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COLLEGE OF ENGINEERING
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STUDY OF PROBLEM AREAS IN OPTICAL COMMUNICATIONS

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G. Hok, M. L. Barasch, P. Lambropoulos
and E. K. Miller

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ABSTRACT

In the problem area of attenuation of electromagnetic radiation in the atmosphere, some additional discussion is presented concerning the effects of rainstorms, clouds, haze , fog, Rayleigh scattering and molecular absorption bands. Further data collected from the literature are tabulated.

The previous over-emphasis on energy states and energy measurements in quantum communication theory is criticized. Recent suggestions of analyzing electromagnetic radiation in terms of 'coherent states' , essentially equivalent to minimum-uncertainty wave packets, are considered.

A binary photon channel with a laser preamplifier is evaluated from the point of view of statistical decision theory. A certain quality parameter proposed in the literature is calculated for the purposes of comparing the potential performance of channels with or without preamplifier.

No work has been carried out during the work period on the third problem area of the contract, concerning passive optical band filters for photon-counter receivers.

I INTRODUCTION

The introduction to the First Interim Report submitted under this contract (Barasch et al, 1964) gave an outline of the problem areas to be studied, the relevant ranges of the quantitative parameters in each case, as well as a tentative plan of various specific topics to be investigated consecutively. It would serve no useful purpose to repeat this material here, we shall only comment briefly on the progress made and the directions indicated for future efforts.

In the Second Interim Report (Hok et al, 1964) we could report the tentative conclusion of the work in one problem area, designated as the third problem area of this project and concerned with highly selective, preferably tunable optical filters for use with photon-counting receivers. The result of the investigation indicated that theoretically such filters are feasible, but that the available materials, the technological state of the art, etc., do not at present permit construction of such filters with reasonable losses, size, operating voltages, etc., for use in satellite communication.

During the present reporting period it has not been possible in a similar way to wrap up and conclude the work on any of the other topics under investigation. Neither has any new topic been initiated, so that the work has been limited to continued attacks on the same problems as previously studied, although in several instances new points of view and new methods of attack have been used in order to gain better insight and to come closer to a complete solution.

In the first problem area, attenuation of electromagnetic radiation in the atmosphere, further discussion of the effects of rainstorms, clouds, haze, fogs, Rayleigh scattering and molecular absorption bands is presented. Additional data collected from the current literature are tabulated.

In the second problem area, the basis of communication theory in quantum physics has been re-examined in the light of some recent publications. Some of the questionable conclusions from earlier presentations appear to be a consequence of undue emphasis on energy states and energy measurements. A less restrictive approach describes electromagnetic radiation in terms of 'coherent states' which are essentially equivalent to minimum-uncertainty wave packets. Another new feature of the application of quantum mechanics to communications is the necessity for repeated observations or measurements on the same system, such as a receiver cavity or an antenna, rather than observations of single events, as is usually the case in basic experiments in quantum physics. The dependence or independence of the result of consecutive measurements requires investigation.

The evaluation of the binary photon channel with a laser preamplifer has been given continued attention during the period. A different approach related to the statistical decision theory, or more specifically the theory of signal detection, has been investigated and is reported in Section III of this report.

Pending the solution of more basic problems, the coding of binary photon channels with very low signal power has not yet been given any attention. Work in this area is expected to start in the near future.

II FIRST PROBLEM AREA: OPTICAL COMMUNICATION UNDER ADVERSE METEOROLOGICAL CONDITIONS

2.1 Introduction

Attenuation in the real atmosphere is attributable to many diverse effects which will be considered separately in this section. One of the present objectives has been discussed in our Interim Reports 1 and 2 (Barasch et al, 1964, Hok et al, 1964), that is, to determine the theoretical limitations on propagation. The large quantity of research now existing on this subject contains many crude approximations presumably made out of a desperate need to obtain a numerical result. In addition, there seems to be some difficulty in specifying input parameters corresponding to disturbed weather. We must stress that, probably for these reasons, most theoretical conclusions offered here should be taken as, at best, internally consistent but not necessarily referring to real conditions in practice.

