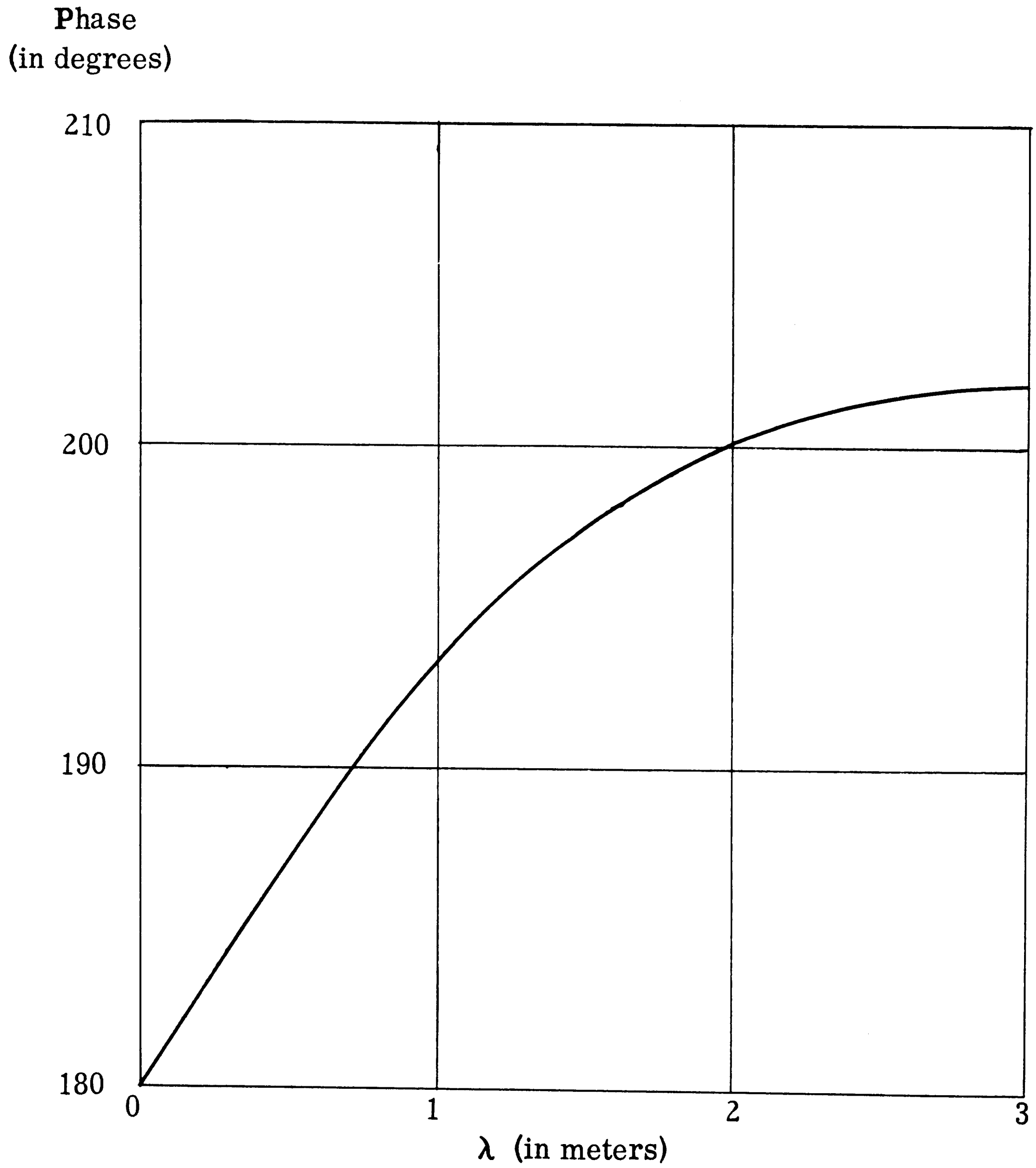

FIG. 4-2: PHASE OF VOLTAGE REFLECTION COEFFICIENT, R .

SECTION V

THE FARADAY EFFECT

The Faraday effect is a rotation of the plane of polarization of a plane-polarized wave on propagation through an ionized region in which there is a magnetic field. The rotation is anti-reciprocal, that is, it is doubled rather than cancelled when the path is traversed twice in opposite directions. Since this rotation is manifested as a power loss, or fade, if linear polarization is used, it must be considered in any discussion of radar reflection from the moon. The ionosphere and the geomagnetic field are the sources of the Faraday rotation in this situation.

For radar frequencies in excess of 100 Mc, the expression for the rotation angle on a one-way traversal may be written as

$$\Psi = \frac{e^3}{2\pi m^2 c^2 \nu^2} \int_0^{\ell} N(\vec{s})\, \vec{H}(\vec{s}) \cdot d\vec{s} \quad , \qquad (1)$$

in which e and m are the charge and mass of the electron, c is the velocity of light, and ν the radar frequency. N is the electron density, and $\vec{H}$ the magnetic field. The integral is taken along the propagation path.

Unfortunately, N is not known very well, so that accurate evaluation of the integral is precluded. As a rough estimate, we might assume (Ref. 6) the value $\int N\, ds = 10^{13}/\text{cm}^2$ for the whole ionosphere. Again, to obtain an order of magnitude value for Ψ, we assume the moon at zenith, so that only the vertical component of $\vec{H}$

is involved. This we take as 0.5 gauss, a representative value for the United States. These values yield a rotation of roughly π at $\nu = 200$ Mc. We see then that the Faraday effect is not negligible at lower frequencies.

Since it might be desirable to operate in such a manner that the Faraday effect is significant, the bistatic Faraday effect has been analyzed. Such information would be of most value if formulated explicitly in terms of the location of the transmitter and receiver, and the position of the moon relative to one or both of these. We now proceed to derive such a result. Since the only complexity of the derivation is geometrical, it will not be presented in great detail.

As seen from Equation (1) the geometrical dependence of the Faraday effect is contained in the integral

$$\int N(\ell)\ \vec{H}(\ell) \cdot \hat{L}\ d\ell \quad ,$$

in which $N(\ell)$ is the free electron density, $\vec{H}(\ell)$ the geomagnetic field, and $\hat{L}$ a unit vector along the propagation direction. The integral is taken along the ray path (i.e., from transmitter to moon for the Faraday effect on the transmitted ray, and from moon to receiver for the rotation on the return path). We want to express this integral in convenient coordinates.

We designate the transmitter and receiver locations by θ_T, ϕ_T and θ_R, ϕ_R. Here θ is the usual polar angle (the colatitude), and ϕ is longitude ($\phi < 0$ is W. Longitude). The position of the moon is defined by angles $\psi_{R,T}$ and $\chi_{R,T}$. χ_T is the elevation above the tangent plant at θ_T, ϕ_T of the line of sight from the transmitter to the moon.

ψ_T is measured from the meridian pointing north clockwise to the projection of this line of sight into the tangent plane. χ_R and ψ_R are defined similarly at the receiver.

One finds that the vector $\hat{L}_T$ for the path of the transmitted ray is given by

$$\hat{L}_T = \hat{x}\left[\sin\theta_T\cos\phi_T\sin\chi_T - \cos\theta_T\cos\phi_T\cos\chi_T\cos\psi_T - \sin\phi_T\cos\chi_T\sin\psi_T\right]$$
$$+ \hat{y}\left[\sin\theta_T\sin\phi_T\sin\chi_T - \cos\theta_T\sin\phi_T\cos\chi_T\cos\psi_T + \cos\phi_T\cos\chi_T\sin\psi_T\right]$$
$$+ \hat{z}\left[\cos\theta_T\sin\chi_T + \sin\theta_T\cos\chi_T\cos\psi_T\right] \quad .$$

According to Mitra (Ref. 9), the main part of the geomagnetic field can be attributed to a dipole of strength $|\vec{P}| = 8.19 \times 10^{25}$ cgs., oriented with N. pole at 78.5°N, 69°W. Then, where $P_x = \sin 11.5^{\circ} \cos 69^{\circ}$, $P_y = -\sin 11.5^{\circ} \sin 69^{\circ}$, $P_z = \cos 11.5^{\circ}$, we may write

$$\frac{\vec{H}(x,y,z)}{|\vec{P}|} = \frac{\hat{x}}{r^3}\left[-P_x + 3P_x\frac{x^2}{r^2} + 3P_y\frac{xy}{r^2} + 3P_z\frac{xz}{r^2}\right]$$

+ similar terms in cyclic order for the other two components.

We want to obtain $\vec{H}(\ell)$ from this. To do this, we first write the position vector $\vec{S}_T(\ell)$ of a point on the transmission path:

$$\vec{S}_T(\ell) = G(\hat{x}\sin\theta_T\cos\phi_T + \hat{y}\sin\theta_T\sin\phi_T + \hat{z}\cos\theta_T) + \left(-G\sin\chi_T + \sqrt{\ell^2 - G^2\cos^2\chi_T}\right)\hat{L}_T.$$

Here G is the radius of the earth, and ℓ runs from G to D, the mean earth-moon distance. Now to get $\vec{H}(\ell)$, we replace r by $|\vec{S}_T(\ell)|$, which is ℓ, and x, y, z by the three components of $\vec{S}_T(\ell)$.

The scalar product $\frac{\vec{H}_T(\ell)}{|\vec{P}|} \cdot \hat{L}_T$ may now be evaluated, yielding

$$\frac{3C_1}{\ell^5} G\sqrt{\ell^2 - G^2\cos^2\chi_T} + \frac{1}{\ell^5}\left[C_1 \sin\chi_T + C_2\cos\chi_T\cos\psi_T + C_3\cos\chi_T\sin\psi_T\right]$$

$$\cdot\left[2\,\ell^2 - 3G^2\cos^2\chi_T - 3G\sin\chi_T\sqrt{\ell^2 - G^2\cos^2\chi_T}\right]$$

in which

$$C_1 = \sin 11.5^o \sin\theta_T \cos(\phi_T + 69^o) + \cos 11.5^o \cos\theta_T$$

$$C_2 = \sin 11.5^o \cos\theta_T \cos(\phi_T + 69^o) + \cos 11.5^o \sin\theta_T$$

$$C_3 = -\sin 11.5^o \sin(\phi_T + 69^o) \;.$$

Finally, $N(\ell)$ may be obtained from some standard tables of atmospheric properties. Such a table would give $N(z)$, where z is altitude above the earth's surface. The replacement $z = \ell - G$ converts this to $N(\ell)$. With the expressions given above, the Faraday rotation between the transmitter and moon can be obtained. Of course, a numerical integration would be required.

Since the rotation between the moon and receiver is the same as if the receiver transmitted to the moon, the rotation on this second segment of the path can be obtained simply by changing all subscripts T to R in the expressions given above. However, the angles χ_R and ψ_R are obviously redundant, and may be expressed in term of θ_T, ϕ_T, θ_R, ϕ_R, χ_T, and ψ_T.

Let $\vec{t} = \vec{S}_T(\ell) - G(\hat{x}\sin\theta_R\cos\phi_R + \hat{y}\sin\theta_R\sin\phi_R + \hat{z}\cos\theta_R)$. Then if $\hat{t}$ is the normalized vector defined from $\vec{t}$, ($\hat{t}$ is the line of sight of the moon from the receiver),

$$\sin\chi_R = \hat{t}\cdot(\hat{x}\sin\theta_R\cos\phi_R + \hat{y}\sin\theta_R\sin\phi_R + \hat{z}\cos\theta_R) \;.$$

Since if the moon is visible at the receiver, it is above the horizon, we know $0 < \chi_R < \pi/2$ and thus the sign of cos χ_R is determined, while $|\cos \chi_R|$ is obtained from sin χ_R. Further,

$$\cos \chi_R \cos \psi_R = \hat{t} \cdot -\hat{\theta}(\theta_R, \phi_R) = -\hat{t} \cdot (\hat{x} \cos\theta_R \cos\phi_R + \hat{y}\cos\theta_R \sin\phi_R - \hat{z}\sin\theta_R).$$

Finally,

$$\cos \chi_R \sin \psi_R = \hat{t} \cdot \hat{\phi}(\theta_R, \phi_R) = \hat{t} \cdot (-\hat{x}\sin\phi_R + \hat{y}\cos\phi_R).$$

Since cos χ_R may first be obtained numerically, sin ψ_R and cos ψ_R may be determined this way. All the quantities required to replace T by R to get the Faraday rotation for the return path are now prescribed.

However, normal diurnal fluctuations in ionospheric electron density, and those associated with magnetic storms, make it pointless to attempt a numerical evaluation of the Faraday integral. To yield a practically useful result, such evaluation would have to be in terms of site coordinate and time and date (corresponding to the moon's position). But such fluctuations would invalidate the table.

For these reasons, we feel the Faraday effect should be circumvented. Receiving two orthogonal polarizations, or transmitting and receiving circular polarization, are schemes for overcoming power loss due to the Faraday effect. Since it varies inversely with ν^2, operation in the KMc range will eliminate it.

In the presence of the Faraday effect, one may ask how the two rotations of the plane of polarization will disturb or alter the determination of the scattering properties of the moon (or an earth satellite). For a given aspect and a fixed frequency, the full description of the cross section of a target is specified by the scattering matrix. The scattering matrix of interest is of course that which would be present in the absence of the ionosphere, not that due to the composite system of scatterer plus ionosphere. The complication is that we can only observe it with the effect of the Faraday rotation superimposed. For simplicity, the discussion is restricted to back-scattering; no new essential features appear in the general case.

