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Final Technical Report

LUNAR SUBSURFACE ELECTROMAGNETIC PROBING

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I. Introduction

By deploying an electrically short dipole antenna on the lunar surface and measuring the antenna electrical impedance at selected frequencies, potentially useful information about the surface and subsurface materials of the moon can be obtained. During the impedance measurement at a particular frequency, a portion of the electro-magnetic field radiated by the antenna will penetrate the lunar surface, interact with the lunar material, and thereby change the antenna impedance from its free-space value. The field penetration is determined by antenna length, operating frequency, and the local lunar conductivity (σ) permittivity (ϵ) and permeability (μ). The presence of a low-altitude lunar ionosphere might also interact with the antenna, but for the purposes of this discussion, the effect will be considered negligible.

It should be noted here that this technique has not been vigorously pursued on the earth because of the availability of more direct methods. Furthermore, the relatively high water content of much of the earth's surface layers results in high soil conductivity and thus penetration of radio frequency energy is limited by the skin-effect. It will be assumed henceforth that the lunar material has a relatively low conductivity. If the lunar conductivity is high however, the field penetration will be limited, but the discovery of highly conductive lunar material will be of prime geological interest in its own right.

While the above technique is simple in concept, interpretation of the impedance versus frequency measurements in terms of absolute values of σ, ϵ, μ, and material density with respect to depth is difficult. This results from the potentially infinite number of combinations of subsurface characteristics and possible material variations with respect to frequency. Also, there is not as yet a satisfactory solution for the problem of the impedance of an antenna located at the interface between free space and a lossy medium.

In spite of these difficulties in obtaining an unambiguous absolute interpretation of the subsurface make-up, the proposed technique appears to be very useful in manned and unmanned exploration of the moon because of its ability to detect changes in lunar subsurface conditions from one place to another. If, for instance, a series of impedance measurements are made at one location, and this same series is repeated at another site, the following conclusions can be drawn from the results:

1. If the measurements at one site differ from those at the other, then at least some of the subsurface conditions at the two sites must be different.

2. If the measurements at both sites are the same, or essentially so, then the probability is high (but not absolute) that the subsurface conditions are likely to be similar. Even the limited amount of information implicit in the above two conditions is of value for instance, in site selection for lunar core drilling. If the object of the drilling is to obtain (if possible) a series of cores of differing characteristics, then whenever criterion number one above is satisfied, the objective will be achieved.

It might also be feasible to detect the presence of crevasses, particularly if they are not deeply buried.

To investigate the sensitivity of the antenna impedance to material variations it will be assumed that the antenna is immersed in a homogeneous medium which has either the properties of the material medium or free space. Then it will be recognized that when the antenna is at the material-space interface, its behavior will be somewhere between these two extremes. The impedance of an antenna placed in an infinite, homogeneous lossy medium which may have a complex permeability, permittivity and conductivity is considered in the following section.

II. DIPOLE IMPEDANCE IN AN INFINITE HOMOGENEOUS MEDIUM

According to King (1956), solutions which apply to free space are correct for the material medium if the values for the electrical constants of the material medium are substituted for those of free space. Thus, an expression for the impedance of an antenna in free space may be transformed into the correct form for the material medium with the appropriate substitution of electrical constants. Attention here will be limited to the electric dipole antenna, and due to the frequency range which is of interest and the practical limitations on the length of the antenna, as well as the theoretical simplification which arises, the antenna will be assumed to be short compared with the wavelength. With this restriction, an expression for the impedance in a material medium of an electric dipole antenna with zero base separation, obtained from the emf method and a first-order solution for the antenna current, is (King, 1956, p. 184):

$$Z = \frac{j(\Omega-2)}{2\pi h\omega\xi}\left[\frac{\Omega-2+j\frac{2}{3}(\omega\sqrt{\xi\mu}\ h)^3}{\Omega-2+2\ln 2+j\frac{1}{3}(\omega\sqrt{\xi\mu}\ h)^3}\right] = R+jX \quad (1)$$

where $2h$ = antenna length

a = antenna radius

$\omega = 2\quad f = 2\pi$ x frequency

$\Omega = 2\ \ln(2h/a)$

$\xi = \epsilon_e - j\frac{\sigma_e}{\omega}$

$\epsilon_e = \epsilon' - \sigma''/\omega$

$\sigma_e = \sigma' + \omega\epsilon''$

$\epsilon = \epsilon' - j\epsilon''$

$\sigma = \sigma'$

$\mu = \mu' - j\mu''$

This expression produces values within 10 percent of those obtained from a second-order solution for antenna lengths in the range $2|k|h = 2h\omega\sqrt{|\mu||\xi|} \le 1.0$ and within 20 percent in the range $1.0 \le 2|k|h \le 2.0$.