2.2 Attenuation in a Rainstorm

For parameters claimed to represent a moderately heavy rainstorm(Deirmendjian, 1963) an approximate calculation of the attenuation due to the raindrops has been performed+. The value obtained for the attenuation coefficient was

$$\beta_{rain} \sim 0.8 \text{ Km}^{-1} \tag{2.1}$$

No consideration of parameters (drop size distributions) for light and severe rainstorms has been found, so that no calculation of the range of variability of β has been possible. Further, it will be necessary to ascertain the concentration of water vapor associated with the presence of liquid water, and to compute the attenuation corresponding to its absorption spectrum.

+This is an unpublished memorandum which will be revised and incorporated in a future report.

2.3 Propagation Through Clouds

A cumulus cloud model is introduced by Deirmendjian (1963) where the attenuation corresponding to the liquid water droplets only for this model is reported to be essentially frequency independent for wavelengths between 0.45μ and 16.6μ at the value

$$\beta_{cumulus} \sim 17 \text{ Km}^{-1} \quad . \tag{2.2}$$

Since the cloud thickness is claimed to be in the range 230 - 2100 m, the one-way transmissivity would be between exp(-3,-4) and exp (-34) or roughly between 0 and 3 per cent. Models for other types of clouds, and the effect of water vapor, remain to be considered. The extension of exact computations to longer wavelengths may be difficult, since values of the index of refraction of water at these wavelengths, which would be required, do not seem to be available.

2.4 Haze and Fog

Derimendjian (1964) has computed the attenuation coefficients for continental and coastal types of haze. These results are tabulated below.

TABLE I: ATTENUATION BY HAZES (Deirmendjian, 1964)

Wavelength (microns)	$\beta_{continental}(\text{Km}^{-1})$ Haze	$\beta_{coastal}(\text{Km}^{-1})$ Haze
0.45	0.1206	0.1056
0.70	0.0759	0.1055
1.61	0.0312	0.0691
2.25	0.0194	0.0424
3.07	0.0289	0.0602
3.90	0.0128	0.0236
5.30	0.0075	0.0112
6.05	0.0129	0.0189
8.15	0.0050	0.0062
10.0	0.0032	0.0045
11.5	0.0064	0.0097
16.6	0.0082	0.0134

For fogs no satisfactory theoretical values of the attenuation coefficient have been located in the existing literature, although some indication of parameters for a fog has been obtained, namely a water content of 10^{-7}g/cm with radius of droplets distributed according to an inverse square law, $n(r)=cr^{-2}$. A calculation has accordingly been attempted here, but this remains incomplete. There has been some difficulty in constructing a satisfactory functional form for approximation to the Mie efficiency factor for extinction which will be valid over the whole range of integration over droplet size, for the index of refraction corresponding to the frequency band of interest, or that part for which the index is available.

2.5 Rayleigh Molecular Scattering and Extinction by the Aerosol Component Associated with the Clear Standard Atmosphere

This subject was discussed in the preceding Interim Report (Hok et al, 1964) and the research of Elterman (1963) cited previously appears to be sufficient and thus requires no further study at this time.

2.6 Molecular Absorption in Bands

The material on this subject which we presented previously is actually only useful quantitatively for those cases where very little absorption is predicted, i.e. short paths or weak lines. For other cases, considerations of line shape, spacing and intensity distribution, may not be ignored. Some efficient methods of introducing these conditions and their variation along propagation paths have been given by Zachor (1961) and Plass (1963). The first of these allows calculation of 'the spectral transmittance between 1 - 8 μ for any atmospheric path ' (p. 6, Zachor, 1961) under certain necessary approximations, while the other tabulates separately the transmittances of CO_2 and of H_2O vapor (wet and dry stratosphere models) between 1,000 - 10,000 cm^{-1} (i.e. 1 μ and 10 μ) for slant paths traversing the entire atmosphere from initial altitudes of 15, 25, 30 and 50 Km and for angles of inclination from vertical to horizontal by 5^o increments.

In subsequent reports we hope to quote some of the results obtained by Zachor and by Plass and report on investigations made to determine whether or not absorption by other atmospheric gases, or by longer wavelength bands of H_2O vapor, may be treated by their methods.

It should be further noted that Altshuler (1961) gives procedures for graphical determination of transmission. These will be reviewed and, if possible, applied to situations of interest in this study.

III SECOND PROBLEM AREA: INFORMATION EFFICIENCY AND CHOICE OF DETECTION SYSTEM.