Let us define the direction of propagation as the z-axis and let $\vec{E}^i$ be the incident field and $\vec{E}^s$ the scattered field. The scattering matrix is then described by

$$\begin{aligned} E_x^s &= S_{11} E_x^i + S_{12} E_y^i \\ E_y^s &= S_{21} E_x^i + S_{22} E_y^i \end{aligned} \qquad (2)$$

where the S_{jk} are complex numbers. By reciprocity, $S_{12} = S_{21}$. An overall absolute phase is irrelevant. There are thus five independent real numbers that determine the S-matrix: The magnitudes of S_{11}, S_{22}, and S_{12}, and the relative phases between them.

Denote the electric polarization vector of the field incident on the scatterer by $(\hat{x} \cos\theta + \hat{y} \sin\theta)$, and let $(\hat{x} \cos\alpha + \hat{y} \sin\alpha)$ be the polarization vector upon leaving the scatterer of a scattered field whose polarization on arrival at the receiver will coincide with that of the receiver. The field measured at the receiver will be

$$\begin{aligned} E^r &= E_x^s \cos\alpha + E_y^s \sin\alpha \\ &= E^t \left[S_{11} \cos\theta \cos\alpha + S_{22} \sin\theta \sin\alpha + S_{12} \sin(\theta + \alpha) \right] . \end{aligned} \qquad (3)$$

Without the ionosphere, θ and α would be the polarization angles of the transmitter and receiver respectively, and the S-matrix would be determined by making five measurements [e.g. for the pairs of values ($\theta = 0$, $\alpha = 0$), ($\theta = \pi/2$, $\alpha = \pi/2$), and ($\theta = 0$, $\alpha = \pi/2$), one obtains $(E^r/E^t) = S_{11}$, S_{22}, and S_{12} respectively, so that if one measures the amplitude of each of the three returns and the two relative phases the S-matrix is determined.]

With the ionosphere in the way, the Faraday effect rotates the polarization vector by an angle Ψ, so that if the radiation is incident with a polarization angle θ it must have started with an angle $\phi = \theta - \Psi$; similarly, to receive the component of the scattered field that leaves the scatterer with polarization angle α one must receive with a polarization angle $\beta = \alpha + \Psi$. Thus, if the transmitter has a polarization ($\hat{x} \cos \phi$ + $\hat{y} \sin \phi$) and the receiver a polarization ($\hat{x} \cos \beta + \hat{y} \sin \beta$), the measured field will be

$$\begin{aligned} E^r = E^t \Big[& S_{11} \cos(\phi + \Psi) \cos(\beta - \Psi) + S_{22} \sin(\phi + \Psi) \sin(\beta - \Psi) \\ & + S_{12} \sin(\phi + \beta) \Big] \\ = E^t \Big[& S_{12} \sin(\phi + \beta) + (1/2)(S_{11} - S_{22}) \cos(\phi + \beta) \\ & + (1/2)(S_{11} + S_{22}) \cos(\phi - \beta) \cos 2\Psi + (1/2)(S_{11} + S_{22}) \sin(\phi - \beta) \sin 2\Psi \Big] \end{aligned} \tag{4}$$

This expression contains functions of ϕ and β which are known for a particular choice of transmitter and receiver polarizations, and the unknown quantities S_{12}, $(S_{11}-S_{22})$, $(S_{11}+S_{22}) \cos 2\Psi$, and $(S_{11}+S_{22}) \sin 2\Psi$. Four pairs of polarization angle settings are now necessary, supplying seven measurements; these suffice to determine the amplitude and phase of the above unknowns. Since the trigonometric functions are real, there is a redundancy in that the phase of the last two unknowns is the same (modulo π). It is not proper to simply discard one phase measurement on this account; the corresponding amplitude is then a quadratic expression which possesses an extraneous root (unless one of the unknowns vanishes). One can, however, use delay line circuitry to superpose the returns without loss of phase before the recording stage in such a way that the resulting linear combinations are the above unknowns; after this is done, only six quantities need be recorded for maximum information about the scattering matrix (discarding the relative sign of cos 2ψ and sin 2ψ is of no importance).

These measurements do not, however, completely determine the scattering matrix. While cos 2ψ (or sin 2ψ) is real, it can be either positive or negative; unless we know which it is, there is an ambiguity of π in the phase of $(S_{11} + S_{22})$. Because of this ambiguity, the measurements yield two scattering matrices to choose between, only one of which is of course correct. (The two are identical for a matrix proportional to the unit matrix such as occurs for the sphere). If we denote the two solutions by superscripts a and b, and fix arbitrarily the overall phase of one with respect to the other, they are related by

$$S_{11}^{a} = S_{22}^{b} \ , \quad S_{22}^{a} = S_{11}^{b} \ , \quad S_{12}^{a} = - S_{12}^{b} \ . \tag{5}$$

The choice of the correct S-matrix can be made only if either of two additional data are available:

1. a determination of the Faraday rotation adequate to ascertain the sign of cos 2ψ (e.g. frequency comparison); or
2. sufficient prior knowledge of the properties of the scatterer to decide which of the two scattering matrices makes sense (e.g. the datum that $|S_{11}| > |S_{22}|$).

SECTION VI

ELECTROMAGNETIC SCATTERING BY LOW-DENSITY METEOR TRAILS

6.1 Scattering by a Low-Density Uniform Line Distribution of Electrons

6.1.1 - Statement of the Problem

As a meteoroid approaches the earth, its interaction with the atmospnere leads to the formation of ionization in its wake. This ionized region will scatter electromagnetic energy incident upon it. Although the trail is electrically neutral (i.e., there are an equal number of electrons and positive ions), the electrons, because their mass is so much smaller than that of the ions, contribute essentially all of the return. The meteoroid itself is too tiny an object to have a significant effect (the frequency of occur-) rence of meteorites is negligible). Thus, the most elementary model of a meteor trail that suggests itself, for the purpose of a calculation of electromagnetic scattering, is a uniform line distribution of electrons.

At radar frequencies, the Compton scattering due to an electron is given by the Thomson cross section

$$d\sigma = (\hat{e} \cdot \hat{e}')^2 \, r_0^2 \, d\Omega \tag{1}$$

where $\hat{e}$ and $\hat{e}'$ are the electric polarization vectors of the incident and scattered fields respectively, r_0 is the classical electron radius (e^2/mc^2), and $d\Omega$ is the solid angle subtended at the detector. If the transmitter power is P and its gain is G, the power density at a distance r from the transmitter will be $PG/4\pi r^2$. The effective collecting area of a receiver of gain G' for an operating wavelength λ is $G'\lambda^2/4\pi$. Hence, the effective solid angle subtended by the receiver located at a distance r' from the scatterer is $G'\lambda^2/4\pi r'^2$. Thus, the scattered power received from one electron is

$$S = PGG' r_0^2 \lambda^2 (\hat{e} \cdot \hat{e}')^2 / 16\pi^2 r^2 r'^2 . \qquad (2)$$

Consider next a uniform line distribution of electrons, α per unit length. Assume α sufficiently small that the field incident on one electron is essentially unaffected by the presence of the other electrons. The amplitude of the scattered field due to a line segment of length ds will then be α ds times that due to a single electron. Its phase, relative to other segments, will be given by $(2\pi/\lambda)(r + r')$. The gains of the transmitter and receiver will, in general, vary along the line; this is most conveniently taken into account by understanding G and G' to be the maximum gains and using a lumped antennae pattern factor f(s) to denote the reduction in amplitude at the point whose position along the line is denoted by s. The scattered power due to the line distribution is then

$$S = (PGG' r_0^2 \lambda^2/16\pi^2) \left| \int ds\, (\hat{e} \cdot \hat{e}')\, \alpha\, f(s)\, e^{i(2\pi/\lambda)(r+r')}/rr' \right|^2 \qquad (3)$$

where α, r, r', and $(\hat{e} \cdot \hat{e}')$ are functions of the position s. We shall take α to be constant, reinterpreting f(s) to include any variation in α over the line. Inasmuch as the altitude of the trail is much larger than its length, the variation in r, r', and $(\hat{e} \cdot \hat{e}')$ over the trail is small and, except in the phase, can be ignored. Letting R and R' represent the distances r and r' measured with respect to a representative point on the trail and $(\hat{e} \cdot \hat{e}')$ the value at that point, the scattered power is

$$S = (PGG' \alpha^2 r_0^2 \lambda^2/16\pi^2 R^2 R'^2)(\hat{e} \cdot \hat{e}')^2 \left| \int ds\ f(s)\ e^{i(2\pi/\lambda)(r+r')} \right|^2 . \qquad (4)$$

The subsequent discussion concerns itself with the evaluation of the integral

$$I = \int ds\ f(s)\ e^{i(2\pi/\lambda)(r + r')} \qquad (5)$$

under various conditions.

6.1.2 - Integration by Parts

It is convenient to rewrite the integral as

$$I = \int ds\ f(s)\ e^{iK\,g(s)} \qquad (6)$$

where $K = (2\pi/\lambda)(R + R')$ (6a)

and $g(s) = (r + r')/(R + R')$. (6b)

For the parameters of interest, K is a very large number, while g is of order unity. The functions f and g are well-behaved.

In any region of s within which g'(s) does not vanish, we can change the variable to g:

$$I = \int_{g(a)}^{g(b)} dg\ (f/g')\ e^{iKg} \qquad (7)$$

For convenience, write

$$G(g) = f/g' \qquad (7a)$$

so that

$$I = \int_{g(a)}^{g(b)} dg\ G(g)\ e^{iKg} \quad . \qquad (8)$$

Integrating by parts repeatedly, we obtain

$$I = (iK)^{-1} \sum_{n=o}^{\infty} (-iK)^{-n} \left[G^{(n)}(g(b))\, e^{iKg(b)} - G^{(n)}(g(a))\, e^{iKg(a)} \right] \tag{9}$$

If $g'(s)$ vanishes at the point $s = s_o$ within a region, we can break up the region into the interval $(s_o-h) \leq s \leq (s_o+h)$ about this point, and the two intervals $a \leq s \leq (s_o-h)$ and $(s_o+h) \leq s \leq b$ for which the above treatment is still valid:

$$\begin{aligned} I = {} & (iK)^{-1} \sum_{n=o}^{\infty} (-iK)^{-n} \left[G^{(n)}(g(s_o-h))\, e^{iKg(s_o-h)} - G^{(n)}(g(a))\, e^{iKg(a)} \right] \\ & + \int_{s_o-h}^{s_o+h} ds\ f(s)\, e^{iKg(s)} \\ & + (iK)^{-1} \sum_{n=o}^{\infty} (-iK)^{-n} \left[G^{(n)}(g(b))\, e^{iKg(b)} - G^{(n)}(g(s_o+h))\, e^{iKg(s_o+h)} \right] \end{aligned} \tag{10}$$

$$\begin{aligned} \text{or } I = {} & (iK)^{-1} \sum_{n=o}^{\infty} (-iK)^{-n} \left[G^{(n)}(g(b))\, e^{iKg(b)} - G^{(n)}(g(a))\, e^{iKg(a)} \right] \\ & - (iK)^{-1} \sum_{n=o}^{\infty} (-iK)^{-n} \left[G^{(n)}(g(s_o+h))\, e^{iKg(s_o+h)} - G^{(n)}(g(s_o-h))\, e^{iKg(s_o-h)} \right] \\ & + \int_{s_o-h}^{s_o+h} ds\ f(s)\, e^{iKg(s)} . \end{aligned} \tag{11}$$

The first term is the same as in the absence of a point s_o at which g' vanishes, while the other two represent the contribution of the neighborhood of s_o. If there were more than one point at which g' vanished, there would be two similar terms due to each. (For a straight line trail there will not be more than one such point; for a curved line there can be several).

To evaluate the remaining integral, expand g(s) in a Taylor series about the point $s = s_o$:

$$g(s) = g(s_o) + \sum_{n=o}^{\infty} \frac{(s-s_o)^n}{n!} g^{(n)}(s_o) \qquad (12)$$

$$\int_{s_o-h}^{s_o+h} ds\, f(s)\, e^{iKg(s)} = e^{iKg(s_o)} \int_{s_o-h}^{s_o+h} ds\, f(s)\, e^{iK\sum_{n=o}^{\infty} \frac{(s-s_o)^n}{n!} g^{(n)}(s_o)} \qquad (13)$$

The first step in examining the evaluation of I is to look at the explicit form of g. Take the z-axis along the trail, and the origin at the point on the trail from which R and R' are measured. Then

$$\vec{s} = \hat{z}\, s$$

$$\vec{R} = R(\hat{x} \sin\theta \cos\phi + \hat{y} \sin\theta \sin\phi + \hat{z}\cos\theta)$$

$$\vec{R}' = R'(\hat{x} \sin\theta' \cos\phi' + \hat{y} \sin\theta' \sin\phi' + \hat{z}\cos\theta') \qquad (14)$$

and

$$\vec{r} = \vec{R} - \vec{s}$$

$$\vec{r}' = \vec{R}' - \vec{s}\ .$$

Evaluating r and r' in terms of the variable s and the constant quantities,

$$r = R\left[1 - \frac{2\, s\cos\theta}{R} + \frac{s^2}{R^2}\right]^{1/2}$$

$$r' = R'\left[1 - \frac{2\, s\cos\theta'}{R'} + \frac{s^2}{R'^2}\right]^{1/2} \qquad (15)$$

so that

$$g(s) = \frac{R}{R+R'}\left[1 - \frac{2s\cos\theta}{R} + \frac{s^2}{R^2}\right]^{1/2} + \frac{R'}{R+R'}\left[1 - \frac{2s\cos\theta'}{R'} + \frac{s^2}{R'^2}\right]^{1/2}\ . \qquad (16)$$

Differentiating,

$$g'(s) = -\frac{1}{R+R'}\left\{\left[1-\frac{2s\cos\theta}{R}+\frac{s^2}{R^2}\right]^{-1/2}\left(\cos\theta-\frac{s}{R}\right)+\left[1-\frac{2s\cos\theta'}{R'}+\frac{s^2}{R'^2}\right]^{-1/2}\left(\cos\theta'-\frac{s}{R'}\right)\right\} \quad (17)$$

$$g''(s) = \frac{1}{R+R'}\left\{\left[1-\frac{2s\cos\theta}{R}+\frac{s^2}{R^2}\right]^{-3/2}\frac{\sin^2\theta}{R}+\left[1-\frac{2s\cos\theta'}{R'}+\frac{s^2}{R'^2}\right]^{-3/2}\frac{\sin^2\theta'}{R'}\right\} \quad (18)$$

$$g'''(s) = \frac{3}{R+R'}\left\{\frac{\sin^2\theta}{R^2}\left(\cos\theta-\frac{s}{R}\right)\left[1-\frac{2s\cos\theta}{R}+\frac{s^2}{R^2}\right]^{-5/2}+\frac{\sin^2\theta'}{R'^2}\left(\cos\theta'-\frac{s}{R'}\right)\left[1-\frac{2s\cos\theta'}{R'}+\frac{s^2}{R'^2}\right]^{-5/2}\right\} \quad (19)$$

In general, successive derivatives decrease in magnitude by a factor of the order of $1/R$.