We have allowed for a complex permeability and permittivity while considering only a real conductivity, since most materials can be represented in this way. It should be noted that these electrical properties of the material may be functions of the frequency, but it will be assumed here that they are constant in value over the frequency interval under investigation.

This expression was programmed for solution on an IBM 7090 computer. A frequency interval of 5×10^4 to 5×10^6 cps was investigated for an antenna length 2h of 20 meters and a radius a of 1 millimeter. The center frequency f_c about which the frequency f is incremented is $f_c = 5 \times 10^5$ cps ($\omega_c = 2\pi f_c$). Typical results for the resistance and reactance as the electric constants of the medium are varied are given in Figs. 2.1 to 2.8.

Figures 2.1 and 2.2 show the results where μ and ϵ are real and separately have values which are 4 times those for free space. The result is to increase the resistance in proportion to the 3/2 power of the increase in permeability and to the square root of the increase in permittivity. The reactance decreases in proportion to the increase in the permittivity and is unaffected by changes in the permeability.

In Figs. 2.3 and 2.4 are shown the results for various real conductivities when $\mu = \mu' = \mu_0$ and $\epsilon = \epsilon' = 2\epsilon_0$. There is a very large increase in the resistance of 4 to 6 orders of magnitude at the lower end of the frequency interval, while at the higher end the values are close to those for zero conductivity. A corresponding decrease of the same order of magnitude is observed in the reactance.

Figs. 2.5 and 2.6 present the impedance when $\sigma = 0$, $\mu = \mu' = \mu_0$, and $\epsilon = \epsilon' - j\epsilon'' = 2\epsilon_0 - j\epsilon''$ for various ratios of ϵ''/ϵ'. Again a large increase in resistance is seen, but there is relatively little decrease in reactance. It is especially interesting to observe in the frequency interval where $dR/df < 0$, that $R \alpha f^{-2}$ for the conducting medium and $R \alpha f^{-1}$ for the medium with a complex permittivity. This characteristic, as well as the great difference in their respective reactance curves for the larger values of conductivity, indicate that frequency swept measurements may provide a method for discriminating between a real conductivity and a complex permittivity.

Finally, Figs. 2.7 and 2.8 show the results obtained for the case where $\sigma = 0$, $\epsilon - \epsilon' = 2\epsilon_0$, and $\mu = \mu' - j\mu'' = 2\mu_0 - j\mu''$ for various values of the ratio of μ''/μ'. There is relatively little change in either the resistance or reactance due to the complex permeability.

On the basis of some radar scattering measurements of the moon, Brunschwig et all (1960) conclude that $\epsilon' = 1.08\,\epsilon_0$ and $\sigma'/\omega_c\epsilon' \approx 11.2$ ($\sigma' = 3.36 \times 10^{4}$ MHOS/METER) if $\mu' = \mu_0$. A study of lunar surface radio communication recently concluded by Vogler (1964) uses $\epsilon' = 2\,\epsilon_0$ and $\sigma = 10^{-4}$ MHOS/METER as the values for the top layer of the lunar surface. Another study of lunar communication by Smith (1964) used $\epsilon' = 4$ and $\sigma' = 4 \times 0^{-4}$ MHOS/METER. The values which are employed for the calculations here are thus representative of those which are currently considered reasonable for the electrical properties of the moon's surface.

More recently, King (1961) followed a new approach to find the impedance of the electric dipole antenna. It involved rearranging the integral equation which was formerly used for the antenna current. The new form of the equation is again solved by an iterative procedure, but it has the advantage that the zeroth order solution produces results whose accuracy lies somewhere between the first- and second-order solutions of the original equation. King presents some results for the impedance of a short dipole antenna immersed in a dissipative medium, obtained from this approach. Some calculations were performed with (1) for some of the parameter values used by King, with the result that impedance values obtained by the two methods agreed to within 10 percent.

