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LOW FREQUENCY SOLUTION OF ELECTROMAGNETIC SCATTERING PROBLEMS

Final Report (1 July 1969 - 1 July 1970)

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The prime focus of our efforts has been on the nature of the leading term in the low frequency expansion of the field scattered by a finite, three dimensional object when illuminated by an electromagnetic wave. It is known that this term is attributable to the electric and magnetic dipole contribution, and in the far field we can write

$$\underline{E} \sim \frac{e^{ikr}}{r}\,\frac{k^2}{4\pi\epsilon_0}\left\{\hat{r}\wedge(\underline{P}\wedge\underline{r}) + \frac{\epsilon_0}{Y}(\underline{M}\wedge\hat{r})\right\}$$

$$\underline{H} \sim \frac{-e^{ikr}}{r}\,\frac{k^2}{4\pi}\left\{\hat{r}\wedge(\underline{M}\wedge\hat{r}) + \frac{Y}{\epsilon_0}(\underline{P}\wedge\hat{r})\right\}$$

where ϵ_0 and Y are the permittivity and intrinsic admittance of free space. $\underline{P}$ and $\underline{M}$ are here the electric and magnetic dipole moments respectively. If the scatterer is a homogeneous body of revolution about (say) the z-axis, it can be shown that

$$\underline{P} = -4\pi\,\epsilon_0\left\{(\ell_1\hat{x} + m_1\hat{y})\,a_1^{(1)} + n_1\hat{z}\,a_1^{(3)}\right\}$$

$$M = -4\,\pi\,Y\left\{(\ell_2\hat{x} + m_2\hat{y})\,b_1^{(1)} + n_2\hat{z}\,b_1^{(3)}\right\}$$

where (ℓ_1, m_1, n_1) and (ℓ_2, m_2, n_2) are the directive cosines of the incident electric and magnetic fields, respectively. The dipole moment coefficients $a_1^{(1)}$, $a_1^{(3)}$ and $b_1^{(1)}$, and $b_1^{(3)}$ are independent of the incident illumination and functions only of the scatterer. It would therefore appear that just 4 geometric quantities are needed to specify the Rayleigh term and these can be obtained from the solutions of the standard potential problems for the geometry in question. If the body is metallic, however, the results for a number of geometries for which exact solutions have been found have shown that

$$b_1^{(3)} = -\frac{1}{2}\, a_1^{(1)} \,,$$

thereby reducing the number of independent unknowns to 3 . One task ahead of us is to verify that this relation holds for all rotationally symmetric perfectly conducting bodies.

It is known that Rayleigh scattering and, hence, the dipole moment coefficients are very weak functions of the geometry of the scatterer. The volume is the dominating factor, and a widely-used approximation to the backscattered far field amplitude is to take it equal to the volume modified by a "shape correction" factor (Siegel, 1958). To provide some specific values for the dipole moment coefficients for a non-trivial shape, the $a_1^{(1)}$ and $b_1^{(1)}$ have been determined (Senior, 1970) for a metallic body consisting of the intersection of a cone and a sphere. The primary dependence on the volume is evident in the data, and this is otherwise apparent from the following general and suggestive representation for the $a_1^{(1)}$ and $b_1^{(1)}$ that can be derived:

$$a_1^{(1),(3)} = \frac{-3V}{4\pi}\left\{1 - \frac{1}{V}\int_{\delta V} \nabla \Phi^{(1),(3)}\, dV\right\},$$

$$b_1^{(1),(3)} = \frac{3V}{8\pi}\left\{1 - \frac{1}{V}\int_{\delta V} \nabla \psi^{(1),(3)}\, dV\right\},$$

where V is the volume of the scatterer, δV is the volume exterior to the scatterer, but interior to the smallest surrounding sphere, and $\Phi^{(1),(3)}$, $\psi^{(1),(3)}$, are the exterior potential functions pertaining to the scattered electric and magnetic fields.

The terms in braces play the role of shape factors and are, apparently, slowly varying functions of the geometry, a property which we hope to exploit.

At frequencies which may be above that for which the leading term in the low frequency expansion is sufficient, an alternative to an analytical treatment is a direct numerical solution of the integral equation. A straightforward procedure based on the Maue integral equation and valid for rotationally symmetric perfectly conducting bodies at axial incidence has been developed by Uslenghi (1970) and applied to a variety of shapes. The procedure is both effective and efficient for small bodies, and works well even at frequencies in the resonance region. It has been used to provide values for the dipole coefficients $a_1^{(1)}$ and $b_1^{(1)}$ for all these shapes, which values support the weak dependence on the specific geometry, and can be used to provide the higher order multipole contributions as well.

There are two particular consequences of this numerical study that are quite exciting. An examination of the computed surface fields at low frequencies shows them to be almost completely functions of the local geometry. Even a small change in radius of curvature produces a minute step in the relevant component of the current. Such a local dependence is characteristic of high frequencies, but it does not seem to have been appreciated that it may be even more applicable at low frequencies. Should it be possible (as seems likely) to determine the surface field dependence on the local geometrical parameters, even to a first order, we could then use an alternative expression for the dipole coefficients in terms of the surface values of potential, e.g.

$$a_1^{(1)} = -\frac{1}{4\pi}\int_S \left(x\,\frac{\partial \Phi}{\partial n} - \hat{n}\cdot\hat{x}\;\Phi\right) dS$$

to produce a purely geometric description of these quantities.

The apparently local character of the low frequency surface fields provides some understanding of why it is that a basically high frequency approach to the surface fields (as Siegel, 1958, invokes) leads to a reasonable estimate of the low frequency scattering. It turns out that the zeroth order approximation to the Neumann series solutions of the integral equations in Uslenghi (1970) is equivalent to that implied by Siegel's approach.

Publications

The following journal articles have resulted from the support provided by this Grant.

P. L. E. Uslenghi "Computation of Surface Currents on Bodies of Revolution"
To appear in Alta Frequenza (RL 487)
Abstract:

The scattering of a plane electromagnetic wave axially incident on a perfectly conducting, rotationally symmetric body of revolution is considered. Maue's vector integral equation for the surface current density is reduced to two coupled scalar Fredholm integral equations of the second kind, that are solved numerically. The surface currents are displayed for a variety of scatterers. The computer program, in FORTRAN IV language, is given.

T. B. A. Senior "Low Frequency Scattering by a Finite Cone"
To appear in Applied Scientific Research (RL 483)
Abstract:

The scattering of a low frequency electromagnetic wave by a metallic cone, whose base is part of a spherical surface centered on the apex of the cone, is analyzed using a mode-matching technique. The dipole contributions to the scattering are obtained in complete generality, and numerical results are presented for a wide range of cone angles. Comparisons of the computed data with the predictions of an empirical formula for the scattering reveal both the strengths and weaknesses of the latter.

Students and Professional Staff Involved

The work carried out under this Grant was directed by Professor T. B. A. Senior with the assistance of Dr. P. L. E. Uslenghi. Other personnel and students who received support from this Grant were Mrs. A. Liepa, Mr. P. M. Wilcox and Mr. (now Dr.) Wei C. Yang.

References

Senior, T.B.A. (1970), "Low Frequency Scattering by a Finite Cone," to appear in Applied Scientific Research (RL 483)

Siegel, K.M. (1958), " Far Field Scattering from Bodies of Revolution," Applied Scientific Research, 7 B, pp. 293-328.

Uslenghi, P.L.E. (1970), "Computation of the Surface Currents on Bodies of Revolution," to appear in Alta Frequenza (RL 487) .