3.1 Introduction

In order to study the theoretical limitations of photon-counter receivers in optical communication with very low signal levels at the receiving end, we have previously pointed out the necessity of discussing critically the quantum-mechanical foundations of such communication. The first two Interim Reports (Barasch, et al 1964, Hok et al 1964) on this contract presented some contributions to such a discussion in addition to the more direct attacks on the specific tasks. Since the completion of the second Interim Report our attention has been directed to a few papers concerned with some of the conceptual difficulties involved in previous presentations of problems in quantum communication theory. Quantum theory has many aspects that clash with our physical intuition, which is based on everyday classical experience; it has of course been accepted on the basis of its remarkable success in interpretation and prediction of the outcome of physical experiments for which the classical theory fails miserably. But whenever the quantum doctrines are applied to a new area, where sufficient experimental guidance may not be available, the interpretations and conclusions to be drawn from these doctrines are not necessarily obvious.

The next section (3.2) will be devoted to the continued discussion of the communication theory of a channel operating in the domain of quantum theory. The main topic is the previous over-emphasis on energy measurements and the recent suggestions of measurements in terms of 'coherent states', i.e. eigenstates of the annihilation operator, rather than energy states.

In Section 3.3 we report on progress in the study of the merits of a laser preamplifier in a binary quantum-counter channel.

3.2 Further Discussion of Quantum Communication Theory

Up to this time a large percentage of all discussions of the potential merits of optical communication systems have used as a theoretical basis for the arguments the rich material presented by Gordon (1962). We have in the previous Interim Reports discussed pertinent ideas presented in Gordon's work and also pointed out some questions which are not answered in a completely convincing manner along the lines of reasoning given. The difficulties are evidently associated with the fact that even Gordon's most general results, the wave entropy or wave capacity, are derived solely from energy states and energy measurements. It was shown in our second Interim Report (Hok et al, 1964) that qualitatively similar results could be obtained classically if energy (amplitude) alone was considered rather than both amplitude and phase.

During the period covered by this report, considerable work has been devoted to finding a more complete and self-consistent view of communication in the optical range. This task has not yet been successfully terminated, but we shall here review some of the approaches suggested in the literature, which may lead to a satisfactory picture. It may be tentatively stated that it does not appear likely that the results previously presented for the binary channel will have to be essentially modified. The study of such channels is consequently continued in Section 3.3 along the same lines as in the second Interim Report on this contract.

A communication system may conveniently be represented by and discussed in terms of an assembly of harmonic oscillators. Reception of a message is realized by consecutive 'observations' or 'measurements' of the state of one of these oscillators, the receiver 'cavity' or 'antenna'. According to quantum mechanics the result of one of these measurements indicates one of the eigenstates of the measured quantity. If energy is measured, the result is an integral number of 'quanta' or 'photons', n. hf. However, the measurement has perturbed the oscillator, so the result is not an unequivocal statement of the condition of the oscillator at the moment the measurement was initiated. Furthermore, a state characterized by a precise energy value leaves the conjugate variable, time (or phase) completely indeterminate, according to the Heisenberg uncertainty principle. Neither one of these two uncertainties can be eliminated, but presumably there exists an optimum way of carrying out the observations which minimizes the total uncertainty. Clearly the optimum receiver in an optical communication system should give simultaneous minimum-uncertainty estimates of two conjugate observables, such as equivalent position and momentum, or electric and magnetic field, for each sample of the signal. This is the closest possible analog of observing both amplitude and phase of a classical electromagnetic wave.

On the basis of concepts and procedures introduced earlier by Wigner and Feynman, Wells (1961) has developed a formalism for studying the propagation of coherent wave fields. The connection between the classical dynamics and the quantum formalism is Wigner's probability density of position and momentum for each harmonic oscillator representing a mode of the field. The appealing features in this approach are the classical treatment of the propagation and the introduction of the quantum mechanics and statistics solely by the Wigner density. There is no need for the separate introduction of discrete energy states and 'photon statistics'. However, it is too early to say whether or not this approach will, in a reasonably simple way, suggest a solution to the simultaneous minimum-uncertainty estimate problem mentioned above.

