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THE DESIGN OF A DUAL POLARIZED SLOTTED WAVEGUIDE LINE SOURCE

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CONTENTS

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FIGURES

ABSTRACT

Variable polarization line sources are generally obtained from pillboxes or narrow horns. In our extension of previous work we have designed an efficient resonant array with a broadside beam and with an absence of grating lobes. Experimental results confirm two theoretical designs that achieve these objectives.

A naive approach would suggest slot separations of one guide wavelength in order to obtain a constant phase front along the array; This is objectionable of course since it leads to grating lobes. The first solution for overcoming this objection is to 'load' the guide with dielectrics to reduce the slot separation to less than λ_0. Modifications are given to standard formulas for this case and a curve is given for the change in resonant length as a function of dielectric material.

An alternate solution utilizes the large resonant conductance and resistance of the crossed slots. This technique employs slots that are alternately longer and shorter than resonant length, thus obtaining decreased conductance or resistance with large imaginary components, and at the same time introducing nearly 180^o phase shift between slots. By using half guide wavelength separation, the radiated signal from each slot is then approximately in phase and one obtains a good impedance match while at the same time eliminating grating lobes.

INTRODUCTION

This study deals with the technique for designing a slot array that is capable of being simultaneously operated with two orthogonal polarizations with a broadside pattern maximum, high efficiency, and large isolation between polarizations. These techniques are an outgrowth of studies leading to the modification of an antenna used in current measurements. An alternative, which was easier, was the one finally selected for this purpose; it is described in a recent report [1]. This slot array design was carried to the point of proving its feasibility not only as a backup study for the modification mentioned above, but also because it seemed to represent an important advance in the art of antenna design.

The organization of this report assumes no prior knowledge of slotted waveguide design on the part of the reader. Those familiar with slot theory may wish to omit the second section which deals with basic slot theory. Those familiar with crossed slots in square waveguide might omit the third section, which presents the concept of mode isolation using square waveguide. The material in the fourth and fifth sections extends this previous work; it is this which is believed to be new and a valuable contribution to the "state of the art."

Section six presents experimental verification of the new material. Since no immediate need exists for the development of an elaborate array, the experimental data is somewhat limited. Section seven offers several conclusions, the main one being that theoretical and experimental agreement has been found.

It is believed that this design approach may offer significant advantages in certain applications requiring rigidity, light weight, or small volume. A requirement of large bandwidth might be better met with reflectors.

2

BASIC SINGLE SLOT THEORY (RECTANGULAR WAVEGUIDE)

The description of the performance of an antenna consisting of slots cut in the wall of a waveguide is essentially a complicated problem in field theory. A popular method of attacking the problem, however, is to split it into two parts. The first step is the characterization of the slotted waveguide as a transmission line loaded at discrete points with T- or π- sections which represent the effect of the slots upon the voltage and current variables of the transmission line. The second step is to consider these impedance sections as radiators driven with the appropriate magnitude and phase of voltage or current, and to use this concept in calculation of antenna patterns. The second step is more or less a routine application of well-developed techniques for pattern calculation, and will be only briefly mentioned in this report. Our interest in this section is in the first step, obtaining the equivalent circuit parameters of a waveguide slot. **This theory is applied to multiple slot arrays in following sections.**

The circuit equivalent of a particular slot, for use in a transmission-line representation, depends on the shape, size, position and orientation of the slot. A resonant slot is one which is approximately a half wavelength long; its circuit equivalent is then a combination of resistances. Depending on the position and orientation of the slot, the slot may be represented as a purely shunt element or as a purely series element; in the general case only a T- or π- section in the line adequately represents the effect of the slot.

Obtaining expressions for the circuit equivalents of slots in rectangular waveguide is of primary interest. There are two particularly useful techniques. The first, which will be called Stevenson's method [2] yields an expression for the resistance or conductance of a resonant-length slot. The second, to be called Oliner's method [2] gives more detailed information, including values of both real and imaginary parts of series or shunt impedances, not only for resonant slots but for nonresonant ones as well. Both techniques have their advantages; while Stevenson's formulas are less powerful than Oliner's in that they give less information, their derivation is more direct.

It seems of little purpose to repeat here all the formulas listed in available references: we will confine our discussion to simple outlines of the methods of interest, including details only as they are pertinent to discussions in this report.

Both methods involve calculations based on functions characterizing the electromagnetic fields inside the waveguide, taking into account the effect of the slot as a scattering element. The effect of the slot on the normal mode of propagation is then interpreted as an equivalent load in the transmission line representation.

The Stevenson method, which is well presented by means of a detailed example and several formulas in Silver's book [4], assumes a resonant-length slot with a cosine field distribution along its length. The resistance of a series slot, or the conductance of a shunt slot, is found in terms of the reflection coefficient Γ in the transmission line equivalent of the TE_{mn} wave (the dominant mode in this rectangular guide). This coefficient Γ is then related to the amplitudes of the forward- and backward-scattered waves. These amplitudes are evaluated by means of an energy balance among incident, reflected, transmitted, and radiated power. The first three power terms are found in terms of the amplitudes of the respective waves, while the radiated power is characterized by a radiation resistance. This resistance is obtained by using Babinet's principle in a form which relates the radiation impedance of a slot to that of the geometrically inverse rectangular metal strip radiating into an infinite half-space. The formulas obtained are quite simple and explicit:

For a centered broadwall series slot, (TE_{10} mode),

$$r = 0.523\left(\frac{\lambda_g}{\lambda}\right)^3 \frac{\lambda^2}{ab} \cos^2 \frac{\pi\lambda}{4a}, \tag{2-1}$$

and for a longitudinal edge-wall shunt slot (TE_{10} mode)

$$g = \frac{30}{73\pi}\left(\frac{\lambda_g}{\lambda}\right)\frac{\lambda^4}{a^3 b} \frac{\cos^2\left(\frac{\pi}{2}\frac{\lambda}{\lambda_g}\right)}{\left[1-\left(\frac{\lambda}{\lambda_g}\right)^2\right]^2}, \tag{2-2}$$

where

r is the equivalent series resistance, R, normalized to the characteristic impedance, $Z_{c,j}$; $r = \frac{R}{Z_c}$,

g is the equivalent shunt conductance, G, normalized to the characteristic admittance, $Y_{c,j}$; $g = \frac{G}{Y_c}$,

λ_g is the guide wavelength and λ the free-space wavelength,

a is the (broadwall) width and b the (narrow-wall) height, inside dimensions of the waveguide. These formulas are from Equations (48b) and (50), respectively, in reference [4], pp 292-293 or Equations (9-3) and (9-6) in Reference [5].

This Stevenson technique is discussed further in Section 4 in considering the modifications caused by dielectrically loading the waveguide.

Stevenson's method gives useful results for resonant-length slots. Oliner's method, on the other hand, while employing more complicated expressions, provides much more information. Expressions are available for the reactances or susceptances, as well as the resistances or conductances, of several types of broadwall slots; both at and also away from resonance. The pair of 1957 articles [3] lists several formulas in detail, as well as outlining their derivation and giving some experimental data for comparison. This technique is also discussed in Section 4 in obtaining the resonant length of slots in dielectrically loaded waveguides.

3

THEORY FOR AN ARRAY OF CROSSED SLOTS IN SQUARE WAVEGUIDE

The use of multiple slots in a waveguide as a linear array to produce directive beams has long been established. Broadside beams are often obtained from resonant slotted sections [4]; a resonant section is one with integral numbers of half guide wavelengths between slots and one which uses a short circuit as the waveguide termination. In certain applications the capability may be desired of transmitting and/or receiving two linear polarizations of signal independently (for example, to detect separately the horizontally and vertically polarized radar echoes of a horizontally polarized transmitted signal). This section presents the basic principle of a design to do this. Sections 4 and 5 present two approaches toward a practical implementation of the design.

The use of square waveguide with two sets of slots, as shown in Figs. 1a and 1b, takes advantage of the orthogonality of the TE_{10} and TE_{01} modes in the waveguide to provide an efficient antenna with two independent polarizations and a broadside beam. For example, the mode denoted by the vector $\vec{E}_V$, showing the direction of the electric field inside the guide, excites the slots V; they appear as edge-wall shunt slots to this mode. The electric field excitation of the slot is across the narrow dimension of the slot (vertical) and the slot radiates vertical polarization; hence the designation V. Slots H are not excited by the V mode in the waveguide (see reference 5, Section 2.3 for a list of various slots in rectangular guide).