If there is a point at which f does not vanish and g' does, we can select this point to be the origin. It then follows that, at this point

$$\cos\theta' = -\cos\theta \quad (20)$$

so that the angle between the trail and either of the two rays (incident and scattered) is the same (specular condition). The azimuthal angles do not appear in the condition for g'(0) vanishing, hence there is no requirement that the trail and the two rays be coplanar. We have then

$$g(0) = 1 \quad (16')$$

$$g'(0) = 0 \quad (17')$$

$$g''(0) = \frac{\sin^2\theta}{RR'} \quad (18')$$

$$g'''(0) = 3\left(\frac{R'-R}{R^2R'^2}\right)\sin^2\theta\,\cos\theta \quad (19')$$

6.1.3 - Constant Antenna Pattern Factor

The special case that first suggests itself is f(s) = 1, $a \leq s \leq b$; f(s) = 0, $s < a$ or $s > b$. This might correspond to a constant uniform line distribution of electrons extending back from the meteoroid, two idealized overlapping sharply defined beams which have a constant intensity within the beamwidth and zero intensity just beyond it, and neglect of diffusion of the trail.

In this case, we have

$$G(g) = \frac{1}{g'} \tag{21}$$

$$G'(g) = \frac{d}{dg}\frac{1}{g'} = \frac{1}{g'}\frac{d}{ds}\frac{1}{g'} = -\frac{g''}{(g')^3} \tag{22}$$

The ratio of the second term in the series of Equation (11) to the first is

$$\left|\frac{G'(g)}{KG(g)}\right| = \frac{g''}{K(g')^2} \tag{23}$$

Outside the neighborhood of a point where g' = 0, g'' is of order R^{-2} and g' is of order R^{-1}, so the ratio is of order $K^{-1} \sim (\lambda/R)$, which is quite small. Successive terms fall off to the same order. Near a specular point (chosen as s = 0), we use a Taylor series expansion whose successive terms are of order (s/R), which is reasonably small:

$$g'(s) = s\, g''(0) + \ldots\ldots \tag{24}$$

$$\left|\frac{G'(g)}{KG(g)}\right| = \frac{\lambda\, g''(0)}{2\pi(R + R')s^2\left[g''(0)\right]^2} = \frac{\lambda RR'}{2\pi(R + R')s^2\sin^2\theta} \tag{25}$$

and a comparable factor for successive terms. We shall accordingly stipulate that

$$h \gg \left[\frac{\lambda RR'}{2\pi(R+R')}\right]^{1/2} \quad \csc\theta \ . \tag{26}$$

This will permit us to discard all but the first term of the series. (Provisionally, consider that $a < -h < h < b$ to streamline the discussion; the other possibilities will be treated later).

For the series in the exponential of Equation (13), the first two terms are

$$K \frac{s^2}{2} g''(0) = \frac{\pi}{\lambda} s^2 \left(\frac{R+R'}{RR'}\right)\sin^2\theta \tag{27}$$

$$K \frac{s^3}{6} g'''(0) = \frac{\pi}{\lambda} s^3 \left(\frac{R'^2 - R^2}{R^2 R'^2}\right)\sin^2\theta\cos\theta \tag{28}$$

and successive terms are of order (s/R) smaller. If the expression of Equation (28) and the higher order terms are small, the integral reduces to a Fresnel integral; otherwise it cannot be evaluated analytically. We shall consequently further require that

$$h \ll \left|\frac{\lambda R^2 R'^2}{\pi (R^2 - R'^2)\sin^2\theta\cos\theta}\right|^{1/3} \tag{29}$$

(If the denominator of Equation (29) is near-vanishing, it may be necessary to state in addition the corresponding inequality for the next order term). This inequality is compatible with Equation (26) provided only that $\lambda \ll R$.

With h chosen, in accordance with the discussion of the last paragraph, so that only the first term of the series in the exponential of Equation (13) is required, the integral reduces to

$$I_1 = \int_{-h}^{h} ds\, e^{iKg(s)} = 2e^{iK} \int_0^h ds\, e^{i\frac{\pi}{2}\left[2\left(\frac{R+R'}{2\lambda RR'}\right)^{1/2} s \sin\theta\right]^2} \tag{30}$$

$$= \left(\frac{2\lambda RR'}{R+R'}\right)^{1/2} \csc\theta\, e^{iK}\, F\left[2\left(\frac{R+R'}{2\lambda RR'}\right)^{1/2} h \sin\theta\right]$$

where F(H) is the Fresnel integral

$$F(H) = \int_0^H dx\, e^{i(\pi/2)x^2} \,. \tag{30a}$$

From the asymptotic expansions (Ref. 10),

$$F(H) = 2^{-1/2} e^{i\pi/4} - i(\pi H)^{-1} e^{i(\pi/2)H^2} - (\pi^2 H^3)^{-1} e^{i(\pi/2)H^2} + \ldots \tag{31}$$

Equation (26) sets a lower limit to H; if we take H as small as $2^{1/2}$, the inequality will hold by a factor of about 2.5 and the ratio of terms of Equation (25) will be $(2\pi)^{-1}$. The last term of Equation (31) can then be neglected, being 5 per cent of the total.

Of the two series in Equation (11) the second is the larger, and we have chosen h such that only the first term of this series is required. This is

$$I_2 = -\frac{1}{iK}\left[\frac{1}{g'(h)} e^{iKg(h)} - \frac{1}{g'(-h)} e^{iKg(-h)}\right] = -\frac{2}{iKhg''(0)} e^{iK\left[g(0)+\frac{1}{2}h^2 g''(0)\right]} \tag{32}$$

$$= \frac{i\lambda RR'}{\pi(R+R')h\sin^2\theta} e^{iK(1+h^2\sin^2\theta/2RR')} = \left(\frac{2\lambda RR'}{R+R'}\right)^{1/2} \csc\theta\, e^{iK}\, i(\pi H)^{-1} e^{i(\pi/2)H^2}$$

where the expansion of the exponent has been cut off, as permitted by the choice of h.

When I_1 and I_2 are combined, the sum reduces to

$$I = \left[\lambda RR'/(R+R')\right]^{1/2} \csc\theta\, e^{i(K+\pi/4)} \tag{33}$$

because I_2 just cancels the contribution of the second term of Equation (31). Since h is a mathematical construct of no direct physical meaning, its disappearance from the final result is to be expected.

The contributions of the end-point series have been neglected; they are of magnitude $(h/2\pi|a|)$ and $(h/2\pi b)$ respectively vis-à-vis Equation (33) (for $H = 2^{1/2}$), and hence will contribute less than $(2\pi)^{-1}$ of I provided $|a|$ and b are both at least twice h. If either end point is closer than this to the specular region, the Fresnel integral on that side should be extended out to it (for this integral, the second term of Equation (31) will now have to be retained), eliminating the integration by parts; for the other side, the cancellation will work as before.

We see then that the chief contribution to I comes from the neighborhood of the point at which $g' = 0$. Conceptually, this is to be expected: The phase exponential is a rapidly fluctuating function, and the fluctuations result in contributions from neighboring intervals approximately cancelling; if there is an interval in which the oscillation is temporarily suspended (as indicated by $g' = 0$), its contribution will remain and will dominate the answer. That is the basis of the principle of stationary phase, and in fact the result could have been obtained by invoking it; this has not been done because the investigation has to include situations in which the conditions for applying the stationary phase evaluation are not satisfied, and the other terms then have to be considered.

Hence, for $a < -h < h < b$ and $f = 1$ between a and b (and zero outside),

$$|I|^2 = \frac{\lambda RR' \csc^2 \theta}{R + R'} \tag{34}$$

and

$$S = \frac{PGG' \alpha^2 r_0^2 \lambda^3 \csc^2 \theta}{16\pi^2 RR'(R+R')} (\hat{e} \cdot \hat{e}')^2 \quad . \tag{35}$$

An equivalent expression is given in Ref. 11 after a heuristic discussion. For back-scattering with transmitter and receiver at the same polarization, we have

$$R = R', \quad \theta = \frac{\pi}{2}, \quad \text{and} \quad \hat{e} = \hat{e}', \text{ so that}$$

$$S = \frac{PGG' \alpha^2 r_0^2 \lambda^3}{32\pi^2 R^3} \tag{36}$$

which is the Lovell-Clegg formula as quoted in Ref. 12 from a very rough argument. (There are numerical discrepancies from this value in Refs. 13 and 14).

Alternatively, if there is no stationary point in the interval (a, b), only the first series will contribute, and its first term will dominate:

$$I = \frac{1}{iK} \left[\frac{1}{g'(b)} e^{iKg(b)} - \frac{1}{g'(a)} e^{iKg(a)} \right]. \tag{37}$$

If the interval (a, b) is far removed from a stationary point, g' will vary slowly so that, choosing s = 0 to lie within (a, b), we can replace it by

$$g'(0) = - \frac{\cos \theta + \cos \theta'}{R + R'} \quad . \tag{17''}$$

The magnitude of the two terms in Equation 37 is then the same. The two phases, Kg, are very large numbers and, consequently, the relative phase of the two terms cannot be adequately estimated. This difficulty is due to the oversimplified model and does not occur in a more realistic view of the physical situation. Actually, the contribution of the back end will be reduced by the effect of diffusion. Anticipating this result, let us take

$$I = - \left[iK\, g'(0)\right]^{-1} e^{iK\, g(0)} \tag{38}$$

so that

$$|I|^2 = \frac{\lambda^2}{4\pi^2 (\cos\theta + \cos\theta')^2} \tag{39}$$

and

$$S = \frac{PGG'\alpha^2 r_o^2 \lambda^4}{64\pi^4 R^2 R'^2 (\cos\theta + \cos\theta')^2} (\hat{e}\cdot\hat{e}')^2 . \tag{40}$$

In particular, for back-scattering at a single polarization

$$S = \frac{PGG'\alpha^2 r_o^2 \lambda^4}{256\pi^4 R^4 \cos^2\theta} \tag{41}$$

If there is no stationary point in (a, b), but there is one near one end of the interval, say a distance p from b, then $|g'(b)| \ll |g'(a)|$, and furthermore $g'(b) = -pg''(0)$ if we choose the stationary point to be s = 0. Thus

$$|I|^2 = \frac{\lambda^2}{4\pi^2 (R+R')^2} \; \frac{R^2 R'^2}{p^2 \sin^4\theta} \tag{42}$$

Another case to examine is the stationary point within the interval but close to one end, say a distance p from b, where $p \lesssim h$. Again let the stationary point be s = 0. Referring back to Equation (10), we now have the first series and the pertinent part of the integral, i.e.

$$I = (iK)^{-1} \sum_{n=o}^{\infty} (-iK)^{-n} \left[G^{(n)}(g(-h))\, e^{iKg(-h)} - G^{(n)}(g(a))\, e^{iKg(a)} \right] + \int_{-h}^{p} ds\, e^{iKg(s)} \tag{43}$$

The evaluation of the sum and the portion of the integral for negative s proceeds as in the original case (see Equations 30-33 and associated discussion). The integral from 0 to p is proportional to a Fresnel integral, as in Equation (30), but it cannot now in general be replaced by the first term of the asymptotic expansion. We thus have

$$I = \frac{1}{2}\left(\frac{\lambda RR'}{R+R'}\right)^{1/2} e^{iK} \csc\theta \left\{ e^{i\pi/4} + 2^{1/2}\, F\left[2\left(\frac{R+R'}{2\lambda RR'}\right)^{1/2} p \sin\theta\right]\right\} \tag{44}$$

If p is sufficiently large, this can be simplified by means of Equation (31).

One more case remains to be mentioned for the sake of completeness, a simpler one the conditions for which have not been met by past experimental parameters. It occurs when the irradiated portion of the trail contains the specular point and is short enough so that the higher terms in the phase can be neglected throughout it (i.e. the expression in Equation (28) is small for $a \leqslant s \leqslant b$, and so are the successive terms). We then perform the integration for the whole region at once, obtaining

$$I = \frac{1}{2}\left(\frac{2\lambda RR'}{R+R'}\right)^{1/2} e^{iK} \csc\theta \left\{ F\left[2\left(\frac{R+R'}{2\lambda RR'}\right)^{1/2} |a| \sin\theta\right] + F\left[2\left(\frac{R+R'}{2\lambda RR'}\right)^{1/2} b \sin\theta\right]\right\} \tag{45}$$

Depending on the magnitude of the argument, it may be possible to approximate one or both of the Fresnel integrals by means of Equation (31).

