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# THE UNIVERSITY OF MICHIGAN

## COLLEGE OF ENGINEERING
### DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING
**Radiation Laboratory**

LOW FREQUENCY SOLUTION OF ELECTROMAGNETIC SCATTERING PROBLEMS

Final Report (15 November 1970 - 15 May 1972)

By
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# THE UNIVERSITY OF MICHIGAN

This is the Final Report on Grant GP-25321 covering the period 15 November 1970 to 15 May 1972.

When a small perfectly conducting body is illuminated by an electromagnetic wave, the far zone scattered field can be expanded in a series of increasing positive powers of the wave number k. For sufficiently small values of kD, where D is a typical dimension of the body, the series has been proved convergent in a variety of special cases, and is believed convergent in general. The leading term is $O(k^2)$ and, as shown by Rayleigh (Phil. Mag. 44, 28-52, 1897), is attributable to the electric and magnetic dipoles induced in the body by the incident field. For many practical purposes, this term alone provides an adequate characterisation of the low frequency scattering behavior of a body and the main focus of our work has been the study of this one term.

The initial breakthrough was the realisation that the electric and magnetic dipole moments can be expressed in terms of polarizability tensors which are functions only of the geometry of the body. These tensors have been widely used in other contexts, and certain of their properties were already known. Their elements are, moreover, expressible as surface or volume integrals of potential functions. In the particular case of a rotationally symmetric body, only 6 of the 18 tensor elements are non-zero and it is self evident that only 4 are independent. In fact, however, there are only 3 independent elements, implying that the entire low frequency scattering behavior of a rotationally symmetric body is determined by three scalar quantities, functions of the geometry alone; and even these are subject to constraints.

These results were developed jointly by the author and Professor R. E. Kleinman of the University of Delaware during the winter of 1970-71, and reported at the URSI Meeting in Washington, D. C. in April 1971. We then learned that Professor J. B. Keller of New York University had developed rather similar results several years earlier, but had not reported them. We therefore invited Keller to accept co-authorship of the paper that we had prepared for the Journal of the Institute of Mathematics and Its Applications, and this he agreed to do.

Meanwhile, Kleinman and the author had pursued further the consequences of this new representation and showed that the properties of the tensor elements enabled bounds to be placed on the electromagnetic scattering under all conditions of illumination and scattering. This theme was developed in a paper to appear in Radio Science.

From a study of the nature of the tensor elements for those few geometries for which exact analytical expressions are available, it was apparent how insensitive they are to the details of the actual geometry. To explore this dependence in more detail, it was necessary to gather data for a much wider range of geometries, and a numerical procedure was clearly appropriate. Since all the elements are expressible in terms of potential functions satisfying certain rather trivial integral equations, it was found possible to devise a computer program for the solution of these equations and for the calculation of the corresponding elements as well as certain other related quantities, such as the electrostatic capacity. The program is rather efficient and computes all these quantities in a few seconds time. Some of the data obtained is reported in the above Radio Science article.

To give publicity to the existence of this program, the Air Force Cambridge Research Laboratories agreed to support the writing up of this material as a Report under Contract No. F19628-68-C-0071. The partial support of the present Grant is acknowledged in that Report, a copy of which is attached hereto. This program has already immeasurably increased our knowledge and understanding of low frequency scattering.

During the initial phases of our work, we became intrigued by the fact that the low frequency surface field seems primarily dependent on the local geometry. Such local dependence is an accepted feature of <u>high</u> frequency surface fields, and this led us to examine in some detail the nature of high frequencies approximations. The most commonly used approximation in the vicinity of a surface singularity is based on the work of Ufimtsev, and an article examining the accuracy of the Ufimtsev theory was written in collaboration with Professor Uslenghi, now at the University of Illinois (Chicago Circle).

## Publications

The following journal articles have resulted from the support provided by this Grant.

J. B. Keller, R. E. Kleinman and T. B. A. Senior "Dipole Moments in Rayleigh Scattering," J. Inst. Maths. Applics 9, 14-22, 1972

Abstract:

The field scattered by a finite, closed, perfectly conducting body irradiated by a low frequency plane electromagnetic wave is expressed in terms of the polarization and virtual mass tensors of the body. These tensors are functions only of the geometry of the body, and expressions for their elements are derived. For a body with an axis of symmetry, just three elements are independent, and these serve to specify the scattering in its entirety.

R. E. Kleinman and T. B. A. Senior "Rayleigh Scattering Cross Sections," Radio Sci. (to appear, July 1972).

Abstract:

The significance of the electric and magnetic polarizability tensors in low frequency scattering is emphasized. In the particular case of perfectly conducting, rotationally symmetric bodies with plane wave illumination, it is shown how the entire Rayleigh scattered field can be expressed in terms of just three tensor elements, functions only of the geometry of the body. Inequalities satisfied by these elements are used to establish optimum lower bounds on the scattering cross sections and, in addition, the elements themselves are examined analytically and computationally for a variety of shapes. Some of the implications of these results are discussed.

T. B. A. Senior and P. L. E. Uslenghi "Comparison Between Keller's and Ufimtsev's Theories for the Strip," IEEE Trans. AP-19, 557-8, 1971.

Abstract:

The high frequency backscattered far field produced by a plane electromagnetic wave obliquely incident on a perfectly conducting infinite strip is considered. A comparison between the results obtained by applying Keller's and Ufimtsev's asymptotic theories is performed. It is shown that Ufimtsev's expansion is incorrect beyond the leading term, but that the discrepancy is numerically small. The implications in caustic matchings for more complicated shapes, such as finite cones and disks, are briefly discussed.

In addition, the numerical procedures developed under this Grant have been written up in the following Report:

T. B. A. Senior and D. J. Ahlgren "The Numerical Solution of Low Frequency Scattering Problems," The University of Michigan Radiation Laboratory Report No. 013630-9-T, February 1972. The partial support of the present Grant is acknowledged in the Introduction (see p. 2).

Abstract:

The low frequency scattering of electromagnetic and acoustic waves by rotationally symmetric bodies is considered. By concentrating on certain quantities such as the normalised component of the induced electric and magnetic dipole moments, it is shown how the first one or two terms in the far zone scattered fields can be expressed in terms of quantities which are functions only of the geometry of the body. Each of these is the weighted integral of an elementary potential function which can be found by solving an integral equation. A computer program has been written to solve the appropriate equations by the moment method, and for calculating the dipole moments, the electrostatic capacity, and a further quantity related to the capacity. The program is described and related data are presented.

Copies of the above material are attached.

Paper Presented at Professional Meeting

R. E. Kleinman and T. B. A. Senior "Dipole Moments in Rayleigh Scattering," Spring URSI Meeting, Washington, D. C., April 1971.

Students and Professional Staff Involved

The work carried out under this Grant was directed by Professor T. B. A. Senior. Other personnel and students who received support from this Grant were Professor P. L. E. Uslenghi, Mr. D. J. Ahlgren and (briefly) Mr. L. Hyland.