Similarly, the horizontal polarization, $\vec{E}_H$, excites slots H, which

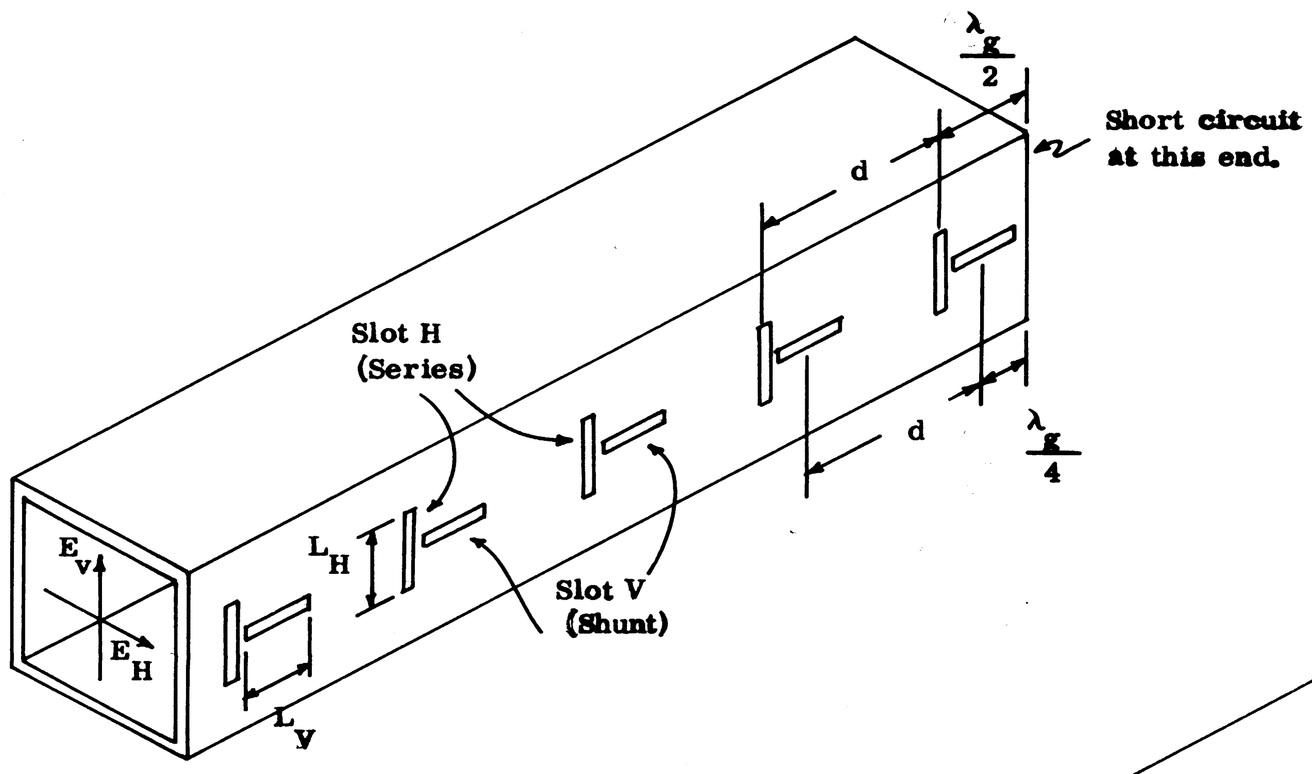


Fig. 1a. Dual-Polarized Slot Array: Separated Slots

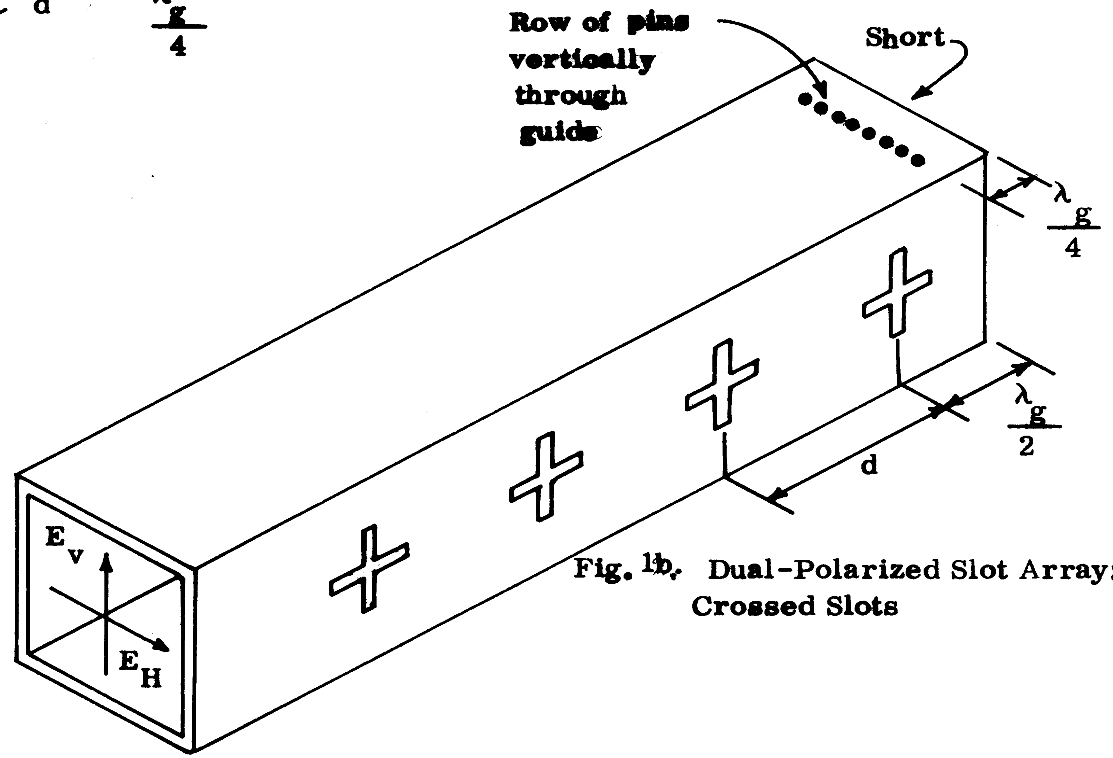


Fig. 1b. Dual-Polarized Slot Array: Crossed Slots

act as broadwall series slots and radiate horizontal polarization. Slots V behave as (non-excited) centered, longitudinal broadwall slots in their lack of response to the horizontal polarization, E_H .

The use of square guide with similar slot configurations for producing an arbitrarily polarized beam has been reported by Hougardy and Shanks [6]. Their crossed-slot design (somewhat like Fig. 1b) provided at least 30 db isolation between modes. Theirs is an end-fire array, terminated in a load; the loss of power into such a load implies an inefficient antenna.

In contrast, the designs presented in this report take advantage of the high efficiency of a resonant array terminated in a short circuit. The series slots are placed at current maxima and the shunt slots at voltage maxima. Since the first current maximum is $\lambda_g/2$ distant from a short circuit and the first voltage maximum $\lambda_g/4$ from the short, the slots in a "resonant" design need not be crossed as suggested by Hougardy and Shanks [6] and shown in Figure 1b, but may be interspaced as in Figure 1a (the slots are still orthogonal, and in that sense will still be called "crossed"). The alternative choice of designing a short circuit that is capable of providing different short positions for each polarization is more difficult. This second scheme is shown in Figure 1b, using vertical pins to simulate a short circuit $\lambda_g/4$ from the last slot only for the vertical polarization and an actual short $\lambda_g/2$ from the last pair of slots. This last approach may be of some advantage in machining the slots.

The set of shunt slots may be designed independently of the series slots. Each shunt slot may be characterized by a conductance, g_n, and a susceptance, b_n (both normalized to the guide characteristic admittance); each series slot by a resistance, r_n, and a reactance, x_n (both normalized to the guide characteristic impedance), as outlined in Section 2. To obtain a good impedance match at the input to the array, the usual procedure is to design all of a set of slots alike and with such characteristics that the total load is exactly the guide admittance (or impedance); that is, $\sum g_n = 1$ and $\sum b_n = 0$ (or $\sum r_n = 1$ and $\sum x_n = 0$). There are other possibilities, however. One is to design resonant slots without regard to the impedance match, and then obtain a good match by use of an impedance transformer between the array and the feed.

Consideration of the effects of mutual coupling among the slots will be for the most part ignored in this report, aside from a few general remarks. Indeed, one practical and often entirely satisfactory approach is an empirical one; to test and evaluate a trial design and then adjust the design parameters to compensate for the effects of mutual coupling, as well as machining errors, roughness, and finite resistive losses in the waveguide, all at once. Our approach is not quite so crude, however. There is evidence [7] to the effect that the mutual coupling effect will be small, if not negligible. The series slots can be expected to have a coupling coefficient with their nearest like neighbors of better than -20 db; the shunt elements, better than -30 db; and the coupling factor between a series element and the nearest shunt element, on the order of -90 db.

An important design objective is that, for each polarization, radiation be normal to the array. This condition was not imposed by Hougardy and Shanks [6] whose slots were spaced $.525\lambda_o/2$ apart (approximately $.4\lambda_g$) with a resultant beam deviation of about 45^o. Usually resonant slot arrays are designed with half guide wavelength separation between slots. With element spacings greater than one free space wavelength, grating lobes (strong off-axis beams) are possible, but they will not occur when $\lambda_g/2 < \lambda_o$. In the usual array design, the half guide wavelength separation is made possible by alternating slot position or rotation angle. In the present study, this alternation is not possible, for inclination or displacement of either type of slot would allow excitation by both types of waveguide modes.

This apparent difficulty can be solved in one of two ways: 1) spacing slots at half guide wavelength intervals with alternating long and short centered slots, or 2) loading the waveguide with dielectric material in order to obtain a spacing $d = \lambda_g < \lambda_g$. These two methods are discussed separately below in Sections 4 and 5.

In summary, the design objectives are:

1) excitation of a single polarization by a single waveguide mode

2) good input impedance match (and reasonable bandwidth and efficiency)

3) a broadside beam with no grating lobes.

4

THEORY OF APPROACH A — ALTERNATING SLOT LENGTHS

One way to meet the three criteria listed at the end of Section 3 is with a design which uses alternately long and short slots. The first criterion, the isolation of the two linear polarizations, is accomplished by the crossed slots. Two methods of satisfying the other two criteria, a good impedance match and a good antenna pattern, will be discussed here and in the next section.

The normalized resonant resistance of a centered series slot, or the conductance of a longitudinal shunt slot, is large. A slot longer or shorter than the resonant length, however, has a smaller resistance (or conductance) as shown in the experimental data of Section 6. For an array of N slots, the lengths can be chosen to give a normalized value of 1/N. The reactance or susceptance will then be much larger than the real part, but by alternating slot lengths a good input match can be achieved. The phase of the radiation from each slot will lag or lead according to the length and type of slot. In order to achieve phase coherence (and thus a beam normal to the array) it is necessary that the slots be spaced $\lambda_g/2$ apart. The large lag and lead angles will then be brought into approximate phase coherence by the 180^o phase reversal every half guide wavelength. The normalized resistances or conductances will sum to a value of one, and the reactances or susceptances will approximately cancel, presenting a matched load at the input to the array. It is estimated that the phase difference between the two types of slots will be between 30^o and 40^o — a value that should not seriously lower the gain or raise the sidelobes; it will not broaden the beam. Also, with $\frac{\lambda_g}{2}$ spacing the pattern will not contain grating lobes, as long as $\lambda_o < 2\lambda$ (λ the free-space wavelength).