Glauber (1962) has defined the concept of 'coherent states' of a harmonic oscillator as eigenstates of the annihilation operator, which is a linear sum of the equivalent position and momentum operators of the oscillator. He also introduces an alternative equivalent representation of the creation and annihilation operators as the negative-frequency and positive-frequency components, respectively, of the electric-field operator, obtained from the Fourier transform of the time-varying field. An expansion of the electromagnetic field in coherent states rather than in energy eigenfunctions has the advantage that the minimum uncertainty permitted by the Heisenberg principle is an intrinsic property of each term in the expansion;

the disadvantage is that the set of states, although complete, is not orthogonal, so that the states do not necessarily represent mutually exclusive events. None-theless Glauber (1963) defines a quasi-probability density over the coherent states which leads to a so-called 'P-representation' of the field, which he shows to be both convenient and significant for the study of the coherence properties of electromagnetic radiation. Evidently there is a certain conceptual kinship between the Wigner density and the P-representation as alternative approaches to statistic descriptions of propagation and interaction of electromagnetic fields. At the time of writing a subjective judgement favors the latter as the most promising path towards a communication theory of coherent and incoherent radiation.

C. Y. She (1964) has attempted to derive the communication capacity of an optical channel of bandwidth Δf and average signal power S in the presence of thermal background radiation of average power N using essentially the same approach as Glauber. The result is

$$C = \Delta f \log \left(1 + \frac{S}{N + \Delta f \cdot hf \,(1 - e^{-\gamma \Delta t})^{-1}} \right) \tag{3.1}$$

where γ is the damping constant in the receiver cavity and Δt is the time between consecutive measurements, i. e. the reciprocal of the bandwidth. The interpretation of this result is the following . Each measurement leaves the receiver cavity in one of the coherent states. The perturbation introduced by a single measurement is equivalent to one quantum of energy hf. The ringing cavity preserves a fraction of this noise energy from one measurement to the next. The total effective noise power represented by the denominator in the expression above is obtained by adding the background radiation and the infinite sequence of attenuated perturbations intro-duced by the measurements.

Equation (3.1) is to be compared with Shannon's classical capacity

$$C = \Delta f \log (1+\frac{S}{N}) \tag{3.2}$$

and Gordon's wave capacity

$$C=\Delta f\left\{\log\left(1+\frac{S}{N+\Delta f\cdot hf}\right)+\frac{S+N}{\Delta f\cdot hf}\log(1+\frac{\Delta f\cdot hf}{S+N})-\frac{N}{\Delta f\cdot hf}\log(1+\frac{\Delta f\cdot hf}{N})\right\} . \tag{3.3}$$

The last two terms in (3.3), which were a consequence of the exclusive consideration of energy states and energy measurements, have disappeared in (3.1). Except for a quantum contribution to the total effective noise, She's capacity has exactly the same form as Shannon's. This agrees with our intuitive expectations and leads to a more satisfactory picture of the ultimate limitations of optical communication.

We have not as yet to our own satisfaction, checked and reproduced She's quantitative evaluation of the quantum noise contribution. We shall hope to do so during this next report period. But we have been completely convinced that the approach followed by Glauber and She leads to a more realistic evaluation of the theoretical limits for the transmission of information in the optical range than has previously been suggested.

3.3 Continued Discussion of a Laser Preamplifier in a Binary Channel

3.3.1 Statistical Separation of Signal and Noise.

The theory of a laser preamplifier was discussed in Hok et al (1964). There, we also obtained an expression for the rate of transmission of information per sample in a binary channel with such a preamplifier. Due to the many parameters involved, the use of simple variational techniques is not feasible and it appears that one will have to resort to numerical calculation. It would be desirable, therefore, to have criteria on the basis of which one could evaluate a laser preamplifer with relatively little computational effort. Such a criterion, based on the notion of the statistical separation between signal and noise, will be discussed here.

Having always in mind the binary channel, we consider a signal accompanied by noise. Signal and noise are statistically independent. Using the notation of Hok et al (1964), let $W^S(m)$, m_s, and σ_s be the probability distribution, the mean, and the standard deviation, respectively, of the number of signal photons, if a signal is present. Similarly, let $W^N(m)$, m_N and σ_N be the corresponding quantities for the noise added to the signal during its transmission. The foregoing quantities refer to the receiving end. A qualitative plot of the distribution will have the form shown in Fig.3-1.