Surveying the results of this section, we find that for f = 1 the return can vary from the specular value given by Equation (35) (or actually somewhat higher at the maximum of the diffraction pattern given by Equation (44)) to the non-specular value given by Equation (40), with a continuous spectrum of intermediate values indicated by Equations (44), (42),

(37) and (38). The extremum values differ by the order of λ/R, with a numerical factor that tends to worsen the gap (at a few meters wavelength, where the meteor trail observations have been made, the ratio of returns might be 10^6). The specular returns have a λ^3 dependence, the non-specular a λ^4 dependence, with an irregular transition in the intermediate region.

The experimental results on the wavelength dependence of meteor trail scattering can be accounted for in terms of these results. Comparing simultaneous returns at 4 and 8 meters from the same trail, Lovell and Clegg (Ref. 13) report the λ^3 dependence characteristic of specular scattering. They state that the sensitivity of their equipment was such that a meteor trail would not be detected at 4 m. unless its aspect were essentially specular. The intensity of the return increases rapidly with the wavelength, so that its detectability improves. The non-specular return varies as λ^4, increasing faster than the specular return. Since there are obviously far more non-specular than specular trails (except possibly for some special conditions of shower observations) the wavelength dependence at longer wavelengths can be expected to become λ^4. This is corroborated by Eckersley's observation (Ref. 15) of a λ^4 increase in total returns for wavelengths between 16 and 32 meters.

6.2 The Effect of Ambipolar Diffusion.

6.2.1 - Local Attenuation

A line distribution of ionization will not maintain itself as a line but will undergo ambipolar diffusion. As a result of this diffusion, electrons change position relative to each other, and hence the relative phase of their contribution to the return also changes. Herlofson (Ref. 16) proposed taking the attenuation due to these phase differences into account by integrating the amplitude of the electron returns over a plane perpendicular to the trail and taking the ratio of this value to that obtained for a line distribution of the same number of electrons. His calculation contains several oversimplifications: he implicitly assumes the meteoroid to have an infinite velocity (taking the diffusion to be identical all along the trail), and he uses a two-dimensional geometry which implies that the transmitter and receiver are lines of infinite extent parallel to the axis of the trail. His idea can be used for a more nearly correct treatment: consider a short element of length of the trail, and carry out the phase integral over a slice perpendicular to this element of length, for a point transmitter and a separate point receiver; then use the result as a factor in f(s) for the integral I.

To start with, we must determine the distribution function due to diffusion. For the moment, we shall assume that the ionization is formed at rest along the line (the consequences of this approximation will be discussed later). For each electron freed at a point at time t = 0, there is a probability density (Ref. 17)

$$C = (4\pi Dt)^{-3/2} \; e^{-\frac{r^2}{4Dt}} \tag{46}$$

for the electron to be at time t at a point r distant from the point of ionization, where D is the diffusion coefficient. Actually, as the meteoroid moves with velocity v, there will be a succession of such electrons originating at different points at different times. Choose the z-axis of a cylindrical coordinate system in the direction of motion of the meteoroid, the origin being the meteoroid's position (the trail extends over positive values of z). If an electron originated at a time t in the past, it must have then been at the point (z = vt, $\rho = 0$). Hence, the electron density due to it at the point (z, ρ) is

$$C = (4\pi Dt)^{-3/2} \, e^{-\frac{(z-vt)^2 + \rho^2}{4Dt}} . \tag{47}$$

We have stipulated before that in traversing a distance dz = vdt the meteoroid produces α free electrons. To find the electron density due to the meteoroid, we must integrate over the path covered, vdt, for all past time:

$$\begin{aligned} N &= \alpha v \int_0^\infty dt\,(4\pi Dt)^{-3/2} \, e^{-\frac{(z-vt)^2+\rho^2}{4Dt}} \\ &= (\alpha v/4\pi D)(z^2+\rho^2)^{-1/2} \, e^{-(v/2D)\left[(z^2+\rho^2)^{1/2} - z\right]} \end{aligned} \tag{48}$$

upon application of Reference 18, for instance. In reality, the trail originated at a finite time in the past, T. The error incurred in taking the upper limit infinite is

$$\begin{aligned} N &= \alpha v \int_T^\infty dt\,(4\pi Dt)^{-3/2} \, e^{-\left[\frac{(z-vt)^2+\rho^2}{4Dt}\right]} \\ &< (2/\pi^{1/2})(\alpha v/4\pi D)(z^2+\rho^2)^{-1/2} \, e^{(vz/2D)-(v^2T/4D)} \left[\int_0^{[(z^2+\rho^2)/4DT]^{1/2}} du\, e^{-u^2} \right] \\ &< (\alpha v/4\pi D)(z^2+\rho^2)^{-1/2} \, e^{-(v/2D)\left[(vT/2) - z\right]} \end{aligned} \tag{49}$$

so that for the approximation to be valid it suffices that $vT/2 > (z^2+\rho^2)^{1/2}$. At a later stage of the calculation, it will be shown that contributions to f(s) from values of $\rho > \rho_c$

can be neglected for the situations of interest, where ρ_c^2 can be kept below $\lambda r/\pi$ and $\lambda r'/\pi$. Hence, within the limitations of the subsequent discussion, Equation (48) can be used if the trail extends as far back again behind the furthest point of interest as the latter is behind the meteoroid.

We now seek to determine the attenuation factor

$$f(z) = e^{-iKg(z,0)} \int_0^{2\pi} d\psi \int_0^{\infty} \rho d\rho \, N \, e^{iKg(z,\rho)} \Bigg/ \int_0^{2\pi} d\psi \int_0^{\infty} \rho \, d\rho \, N . \quad (50)$$

The denominator is simply α, the electron density per unit length of trail, as expected.

For the phase, for fixed z we can express the range to a point in terms of the range to the axis and the distance off the axis, i.e.

$$\vec{r}(\rho) = \vec{r}(0) - \vec{\rho} \quad (51)$$

where

$$\vec{\rho} = \rho(\hat{x}\cos\psi + \hat{y}\sin\psi) \quad (51a)$$

Temporarily taking the origin at the point on the axis, $\vec{r}(0)$ and $\vec{r'}(0)$ are given by Equation (14), and we find

$$\begin{aligned} r^2(\rho) &= r^2(0) + \rho^2 - 2r(0)\,\rho\sin\theta(\cos\phi\cos\psi + \sin\phi\sin\psi) \\ r'^2(\rho) &= r'^2(0) + \rho^2 - 2r'(0)\,\rho\sin\theta'(\cos\phi'\cos\psi + \sin\phi'\sin\psi) \end{aligned} \quad (52)$$

so that (dropping the zeros for convenience)

$$\begin{aligned} Kg = (2\pi/\lambda) \Big\{ & r\left[1 - 2(\rho/r)\sin\theta\cos(\psi - \phi) + \rho^2/r^2\right]^{1/2} \\ & + r'\left[1 - 2(\rho/r')\sin\theta'\cos(\psi - \phi') + \rho^2/r'^2\right]^{1/2} \Big\} . \end{aligned} \quad (53)$$

Break up the ρ - integration in Equation (50) into two portions: $0 \leq \rho \leq \rho_c$ and $\rho_c < \rho < \infty$. Take ρ_c small enough so that the first two terms in the binomial expansion of the radicals suffice for $\rho \leq \rho_c$, i.e $\rho_c^2 << \lambda r/\pi$ and $\lambda r'/\pi$. Then

$$Kg = (2\pi/\lambda) \left\{ r + r' - \rho \left[\sin\theta \cos(\psi - \phi) + \sin\theta' \cos(\psi - \phi') \right] \right\} . \tag{54}$$

With a little trigonometric manipulation, the coefficient of ρ can be rewritten in the form

$$\sin\theta \cos(\psi - \phi) + \sin\theta' \cos(\psi - \phi') = \gamma \cos(\psi - \delta) \tag{55}$$

where

$$\gamma^2 = \sin^2\theta + \sin^2\theta' + 2 \sin\theta \sin\theta' \cos(\phi - \phi') \tag{55a}$$

and

$$\tan\delta = (\sin\theta \sin\phi + \sin\theta' \sin\phi')/(\sin\theta \cos\phi + \sin\theta' \cos\phi'). \tag{55b}$$

Instead of using the azimuthal angles, γ can be expressed in terms of the scattering angle β

$$\cos\beta = \vec{r} \cdot \vec{r}'/r\,r' = \cos\theta \cos\theta' + \sin\theta \sin\theta' \cos(\phi - \phi') \tag{56}$$

namely

$$\gamma^2 = 4\cos^2(\beta/2) - (\cos\theta + \cos\theta')^2 \tag{57}$$

The angular integration can now be carried out, just as in Herlofson's case:

$$\int_0^{2\pi} d\psi \; e^{iKg} = e^{i(2\pi/\lambda)(r+r')} \int_0^{2\pi} d\psi \; e^{-i(2\pi/\lambda)\, \gamma \rho \cos(\psi - \delta)}$$
$$= e^{i(2\pi/\lambda)(r+r')} \; 2\pi J_0(2\pi \gamma \rho / \lambda) . \tag{58}$$

Thus, the attenuation factor can be written as

$$f(z) = (v/2D)\, e^{vz/2D} \int_0^\infty \rho\, d\rho\, (z^2+\rho^2)^{-1/2}\, e^{-(v/2D)(z^2+\rho^2)^{1/2}}\, J_0(2\pi\gamma\rho/\lambda)$$

$$- (v/4\pi D)\, e^{vz/2D} \int_0^{2\pi} d\psi \int_{\rho_c}^\infty \rho\, d\rho\, (z^2+\rho^2)^{-1/2}\, e^{-(v/2D)(z^2+\rho^2)^{1/2}}\, e^{-i(2\pi/\lambda)\gamma\rho\cos(\psi-\delta)} \tag{59}$$

$$+ (v/4\pi D)\, e^{vz/2D}\, e^{-i(2\pi/\lambda)(r+r')} \int_0^{2\pi} d\psi \int_{\rho_c}^\infty \rho\, d\rho\, (z^2+\rho^2)^{-1/2}\, e^{-(v/2D)(z^2+\rho^2)^{1/2}}\, e^{iKg(s,\rho)} .$$

Provided that ρ_c can be chosen sufficiently large, subject to the earlier condition, the last two terms can be dropped. Before deciding this point, we must know the first term.

If we apply to the first term the coordinate change

$$t = (2\pi/\lambda)\, \gamma\, (z^2+\rho^2)^{1/2} \tag{60}$$

it can be evaluated as a Laplace transform (Ref. 19)

$$f(z) = (v/2D)\, e^{vz/2D} \int_0^\infty \rho\, d\rho\, (z^2+\rho^2)^{-1/2}\, e^{-(v/2D)(z^2+\rho^2)^{1/2}}\, J_0(2\pi\gamma\rho/\lambda)$$

$$= (v\lambda/4\pi D\gamma)\, e^{vz/2D} \int_{2\pi\gamma z/\lambda}^\infty dt\, e^{-v\lambda t/4\pi D\gamma}\, J_0\left[(t^2 - 4\pi^2\gamma^2 z^2/\lambda^2)^{1/2}\right] \tag{61}$$

$$= (v\lambda/4\pi D\gamma)\left[(v\lambda/4\pi D\gamma)^2+1\right]^{-1/2} e^{-(2\pi/\lambda)\gamma z\left[(v\lambda/4\pi D\gamma)^2+1\right]^{1/2} + (vz/2D)} .$$

In all cases of interest, the expression simplifies because $(v\lambda/4\pi D\gamma) \gg 1$ (taking typical values v = 40 km/sec and D = 4 m^2/sec, and $\gamma = 2$, this is true provided $\lambda \gg 2.5$ mm;

even for a very unfavorable combination of parameters, this lower limiting value will not exceed 4 cm.). Hence, upon expanding the exponent

$$f(z) = e^{-4\pi^2 \gamma^2 Dz/v\lambda^2} \qquad (62)$$

An upper bound on the last two terms of Equation (59) can be obtained by replacing the integrands by their absolute value:

$$\delta f(z) = (v/4\pi D)\, e^{vz/2D} \int_0^{2\pi} d\psi \int_{\rho_c}^{\infty} \rho\, d\rho\, (z^2+\rho^2)^{-1/2}\, e^{-(v/2D)(z^2+\rho^2)^{1/2}}$$

$$= e^{-(v/2D)\left[(z^2+\rho_c^2)^{1/2} - z\right]} \qquad (63)$$

In order for these terms to be negligible, it is necessary that the exponent of Equation (63) be large and that it be larger than that of Equation (62). Solving the resulting inequalities for ρ_c, and recapitulating the previous condition, we have

$$\frac{\lambda r}{\pi} \gg \rho_c^2 \gg (4D^2/v^2) + (4Dz/v) \qquad (64)$$

$$\frac{\lambda r}{\pi} \gg \rho_c^2 \gg (4\pi\gamma Dz/v\lambda)^2 \left[1 + (2\pi\gamma D/v\lambda)^2\right] . \qquad (65)$$

Even for the least favorable situation, the first term on the right side of Equation (64) is less than 10^{-4} m^2; the second term, taking z as large as 20 km, is no more than some 100 m^2. Even at the minimum range, Equation (64) can be satisfied provided only that λ be more than 3 mm. In Equation (65), the second term in the bracket is small compared with the first (see comment below Equation 61). The remaining inequality becomes

$$\lambda^3 \gg 16\pi^3 \gamma^2 D^2 z^2/v^2 r . \qquad (66)$$

For the typical values given above, with the most unfavorable range and trail length, this reduces to $\lambda \gg 0.5m$; for the unfavorable set of parameters it is $\lambda \gg 2m$.