The above discussion is schematically illustrated in Figure 2. A side and an end view of the square guide, showing all four kinds of slots, long and short series and shunt slots in the configuration of Figure 1b, are shown in the center of the figure. Schematic representations of the performance of the series slots and of the shunt slots are shown above and below the center, respectively. First above the central view is an indication of the variation in the electric and magnetic field amplitudes, E_{YH} and H_{XH}, which excite the series slot. Above this, the short slot is shown as having a positive reactance (inductive) and the long slot a negative reactance (capacitive) in the series element transmission line representation. At the top is an illustration of the phase relationship of the contribution to the far field from each kind of slot. The field from the short slot leads the current and the field from the long slot lags; the current is 180^o out of phase between elements, however, because of their $\lambda g/2$ spacing. The components from the long and short slots add vectorially, as depicted. Below the center of the figure, the corresponding story is illustrated for the shunt slots. The magnitudes of their excitation are shown as E_{XV} and H_{ZV}. The short slots have positive susceptance (capacitive) and the long slots negative susceptance (inductive); the corresponding phase relations and vector addition for the far field are depicted at the bottom of the figure.

A logical procedure for designing an array of non-resonant series and shunt slots such as is discussed in this section consists of the following steps:

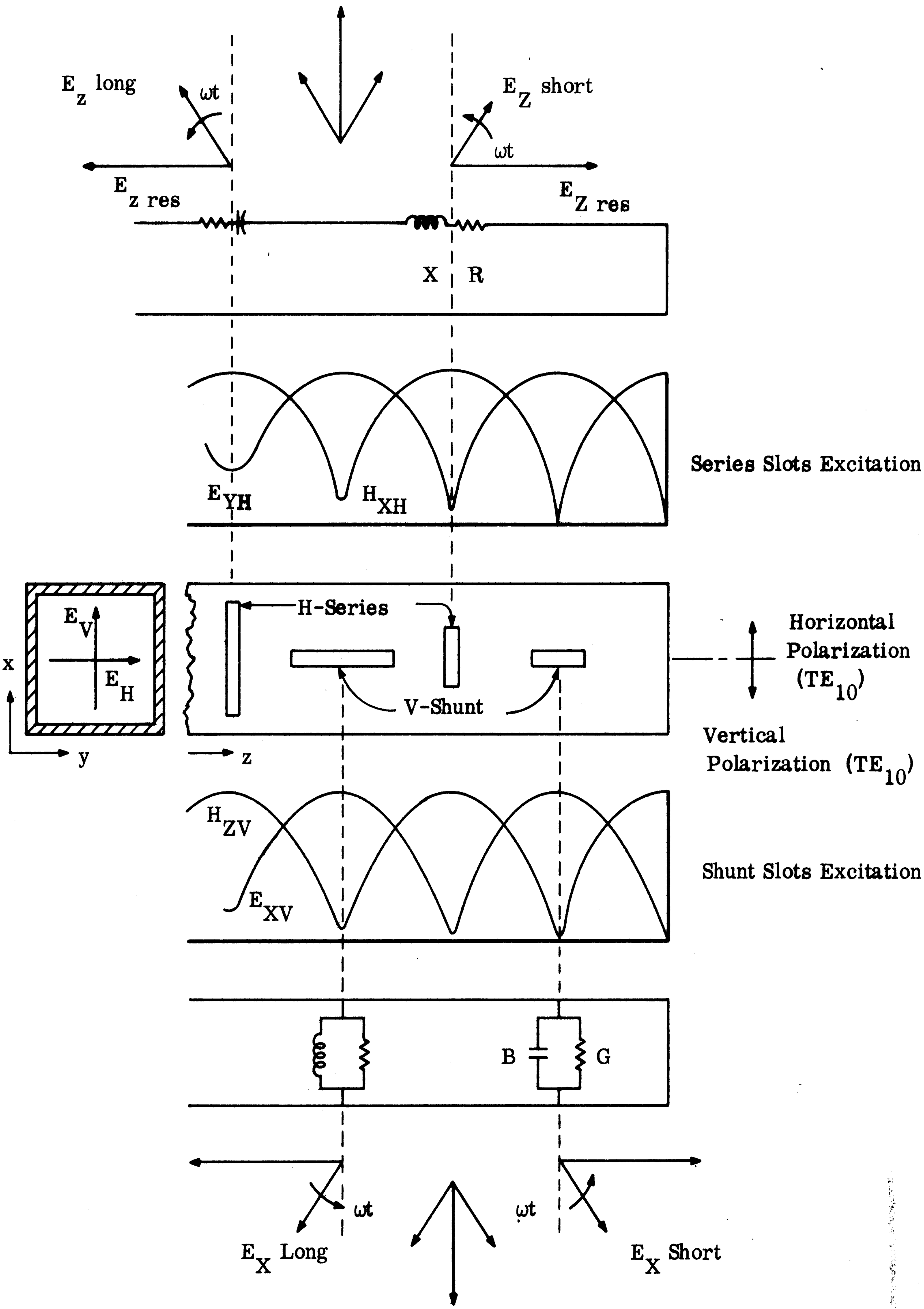


FIGURE 2: SCHEMATIC OF ALTERNATING SLOT LENGTH APPROACH

1. Specify guide dimension, a, operating frequency, f, and desired array length, L.

2. Determine guide wavelength, λ_g, and from this the number of slots, $N \simeq 2L/\lambda_g$; since the percentage bandwidth will be about 25/N, a respecification of array length may be necessary to achieve a given bandwidth.

3. Determine the desired normalized incremental resistance and conductance, 1/N, and the resonant values from equations (2-1) and (2-2).

4. Using either the experimental curves of Jasik[5] or Oliner[3] (or possibly the theoretical curves of Oliner), estimate the amount of shortening and lengthening that should be used to obtain the incremental conductance 1/N from each slot. One must be careful to use normalized values when entering these curves.

5. The actual lengths are obtained from the length ratios obtained from step 4, and use of an experimental or theoretical resonant length. For slots with round ends the resonant length for centered slots is about .485λ; with square ends, Oliner suggests using values that are 2% lower.

6. Slot locations are specified according to the choice of crossed or separated slots.

Experimental data on a design using this approach appears in Section 6; Section 5 presents the basis for the approach using a dielectric-filled waveguide.

THEORY OF APPROACH B -- DIELECTRIC LOADING

5.1 Introduction

This section presents theoretical formulas for the complete design of slot arrays in dielectrically loaded waveguides. There are apparently no prior studies of this topic; at least, communications with two leading authorities [8] suggests that this is so. To repeat the background given in Section 3, this is a necessary design approach to achieve uniform slot phasing without grating lobes, since the preceding design only approximates uniform phase. The material in this chapter, however, is quite general, since it is applicable to any type of slot design in dielectrically loaded waveguides.

There are three groups of quantities that need to be determined in this section for each type of slot:

a) values of resistance or conductance

b) values of reactance or susceptance

c) resonant lengths of slots

The first group can be obtained fairly simply; it is given first. The second is more difficult and the third is obtained from the second. In our approach, we obtain the resonant length from the complete formulas for the susceptance; a few calculations suffice to present a complete curve. As discussed in Chapter 2, there are two approaches to slot theory. In Section 5.2 we give the resonance formulas for both approaches; they are shown to be equivalent. This presentation is followed by a discussion of the susceptance formulas, which can only be obtained from the Oliner approach. It is likewise found that the modifications of his analysis for the case of dielectrically loaded guide are not difficult. The first resonant lengths are then obtained as those lengths that give zero susceptance.

5.2 Resonant Resistance or Conductance

The first method to be discussed in obtaining the modifications to the resonant resistance and conductance expressions (as in equations (2.1) and (2.2)) is that of Stevenson. The Oliner approach is similar and is briefly discussed next. Unfortunately, the Stevenson formulas are presented in cgs units and it is therefore advantageous to combine his work with the summary given by Silver. The basic approach is to equate the radiated power to the difference between incident power and transmitted power. The Stevenson-Silver results are limited to the case $a' = \lambda_o/2$; the Oliner results reduce to the Stevenson-Silver results in that special case.

Our modifications of the Stevenson-Silver results can begin with an intermediate equation for the resonant conductance of the shunt slot when the slot length is not restricted to being a half wavelength long and with any interior dielectric:

$$\frac{G}{Y_o} = \frac{1}{R_{dip}} \cdot \frac{30}{\pi}\left(\frac{\lambda g}{\lambda_o}\right)\left(\frac{\lambda_o^4}{a^3 b}\right)\left(\frac{2a'}{\lambda_o}\right)^2 \left|\frac{\sin\theta\cos\left(\frac{\pi a'}{\lambda g}\sin\theta\right)}{1-\left(\frac{2a'}{\lambda g}\right)^2\sin^2\theta}\right|^2 \qquad (5.1)$$

This slight modification is appropriate because Silver's "transformation ratio" is easily changed when $a' \neq \lambda_o/2$ and because the characteristic admittance can be expressed as

$$Y_o = \sqrt{\frac{\epsilon_o}{\mu_o}} \cdot \frac{\lambda_o}{\lambda g} \qquad (5.2)$$

where

$$\lambda_g/\lambda_o = \frac{1}{\sqrt{\epsilon_r - (\lambda_o/2a)^2}} \qquad (5.3)$$

In Equation (5.1), Babinet's principle has been used exactly as it was by Silver; the justification for this step follows: The slot has already been assumed "resonant" at this stage in order to allow the simplification that follows from having a real reflection coefficient. The reader is referred to Silver for that discussion+ so that the discussion here may be confined to the modifications that follow from using an interior dielectric.

