RADAR REFLECTIVITY OF MORTAR SHELLS

by

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on
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INTRODUCTION

This report describes measurements made and presents data obtained on Contract Nonr 2200(03)X with the Office of Naval Research and The Naval Research Laboratory. The period of performance of the contract was from 1 January 1959 to 28 February 1959. The contract calls for measurements only - with no requirements for data analysis. The results are, however, as were expected from cursory theoretical analysis.

Patterns have been taken on five mortar models, showing radar cross section areas versus aspect. The models were provided by Dr. Rufus Wright of the Naval Research Laboratory. Figure 1 is a photograph of the five models; number one is a more or less standard mortar; model two has been covered with a layer of plastic material designed to absorb S-band radar energy; model three has been covered with a rubber like layer designed to absorb K_u radar waves; model four was also covered with a rubber like layer; and model five had been sprayed with a number of layers of black "paint". The coatings on models four and five were designed to absorb X-band radar waves. For further information on the type of absorbers, the reader is referred to Dr. Wright of NRL. All models were approximately 20
inches long, about 3 inches in diameter and shaped somewhat like a prolate spheroid, except for the addition of the tail section. The tail section had fins spaced at 60° intervals.

The patterns were taken to compare the reflectivity between the coated and uncoated models at the design frequencies.

MEASUREMENT RANGE AND EQUIPMENT

All patterns were taken in a large room, about 30 feet by 80 feet by 15 feet high. Four foot by eight foot plywood panels covered with absorbing materials were arranged in the form of a horse-shoe - with additional absorber sections forming a ceiling and with some absorber sections on strategic spots on the floor. The area enclosed by absorbers started at one end of the room and extended well toward the other end. Figure 2 shows the arrangement of the absorber screens and the positioning of the equipment.

FIGURE 2.
The system used is patterned after the conventional continuous wave radar cross section measuring equipment first used at Ohio State University. The system uses a common transmit and receive horn. The transmitted signal is separated from the received signal by a balanced hybrid tee. To maintain the necessary stability and isolation, it is important to have an oscillator with good stability in frequency. At S-band, a crystal controlled oscillator was used. The Laboratory For Electronics "Ultra Stable Oscillator"—Model 814-X-21 was used at X-band and a free running Varian klystron, Model V-94, powered by the FXR Model Z815B supply was used at K_u-band. Equipment used which was common to all three frequency bands was the Scientific Atlanta "Wide Range Receiver", Model 402, the Model SA 121-B recorder and the Model SA 411 target support pedestal. The models were supported on a poly-foam cone, about 5 feet tall. The cone rested on the pedestal.

The patterns required by the contract were as follows:
(a) S-band, (2,870 mc/s) on models 1 and 2 to be taken with both horizontal and vertical polarizations.
(b) X-band (9,400 mc/s) on models 1, 4 and 5 with both horizontal and vertical polarizations.
(c) K_u-band (16,500 mc/s) on models 1 and 3 with both horizontal and vertical polarizations.
As is the case with most microwave free space rooms, the temporary free space room used for these measurements was not perfect. Although the effect of the background could be balanced out before the model is placed on the pedestal, the background and side walls contribute to the back scatter pattern in the multiple bounce paths that are created when the model is introduced and rotated.

For this reason, a second pattern was taken at a range which differs from the first by a quarter of a wavelength. The average of these two patterns, where all conditions remain the same except for the change in range, is a more realistic picture of the true free space pattern. Twenty-eight patterns instead of fourteen, are therefore presented. It will be noted that the effect of multiple scattering is more pronounced in the low signal areas - as would be expected. Also, as would be expected, the effect is less troublesome as the range between the antenna and target is decreased.

An accepted criterion for the range separation between the antenna and the model is $2d^2/\lambda$ where $d$ is the maximum dimension of the target and $\lambda$ is the wavelength. This assumes that the antenna aperture is not greater than $d$. For the 20 inch model of these experiments, a range of about 17 feet, 53 feet and 93 feet would be required for S, X and K$_u$ bands respectively. As is noted on the
patterns, the ranges used were 17 feet, 27 feet and 17 feet for the
S, X and K_u bands. The size of the measurement room, the temporary
nature of the absorber set-up and the lack of transmitter stability
and receiver sensitivity necessitated the use of shorter ranges
\((d^2/\lambda)\) for X-band and \(d^2/3\lambda\) for K_u-band). Since the main objective
of the experiments is to determine the amount of the decrease in the
radar back scatter cross section area when the coated model is com-
pared to the standard model and since both models are examined at
the same range, we believe that meaningful data was obtained even
at the short range. Studies made at Ohio State University Research
Foundation\(^1\) concerned the effect of range as range was decreased to
as short as \(d^2/(13\lambda)\). The results led them to the conclusion that
there is little range sensitivity for round trip phase deviations
of as much as \(6.6\pi\) between the horn and the target center and between
the horn and the end of the target, (with target in broadside position).
For a 20 inch model at K_u-band and a range of 17 feet, the round trip
phase deviation is \(1.4\pi\).