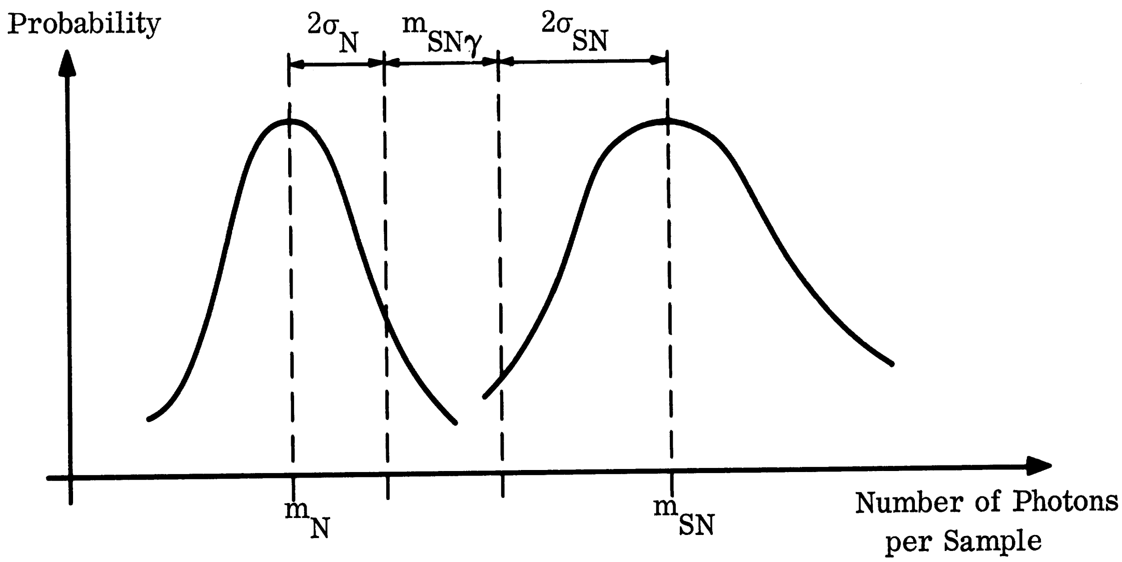

FIG. 3-1.

The quantities $W^{SN}(m)$, m_{SN} and σ_{SN} represent the probability distribution, the mean, and the standard deviation, respectively, of photons due to the signal plus the added noise, per sample in the presence of a signal. As discussed in our second Interim Report (Hok et al, 1964) if we know $W^S(m)$ and $W^N(m)$, $W^{SN}(m)$ is given by

$$W^{WN}(m) = \sum_{n=0}^{m} W^S(n)W^N(m-n) \quad . \tag{3.4}$$

Following Steinberg (1964), we define a quality parameter

$$\gamma \equiv \frac{(m_{SN}-2\sigma_{SN})-(m_N+2\sigma_N)}{m_{SN}} \quad . \tag{3.5}$$

The meaning of γ is evident; as it becomes larger, the discrimination between signal and noise improves and, hence, the rate of errors will be smaller. As defined above, γ corresponds to the case in which we have at the receiving end a simple detector (e.g. a photon counter). If a laser amplifier is placed in front of the detector, the probability distribution, the means and the standard deviation will be modified as discussed in our last interim report. Thus, in the absence of signal, at the amplifier output we shall have the distribution

$$W^{NA}(m) = \sum_{n=o}^{m} W^N(n) W^A(n;m) \quad , \tag{3.6}$$

where $W^A(n;m)$ is the probability that m photons will appear in the output of the amplifier, when it is known that exactly n photons entered the input (cf, Equations (2.30) and (2.33) of Hok et al, 1964).

The corresponding mean and standard deviation will be denoted by m_{NA} and σ_{NA}, respectively. In the presence of signal, the probability distribution will be

$$W^{SNA}(m) = \sum_{n=o}^{m} W^{SN}(n) W^A(n;m) \quad , \tag{3.7}$$

with mean m_{SNA} and standard deviation σ_{SNA}. Thus, if the amplifier is used, we shall have a new γ which we shall denote by γ_A, that is

$$\gamma_A \equiv \frac{(m_{SNA}-2\sigma_{SNA})-(m_{NA}+2\sigma_{NA})}{m_{SNA}} \quad . \tag{3.8}$$

Clearly, the use of the amplifier will improve the separation of the signal from the noise if $\gamma_A > \gamma$. When the inequality is reversed the amplifier will be a disadvantage. Which of the two will happen, depends on the relative effect of the spontaneous emission and the signal amplification.