It should be noted that the limit $2|k|h \leq 2.0$ is satisfied over the entire frequency range when the free space electrical properties are used. When, however, $\epsilon' = 2\epsilon_0$ or $\mu' = 2\mu_0$ then $2|k|h$ exceeds this limit for frequencies of $f > 4 \times 10^6$ cps, which could result in errors larger than 20 percent in the calculated impedances. This is a small portion of the total frequency interval however, and does not invalidate the results. The dashed portion of the curves indicates the range where the above limit does not hold.

There is an additional manifestation of the approximate character of (1). When a complex permeability is used, the resistance can be shown to become negative when the imaginary part of the permeability sufficiently exceeds the real part. It is unlikely that any real medium would possess such a permeability.

In Figs. 2.9 and 2.10 the results are rearranged to illustrate the changes in resistance and reactance as σ and ϵ vary at selected frequencies. These curves demonstrate that adequate sensitivity to material parameter variations exists, even when allowance is made for the fact that these data are for the completely immersed antenna.

III. DIPOLE IMPEDANCE NEAR A FREE SPACE-MATERIAL MEDIUM INTERFACE

The original study of the dipole antenna over an infinite conducting half-space was carried out by Sommerfeld (1909). There have been various treatments of this problem by many authors since then. Sommerfeld and Renner (1942) extended Sommerfeld's original study to a half-space of arbitrary properties and found the radiation resistance from the emf method for horizontal (HED) and vertical electric Hertzian dipoles (VED) as a function of height above the interface. The surprising result is that when the half-space has finite conductivity, the radiation resistance of both antennas becomes infinite when their height above the interface decreases to zero. When, however, their half-space is infinitely conducting or non-conducting, the radiation resistance is finite at the interface. This is a somewhat perplexing double limit process which King (1956) sidesteps by noting that since a physical antenna has a non-zero thickness the limit of the antenna height decreasing to zero is meaningless.

There is an interesting comparison which can be made between the HED at the free space boundary and the same antenna immersed in an infinite material medium which has the properties of the half-space. In both cases, the antenna radiation resistance becomes zero when the conductivity of the material medium goes to infinity. On the other hand, when the material medium has a finite conductivity and then the frequency is allowed to approach zero, which is equivalent to letting the antenna height above the interface become zero, different results for radiation resistance are obtained. The radiation resistance of the antenna in the infinite medium becomes a constant inversely proportional to the conductivity, while that of the HED, as mentioned above, becomes infinite. It seems intuitively obvious that the antenna impedance would be most affected by the material medium when it is surrounded by it, and when the antenna is at the interface between the material medium and free space, the effect should be smaller. Also, since an infinite radiation resistance is not acceptable on physical grounds, it is apparent that the present solution to the interface problem fails when the antenna is at, or near, the interface. Some experimental measurements by Proctor (1950) indicate that the theory is quite good in predicting the actual radiation resistance for an antenna height down to 6×10^{-4} wavelengths. The theory must fail for heights less than this for a lossy medium however.

With this limitation in mind, curves have been derived from results recently given by Vogler (1964) for the HED as a function of frequency. His results were obtained using the emf method for calculating the impedance of the Hertzian dipole. Fig. 3.1 shows the radiation resistance over the same frequency interval for a Hertzian HED with same length, 2h = 20 METERS, as was used for the antenna in the previous calculations. For the parameters considered, 50 meters is as close as we can approach the interface and still read the Vogler's curves. Two curves are presented for finite conductivities, and an antenna height of 50 meters, both of which resemble those obtained for the infinite medium except that the increase in resistance is not so pronounced. Also shown is the resistance when the conductivity of the half-space is zero and the antenna is at the interface. The increase in resistance then is not quite equal to the square root of the increase in permittivity as was the case for the infinite medium. No curve is shown for the reactance at the 50 meter height since it is relatively unaffected by the half-space. When the antenna height is zero, the reactance is infinite according to this theory.