NOTE: Prior to bidding on this contract, the expected difficulties
with the K_u range were made known to Dr. Wright.

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   Echo Measurements", OSURF Report 475-17; 30 June 1954, Contract
   AF 18(600)-19.
It should also be noted that the \( d \) used to obtain the \( 2d^2/\lambda \) range is the full 20 inch length of the model. At aspects other than broadside, the projected horizontal dimension as seen from the horn may be used for the critical dimension to determine the required range. For aspect angles within plus or minus 25° of the nose on, or tail on aspect the projected horizontal dimension is short enough to be within the \( 2d^2/\lambda \) criterion at the 17 foot range.

K. M. Siegel's analysis of this problem is given in the appendix.

**PATTERNS**

Patterns were taken at a single frequency in the three bands (2,870 mc/s; 9,400 mc/s; and 16,500 mc/s). All patterns taken represent the return as the models rotate about a vertical axis, with their longitudinal axis in the horizontal plane. In all patterns, one set of the tail fins were in the horizontal plane. Early in the experiments, it was noticed that the slots on either side and at the rear of the nose section on model 1 effected the shape of the nose of the pattern. These slots were maintained in the vertical position for all patterns with the possible exception of the vertical patterns at the X-band frequency.
All patterns show the nose at $0^\circ$, with the tail at $180^\circ$ and the nose again at $360^\circ$ ($0^\circ$). The trace of a sphere, 3.939 inches in diameter is shown on all patterns. Variations in this trace in some of the patterns is the result of multiple reflections in the room or possibly, in some cases, the result of transmitter instability. The amount of variation was about 1/2 db at S-band and as much as 1.5 db at X and K_u-band. The cross section of the sphere in db above a square wavelength is indicated on all patterns. A dual trace is shown on some of the patterns to show the degree of repeatability in plotting a pattern of the model, of a sphere, of the model again and then of the sphere. The balance was always checked at the beginning and at the end of a pattern. The pattern was discarded in case of a serious unbalance. In several patterns a separate trace shows the noise of the system.

The ordinate of the patterns shows the power received of the backscatter signal in decibels. Checks on the linearity of the system, made with calibrated waveguide attenuators, showed that the pen movement was accurate over the 40 decibel range to about $\pm 0.5$ db.

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1. The note on the pattern indicating relative power one way pertains to antenna patterns - not to the radar backscatter patterns presented here.
RESULTS

To comment briefly on the results – the patterns show that the addition of the camouflage material to the mortars gives a general reduction in radar cross section of from 10 to 15 db. The reduction is frequently more and sometimes less than this amount. The reader should understand that the tail aspect was not to be camouflaged. In fact, all models had a copper penny in the tail except model 5. The amount of cross section reduction in models 2, 3 and 4 is not highly sensitive to whether the incident radiation is horizontally or vertically polarized.

ACKNOWLEDGMENTS

Appreciation is expressed to Professor K. M. Siegel for his analysis of the range requirement given in the appendix, to Professor A. Olte for his helpful suggestions and to Mr. Donald Pepper, Mr. Theodore Hon and Mr. Eugene Knott for the patient and persistent effort they expended to obtain the patterns.
The back scattered field from a sphere (augmented to hold for an imperfect conductor) is\(^1\):

\[ E^s = e^{i(kr-2ka)} R \left\{ \frac{a}{2r-a} \right\} \left\{ 1 + 0 \frac{\lambda}{a} \right\} \]  

\hspace{10cm} (1)

where \( R \) is the voltage reflection coefficient, \( a \) is the radius of the sphere, \( k = 2\pi/\lambda \), \( \lambda \) = wavelength, and \( r \) = distance from center of sphere to field point. The radar cross section is

\[ \sigma = 4\pi |f|^2 \]  

\hspace{10cm} (2)

where \( E^s = \frac{e^{ikr}}{r} f \). \hspace{10cm} (3)

\[ \therefore \] \hspace{10cm} \text{when } \lambda \ll a \text{ and } r \gg a \text{ then from (1)}

\[ E^s = \frac{e^{i(kr-2ka)}}{r} R(a/2) \]  