First, however, we examine the use of Babinet's principle, which in the form we use expresses the relationship between the radiation impedances of a slot in a metal plate, and of the geometrically inverse metal strip antenna (as illustrated in Figure 3):

$$Z_{r(slot)}\, Z^*_{r(strip)} = \frac{1}{4}\,\frac{\mu_o}{\epsilon_o}\,. \qquad (5.4)$$

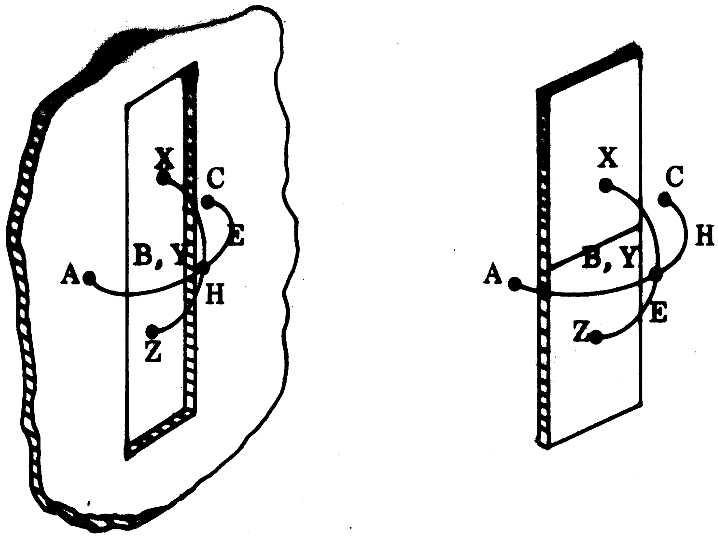
If the metal strip is assumed to be resonant, $Z_{r(strip)} = R_{(dip)}$, $Z_{r(slot)}$ is also real ($= R_{slot}$), and the resistance of the slot is:

$$R_{slot} = \frac{1}{R_{dip}} \cdot \frac{\mu_o}{4\epsilon_o}\,. \qquad (5.5)$$

This is precisely the result used in Silver's work leading to his equivalent of (5.1) above.

+An error that should be noted occurs in equation (51b) on page 293 of Reference [4] which should read:

$$S_{10} = Y^{(0)}_{10}\,\frac{ab}{2}\,\left(\frac{\beta_{10}}{k}\right)^2$$



a) Slot b) Complementary Dipole

FIGURE 3: ILLUSTRATION OF BABINET'S PRINCIPLE

The main feature of the analysis of the slot by using Babinet's principle is to analyze a dipole with complementary fields. The configuration and nomenclature is that used by Booker [8] ; it is apparent from a consideration of the fields that one must use a dipole entirely in air in order to analyze the slot radiating into a half-space even when the waveguide is filled with a dielectric. The magnetic field of the dipole can only be the complement of the slot electric field if the dielectric is ignored in the case of the dipole. Another way of stating this is that the radiated power is determined by the slot voltage alone; the slot voltage is not influenced by the dielectric.

Once we have determined that the resistance of a dipole in air should be used in Equation (5.5) for the resistance of the resonant slot in a dielectric-loaded guide, we must next evaluate this resistance, R_{dip} for a length, a', not equal to $\lambda_o/2$. It is obtained by the Poynting vector method. The far field of a dipole can be written as:

$$E_\theta = j\frac{60\pi}{\lambda_o r_o} e^{-j\beta_o r_o} \sin\theta \cdot I, \tag{5.6}$$

where

$$I = \int_{-a'/2}^{+a'/2} \cos\frac{\pi z'}{a'} e^{j\beta_o z' \cos\theta} I_o\, dz' = \frac{2\frac{\pi}{a'} \cos\left(\beta_o \frac{a'}{2} \cos\theta\right)}{\left(\frac{\pi}{a'}\right)^2 - \beta_o^2 \cos^2\theta} I_o, \tag{5.7}$$

obtained from the variation of the principal-mode fields along the guide; the dipole must have the same excitation in order to provide complementary fields, as required by Babinet's principle.

Next, we obtain the radiated power as:

$$P = \int_0^\pi \int_0^{2\pi} \frac{1}{2} R_e \left[E_\theta H_\phi^*\right] r_o^2 \sin\theta\, d\theta\, d\phi$$

$$= -30\, I_o^2 a_1^2 \int_0^\pi \frac{\sin^2\theta}{(1 - a_1^2 \cos^2\theta)^2} \cos^2\left(\frac{\pi}{2} a_1 \cos\theta\right) \sin\theta\, d\theta,$$

where $a_1 = \beta_o a'/\pi = 2a'/\lambda_o$. (5.8)

This can be integrated by using partial fractions and integration by parts; from the expression $P = \frac{1}{2} I_o^2 R_{dip}$, one obtains:

$$R_{dip} = \frac{120}{a_1} \left\{ \frac{(a_1^2 + 1)}{8} \left[\mathrm{Cin}\left((1+a_1)\pi\right) - \mathrm{Cin}\left((1-a_1)\pi\right)\right] + \frac{a_1}{2} \cos^2 \frac{\pi}{2} a_1 + \left(\frac{a_1^2 - 1}{8}\right) \pi \left[\mathrm{Si}\left((1 + a_1)\pi\right) - \mathrm{Si}\left((1-a_1)\pi\right)\right]\right\}. \tag{5.9}$$

When $a_1 = 2a'/\lambda_o = 1$, this reduces to $R = 30\, \mathrm{Cin}\,(2\pi) = 73.2$.

This expression has been calculated and the plot of the calculated points can be approximated well for $0.35 < \frac{a'}{\lambda_o} < 0.5$ by:

$$R_{dip} = 79.75\,(1 - .348(\frac{a'}{\lambda_o})^2)\,(\frac{2a'}{\lambda_o})^2 \qquad (5.10)$$

for $a'/\lambda_o = 1/2$, $R_{dip} = 73.2\,\Omega$ as given by Stevenson. It is also similar to the admittance expression used by Oliner, G_{rj} proportional to:

$$80(1 - .374\,(\frac{a'}{\lambda})^2 + .130\,(\frac{a'}{\lambda})^4) \qquad (5.11)$$

For future calculations, Oliner's values will be used because of the excellent agreement.

The conclusion is that the Stevenson formulas need be modified only to take account of the change in guide wavelength, input admittance and resonant slot length. The Oliner formulas then result with the addition of the edge shunt slot formula given above and repeated here in a slightly different form:

$$\frac{R}{Z_o} = \frac{8\pi}{3}\,(\frac{a^3 b}{\lambda_o^3 \lambda_g})\,\frac{\left[1 - (\frac{2a'}{\lambda_g}\sin\theta)^2\right]\left[1 - .374(\frac{a'}{\lambda_o})^2 + .130\,(\frac{a'}{\lambda_o})^4\right]}{\sin^2\theta\,\cos^2(\frac{\pi a'\sin\theta}{\lambda_g})} \qquad (5.12)$$

which looks much like the broadwall shunt slot expression given by Oliner in his equation (20) of Part I.

5.3 Suspeptance or Reactance Calculations for a Centered Series Slot

The basic Oliner approach [3] is to obtain an equivalent circuit for the slot as shown in Figure 4. The radiating centered series slot is characterized as an E-plane Tee junction (series susceptance B_j and transformer ratio n_j:1), a short section of small waveguide (characteristic admittance Y_o'), and a radiating junction (admittance $G_{rj} + jB_{rj}$).