Actually, the restriction implied by Equation (66) can be further limited. A large value of z can have such a strong attenuation factor connected with it as to be of no interest. A very safe criterion for the maximum relevant z is a value for which the attenuation exceeds the amplitude reduction from specular to non-specular scattering by, say, a factor of 10. This reduction (taking Equations 36 and 41 with $\cos\theta = 1$ for simplicity) is

$$(S_{non-s}/S_s)^{1/2} = (2\pi)^{-1} (\lambda/r)^{1/2} \quad . \qquad (67)$$

Thus the maximum z we need consider will be given by

$$e^{-4\pi^2 \gamma^2 Dz/v\lambda^2} = (20\pi)^{-1} (\lambda/r)^{1/2} \qquad (68)$$

which yields

$$z = (v\lambda^2/4\pi^2 \gamma^2 D) \left[\ln 20\pi - (1/2)\ln\lambda + (1/2)\ln r \right] \quad . \qquad (69)$$

As λ decreases, z_{max} decreases faster, so that the sense of the inequality in Equation (66) gets reversed, i.e. if Equation (69) is substituted into Equation (66) there results

$$\lambda \ll \pi \gamma^2 r \left[\ln 20\pi - (1/2)\ln\lambda + (1/2)\ln r \right]^{-2} , \qquad (70)$$

which reduces to

$$\lambda \ll (\gamma^2/6) r \quad . \qquad (70')$$

Thus, for any z of conceivable interest, f(z) will be given by Equation (62) provided only that

$$(\gamma^2/6) r \gg \lambda \gg 4\pi \gamma D/v \quad . \qquad (71)$$

The lower limit, as pointed out above, is no larger than 4 cm.

As a special case of Equation (62), if the point on the axis happens to be the specular point, γ simplifies and

$$f(z) = e^{-16\pi^2 (Dz/v\lambda^2)\cos^2(\beta/2)} \quad . \qquad (62')$$

This expression was obtained by Eshleman (Ref. 11) by a slight extension of Herlofson's calculation (Ref. 16). Herlofson's own result applies to the further specialization of normal incidence with specular scattering:

$$f(z) = e^{-16\pi^2 Dz/v\lambda^2} \quad . \qquad (62'')$$

6.2.2 - Scattering by the Trail

Equation (62) specifies the amplitude attenuation due to ambipolar diffusion in a plane perpendicular to the trail axis (within the wavelength limitations specified). The effect of this attenuation upon the return from the whole visible trail remains to be examined.

If we assume that the specular point $z = z_0$ lies within the visible region for sharply-defined overlapping beam patterns, and if we measure s from the specular point, the pattern factor becomes now

$$f(s) = e^{-4\pi^2 \gamma^2 Dz/v\lambda^2} = e^{-4\pi^2 \gamma^2 Dz_0/v\lambda^2} \, e^{-4\pi^2 \gamma^2 Ds/v\lambda^2} \quad . \qquad (72)$$

Instead of Equation (30), there occurs

$$I_1 = e^{iK} e^{-4\pi^2 \gamma^2 Dz_0/v\lambda^2} \int_{-h}^{h} ds \; e^{i\frac{\pi}{2}\left[2\left(\frac{R+R'}{2\lambda RR'}\right)^{1/2} s \sin\theta\right]^2} e^{-4\pi^2 \gamma^2 Ds/v\lambda^2} \quad . \qquad (73)$$

We next make the changes of variable

$$u = 2\left[(R+R')/2\lambda RR'\right]^{1/2} s \sin\theta \tag{73a}$$

$$H = 2\left[(R+R')/2\lambda RR'\right]^{1/2} h \sin\theta \tag{73b}$$

$$A = \left[(R+R')/2\lambda RR'\right]^{1/2} v\lambda^2 \sin\theta/2\pi^2\gamma^2 D \tag{73c}$$

so that Equation (73) is rewritten as

$$I_1 = e^{iK} e^{-4\pi^2\gamma^2 Dz_0/v\lambda^2}\left[\lambda RR'/2(R+R')\right]^{1/2} \csc\theta \int_{-H}^{H} du\, e^{i(\pi/2)u^2} e^{-u/A} . \tag{73'}$$

In this form, the various parameters that affect the integral are lumped into the two combinations H and A; it will be convenient below to refer to values of H and A rather than run through various combinations of the larger set of parameters.

The integrand can be put into a standard form by resort to the complex variable

$$w = u + i/\pi A . \tag{74}$$

Then,

$$\int_{-H}^{H} du\, e^{i(\pi/2)u^2 - u/A} = e^{i/(2\pi A^2)} \int_{-H+i/\pi A}^{H+i/\pi A} dw\, e^{i(\pi/2)w^2}$$

$$= e^{i/(2\pi A^2)}\left[\int_0^{H+i/\pi A} dw\, e^{i(\pi/2)w^2} + \int_0^{H-i/\pi A} dw\, e^{i(\pi/2)w^2}\right] . \tag{75}$$

The last integrals are Fresnel integrals of complex argument, in terms of which Equation (73') becomes

$$I_1 = e^{i\left[K+(1/2\pi A^2)\right]} e^{-4\pi^2 \delta^2 Dz_0/v\lambda^2} \left[\lambda RR'/2(R+R')\right]^{1/2} \csc\theta \left[F(H+i/\pi A)+F(H-i/\pi A)\right]. \qquad (76)$$

The introduction of f(s) from Equation (72) also changes Equation (32) to

$$I_2 = e^{iK} e^{-4\pi^2 \gamma^2 Dz_0/v\lambda^2} \left[\lambda RR'/2(R+R')\right]^{1/2} \csc\theta \,(2i/\pi H)\, e^{i(\pi/2)H^2} \cosh(H/A) \ . \qquad (77)$$

The expansion of the Fresnel integral given in Equation (31) is valid for complex argument, U, provided that

$$-\pi/4 < \arg U < 3\pi/4 \qquad (78)$$

as can be verified, for instance, from the confluent hypergeometric function representation of the Fresnel integral (Ref. 20). Equation (78) is satisfied for both Fresnel integrals of Equation (76) provided that

$$\pi A H > 1 \quad . \qquad (79)$$

In that case, Equation (31) yields

$$F(H + i/\pi A) + F(H - i/\pi A) = 2^{1/2} e^{i\pi/4} - (2i/\pi)\, e^{i(\pi/2)H^2 - i/2\pi A^2} \left[H^2+(\pi A)^{-2}\right]^{-1} \left[H \cosh(H/A)+(i/\pi A)\sinh(H/A)\right] \qquad (80)$$

if terms beyond the second in Equation (31) are neglected. Unlike the situation in the absence of diffusion, adding Equation (77) to Equation (76) does not now simplv cancel the second term in the expansion of the Fresnel integral. The condition that the remainder be at least a factor $(2\pi)^{-1}$ smaller than the first term of Equation (80) is expressed by

$$\left| 2A\, e^{i(\pi/2)H^2} \left[1+(\pi AH)^2\right]^{-1} \left[\sinh(H/A)+i(\pi AH)^{-1}\cosh(H/A)\right] \right| \leqslant 2^{1/2}(2\pi)^{-1} \qquad (81)$$

We must re-examine the requirements imposed by the sequence of integrations by parts and its truncation. We now have

$$G'(g) = \frac{d}{dg}\frac{f}{g'} = \frac{1}{g'}\frac{d}{ds}\frac{f}{g'} = \frac{f'g' - fg''}{(g')^3} \qquad (82)$$

so that Equation (25) is replaced by

$$\left|\frac{G'(g)}{KG(g)}\right| = \frac{|g'' + g'(-f'/f)|}{K(g')^2} = \frac{1+(H/A)}{\pi H^2} \qquad (83)$$

and we now stipulate that

$$(\pi H^2)^{-1}\left[1+(H/A)\right] \leqslant (2\pi)^{-1} \quad . \qquad (84)$$

Equations (81) and (84) each set a lower bound on A, expressed as a function of H. Inasmuch as this bound increases with H in the first case and decreases with H in the second (in both instances monotonically), the minimum acceptable value of A is obtained by requiring the equality to hold in each case and solving simultaneously. The result is A = 1, H = 2.8. With these values, the neglect of the higher order terms in the expansion of the Fresnel integrals in Equation (80) is justified. Furthermore, the phase $(2\pi A^2)^{-1}$ can be omitted. The combination of Equations (76) and (77) then leads to

$$I = e^{i\left[K+(\pi/4)\right]} e^{-16\pi^2(Dz_0/v\lambda^2)\cos^2(\beta/2)} \left[\lambda RR'/(R+R')\right]^{1/2} \csc\theta \tag{85}$$

which is just the stationary-phase value.

The discussion so far has left out of consideration the contributions of the end-point series. Because of the exponential factor introduced by consideration of diffusion, the front-end contribution ("head echo") is now enhanced relative to the specular value and may become competitive with it in magnitude. It yields

$$I' = i\left[\lambda RR'/2\pi(R+R')\, a \sin^2\theta\right] e^{iKg(a)} e^{-4\pi^2\gamma^2 Dz_0/v\lambda^2} e^{-4\pi^2\gamma^2 Da/v\lambda^2} \tag{86}$$

$$= -i\, e^{iKg(a)} e^{-16\pi^2(Dz_0/v\lambda^2)\cos^2(\beta/2)} \left[\lambda RR'/(R+R')\right]^{1/2} \csc\theta\, 2^{-1/2} (\pi L)^{-1} e^{L/A}$$

where

$$L = -2\left[(R+R')/2\lambda RR'\right]^{1/2} a \sin\theta \ . \tag{86a}$$

The validity of Equation (86) is limited by the fact that it represents the first term in a sequence of integrations by parts. Corresponding to Equation (84), there is therefore the requirement that

$$(\pi L^2)^{-1}\left[1 + (L/A)\right] \lesssim (2\pi)^{-1} \tag{87}$$

for Equation (86) to be a good approximation.

The relative magnitude of the head echo as against the specular return is

$$\left|I'/I\right| = 2^{-1/2} (\pi L)^{-1} e^{L/A} \ . \tag{88}$$

The minimum value of this expression for a given A occurs for L = A. Hence, if the ratio is to be less than $(2\pi)^{-1}$ for <u>any</u> L at all, A must be at least $2^{1/2}e$. If it is to remain that small over a significant range of L, a larger A is required. Obviously, the larger the L that has to be considered, the greater A must be before the head echo can be neglected. At the other end, Equation (86) alone suffices provided that the ratio of Equation (88) is greater than 2π and Equation (87) is satisfied. It should be noted that, if these conditions are met, Equation (86) can be used for $A < 1$, since for smaller A the specular return will be less than the value given by Equation (85). If Equation (87) is satisfied and the ratio of Equation (88) has an intermediate value, we need the sum of Equations (85) and (86):

$$I_{tot} = e^{-16\pi^2(Dz_0/v\lambda^2)\cos^2(\beta/2)} \left[\lambda RR'/(R+R')\right]^{1/2} \csc\theta \times \left[e^{i\left[K+(1/2\pi A^2)+(\pi/4)\right]} - i\,e^{iKg(a)}\, 2^{-1/2}\, (\pi L)^{-1}\, e^{L/A}\right] . \qquad (89)$$

Equation (89) holds up to terms smaller by a factor $(2\pi)^{-1}$ with no other restriction on A than Equation (87); the additional constraints due to the limitations on Equation (85) can be consistently relaxed because of the contribution of the head-echo term. The realm of validity of Equations (85), (86), and (89) in terms of values of A and L is displayed graphically in Figure 6-1.

Let us next consider the case where the front end of the visible trail is located at the specular point (if this front end is defined by the meteoroid, and not by a beam cut-off, z_0 is zero). Instead of Equation (76) there now occurs

$$I_1 = e^{i\left[K+(1/2\pi A^2)\right]}\, e^{-16\pi^2(Dz_0/v\lambda^2)\cos^2(\beta/2)} \left[\lambda RR'/2(R+R')\right]^{1/2} \csc\theta \left[F(H+i/\pi A) - F(i/\pi A)\right] \qquad (90)$$