Figure 3.2 shows the resistance as a function of height above a half-space with $\sigma' = 0$, $\mu' = \mu_0$ and $\epsilon' = 4\epsilon_0$, and a frequency of 5×10^5 cps. Since, for the infinite medium of zero conductivity and real permittivity ϵ and permeability μ

$$R = F(h, a, \omega)\ \sqrt{\epsilon\mu^3} \qquad (2)$$

where F is a function of antenna length and radius and of frequency, it is reasonable to write for the same antenna in free space near the half-space

$$R = F(h, a, \omega)\ G(H, \epsilon, \mu)\sqrt{\epsilon_0\ \mu_0^3} \qquad (3)$$

where G is a function of the antenna height H and shows the results of the half-space properties on the antenna resistance. This form is correct for the half-space problem and G(H) is given by, (Sommerfeld, 1949).

$$G(H) = \frac{3}{2}\left[\frac{2}{3} - \frac{\sin 2k_o H}{2k_o H} + \frac{\sin 2k_o H - 2k_o H \cos 2k_o H}{(2k_o H)^3}\right. \quad (4)$$

$$\left. + \frac{1}{k_o^3} R_e\left\{ i \int_0^\infty e^{-2H\sqrt{\lambda^2 - k_o^2}} \left[\frac{2\sqrt{\lambda^2-k_o^2}\sqrt{\lambda^2-k^2} - \lambda^2}{\mu\epsilon\sqrt{\lambda^2-k_o^2}+\sqrt{\lambda^2-k^2}}\right]\lambda\, d\lambda\right\}\right]$$

where $k_o = \omega\sqrt{\mu_o \epsilon_o}$ in the propagation constant of free space and $k = \omega\sqrt{\mu\epsilon}$ is the propagation constant of the material medium half space. Vogler has given an asymptotic form for G in the limit $H\sqrt{\epsilon/\epsilon_o} << 1$, which is

$$G(H, \epsilon . \mu_o) = 1 + \frac{3}{8}\left(\sqrt{\epsilon/\epsilon_o} + 1\right)\left[F_1(o) + \frac{\sqrt{\epsilon/\epsilon_o}+1}{2} F_2(D)\right] \quad (5)$$

and is seen to be independent of H. F_1 and F_2 are given by

$$F_1(x) = x + \left[1 - x(1+x)\right] d + \frac{1}{3}(1+x)(1-x^2)d^2$$

$$F_2(x) = -\frac{x}{3} + \left[1 + x(1-x)\right] d + \left[1 - x(1+x)(2-x)\right] d^2$$

$$+ \frac{1}{3}\left[1 + x(1-x)(2-x^2)\right] d^3 - \frac{1}{5}(1-x)(1-x^2)^2 d^4$$

with

$$d = \frac{\sqrt{\epsilon/\epsilon_o} - 1}{\sqrt{\epsilon/\epsilon_o} + 1}$$

$$D = \frac{\epsilon/\epsilon_o - 1}{\epsilon/\epsilon_o + 1}$$

Figure 3.3 shows (5) as a function of $\sqrt{\epsilon/\epsilon_o}$. It is apparent that G is slightly less than $\sqrt{\epsilon/\epsilon_o}$ but that as $\sqrt{\epsilon/\epsilon_o}$ increases, G approaches $\sqrt{\epsilon/\epsilon_o}$ or the resistance then approaches the infinite medium value. Thus, the theoretical indication is that the antenna radiation resistance for the HED located at the interface between free space and a material medium half-space approaches that value which would result when the same antenna is immersed in the material medium if the permittivity of the material medium is greater than about 10 times that of free space. It should be recalled, however, that the treatment followed by Vogler indicates an infinite reactance for the same antenna when at the interface, even for a lossless medium.

It may be of interest to include some experimental results obtained by Proctor (1950), Iizuka (1964) and Seeley et al (1964) for impedance measurements of linear antenna near the interface between free space and a material medium half space. The work of Seeley (1964) et al is especially interesting, in that measurements were made on a dipole antenna 3.25 miles long laid across an island, at frequencies between 3 Kc and 45 Kc. The experiment was carried out to determine whether an island could be made to radiate as a slot radiator. Table I presents the results of some of these measurements.