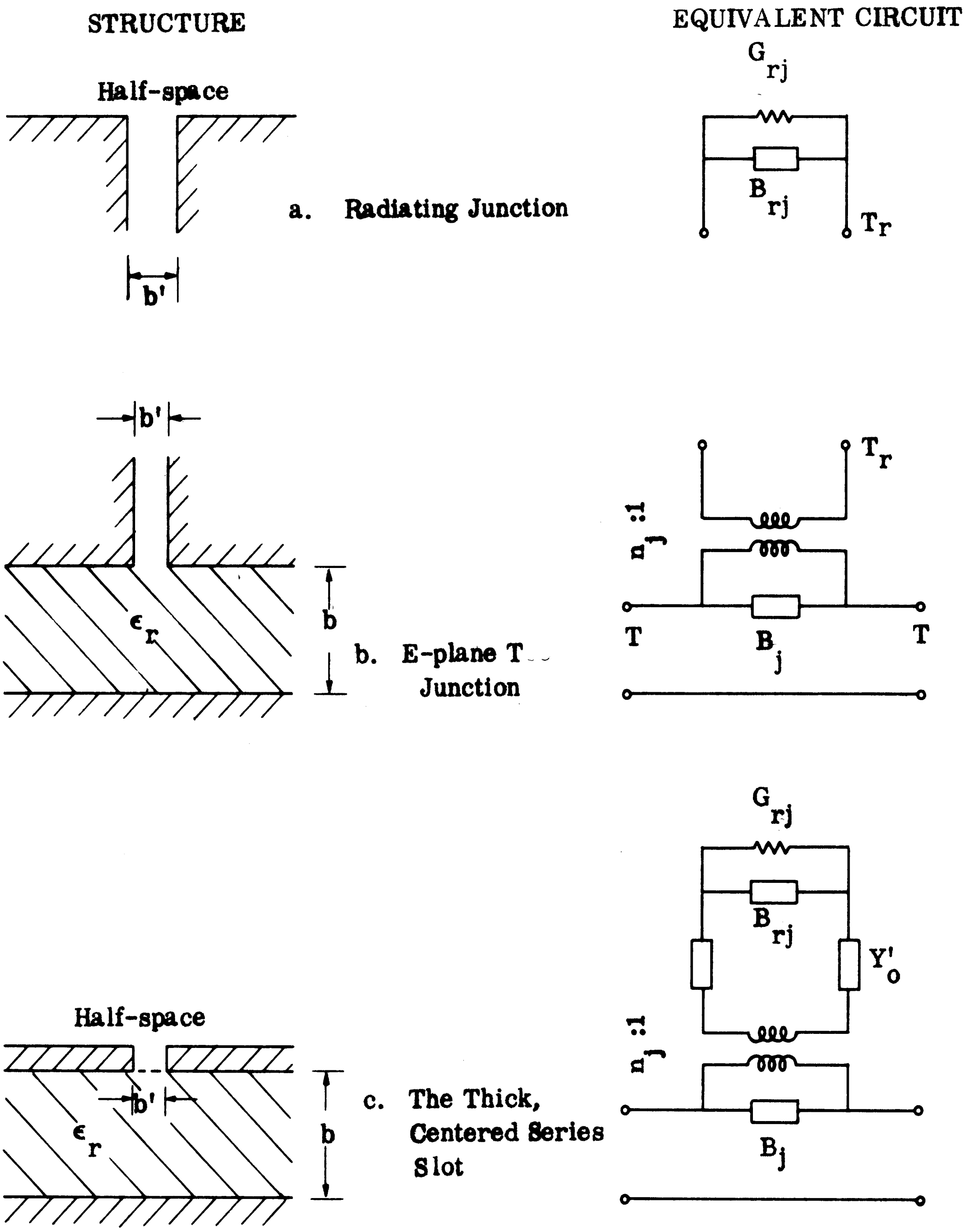


FIGURE 1: SLOT STRUCTURES AND THEIR EQUIVALENT CIRCUITS

It is obvious that in this circuit the parameters associated with the radiating junction (Figure 4a) are unaffected by properties internal to the waveguide. This idea corresponds to the use in the preceding section of Babinet's principle with a dipole radiating in free space rather than one radiating at an air dielectric interface. Thus, the values of B_{rj} and G_{rj} are assumed to be unchanged by the presence of the dielectric. Similarly, the section of transmission line characterized in Figure 4c by Y'_o is assumed to be unchanged. However, the parameters of the E-plane Tee Junction shown in Figure 4b must be carefully reevaluated to determine how they will be altered. This method will give the results in the preceding section on conductance as a by-product, but our main interest is in the effect on the susceptance.

Oliner does not derive formulas for n_j and B_j in Reference [3], but notes the results of an earlier study [10] for which he was group leader. This earlier report in turn draws upon still earlier material by Marcuvitz [11 and 12]. In the following we shall attempt to describe how the analysis is obtained from all of these sources and how it is modified by the presence of a dielectric. Fortunately, this discussion can be limited to n_j and B_j only.

In reference [10], Oliner's slot derivations for the n_j and B_j expressions are for an E-plane slot, but they draw heavily on prior material for transverse radiating slots and E-plane Tees, which in turn are derived from the results for transverse coupling apertures. In this study, for the above reasons, we can limit ourselves to the E-plane tee which is discussed in Reference [10] on page 91-94 and page 125-127.

Before proceeding to describe the modifications in these derivations due to dielectric loading, several assumptions must be described that greatly simplify the work. First, Oliner [3] discusses on pages 17 and 20 the justification of ignoring the shunt elements in the usual pi equivalent circuit. This is done in his theoretical work because the experimental results showed it to be a valid approximation. With this simplification, it is not necessary to consider the transformation to an invariant representation; derivations may start with this equivalent form shown in Figure 4c. Also on page 20 of [10] we find that the usual X_c and X_d of the tee representation given by Marcuvitz in his Waveguide Handbook [12] (page 336) can be dropped. We therefore shall assume that these same simplifications will hold in our case; for further justification we shall only refer to the theoretical-experimental agreement discussed in the next chapter.

The derivation of n_j^2 as given by Oliner follows that of Marcuvitz [11] on pages VI-14 ff. The actual expression is equation (VI-35) on page VI-19. Although Oliner uses this result, he is not using the same circuit. That is, Marcuvitz uses $Z_{22} = n^2 Z_{11}$ (page 18) whereas Oliner would have $Z_{11} = n^2 Z_{22}$. Marcuvitz gives an answer valid for the Oliner circuit rather than for his own - for a step-down rather than a step-up transformer. It is our guess that the explanation of n_j^2 followed the derivation and was put in incorrectly; in any case, the result given by Marcuvitz is correct for the Oliner material.

When the main guide is filled with a dielectric, n_j^2 does not change. That this is so follows from the derivation wherein we now define:

$$n_j^2 Z_{22} = Z_{11}$$

so

$$n_j = \frac{\iint_{ap} \hat{n} \times \vec{E}(x,y) \cdot \vec{h}(x,y)\, dx\, dy}{\iint_{ap} \hat{n} \times \vec{E}(x,y) \cdot \vec{'h}(x,y)\, dx\, dy} \tag{5.13}$$

Here and in the rest of the formulas in this section, symbols will have the same meanings as in the references, unless defined otherwise.

In the Oliner-Marcuvitz normalization scheme, we use (for the E-plane tee shown in Figure 4b):

$$\hat{n} \times \vec{E}(x,y) = \hat{x}_o \cos \frac{\pi x}{a'} \tag{5.14}$$

$$\vec{h}(x,y) = \hat{x}_o \sqrt{\frac{2}{a'b'}} \cos \frac{\pi x}{a} \tag{5.15}$$

$$\vec{'h}(x,y) = \hat{x}_o \sqrt{\frac{2}{a'b'}} \cos \frac{\pi x}{a'} \tag{5.16}$$

When equations (5.14) through (5.16) are used with equation (5.13), we obtain the value given by Marcuvitz and Oliner:

$$\frac{1}{n_j^2} = \frac{ab}{a'b'} \left[\frac{\pi}{4} \; \frac{1-(a'/a)^2}{\cos(\pi a'/2a)} \right]^2 \tag{5.17}$$

which is independent of the guide dielectric as stated above.

The value of B_j shown in Figure 4b was given by Oliner [3] as:

$$\frac{B_j}{Y_o} = \frac{1}{2} \frac{B_t}{Y_o} + \frac{2b}{\lambda g} \left[\ln 2 + \frac{\pi}{6} \frac{b'}{b} + \frac{3}{2} \left(\frac{b}{\lambda g}\right)^2 \right] \tag{5.18}$$

where

$$\frac{B_t}{Y_o} = \frac{4b}{\lambda g}\left[\ln \csc \frac{\pi b'}{2b} + \frac{1}{2}\left(\frac{b}{\lambda_g}\right)^2 \cos^4\left(\frac{\pi b'}{2b}\right)\right]$$

$$- \frac{4b}{\lambda g}\left(\frac{\lambda_g}{\lambda_{g3}}\right)^2 \left[\frac{\cos\frac{3\pi a'}{2a}}{\cos\frac{\pi a'}{2a}} \cdot \frac{1-\left(\frac{a'}{a}\right)^2}{1-9\left(\frac{a'}{a}\right)^2}\right]^2 \left[1+\left(\frac{\pi b'}{2\lambda_{g3}}\right)^2\right] \ln\left|\frac{4\lambda_{g3}}{\pi\gamma b'}\right| \tag{5.19}$$

with

$$\gamma = 1.781, \text{ and}$$

$$\lambda_{g3} = \left|\frac{\lambda}{\sqrt{1-\left(\frac{3\lambda}{2a}\right)^2}}\right| .$$

These forms were taken from reference [10], where equation (5.19) above was given as equation 3.129 (page 120):

$$\frac{B_\ell}{Y_o} = \left(\frac{B_b}{Y_o} + \frac{B_a}{2Y_o}\right) - \frac{B_a}{2Y_o} - \frac{1}{n_c^2}\frac{B_t}{2Y_o} \tag{5.20}$$

with the circuit shown in Figure 5. For our case, as mentioned above and in reference [10] on pages 20 and 127, both X_{cc} and B_s are zero, and n_{cs} reduces to the expression for n_j given in equation (5.5) as long as b' is small and

$$J_o\left(\frac{\pi b'}{\lambda g}\right) \simeq 1 \tag{5.21}$$

The expression for $\frac{B_b}{Y_o} + \frac{B_a}{2Y_o}$ was given in equation 3.122 of [11] for our range of parameters as:

$$\frac{B_b}{Y_o} + \frac{B_a}{2Y_o} = \frac{B_t}{Y_o} + \frac{2b}{\lambda g}\left[\ln 2 + \frac{\pi b'}{6b} + \frac{3}{2}\left(\frac{b}{\lambda g}\right)^2\right] \tag{5.22}$$

where a correction noted on the errata sheet has been inserted. Combining (5.21) and (5.23) gives (5.19).

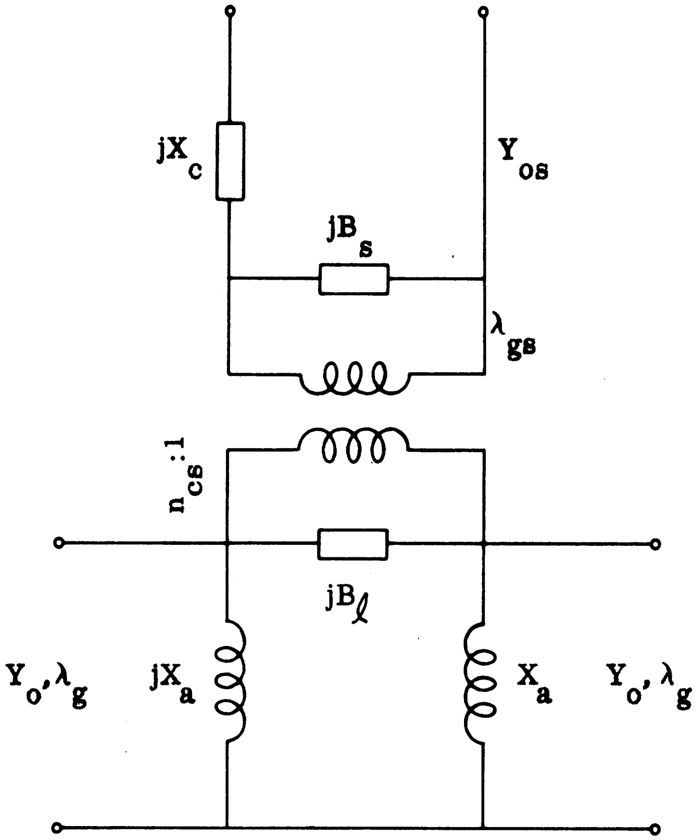


FIGURE 5: COMPLETE EQUIVALENT CIRCUIT FOR AN E-PLANE TEE

The expression for B_t/Y_o was given in [10] as equation 3.107 (page 114), with a deletion given in equation (5.20) above that was noted in their errata sheet. The second part of the expression for B_t/Y_o was given in equation (3.51) of [10]; the first part was stated on page 83 to be the susceptance of a capacitive iris and the variational expression is given there. Unfortunately, none of the derivations appear in reference [10], although certain ones appear in [11]; they are developed as follows. Before proceeding, we should note again that we only need obtain B_b and B_t now since we have assumed that B_a can be ignored.