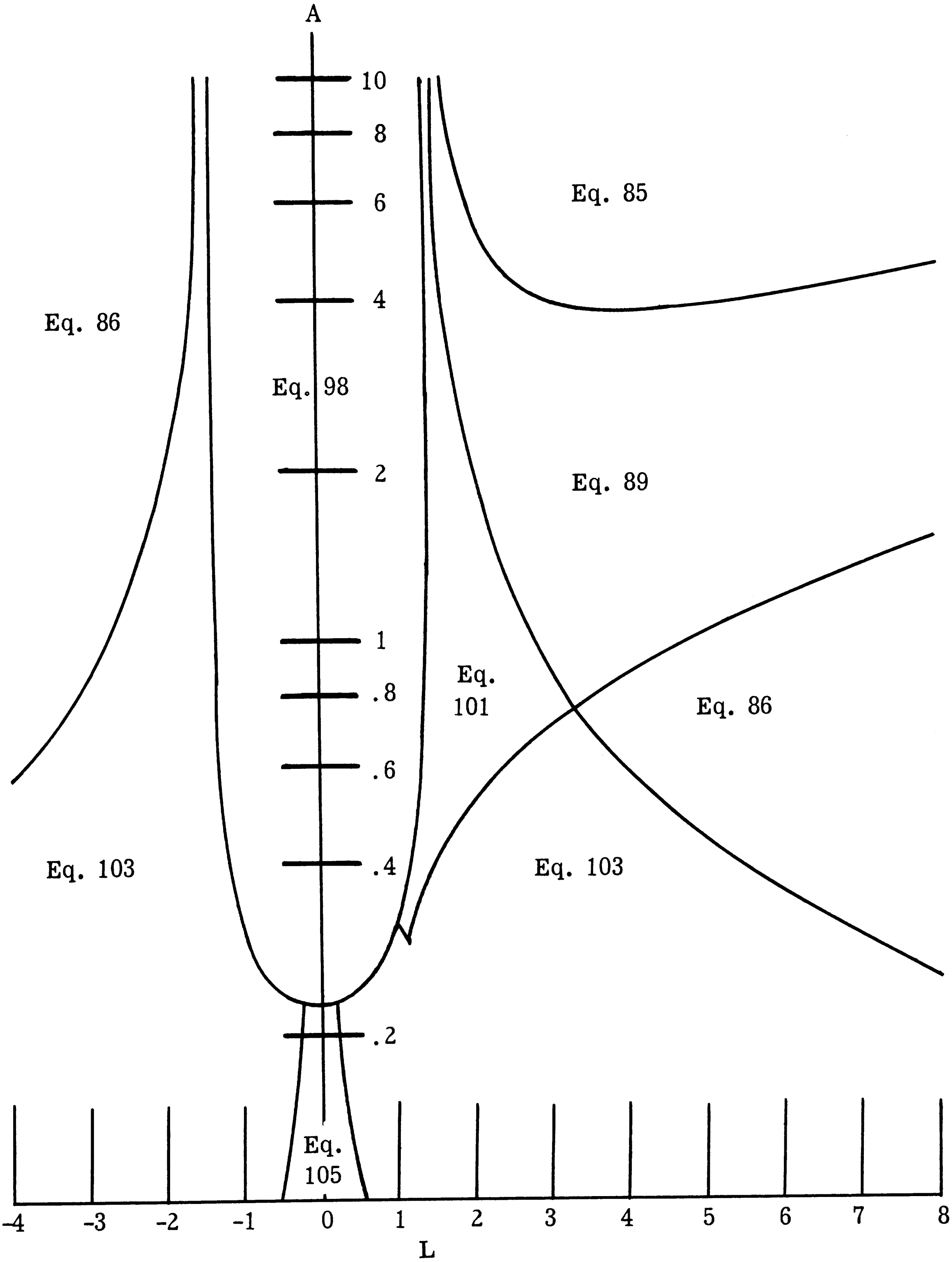

FIG. 6-1: REALMS OF VALIDITY OF EQUATIONS

and instead of Equation (77)

$$I_2 = e^{iK}\, e^{-16\pi^2(Dz_0/v\lambda^2)\cos^2(\beta/2)} \left[\lambda RR'/2(R+R')\right]^{1/2} \csc\theta\,(i/\pi H)\, e^{i(\pi/2)H^2}\, e^{-H/A} \; . \tag{91}$$

We can now always choose H sufficiently large that the first Fresnel integral can be replaced by the first term of its asymptotic expansion and that the contribution of Equation (91) can be neglected . (For small A, the negative exponential predominates strongly enough to insure this despite the near-cancellation of the two terms in Equation 90).

Thus, we have

$$I = e^{i\{K+(1/2\pi A^2)\}}\, e^{-16\pi^2(Dz_0/v\lambda^2)\cos^2(\beta/2)} \left\{\lambda RR'/2(R+R')\right\}^{1/2} \csc\theta \left[2^{-1/2}\, e^{i\pi/4} - F(i/\pi A)\right] . \tag{92}$$

For A sufficiently large, the argument of the Fresnel integral in Equation (92) is small, and the Fresnel integral is approximately equal to its argument. In order for the higher terms in the power series to be negligible to order $(2\pi)^{-1}$, we must have $A > .67$.

Then

$$I = e^{i\left[K+(1/2\pi A^2)\right]}\, e^{-16\pi^2(Dz_0/v\lambda^2)\cos^2(\beta/2)} \left[\lambda RR'/2(R+R')\right]^{1/2} \csc\theta \left[2^{-1/2}\, e^{i\pi/4} - i/\pi A\right] . \tag{93}$$

If $A > 2^{3/2}$, the contribution of $F(i/\pi A)$ can in fact be completely neglected, and the phase factor $(2\pi A^2)^{-1}$ can be ignored, leaving

$$I = (1/2)\, e^{i\left[K+(\pi/4)\right]}\, e^{-16\pi^2(Dz_0/v\lambda^2)\cos^2(\beta/2)} \left[\lambda RR'/(R+R')\right]^{1/2} \csc\theta \tag{94}$$

which, except for the negative exponential factor, is the value obtained without consideration of diffusion.

At the other end, for small A the asymptotic expansion holds for the Fresnel integral in Equation (92), namely

$$F(i/\pi A) = 2^{-1/2}\, e^{i\pi/4} - A\, e^{-i/2\pi A^2} - i\pi A^3\, e^{-i/2\pi A^2} + \ldots \quad . \tag{95}$$

If the third term is a factor $(2\pi)^{-1}$ smaller than the second, i.e. if $A \lesssim 2^{-1/2}\pi^{-1}$, Equation (92) can be reduced to

$$I = A\, e^{iK}\, e^{-16\pi^2(Dz_0/v\lambda^2)\cos^2(\beta/2)} \left[\lambda RR'/2(R+R')\right]^{1/2} \csc\theta \quad . \tag{96}$$

This is the value that would be obtained by pulling the phase factor out of the integral. It thus represents a situation in which the negative exponential dominates the behavior and overrides the phase.

So far, the discussion has covered the front of the visible trail being well ahead of the specular point (i.e. Equation (87) being satisfied) or being right at it. Next, we cover the intermediate situation. This involves adding to Equation (92)

$$I' = e^{iK}\, e^{-16\pi^2(Dz_0/v\lambda^2)\cos^2(\beta/2)} \left[\lambda RR'/2(R+R')\right]^{1/2} \csc\theta \int_{-L}^{0} du\, e^{i(\pi/2)u^2}\, e^{-u/A}$$

$$= e^{i\left[K+(1/2\pi A^2)\right]}\, e^{-16\pi^2(Dz_0/v\lambda^2)\cos^2(\beta/2)} \left[\lambda RR'/2(R+R')\right]^{1/2} \csc\theta \left[F(i/\pi A) - F(-L+i/\pi A)\right] \tag{97}$$

to obtain

$$I = e^{i\left[K+(1/2\pi A^2)\right]}\, e^{-16\pi^2(Dz_0/v\lambda^2)\cos^2(\beta/2)} \left[\lambda RR'/2(R+R')\right]^{1/2} \csc\theta \left[2^{-1/2}\, e^{i\pi/4} - F(-L+i/\pi A)\right] \tag{98}$$

or alternatively, since F is an odd function,

$$I = e^{i\left[K+(1/2\pi A^2)\right]}\, e^{-16\pi^2(Dz_0/v\lambda^2)\cos^2(\beta/2)} \left[\lambda RR'/2(R+R')\right]^{1/2} \csc\theta \left[2^{-1/2}\, e^{i\pi/4} + F(L-i/\pi A)\right] . \tag{99}$$

An expression resembling this is apparently used by Loewenthal (Ref. 21), though the approximations he made in obtaining it are not sufficiently justified, and multiple typographical errors obscure the results.

For L sufficiently large, or A sufficiently small, F can be replaced by its asymptotic expansion. As the criterion for the usefulness of this expansion, we impose the requirement that the magnitude of the third term be no more than $(2\pi)^{-1}$ of the second, whence

$$L^2 + (\pi A)^{-2} \gg 2 \quad . \tag{100}$$

If $\pi AL > 1$, Equation (99) then yields

$$I = e^{-16\pi^2 (Dz_0/v\lambda^2)\cos^2(\beta/2)} \left[\lambda RR'/(R+R')\right]^{1/2} \csc\theta$$

$$\times \left\{ e^{i\left[K+(1/2\pi A^2)+(\pi/4)\right]} - ie^{i\left[K+(\pi/2)L^2\right]} 2^{-1/2} \left[\pi(L-i/\pi A)\right]^{-1} e^{L/A} \right\} \tag{101}$$

which looks just like Equation (89) except that the phase factor of the second term has been expanded; and $(L-i/\pi A)$ appears instead of L in the denominator. If $\pi AL > 2\pi$, as it must be whenever Equation (87) is satisfied, this latter difference can be neglected. Thus, Equations (89) and (101) are mutually consistent. The specular term cannot dominate the head-echo term by a factor 2π for any value of the parameters for which Equation (87) does not hold. On the other hand, the special case does occur for which the head-echo term dominates by a factor 2π, i.e.

$$\left[L^2 + (\pi A)^{-2}\right]^{-1/2} e^{L/A} \gg 2^{3/2}\, \pi^2 \quad . \tag{102}$$

When this happens,

$$I = i\, e^{i[K+(\pi/2)L^2]}\, e^{-16\pi^2(Dz_0/v\lambda^2)\cos^2(\beta/2)} \left[\lambda RR'/2(R+R')\right]^{1/2} \csc\theta \left[\pi(-L+i/\pi A)\right]^{-1} e^{L/A} \quad . \tag{103}$$

If $\pi AL < 1$ and Equation (100) holds, the asymptotic expansion can be applied to the Fresnel integral of Equation (98). When this is done, Equation (103) emerges again. Pushing on further, if

$$\pi AL \lesssim (2\pi)^{-1} \quad , \tag{104}$$

Equation (103) simplifies to

$$I = e^{i[K+(\pi/2)L^2]}\, e^{-16\pi^2(Dz_0/v\lambda^2)\cos^2(\beta/2)} \left[\lambda RR'/2(R+R')\right]^{1/2} \csc\theta\, A\, e^{L/A} \quad . \tag{105}$$

In Figure 6-1, the realm of validity in terms of values of A and L of the successively more general Equations (105), (103), (101), and (98) is displayed graphically.

The extension to the front of the visible trail falling somewhat short of the specular point is now very simple. Equation (98) applies with L negative. If Equation (100) is satisfied, this reduces to Equation (103). If in addition

$$\pi A\,|L| \lesssim (2\pi)^{-1} \quad , \tag{106}$$

it reduces further to Equation (105). On the other hand, if

$$(\pi L^2)^{-1} \left[1 - (L/A)\right] \lesssim (2\pi)^{-1} \, , \tag{107}$$

Equation (86) can be used. The realm of validity of the various equations for L negative is also depicted in Figure 6-1.

For very narrow beams including the specular point, there would occur the generalization of Equation (45)

$$I = e^{i\left[K+(1/2\pi A^2)\right]} e^{-16\pi^2(Dz_0/v\lambda^2)\cos^2(\beta/2)} \left[\lambda RR'/2(R+R')\right]^{1/2} \csc\theta \left[F(L+i/\pi A)+F(L'-i/\pi A)\right] \tag{108}$$

where

$$L' = 2\left[(R+R')/2\lambda RR'\right]^{1/2} b \sin\theta \tag{108a}$$

and the F's may or may not be expandable.

Wherever in the foregoing the exact Fresnel integral of complex argument has to be resorted to, numerical values for it can be obtained from Reference 22. In that table, the function computed is

$$\Phi(z) = 2\pi^{-1/2}\, e^{z^2} \int_z^{\infty} e^{-t^2}\, dt \; . \tag{109}$$

In terms of this function, the Fresnel integral is given by

$$F(U) = 2^{-1/2}\, e^{i\pi/4} \left\{ 1 - e^{i(\pi/2)U^2}\, \Phi\left[(\pi/2)^{1/2}\, e^{-i\pi/4}\, U\right]\right\} . \tag{110}$$

For completely non-specular scattering, we parallel closely the treatment for f = 1 above, except that the negative exponential reduces the contribution from the back end of the trail relative to that from the front, thus eliminating the first of the two terms in Equation (37) and with it the phase difficulty. Instead of Equation (38), there now occurs

$$I = i\left[\lambda/2\pi\,(\cos\theta + \cos\theta')\right] e^{iK(a)}\, e^{-4\pi^2 \gamma^2 Dz(a)/v\lambda^2} \; . \tag{111}$$

The validity of this expression is limited by the requirement

$$\left|\frac{G'(g)}{KG(g)}\right| = \frac{|g''+g'(-f'/f)|}{K(g')^2} < \frac{1}{2\pi} \; . \tag{112}$$

As the first term of Equation (112) cannot threaten the inequality for non-specular scattering, this simplifies to

$$(Kg')^{-1}(-f'/f) = \left[\lambda/2\pi(\cos\theta + \cos\theta')\right]\left[4\pi^2\gamma^2 D/v\lambda^2\right] < (2\pi)^{-1} \tag{113}$$

or

$$\lambda > 4\pi^2\gamma^2 D/v\ (\cos\theta + \cos\theta') \quad . \tag{114}$$

If we set $D = 4\ m^2/sec$, $v = 40\ km/sec$, and ignore the angular factors (both in γ^2 and in the denominator), this requires $\lambda > 1.6$ cm.

The fact that Equation (111) holds down to quite small wavelengths is not surprising. Away from the specular point, the oscillations of the phase factor become more violent and are able to override a stronger damping factor in dominating the characteristics of the scattering.

Having obtained results for the scattering by the trail in various positions, we can now relate these results to map out the intensity variation as the trail moves across the field of view. It has been assumed in the literature until now that, if the specular point is in the field of view, this variation is given by an exponential decay beyond the specular point (Equation 85). As we have seen, this is true only in the large A (i.e. large wavelength limit. As A decreases, the variation becomes less strong than the exponential, and in fact reaches a constant in the limit of small A. To illustrate this behavior, Figure 6-2 exhibits the scattering intensity as a function of L for various A's; for convenience, the received power is expressed in units of the no-diffusion specular return (Equation 35).