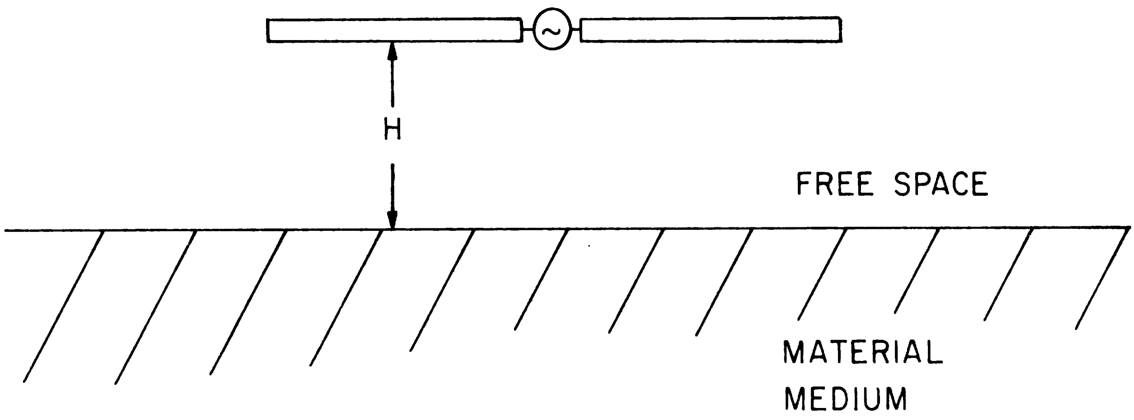
The smaller values of H/λ for Iizuka's and Seeley's work represent the case when the antenna was actually touching the interface. It is interesting to observe that the resistance and reactance, in all cases but one, exhibit a decrease in magnitude when the antenna is brought into contact with the interface with respect to the values just above the interface, contrary to the theoretical calculations from the Hertzian dipole theory. These results do show the possibility for a large increase in resistance over that free space. At the same time, they indicate the limitations of the available theory which predict infinitely large impedances in such situations.

IV. FURTHER THEORETICAL WORK

In a uniform material medium we may calculate the impedance of electrically short dipole antenna with relative ease and reasonably good accuracy largely because of the theoretical work of King and his students. The medium may be loss-free or lossy. The complexity of computations in the lossy medium are greater than in the loss-free medium but as shown above, they are still managable. The expressions for the impedance rest on a solution of an integral equation for the current of the linear antenna. The linear antenna consists of perfectly conducting cylinders joined by an ideal voltage generator.

When the space consists of a material half-space and the rest vacuum (or air), then the only calculations for the dipole impedance which exist are based on the EMF method and the Hertzian dipole fields. This procedure was first formulated by Sommerfeld at the turn of the century and since then extended by himself, his students and other workers. Experience indicates that this method gives useable results when the half-space is either loss-less and the medium electromagnetic parameters finite, or when it is of infinite conductivity. When the half-space is lossy, then the impedance results are of sufficient accuracy only as long as the dipole does not approach too close to the lossy interface. When the separation distance becomes less than one-half wavelength, it is clear that the error is bound to increase and, for example, as the horizontal Hertzian dipole approaches the lossy interface the real part of the dipole impedance becomes infinite: a result that is completely wrong. How this comes about is easy to see. The Hertzian dipole is an oscillating point current. The electric field of this source becomes infinite at the rate of d^{-3} for $d \ll \lambda$ where d is the distance of the observer from the source. As we let the horizontal Hertzian dipole approach the lossy interface, we impress on the finitely conducting interface an infinitely large electric field which gives rise to non-integrable singularity in the power absorption by the medium. Such a situation will never arise if one considers a linear antenna as discussed in the first paragraph of this section. We are forced to conclude that the method of computing the dipole impedance which rests on the EMF method and the Hertzian dipole fields leads to incorrect results when the dipole is chose to the material interface.

Thus there is a need to formulate an integral equation for the current of a linear antenna that is horizontal to a lossy interface, as shown below, as was done by King for the homogeneous lossy medium.



From a solution of this integral equation one can obtain antenna currents that will lead to useful dipole antenna impedances close to the interface. This problem is involved and tedious mathematically, but the present need for a good solution and availability of high speed computers should combine to solve this problem. Both of these factors were missing in the earlier studies.

V. CONCLUSIONS AND RECOMMENDATIONS

The preceeding sections have demonstrated that the proposed method for lunar subsurface electromagnetic probing should provide useful results, particularly when advantage is taken of the ability to make relative measurements from one lunar site to another. Variations in both permittivity and conductivity can be detected readily. Permeability changes have only minor effects on the electric dipole. An investigation of the loop antenna might be undertaken in future work if sensitivity to permeability change is desired.

In order to obtain the maximum amount of information from the measurements, it will be necessary to carry out the analysis of an antenna at a material-free space boundary and then experimentally verify the theoretical predictions. It will then be possible to conduct an error analysis of the effect of lack of contact of the antenna with the interface at all points along its length. This will assist in specifying the care with which the antenna must be deployed prior to measurement. It will also allow an estimate of the feasibility of mounting an antenna on a traverse vehicle to obtain continuous readings during vehicle operation.