Oliner's basic premise for the E-plane junction is that the element B in Figure 5 is the only significant element. Changes in stub guide cross-section affect Y'_{os} and n_{cs} but not B_ℓ since the rate of change of energy storage is assumed (in this variational approach) to be independent of the stub guide. The value of B_ℓ can thus be found from known results for any slot-coupled junction, in particular for the case when a = c and b = d. It is given by

$$\frac{B_\ell}{Y_o} = \left.\frac{B_b}{Y_o}\right|_{\substack{a=c\\ b=d}} - \left.\frac{1}{n_c^2}\,\frac{B_t}{2Y_o}\right|_{\substack{a=c\\ b=d}} \tag{5.23}$$

Since the variational expression for $\frac{B_b}{Y_o}$ differs from that for B_t/Y_o only by the inclusion of image terms in the latter case we have

$$\frac{B_b}{Y_o} = \frac{B_t}{Y_o} + \Delta \quad (\Delta \text{ is our nomenclature})$$

so

$$\frac{B_\ell}{Y_o} = \frac{B_t}{2Y_o} + \Delta \tag{5.24}$$

as long as $n_c \approx 1$ (which is true in our case).

$$\frac{B_t}{Y_o} = \frac{B_t}{Y_o}\bigg|_{a'/a = 1} + \frac{B_{corr}}{Y_o} \tag{5.26}$$

The value of Δ is obtained by Oliner by comparison of the B_b/Y_o and B_t/Y_o values given by Marcuvitz in his <u>Waveguide Handbook</u> [12], pages 218 and 351. The value that we obtain in this comparison differs only slightly from that given by Oliner; but his value is used in the following calculations.

When a dielectric is present the value of B_ℓ is obviously obtainable in the same way, i.e.

$$\frac{B_\ell}{Y_{o\epsilon}} = \frac{B_b}{Y_{o\epsilon}} - \frac{B_t}{2Y_{o\epsilon}} \frac{1}{n_c^2} , \tag{5.27}$$

where $Y_{o\epsilon}$ signifies the guide characteristic admittance when dielectrically loaded. Fortunately, as shown above, n_c is independent of the value of ϵ in either guide, but it should be used here as though the stub guide were filled with dielectric. Thus using

$$\frac{B_b}{Y_{o\epsilon}} = \frac{B_t}{Y_{o\epsilon}} + \Delta_\epsilon \tag{5.28}$$

we obtain the result given above:

$$\frac{B_\ell}{Y_{o\epsilon}} = \frac{B_t}{2Y_{o\epsilon}} + \Delta_\epsilon \tag{5.29}$$

The values given by Marcuvitz quoted above are given in terms of λ_g so it is only necessary to use:

$$\lambda_{gm} = \frac{\lambda_o}{\sqrt{\epsilon_r - \left(\frac{m\lambda_o}{2a}\right)^2}} \tag{5.30}$$

since none of the expressions or derivations were based specifically on air dielectrics. This is also seen in the work by Lewin [18]. The expressions given by Oliner ([1]) thus can be used directly in the analysis of loaded guides as long as (denoting by $\Rightarrow$ the replacement of a symbol by the one to be used when $\epsilon_r \neq 1$ inside the guide):

$\lambda_g \Rightarrow \lambda_{g\epsilon}$

$\lambda_{g3} \Rightarrow \lambda_{g3\epsilon}$

$\lambda \Rightarrow \lambda_o$

K stays the same (not so if the dielectric extends up into the slot).

5.4 Resonant Length of a Broadwall Series Slot in a Dielectrically Loaded Waveguide

There does not seem to be any more direct method of obtaining the resonant length of a slot in a dielectrically filled waveguide than that of obtaining the slot length a' which gives a total susceptance $B_{ce} = 0$ for each ϵ_r. For RG-52/U waveguide and a slot that is resonant at f = 10.70 Gc (a' = .532 inches) when $\epsilon_r = 1.0$, a plot is given in Figure 6 of the variation in a'/λ_o with ϵ_r. This variation is obtained from numerical calculations at lower frequencies with $\epsilon_r = 1.6$, 2.25, and 3.0, using the Oliner formulas as modified in the previous section. It is seen that a nearly linear variation occurs that can be approximately expressed as:

$$2a'/\lambda_o = -.108\,\epsilon_r + 1.04 \tag{5.31}$$

This approximate empirical formula is plotted for comparison with the zero of Oliner's susceptance formulas and with experimental results. Equation (5.31) is seen to give a good starting point for trial and error calculations, using Oliner's B_c formula, in search of a more exact resonant point.

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PAGE

6

EXPERIMENTAL RESULTS

In this section, experimental results are presented which verify the theory presented in the two previous sections. This data was obtained at the Willow Run facilities of the University of Michigan Radiation Laboratory. The radiation patterns were obtained on an indoor antenna range. The Smith chart plots were obtained by the standard method of shift of null positions using precision slotted lines.

The waveguides used in these tests were brass; the rectangular guide used in the dielectric loading tests was standard RG-52/U (.900" x .400" x .050"). The slots were of two types, each machined in a different way. The slots in the square waveguide had square ends and were placed in the guide by electrical discharge machining. The slots in the rectangular waveguide (used with tests of the dielectric loading) had rounded ends and were cut by a standard milling operation.

In each of the two following sub-sections, experimental verification is presented for the two theoretical discussions given in the preceding sections.

6.1 ALTERNATING SLOT LENGTHS

The important feature of this approach is the demonstration of adequate radiation patterns. These are shown in Figure 7 where the vertical, (edge shunt slots) and cross-polarization patterns are shown with the frequency and number of slots indicated. The design tested was similar to that given in section 4.

It can be seen that adequate isolation and reasonably good patterns were obtained for both polarizations.

Actually, with this design, the slot lengths were not correct; the conductance was too large, especially for the longer slots. Nevertheless the radiation patterns of Figure 7 demonstrate the essential validity of this approach.

The admittance plot of Figure 8 demonstrates the behavior of the admittance of the edge shunt slots as the number of slots is increased. Essentially the same results are seen for the impedance of the broadwall series slots. The excessively large conductance or admittance of the longer slots should be noted, indicating the need for lengthening them.

The input VSWR for the edge shunt slots is shown in Figure 9 as a function of frequency; with the number of slots as a parameter. It can be seen that a good match is achieved at frequencies well above and below the resonant frequency; this corresponds to the short and long slots nearest the generator respectively approaching their resonances. It is believed that the intermediate low VSWR regions correspond to the slots near the short circuit respectively approaching their resonance. Similar results are obtained for the series slot cases, but with larger mismatches.

These figures are believed to indicate the validity of the approach although it is obvious that a better match could be made.

6.2 DIELECTRIC LOADING

Because of the more practical nature of the previous approach, this approach was not tested with a practical array; only single slot data is presented here. By taking impedance data, the resonance length of a broadwall series slot was obtained and was given in the preceding section along with the theoretical curve.

Figure 10 gives both theoretical and experimental plots of the resistance and reactance of slots in a waveguide loaded with a dielectric with $\epsilon_r = 1.60$. The agreement is seen to be very good. This data is believed to verify the correctness of the theory presented in section 5.3. Practical arrays should be possible but have not been constructed because of the success of the alternating slot approach and the fact that the work was performed only to illustrate the practicability of a dual polarized slot array.