The analysis presented in our Interim Report No. 2 required the full knowledge of the probability distributions. The analysis to be presented here requires only the knowledge of the means and the standard deviations and these can be determined relatively easily.

3.3.2 Means and Standard Deviations

With the case of a weak signal in mind, we shall take the probability distribution $W^S(n)$ to be a Poisson distribution, that is

$$W^S(n) = \frac{m_S^n e^{-m_S}}{n!} \quad . \tag{3.9}$$

Also the noise probability distribution shall be assumed to be a Poisson distribution, that is

$$W^N(n) = \frac{m_N^n e^{-m_N}}{n!} \quad . \tag{3.10}$$

Due to the additivity property of the Poisson distribution, we shall have

$$W^{SN}(n) = \frac{m_{SN}^n e^{-m_{SN}}}{n!} \quad , \tag{3.11a}$$

where

$$m_{SN} = m_S + m_N \quad . \tag{3.11b}$$

Let us consider now, an amplifier with n_2 atoms in the upper state and n_1 atoms in the lower state. The gain of the amplifier is

$$G = e^{a(n_2 - n_1)L} \quad , \tag{3.12}$$

where a is a constant characteristic of the material and L is the length of the active material travelled by the photon beam. Shimoda et al (1957) have shown that if the mean at the input of such an amplifier is m(o) , the mean at the output will be

$$m(L) = Gm(o)+(G-1)\frac{n_2}{n_2-n_1} \quad . \tag{3.13}$$

Introducing

$$b \equiv \frac{G-1}{n_2-n_1} \quad , \tag{3.14}$$

and applying (3.13) for $m(o)=m_N$ and $m(o)=m_{SN}$, we obtain, respectively

$$m_{NA}=Gm_N+bn_2 \quad , \tag{3.15a}$$

and

$$m_{SNA}=Gm_{SN}+bn_2 \quad . \tag{3.15b}$$

Moreover, it is shown by Shimoda et al (1957) that if the mean and variance at the input are m(o) and $\sigma^2(o)$, respectively, the variance at the output is

$$\sigma^2=G^2\sigma^2(o)+m(o)Gb(n_2+n_1)+n_2b(n_2b+1) \quad , \tag{3.16}$$

where we have made use of (3.14). For our purposes, this equation is more conveniently written in the form

$$\sigma^2=G^2\left\{\sigma^2(o)-m(o)\right\}+m(o)G(2n_2b+1)+n_2b(n_2b+1) \quad . \tag{3.17}$$

Note that equations (3.13) and (3.16) are quite general and do not depend on the particular form of the input distribution. For the case we are considering here, the input will be a Poisson distribution and we shall have $\sigma^2(o)=m(o)$. Then, (3.17) yields

$$\sigma^2_{NA} = m_N G(2n_2b+1)+n_2b(n_2b+1) \tag{3.18a}$$

and

$$\sigma^2_{SNA}=m_{SN}G(2n_2b+1)+n_2b(n_2b+1) \quad . \tag{3.18b}$$

Thus far, we have not specified the method of detecting photons arriving at the receiving end. Let us assume that the photons are counted by means of our ideal detector which, however, is not 100 per cent efficient. By ideal, we mean that the detector does not introduce additional noise nor does it change the statistics of the beam. Let η be the quantum efficiency of this detector. If the mean and the variance at the input are known, the corresponding quantities at the output of such a detector are given by (3.13) and (3.17) for $n_2=0$ and for $G=\eta$.

The output of the detector, if the amplifier is not used and the input is noise, will have mean and variance given by

$$m_{Nd}=m_N\eta \,, \tag{3.19a}$$

and

$$\sigma^2_{Nd}=m_N\,\eta \,. \tag{3.19b}$$

The output of the detector, if the amplifier is not used and the input is signal plus noise, will have mean and variance given by

$$m_{SNd}=m_{SN}\eta \,, \tag{3.20a}$$

and

$$\sigma^2_{SNd}=m_{SN}\eta \,. \tag{3.20b}$$

In both cases the Poisson distribution has been used. The output of the detector, if the amplifier is used and the input to the amplifier is noise, will have mean and variance given by

$$m_{NAd}=\eta m_{NA}=\eta\,(Gm_N+bn_2\,) \tag{3.21a}$$

and

$$\sigma^2_{NAd}=(\sigma^2_{NA}-m^2_{NA})\eta^2+m_{NA}\eta \,,$$

which upon using equations (3.15a) and(3.18a) and performing some manipulations becomes