As presently envisioned, the instrument would consist of a box containing spring-loaded reels for antenna storage and all the electronics necessary to make both resistive and reactive measurements of the antenna at a minimum of three frequencies. Instrument volume should be less than 1/2 cubic foot and it should weigh less than 6 pounds. During operation, less than 2 watts of power would be required. The data output would be in the form of analog voltages which would displayed on self-contained voltmeters and also would be properly conditioned for tape recorder or telemetering inputs.

FIG. 2.1. Resistance versus Frequency for Dipole Antenna of Length 2h = 20 Meters, Radius a = 1 mm and $\sigma = 0$.

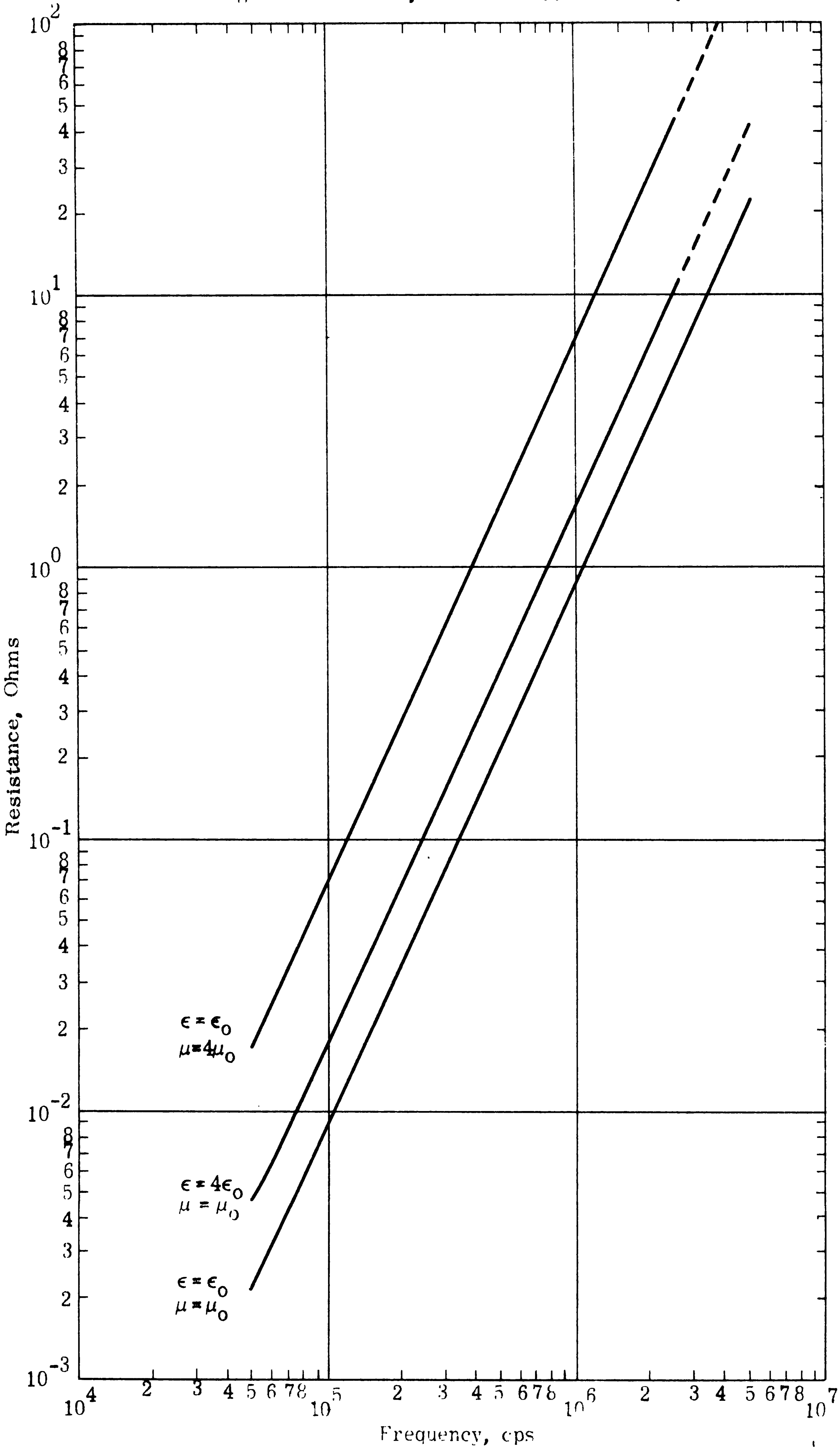


FIG. 2.2. Reactance versus Frequency for Dipole Antenna of Length $2h = 20$ Meters. Radius $a = 1$ mm and $\sigma = 0$.