For purposes of quick orientation, Table I lists the A values corresponding to some representative situations. In part a), specular backscattering is considered; in part b), specular "forward" scattering for a horizontal trail coplanar with the transmitter and the

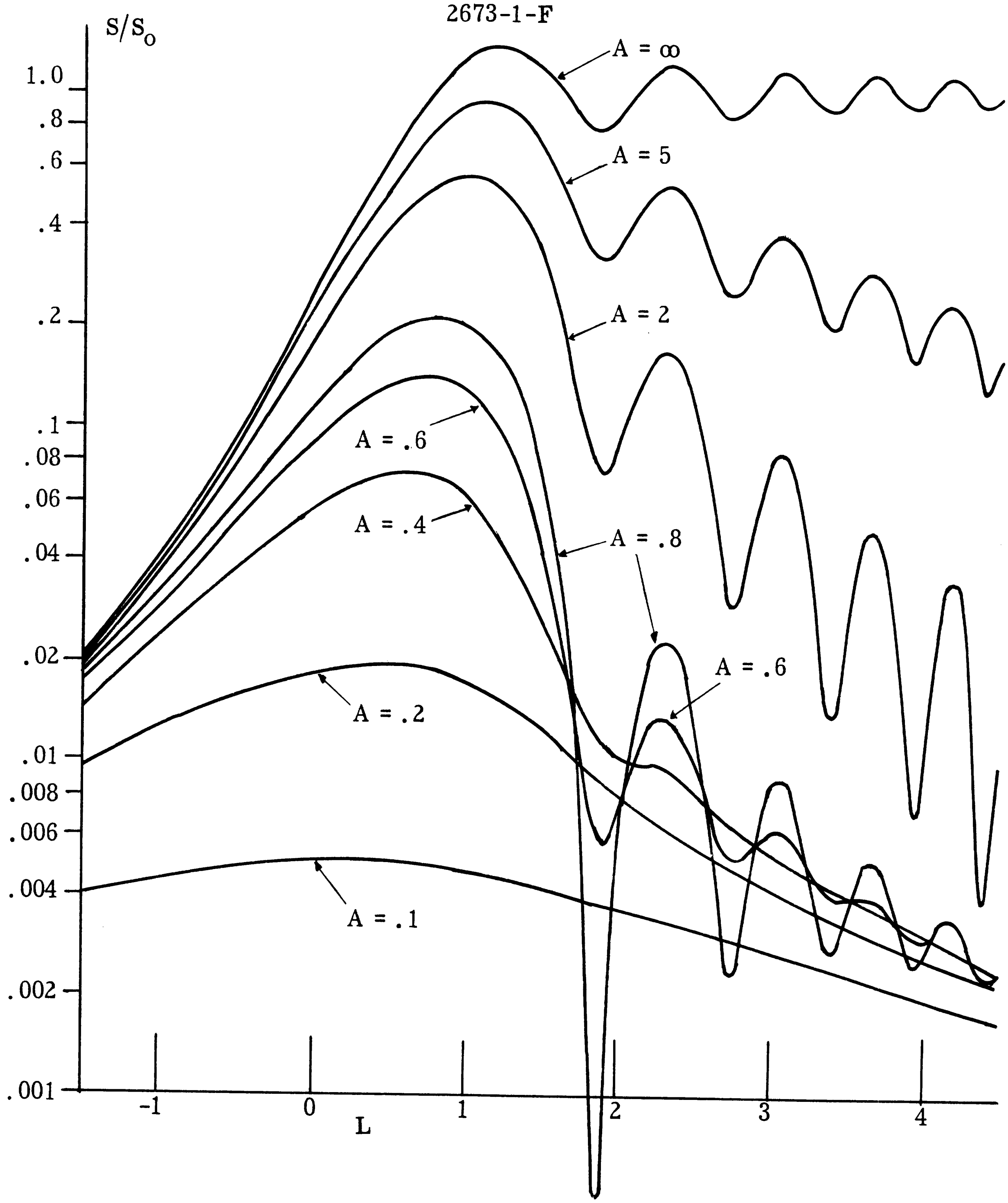

FIG. 6-2: RELATIVE SCATTERING INTENSITY

Table I

Values of A as a Function of Range (R) and Wavelength (λ)

(assuming v = 40 km/sec, D = 4 m^2/sec)

a) Specular Backscattering

λ(m) / R(km)	10	8	6	4	2	1
90	13	9.6	6.2	3.4	1.2	.42
200	8.9	6.4	4.2	2.3	.80	.28
300	7.3	5.2	3.4	1.9	.65	.23
600	5.2	3.7	2.4	1.3	.46	.16
1000	4.0	2.9	1.9	1.0	.36	.13

b) Specular "Forward" Scattering

(for a horizontal trail, at an altitude of 90 km, coplanar with the transmitter and receiver)

λ(m) / R(km)	10	8	6	4	2	1
90	13	9.6	6.2	3.4	1.2	.42
200	19	14	9.0	4.9	1.7	.61
300	23	16	11	5.7	2.0	.72
600	26	19	12	6.7	2.4	.84
1000	24	17	11	6.1	2.2	.76

[Transmitter-receiver separation (△) along great-circle path corresponding to "forward" scattering ranges:

R(km)	90	200	300	600	1000
△ (km)	0	350	530	1200	2000

]

receiver. Typical values of the parameters v = 40 km/sec and D = 4 m^2/sec have been assumed for a trail altitude of 90 km. The reader can easily find A from Equation (73c) for other geometries, different wavelengths and ranges, and alternative values of the velocity and the diffusion constant.

6.3 Additional Refinements

6.3.1 - Initial Stage of Trail Formation

In the ambipolar diffusion treatment above, there is implicit the assumption that the ionization is formed at rest at the site of the impact. Actually, the high velocity of the meteoroid implies that the particles will start with a large kinetic energy, and hence the ionization will initially spread out much more rapidly. This first stage of dissemination (until the ionization is slowed down to thermal velocities) is very brief, so that effectively the ambipolar diffusion starts from an instantaneous distributed source instead of a point source. The longitudinal component of the initial penetration (along the axis of the trail) primarily just displaces the head of the trail relative to the position of the meteoroid. The transverse component has been studied by Öpik (Ref. 23), who first pointed out this phenomenon. He obtains a Gaussian distribution and quotes the results of numerical calculations of its width for different meteor velocities and compositions; the width is inversely proportional to the atmospheric density, and hence is highly altitude-dependent.

To account for this effect, the instantaneous point source expressed by Equation (47) must be modified by shifting the source point off the trail axis, i.e. making the change

$$\rho^2 \rightarrow |\rho - \rho'|^2 = \rho^2 + \rho'^2 - 2\ \rho\rho' \cos\ \psi' \qquad (115)$$

and then integrating with respect to ρ' and ψ' over the normalized distribution function

$$(\pi a^2)^{-1} \quad e^{-(\rho'/a)^2} \quad . \tag{116}$$

Thus instead of the C of Equation (47) there will occur

$$C' = (4\pi Dt)^{-3/2}(\pi a^2)^{-1} \int_0^\infty \rho' d\rho' e^{-(\rho'/a)^2} \int_0^{2\pi} d\psi' \, e^{-\left[(z-vt)^2+\rho^2+\rho'^2-2\rho\rho'\cos\psi'\right]/4Dt} \quad . \tag{117}$$

The ψ' integral is similar to Equation (58)

$$\int_0^{2\pi} d\psi' \quad e^{\rho\rho'\cos\psi'/2Dt} = 2\pi \, I_0\,(\rho\rho'/2Dt) \quad . \tag{118}$$

The remaining ρ' integral is Weber's integral (Ref. 24)

$$\int_0^\infty \rho' d\rho' \; e^{-\rho'^2\left[(1/a^2)+(1/4Dt)\right]} \; I_0\,(\rho\rho'/2Dt)$$

$$= \frac{1}{2}\left(\frac{1}{4Dt} + \frac{1}{a^2}\right)^{-1} e^{(\rho/2Dt)^2/4\left[(1/a^2)+(1/4Dt)\right]} \tag{119}$$

so that

$$C' = \pi^{-1} \; (4\pi Dt)^{-1/2} \; (4Dt+a^2)^{-1} \; e^{-(z-vt)^2/4Dt} \; e^{-\rho^2/(4Dt+a^2)} \quad . \tag{120}$$

With C' replacing C, the computation of f(z) proceeds along the same lines as before. Subject to the additional condition

$$\rho_c \gg a \tag{121}$$

which can be easily satisfied for Öpik's values of a, the analysis is essentially identical. The attenuation factor is now

$$f(z) = \int_0^{2\pi} d\psi \int_0^\infty \rho\, d\rho \int_0^\infty dt\, C' \, e^{-i(2\pi/\lambda)\,\gamma\rho\,\cos(\psi-\delta)} \Bigg/ \int_0^{2\pi} d\psi \int_0^\infty \rho\, d\rho \int_0^\infty dt\, C' \tag{122}$$

It is convenient to defer the t integration. The ψ integration in the numerator has already been performed (Equation 58), and the subsequent ρ integral is again Weber's integral (Ref. 24)

$$\int_0^\infty \rho d\rho \; e^{-\rho^2/(4Dt+a^2)} J_0(2\pi \gamma \rho / \lambda) = (1/2)(4Dt+a^2) \, e^{-\pi^2 \gamma^2 (4Dt+a^2)/\lambda^2} \qquad (123)$$

The ρ integration in the denominator is trivial:

$$\int_0^\infty \rho d\rho \; e^{-\rho^2/(4Dt+a^2)} = (1/2)(4Dt+a^2) \qquad (124)$$

There remains

$$f(z) = \frac{e^{-\pi^2 \gamma^2 a^2/\lambda^2} \int_0^\infty dt \; t^{-1/2} e^{-(z^2/4Dt)-(v^2t/4D)\left[1 + (4\pi \gamma D/v\lambda)^2\right]}}{\int_0^\infty dt \; t^{-1/2} e^{-(z^2/4Dt)-(v^2t/4D)}} \qquad (125)$$

These integrals are given by Reference 25:

$$\int_0^\infty dt \; t^{-1/2} e^{-(z^2/4Dt)-(v^2t/4D)\left[1 + (4\pi \gamma D/v\lambda)^2\right]} \qquad (126)$$

$$= (4\pi D)^{1/2} v^{-1} \left[1+(4\pi \gamma D/v\lambda)^2\right]^{-1/2} e^{-(vz/2D)\left[1+(4\pi\gamma D/v\lambda)^2\right]^{1/2}}$$

$$\int_0^\infty dt \; t^{-1/2} e^{-(z^2/4Dt)-(v^2t/4D)} = (4\pi D)^{1/2} v^{-1} e^{-vz/2D} \quad . \qquad (127)$$

Hence

$$f(z) = e^{-\pi^2\gamma^2 a^2/\lambda^2} \, e^{-(vz/2D)\left\{\left[1+(4\pi\gamma D/v\lambda)^2\right]^{1/2} - 1\right\}} \qquad (128)$$

$$= e^{-\pi^2\gamma^2 a^2/\lambda^2} \, e^{-4\pi^2\gamma^2 Dz/v\lambda^2}$$

where, in the last step, the integral has been expanded just as was done in going from Equation (61) to Equation (62).

As a result of the rapid initial spread, there is thus a uniform reduction of the return all along the trail, compared with the ambipolar diffusion result. Alternatively, if we use the ambipolar diffusion treatment we must re-interpret α to be an effective electron density, related to the true electron density α_o by

$$\alpha = \alpha_o \, e^{-\pi^2 \gamma^2 a^2 / \lambda^2} \qquad (129)$$

The exponent is a function of the wavelength, orientation, altitude, and composition of the meteoroid, and can be large for parameters of interest.

6.3.2 - Antenna Patterns

The effect of different antenna patterns can be visualized rather simply. Apart from the case in which the front end of the visible trail is near the specular point, the scattering amplitude is in general a sum of two terms - one of which represents the contribution of the specular region (if it lies within the overlap of the beams) and the other that of the front end of the visible trail.. Provided that the lumped pattern factor does not have any discontinuities (or near-discontinuities), except for the cut-off at the ends, the only modification that is necessary is to multiply each of these two terms by the value of the pattern factor at the relevant point (specular point or front end). When the front end is near the specular point, a variable antennae pattern factor still presents no problem as long as the variation is small over the distance between the two points (i.e. as long as a Taylor expansion applied to this factor transforms the scattering integral into a rapidly convergent series of integrals). In effect, it would take antenna patterns fluctuating at a rate comparable with the phase oscillations to change the picture qualitatively. For any less pathological case, the adjustment of the results to take into account variation in the antenna patterns is very simple - provided that the overlapping patterns are adequately mapped and there is a means of fully locating the trail within them.

The question of the significance of the sharp cut-off is a trifle more subtle. If the meteoroid is within the beams, it provides the cut-off and there is no problem. If the antennae pattern has an abrupt edge, this edge takes over as the cut-off for the visible trail as the meteoroid crosses it. If there is an area of decline before the edge of the beams is reached, this decline will be reflected in the contribution of the front end as the meteoroid approaches the edge, and the edge will still take over as cut-off as the meteoroid crosses it. On the other hand, if the pattern tapers off gradually and there is no sharp boundary (e.g. a Gaussian), the meteoroid remains indefinitely in sight, providing a continuously attenuated front-end contribution. Of course, if the meteoroid burns out during the observation, the cut-off is quite unambiguous. We see then that the gradualness with which the pattern terminates has no effect as long as the meteoroid is squarely in sight, but can reduce the head-echo contribution after the meteoroid has passed into the boundary region. For small A (small wavelengths), this has no importance because diffusion does not allow the return to persist long behind the meteoroid. For large A (large wavelengths), it leaves the specular return unchanged, but affects the behavior of a non-specular return subsequent to the passage of the meteoroid through the main part of the antennae pattern.