MODEL

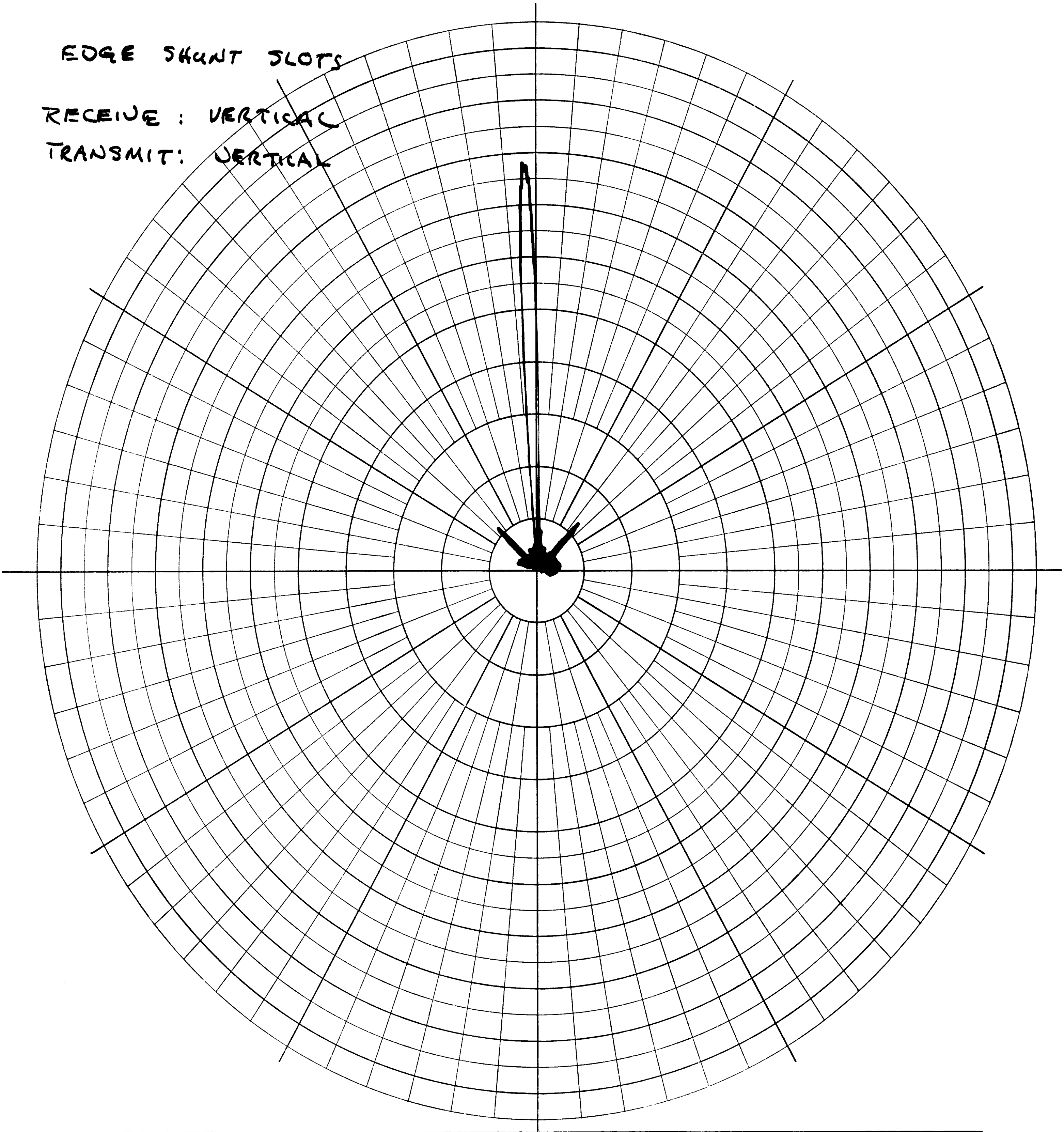
PAGE

FIG. 72 Radiation Pattern: ALTERNATING SLOT LENGTHS

EDGE SHUNT SLOTS

RECEIVE: VERTICAL

TRANSMIT: VERTICAL



ANTENNA TYPE LOCATION USE

TEST MODEL: SQUARE GUIDE FREQUENCY: 9375 MCS

MODEL SCALE: SCALE FREQUENCY: ______ MCS

CONDITIONS: 18 SLOTS POLARIZATION:

CURVES PLOTTED IN: E θ:

VOLTAGE: E ϕ:

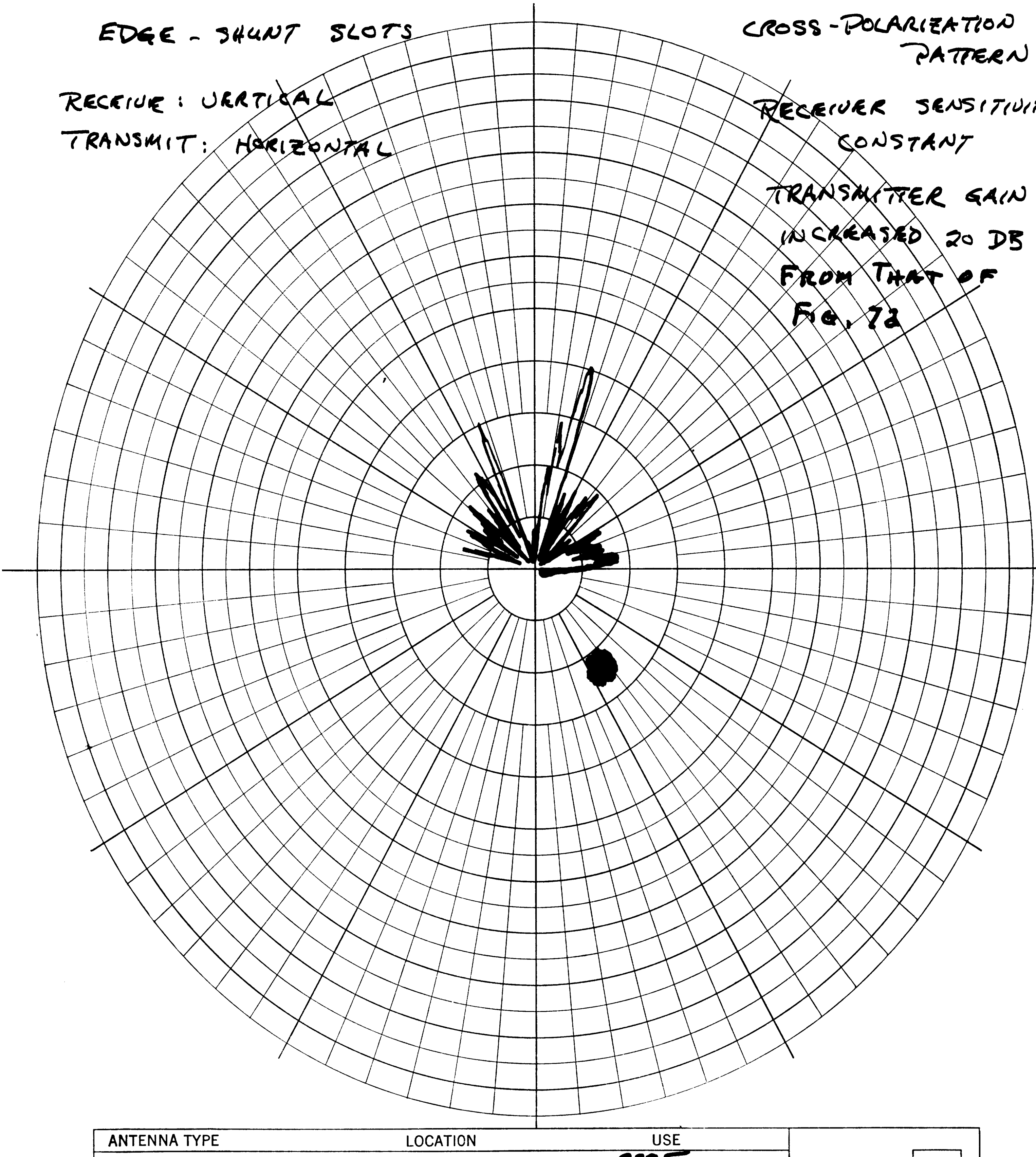
POWER: ✓ PATTERN AREA:

ENGINEER OPERATOR FILE NO. R7 DATE 8-13-64

MODEL

PAGE

Fig 76, Radiation Pattern, cont'd: ALTERNATING SLOT LENGTH



ANTENNA TYPE LOCATION USE

TEST MODEL: SQUARE GUIDE FREQUENCY: 9375 MCS

MODEL SCALE: SCALE FREQUENCY: MCS

CONDITIONS: 18 SLOTS POLARIZATION:

CURVES PLOTTED IN: E θ:

VOLTAGE: E ϕ:

POWER: ✓ PATTERN AREA:

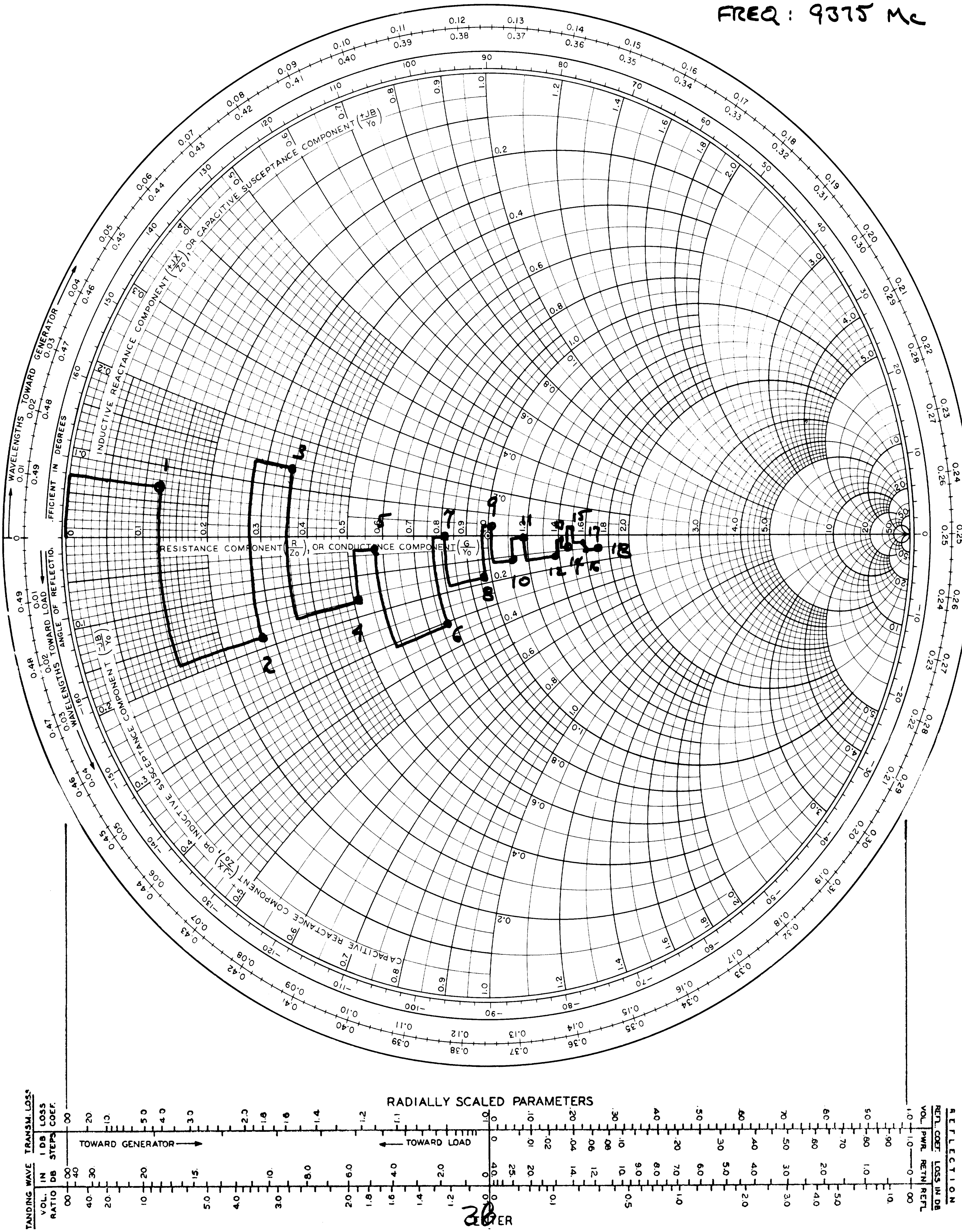
ENGINEER OPERATOR FILE NO. 26 DATE 8-13-64

NAME	TITLE	DWG. NO.
Figure 8	Alternating Length Slots : Admittance	DATE
SMITH CHART FORM 82BSPR (2-49)	KAY ELECTRIC COMPANY, PINE BROOK, N.J. ©1949 PRINTED IN U.S.A.	