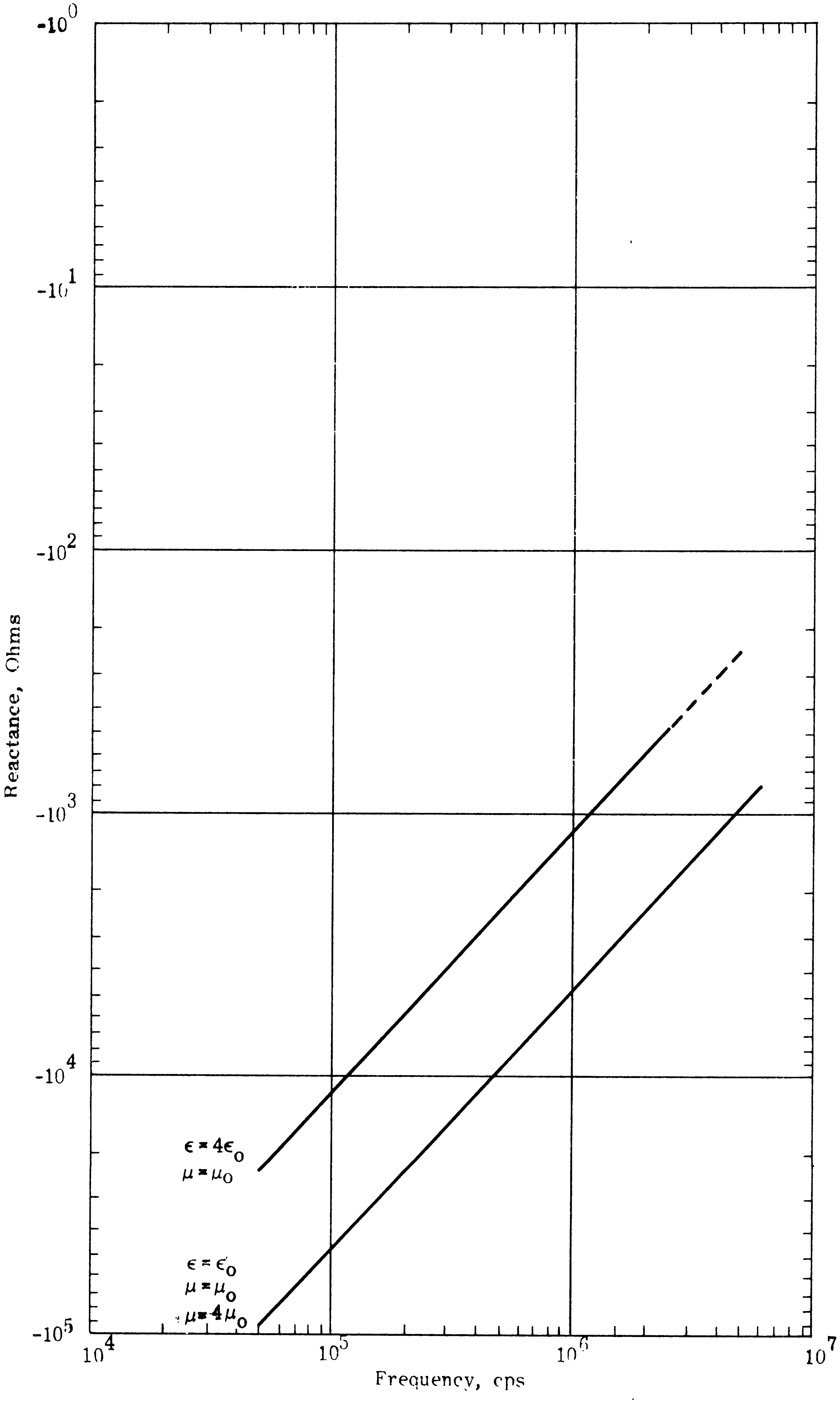


FIG. 2.3. Resistance versus Frequency for Dipole Antenna of Length 2h=20 Meters; Radius a=1 mm; and $\mu=\mu_0$, $\epsilon=2\epsilon_0$.