6.4 Meteor Velocity Measurements

The predominant technique for measuring the velocity of radio meteors is based upon the distance between maxima of the diffraction pattern on an amplitude-time record. Since only those returns that show the initial rise of Figure 6-2 are used, the orientation of the trail is essentially specular. The measurements are made for back-scattering . (For "forward" scattering, the sin θ factor in L would introduce some ambiguity - θ is not completely determined by the scattering angle.)

In interpreting the data, the Fresnel integrals of real argument (corresponding to $A = \infty$) have been used. It is of interest, therefore, to examine the effect of taking a finite A upon the validity of the results (i.e. to determine to how small a wavelength they can be trusted). For this purpose, Table II lists the locations of the successive maxima in the diffraction pattern for various A's. It turns out that the location of all the maxima but the first is very little affected by the value of A, and that the error incurred in using the values obtained for the Fresnel integrals of real arguemnt is always less than one per cent. Thus, the technique is theoretically applicable to small wavelengths without modification. The only limitation is the practical one that the oscillations damp out with decreasing wavelength, and ultimately become indiscernible.

6.5 Meteor Trails as Interference to Radio Communication via the Moon

In Section IV, the data on the return from the moon at different frequencies have been correlated. In the present section, the return to be expected from a meteor trail has been computed. Combining these results, we are now in a position to examine the

Table II

Location of Maxima of Diffraction Pattern

A	L_1	L_2	L_3	L_4	L_5	L_6
∞	1.23	2.33	3.07	3.67	4.18	4.64
5	1.13	2.31	3.06	3.66	4.17	4.63
2	1.02	2.31	3.06	3.66	4.17	4.63
1.5	0.97	2.31	3.06	3.66	4.17	4.63
1.0	0.88	2.30	3.06	3.66	4.17	4.63
0.9	0.86	2.30	3.06	3.66	4.17	4.63
0.8	0.82	2.30	3.06	3.66	4.17	4.63
0.6	0.74	2.30	3.05	*		

* No maxima beyond this point.

competition of the two processes. As the most convenient basis for comparison, Table III lists the value of α (obtained with Table Ia and consequently implying a choice of typical parameters) at which vertical backscattering by a meteor trail will give a return equal to that from the moon, and also the value of α_0 (using a typical value of a from Öpik for an altitude of 90 km), for a number of representative frequencies. Various lines of argument show that the underdense-trail formalism is valid up to a line density somewhat below 10^{12} electrons/cm, and that above that value the return continues to rise, but at an increasingly reduced rate. It is thus obvious that meteor returns far in excess of the moon returns are possible, and that at the least some form of signal limitation or range gating in the receiver is desirable as protection against them, unless a frequency above 500 Mc is used. The exponential due to Öpik's correction has the effect of sharply suppressing meteor signals at higher frequencies.

The other question of interest is how often an interruption of radio communication via the moon can be expected because of meteor trail interference. For this purpose, we refer to a Stanford study of meteor rates (Ref. 26) carried out at 23 Mc with a fan-shaped beam. They quote a maximum count of 5000 meteors per hour for equipment which (on the basis of the Lovell-Clegg formula, which is adequate for this low frequency) can detect trails down to an electron density of 10^8/cm. They also found that the count varied linearly with the transmitter current. This implies that the number of meteors varies inversely as the cut-off value of electron density. Neglecting any effect of a smaller beamwidth in reducing the count, Table III lists the expected maximum count on these assumptions.

Table III

Meteor Trails which Scatter as Strongly as the Moon

wavelength	10 m	3 m	1.5 m	1 m	75 cm	60 cm
freq. (Mc)	30	100	200	300	400	500
α	$2x10^7$	$1.4x10^8$	$4x10^8$	$1.1x10^9$	$2x10^9$	$4x10^9$
α_o	$2x10^7$	$2.3x10^8$	$4x10^9$	$1.3x10^{11}$	$1.1x10^{13}$ *	$3x10^{15}$ *
maximum number per hour	20,000	2000	140	4	$<\frac{1}{20}$	0

* These values correspond to overdense trails, so that the use of the equations above to obtain them is not adequate. Actually, a higher electron density is necessary to yield the desired signal strength.

We conclude that meteor trails need not be considered for any moon communication system operating significantly above 500 Mc.

SECTION VII

COMMUNICATION CONSIDERATIONS

The previous chapters have given detailed consideration to the conditions which affect communication between points on the earth's surface by using the moon as a parasitic antenna. The considerations help us to determine the parameters which occur in the design of a communication system.

Although there is variation in the pulse-to-pulse return from the moon, the pulses exhibit a relatively sharp rise in the leading edge. As discussed in Section II, the shape of the pulse behind the leading edge can depend upon the effect of returns from portions of the moon other than the specular point. The sharp rise in the leading edge of the pulse is clearly seen when sufficient power is transmitted to bring the return above the noise, or when pulse integration or superposition is used.

The primarily specular nature of the return implies that only a small part of the lunar surface contributes most of the detected energy. Estimates of the size of the main reflecting area have ranged from such gross descriptions as less than one-third of the surface to less than the first five miles of depth. However, the specular portion of the return, which is evident in the leading edge of the pulse, need come from an area only a few multiples of wavelength in diameter, and the size of this area will depend upon the characteristics of the illuminated portion of the moon surface. If the specular region is in a large crater, the main part of the pulse may come from a very small area.

A communication system which takes advantage of the characteristics of the lunar echo need not use the entire return pulse. It is not necessary to wait until all of the pulse has been detected, since a major part of the received energy (the specular return) is contained in a small initial portion of the pulse. Such a communication system could, for example, transmit in two alternate frequencies. The receiver would detect only the leading edge of a pulse transmitted in one frequency and then switch to detection of the subsequent pulse transmitted in the second frequency.

Losses due to dense meteor trails should be an infrequent occurrence. Losses due to double fades have been observed, but the frequency and duration of such fades has not yet been established by experimentors. It may be desirable to repeat a message when necessary to overcome loss of information through meteor scattering and double fades. The frequency of such occurrences will depend on geography and time of year and will have to be determined experimentally. At high frequencies, they are not expected to be serious.

The range of useful frequencies for moon communication is bounded on one side by limitations imposed by ionospheric attenuation and meteor scattering, and on the other side by attenuation due to losses from atmospheric oxygen and water vapor. Theoretical values for oxygen and water vapor attenuation,as given by Kerr (Ref. 27), show that for wavelengths shorter than 3 cm the loss due to these factors is largest. Thus a peak frequency bound is about 10,000 megacycles. Using a value of maximum electron density, N, of 5×10^6 electrons/cm^3 obtained from Mitra (Ref. 9) and using the relationship for critical frequency,

$$f_c^2 = \frac{Ne^2}{\pi m} ,$$

a value of 20 megacycles is obtained. However, a frequency larger than the critical frequency should be chosen as a low frequency limit because a more conservative estimate is necessitated by uncertainties in the value of N. Thirty megacycles appears to be a reasonable lower limit in order to avoid ionospheric absorption.

A further consideration involved in choice of an appropriate frequency range is that of the beam width achievable with the antennas available. The moon subtends an arc of approximately 0.5 degrees from the earth. If it were necessary to illuminate the whole moon, the high frequency cutoff with the 60-foot antenna would be 2300 Mc, and 6900 Mc with the 20-foot antenna. However, as indicated earlier, a very much narrower beam would be sufficient for the observed specular reflection. The approximate relation between beam width in degrees at the half power points, wavelength, and antenna diameter is given by the well-known formula (Ref. 28 for example),

$$\theta = 70 \frac{\lambda}{D} .$$

For antenna diameters of 20 feet and 60 feet, the narrowest obtainable beam widths for the shortest feasible wavelength, 3 cm, are 0.3 degrees and 0.1 degrees, respectively. Use of the narrow beam will, of course, conserve power since it is not necessary to illuminate the whole moon to obtain a usable echo. Because of the desirability of a narrow beam, a larger diameter antenna is preferable to a smaller diameter antenna (subject to expense and mechanical requirements, particularly in steering), and a shorter wavelength is preferable to a longer wavelength. Moreover, as shown in Section IV, the radar cross section of the moon drops off very little as wavelength decreases, so there is no drawback on this score to going down to a wavelength of 3 cm for communication. Finally, we note that the Faraday rotation for a wavelength of 3 cm would be considerably less than one degree. It appears from all this that 3 cm is the best wavelength to use in moon communication.

Consideration should be given to automatic methods of tracking the moon. This is especially needed when small beam widths are employed. A computer operating with a predetermined lunar orbit and angular tracking rates adjusted for a particular site may be desirable. The limited times when the moon is visible simultaneously from both receiver and transmitter must also be computed (Ref. 29).

It is not feasible to achieve partial communication security by narrowing the beam so that the signal reflected from the moon will irradiate only part of the hemisphere of the earth facing the moon. The angle of the transmitted beam which will, upon specular reflection from the moon, completely cover the hemisphere of the earth facing the moon is given by

$$\theta \approx \frac{da}{r^2}$$

where d is the radius of the earth, a is the radius of the moon, and r is the distance from the earth to the moon. This beam angle is approximately three seconds of arc. To obtain such a beamwidth, the ratio of antenna diameter to wavelength would have to be of the order of 10^5, which is impractical. Even if such a beam width were achievable, individual ionospheric irregularities would then disturb the transmission.

The receiver noise and the desired information rate place limits on the useful band width for a communication system. Receiver noise power N is given by the following relationship (Ref. 30):

$$N = k\,T\,B(\overline{NF} - 1)\,g$$

where k is Boltzmann's constant, T is 290 degrees Kelvin, B is the bandwidth, NF is the noise figure of the reciever, and g is receiver gain.

An upper bound for the maximum usable band width is that the receiver output noise power be equal to the power of the minimum desired signal at the output of the receiver. The desired signal in this case is moon echo. This minimum signal power is obtained from the range equation:

$$S_{min} = \frac{P_t\, G^2\, \sigma\, \lambda^2}{(4\pi)^3\, r^4}$$

where: P_t is taken to be 250 kilowatts for purposes of illustration,

G for the 60-foot antenna is taken to be 2.6×10^6,

λ is 3 cm,

σ is $4.5 \times 10^{11}\, m^2$, and

r^4 is $2.5 \times 10^{34}\, m^4$.

The cross section of the moon at 3 cm is computed using a value for $|R|^2 = .047$ from Section IV,(Fig. 4-1). The minimum signal power obtained with these values is approximately 1.4×10^{-11} watts.

The band width can now be computed for the representative case in which S_{min} is 1.4×10^{-11} watts. We equate $g\,S_{min}$ to N in order to determine the bandwidth. In this calculation, a noise figure of 11 is taken to be representative of receivers working in the 10,000 megacycle range. The allowable band width is then computed to be 350 Mc. For a reliable moon communication system a signal-to-noise ratio of 40 db would be a more likely value. This choice would result in a decrease in the band width to 35 Kc in the above illustration. Since the effective pulse length is approximately the reciprocal of the band width, the shortest pulse length permitted by these considerations would be 29 μ sec.

If the sun is in the radar beam, solar noise, in addition to the receiver noise, is superposed on the signal. For a quiescent sun, this is mainly thermal noise. (Pawsey and Bracewell, Ref. 31). The band width is then

$$B = \frac{S_{min}}{k\left[(F-1)\, T_0 + \alpha\, T_s\right]}$$

where: T_s = sun temperature ($\sim$18,000^o K for λ=3 cm)

α = a factor which depends on the radar beam pattern and positions of the sun and moon in the beam.

For the narrow beam recommended in this report, α should be so small that B is essentially unaffected by solar radiation. For a very wide beam receiver this would not be true, and solar radiation could become of the same order of magnitude as the receiver noise.

The required information rate will depend upon the use to which the moon communication system is put. For example, the message rate for a 16-channel military teletype may be 750 bits/sec., or approximately 50 bits/channel/sec. The maximum allowable pulse length for all 16 channels transmitting simultaneously would be 1300 microseconds. Meteor communication systems acquire information at relatively slow teletype rates, and store the information for transmission at about 100 times the acquisition rate when conditions for transmission are right. This transmission rate may be 10,000 bits/sec. The maximum allowable pulse length for such a system would be 100 microseconds. The 29 microsecond pulse length of the example of the previous paragraph would be obviously more than adequate for either type of teletype transmission. For a final design, therefore, less power than the 250 kilowatts of this example would be adequate for this type of communication. However, the signal-to-noise ratio could be raised to 45 db if 100 microsecond pulses were used rather than 29 microsecond pulses. In a similar fashion, the values of parameters for other types of communication, and other information rates, can be worked out.

In the course of using frequency shift transmission, frequencies over a band width of say 100 Kc may be considered. It is of interest to note that no appreciable variation in phase of the frequency components of the received signal will be obtained over such a band. The chief source of such changes would be in the variation of phase with frequency on reflection. For bands of 100 Kc, Figure 4-2 shows the change in phase shift over the band is negligible.

SECTION VIII

CONCLUSIONS

Due consideration should be given to the design of a frequency-modulated pulsed communication system with approximately the following characteristics:

frequency	10,000 Mc
transmitted power	250 Kw
band width	30 Kc
pulse length	30 μsec.

The information rate could be as large as that attained during transmission bursts with meteor scatter systems, but now, of course, uninterrupted communication is possible for as long as the moon is visible to both the transmitter and the receiver.

The theory developed in Sections II and IV shows the feasibility of predicting possible constituents of the moon's surface. For this purpose more accurate data on the power reflected from the moon should be obtained, with particular emphasis on the initial part of the returned pulse. This will require echoes of the clarity obtained, for example, by the Royal Radar Establishment (Ref. 3).

Finally, it is worth remarking that more refined radar techniques might well allow a limited form of geological survey of parts of the moon surface.

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