$$\sigma^2_{NAd}=m_N G\eta(2n_2b\eta+1)+n_2b\eta(n_2b\eta+1) \,. \tag{3.21b}$$

Finally, the output of the detector, if the amplifier is used and the input to the amplifier is signal plus noise, will have mean and variance given by

$$m_{SNAd}=\eta m_{SNA}=\eta(Gm_{SN}+bn_2) \ , \tag{3.22a}$$

and

$$\sigma^2_{SNAd}=m_{SN}G\eta(2n_2b\eta+1)+n_2b\eta(n_2b\eta+1) \quad . \tag{3.22b}$$

The quantities we now wish to calculate and compare are:

$$\gamma = \frac{(m_{SNd}-2\sigma_{SNd})-(m_{Nd}+2\sigma_{Nd})}{m_{SNd}} \ , \tag{3.23a}$$

and

$$\gamma_A = \frac{(m_{SNAd}-2\sigma_{SNAd})-(m_{NAd}+2\sigma_{NAd})}{m_{SNAd}} \quad . \tag{3.23b}$$

3.3.3 Ideal Amplifier with High Gain

Let us consider the case of an ideal amplifier in which the lower state is empty, that is $n_1=0$. Moreover, let it be assumed that the amplifier has high gain, that is, $G \gg 1$. Then

$$n_2b = n_2\frac{G-1}{n_2-n_1} = G-1 \cong G \quad . \tag{3.24}$$

Upon using this approximation (3.21)and (3.22) become

$$m_{NAd}=\eta\, Gm_N+\eta\, G \tag{3.25a}$$

$$\sigma^2_{NAd}=m_NG\eta(2G\eta+1)+ G\eta(G\eta+1)=G\eta\left[m_N(1+2G\eta)+(1+G\eta)\right] \tag{3.25b}$$

$$m_{SNAd}=\eta\, Gm_{SN}+\eta\, G, \tag{3.25c}$$

and

$$\sigma^2_{SNAd}=G\eta\left[m_{SN}(1+2G\eta)+(1+G\eta)\right] \quad . \tag{3.25d}$$

Substituting into equations (3.23) we obtain

$$\gamma = \frac{m_{SN}-m_N}{m_{SN}} - \frac{2}{m_{SN}\sqrt{\eta}}\left(\sqrt{m_{SN}}+\sqrt{m_N}\right) \tag{3.26a}$$

and

$$\gamma_A = \frac{m_{SN}-m_N}{m_{SN}+1} - \frac{2}{\sqrt{G\eta}\,(m_{SN}+1)}\left\{\sqrt{m_{SN}(1+2G\eta)+(1+G\eta)}+\right.$$

$$\left.+\sqrt{m_N(1+2G\eta)+(1+G\eta)}\right\} \quad . \tag{3.26b}$$

Due to the additivity of the Poisson distribution, we have $m_{SN}=m_S+m_N$. Thus, (3.26) become

$$\gamma = \frac{m_S}{m_S+m_N} - \frac{2}{\sqrt{\eta}}\,\frac{\sqrt{m_S+m_N}+\sqrt{m_N}}{m_S+m_N} \tag{3.27}$$

and

$$\gamma_A = \frac{m_S}{m_S+m_N+1} - \frac{2}{\sqrt{G\eta}\,(m_S+m_N+1)}\left\{\sqrt{(m_S+m_N)(1+2G\eta)+(1+G\eta)}\right.$$

$$\left.+\sqrt{m_N(1+2G\eta)+(1+G\eta)}\right\} \quad . \tag{3.28}$$

If the signal plus noise power is large, so that $m_S+m_N \gg 1$, (3.28) can be approximated by

$$\gamma_A \cong \frac{m_S}{m_S+m_N} - \frac{2\left(\sqrt{m_S+m_N}+\sqrt{m_N}\right)}{\sqrt{\eta}\,(m_S+m_N)}\sqrt{\frac{1+2G\eta}{G}} \quad . \tag{3.29}$$

Assuming that we had $\gamma > 0$, it is easily seen that in order to have $\gamma_A > \gamma$ we must have

$$\left(\frac{1+2G\eta}{G}\right)^{1/2} = \left(\frac{1}{G}+2\,\eta\right)^{1/2} < 1 \quad . \tag{3.30}$$

If $\eta < 1/2$, this condition can be satisfied by choosing G large enough. If on the other hand, $\eta > 1/2$ no amplifier can improve the discrimination. Thus, it is seen that for a signal of a relatively large power (i. e. number of received photons per sample much larger than one), the amplifier does not improve the detection as long as the efficiency of the detector is better than 50 per cent. This case has been investigated by Steinberg (1964). We now turn to the case of weak signals.