IMPEDANCE OR ADMITTANCE COORDINATES

EDGE SHUNT SLOTS

FREQ: 9375 Mc



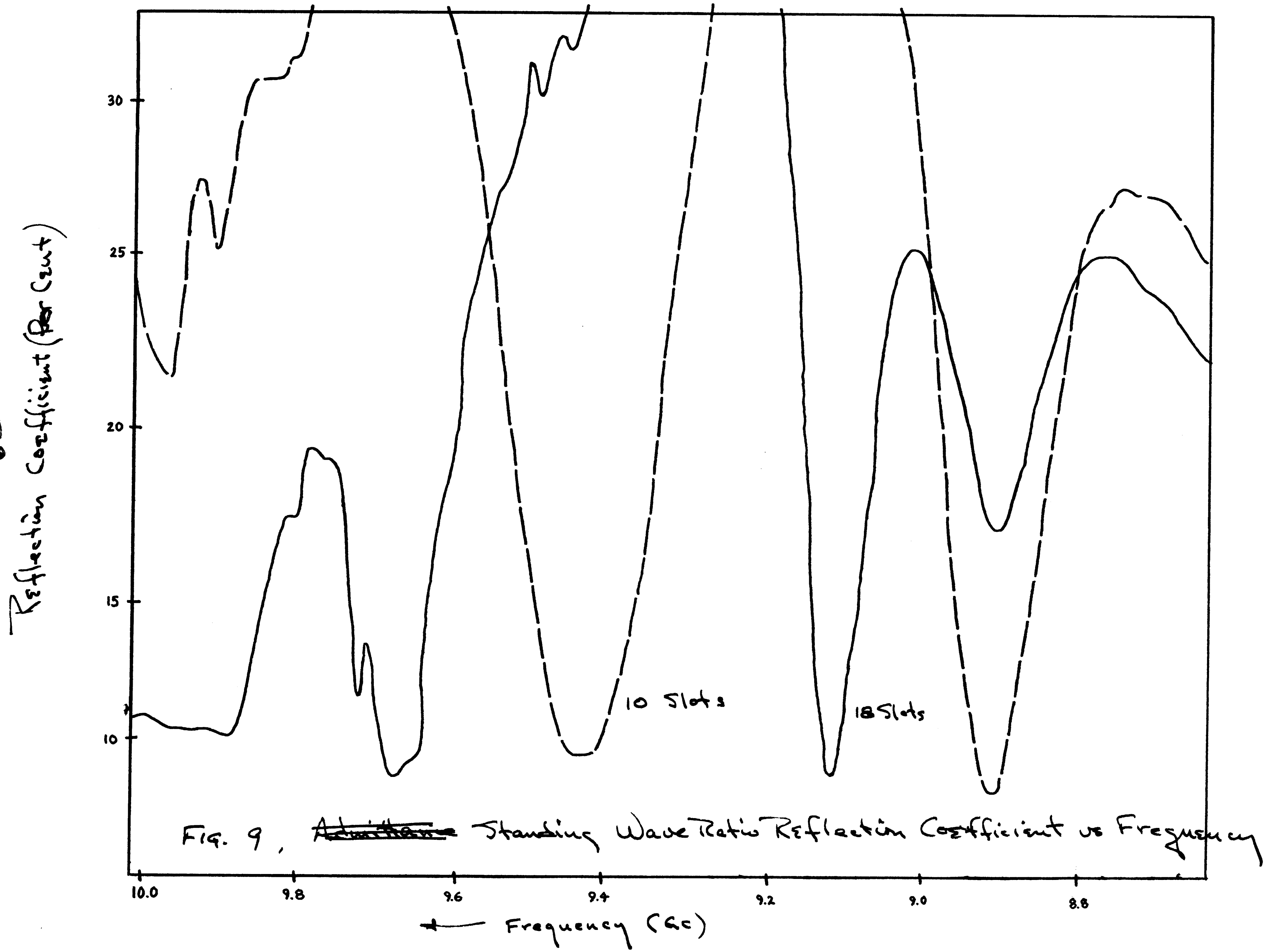


Fig. 9, ~~Admittance~~ Standing Wave Ratio Reflection Coefficient vs Frequency

NAME	TITLE	DWG. NO.
Figure 10	Dielectric Loading : Impedance	DATE
SMITH CHART FORM 82BSPR (2-49)	KAY ELECTRIC COMPANY, PINE BROOK, N.J. ©1949 PRINTED IN U.S.A.	

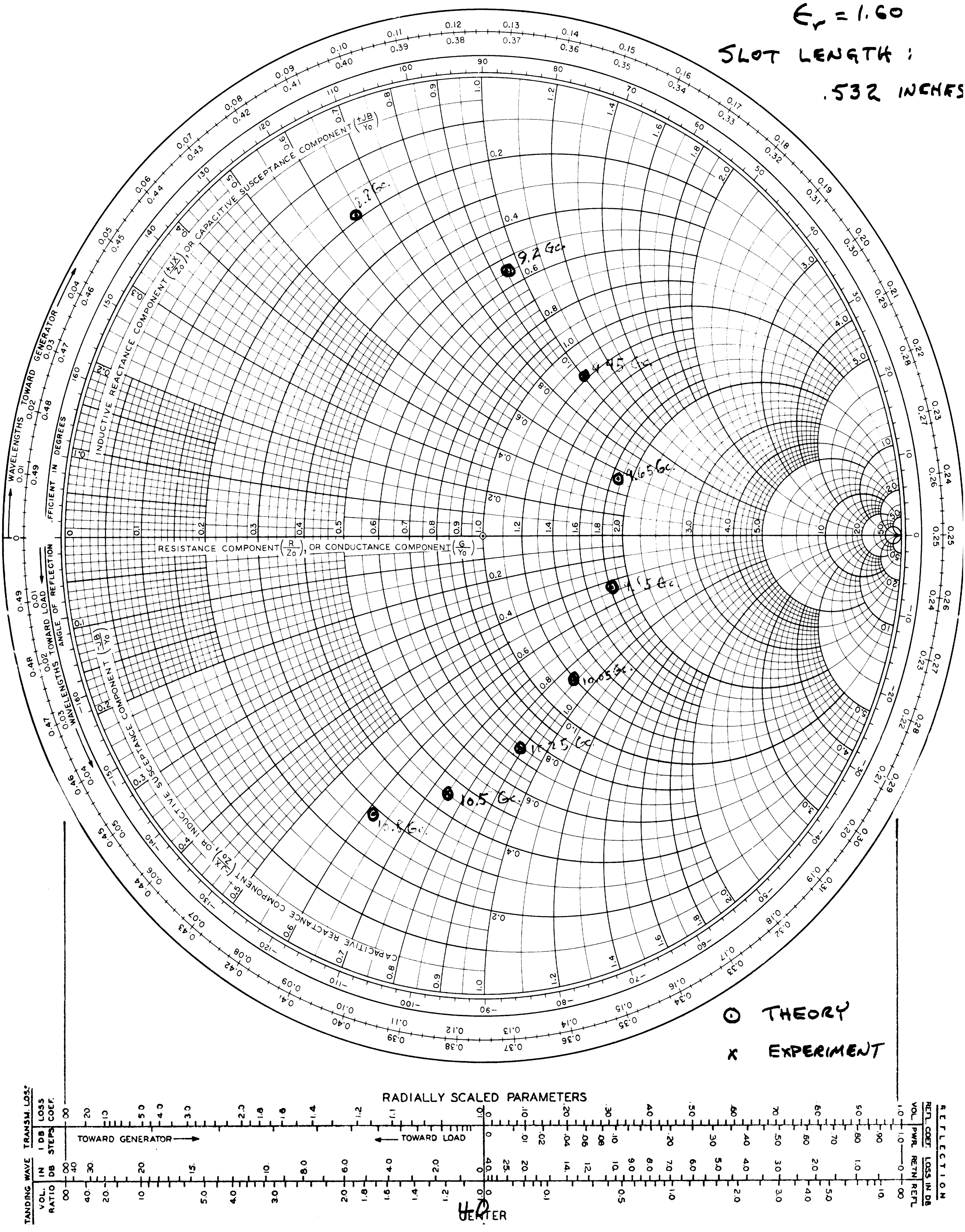
IMPEDANCE OR ADMITTANCE COORDINATES

INTERIOR DIELECTRIC:

$\epsilon_r = 1.60$

SLOT LENGTH:

.532 INCHES



⊙ THEORY

x EXPERIMENT

7. CONCLUSIONS

It has been shown that efficient, broadside, dual-polarized beams can be obtained from "crossed" slots in a single square waveguide. Two new approaches have been developed and verified; neither has been carried through to completion since the present results are believed to fulfill the interests of the contract. Nevertheless, the results are believed to be an important contribution to the state of the art and should prove useful in future requirements for variable polarization.

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