\hspace{10cm} (4)

\[ \therefore f = R(a/2) \]  

\hspace{10cm} (5)

\[ \sigma = \pi a^2 |R|^2 \text{ as expected.} \]  

\hspace{10cm} (6)

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1. Studies in Radar Cross Sections XXXIII - "Exact Near Field and Far Field Solution for the Back Scattering of a Pulse From a Perfectly Conducting Sphere", by V. H. Weston (2778-4T, March 1959). UNCLASSIFIED.
Now let us lift the far field requirement. The worst case would be at the sphere's surface, then \( r = a \) from (1)

\[
E^s = e^{i(kr - 2ka)} R \frac{a}{2a - a} + O(\lambda/a)
\]

(7)

\[
E^s = \frac{e^{i(kr - 2ka)}}{a} aR
\]

\[\Sigma = 4\pi |aR|^2\]

\[= 4\pi a^2 |R|^2\]

(8)

Thus the increase in \( \Sigma \) even at the body for a sphere large in respect to the wavelength is only a factor of \( 4\).

Now consider the 20 inch mortar problem, namely, what is the difference in cross section for

\[
r = \frac{d^2}{\lambda}\]

(a)

and

\[
r = \frac{d^2}{3\lambda}\]

(b)

(9)

where the wavelength is 1.8 centimeters.
Rewriting (1)

\[ E^S = e^{i(kr-2ka)} R \left\{ \frac{a}{2r-a} \right\} \]

\[ = \frac{e^{i(kr-2ka)}}{r} R \left\{ \frac{a}{2-(a/r)} \right\} . \]

Now consider case (a) where \( d = 2a \).

\[ E^S = \frac{e^{i(kr-2ka)}}{r} R \left\{ \frac{a}{2-[a\lambda/(4a^2)]} \right\} \tag{10} \]

Now let us analyze the error term.

Case (a):

\[ \sigma = \pi a^2 |R|^2 \left| \frac{1}{1-(\lambda/8a)} \right|^2 = \pi a^2 |R|^2 \left| 1 + (\lambda/8a) \right|^2 \]

\[ \sigma = \pi a^2 |R|^2 \left[ 1 + (\lambda/4a) \right] . \]

Case (b):

\[ \sigma = \pi a^2 |R|^2 \left[ 1 + (3\lambda/4a) \right] . \]
Thus the error is of order $\lambda/a$ which is the order of the remainder in (1) and since $a > \lambda$ there is no problem as in the worst case the change in cross section is about 5%.

Whenever an object and all its dimensions, including curvature, are very large in respect to the wavelength, the far field distance is determined by the antenna dimensions and not the model's dimensions.

The concept of $d^2/\lambda$ comes from the following picture:

![Diagram](image)

Phase of wave at 1 is $kr$.

Phase of wave at 2 is $k\sqrt{(r)^2 + (d/2)^2}$.

It was thought allowable phase error at the model was $\pm \pi/\lambda$.

$$k\sqrt{(r)^2 + (d/2)^2} - (kr) < \pi/\lambda$$

$$k^2(d/2)^2 < (\pi^2/16) + 2kr(\pi/\lambda)$$
2 kr(π/λ) > k^2(d/2)^2

r > (2/π)(2π/λ)(d/2)^2 or r > (d^2/λ).

Now an error in phase may or may not make a big change in the amplitude. In the case of an object (d \gg λ) whose return is dominated by one specular point the phase change across the object is a meaningless criterion (as shown by normal incidence) for back scattering from any flat or slightly curved plate or for scattering at all angles from a sphere as was done for back scattering in the above example. The criterion in such cases then is given by the d_1 of the antennas and not the model. In the case of models with many specular points, one runs into difficulty if and only if the dominant return comes from more than one specular point and if these specular points are at different distances from the axis of the main lobe of the antennas. Then it is the distance between dominant specular points and not outside model dimensions which determine d. Thus large models often require no greater separation between antenna and model than smaller objects if in both cases d_1 \gg λ. Sometimes a smaller object can require greater separation than a larger model namely d \gg λ for the larger and d_1 \approx λ for the smaller object.
Of course, if there is plenty of space available and isolation and sensitivity requirements can be met, it is still desirable to use the ideal criterion

\[ r > \frac{2(d+m)^2}{\lambda} \]

where \( d \) is the maximum model dimension and \( m \) is maximum dimension of the receiver and/or antenna aperture.