Let us assume that signal and noise are at such a level that

$$m_{SN} = m_S + m_N \ll 1 \; . \tag{3.31}$$

Of course, we shall always have

$$m_N < m_{SN} \; . \tag{3.32}$$

We consider γ as given by (3. 26a) and write it in the following form:

$$\gamma = \frac{1}{\sqrt{m_{SN}}} \left(1 + \sqrt{\frac{m_N}{m_{SN}}}\right) \left[(\sqrt{m_{SN}} - \sqrt{m_N}) - \frac{2}{\sqrt{\eta}}\right] . \tag{3.33}$$

The efficiency η will always be a number smaller than unity. Here, we shall assume that it is not much smaller than unity. For example, $\eta = 0.8 - 0.9$. Then , $2/\sqrt{\eta}$ will be a number larger than 2 . Due to inequalities (3. 31) and (3. 32) the quantity $(\sqrt{m_{SN}} - \sqrt{m_N})$ will be smaller than unity (not necessarily much smaller). Consequently the quantity inside the square bracket will be negative and typically of order unity. Also, the quantity,

$$\left(1 + \sqrt{\frac{m_N}{m_{SN}}}\right) ,$$

is of order unity, while $1/\sqrt{m_{SN}}$ is larger or much larger than unity. We conclude therefore, that for a weak signal (in the sense of inequality (3. 31)), γ is negative and its absolute value is larger or much larger than unity. Note that γ ought to be expected to be negative in view of the Poisson distributions involved.

We have already assumed that G >> 1. For the range of values of η assumed earlier, we shall also have $G\eta \gg 1$. Then, (3.26b) can be replaced by

$$\gamma_A = \frac{m_{SN}-m_S}{m_{SN}+1} - \frac{2}{(m_{SN}+1)}\left\{\sqrt{2m_{SN}+1} + \sqrt{2m_N+1}\right\} \quad . \tag{3.34}$$

Moreover, by virtue of inequalities (3.31) and (3.32), this can be approximated by

$$\gamma_A = (m_{SN}-m_N)-4 \cong -4 \quad . \tag{3.35}$$

Note that the result is independent of G and η due to the assumptions made.

As pointed out earlier we shall have $|\gamma| \gg 1$, if $\sqrt{m_{SN}} \ll 1$. When this is the case, we shall also have $|\gamma_A| \ll |\gamma|$, and since both γ and γ_A are negative, it follows that the amplifier has improved the discrimination. We thus conclude that, if the number of signal plus noise photons per sample is much smaller than unity, the amplifier has sufficiently large gain, and the detector has sufficiently good efficiency, then, the amplifier invariably improves the discrimination between signal and noise. Note that the conclusion is independent of whether the signal photons per sample are more or less than the noise photons, that is, whether $m_S > m_N$ or not.

We have also investigated a third special case, namely the case in which $G\eta \gg 1$, and m_{SN} is of order unity. This case does not lend itself to a simple analytical study. By studying a few numerical examples, however, we have found that the effect of the amplifier can be either improvement or deterioration depending on the values of the parameters involved. We refrain from reproducing the calculations here, and we shall only give two examples.

For m_S=0.5, m_N= 0.5 we have $\gamma \leqslant -2.9$, $\gamma_A = -2.85$

For m_S = 1, m_N= 0.5, we have $\gamma \leqslant -1.91$, γ_A = -0.31 .

The three cases discussed here do not exhaust the possibilities. There are several parameters that one can vary, and many more special cases can be constructed.

Qualitatively, large separation between signal and noise implies a small number of errors. In order to calculate the exact number of errors, however, one must know the precise probability distributions at every stage. Recall that the knowledge of these distributions was also indispensable in the calculation of the information efficiency of a binary channel incorporating a laser preamplifier (Interim Report No. 2 on this contract). These distributions have been investigated by several authors to a greater or lesser extent. Although, in general, they are extremely complicated, some analysis on them will be presented in a future report.

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