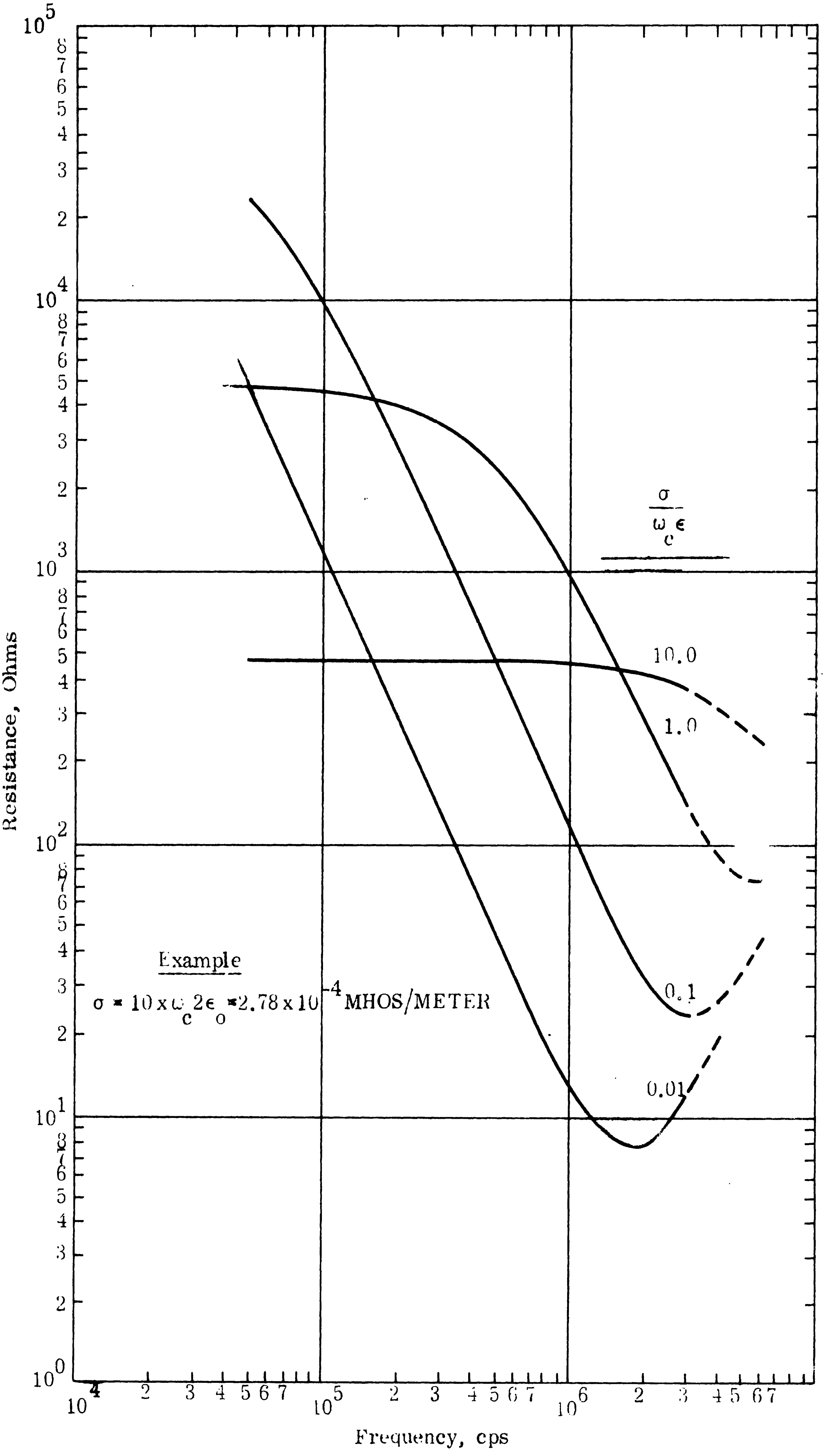


FIG. 2.4. Reactance versus Frequency for Dipole Antenna of Length $2h=20$ Meters, Radius $a=1$ mm and, $\mu=\mu_0$, $\epsilon=2\epsilon_0$

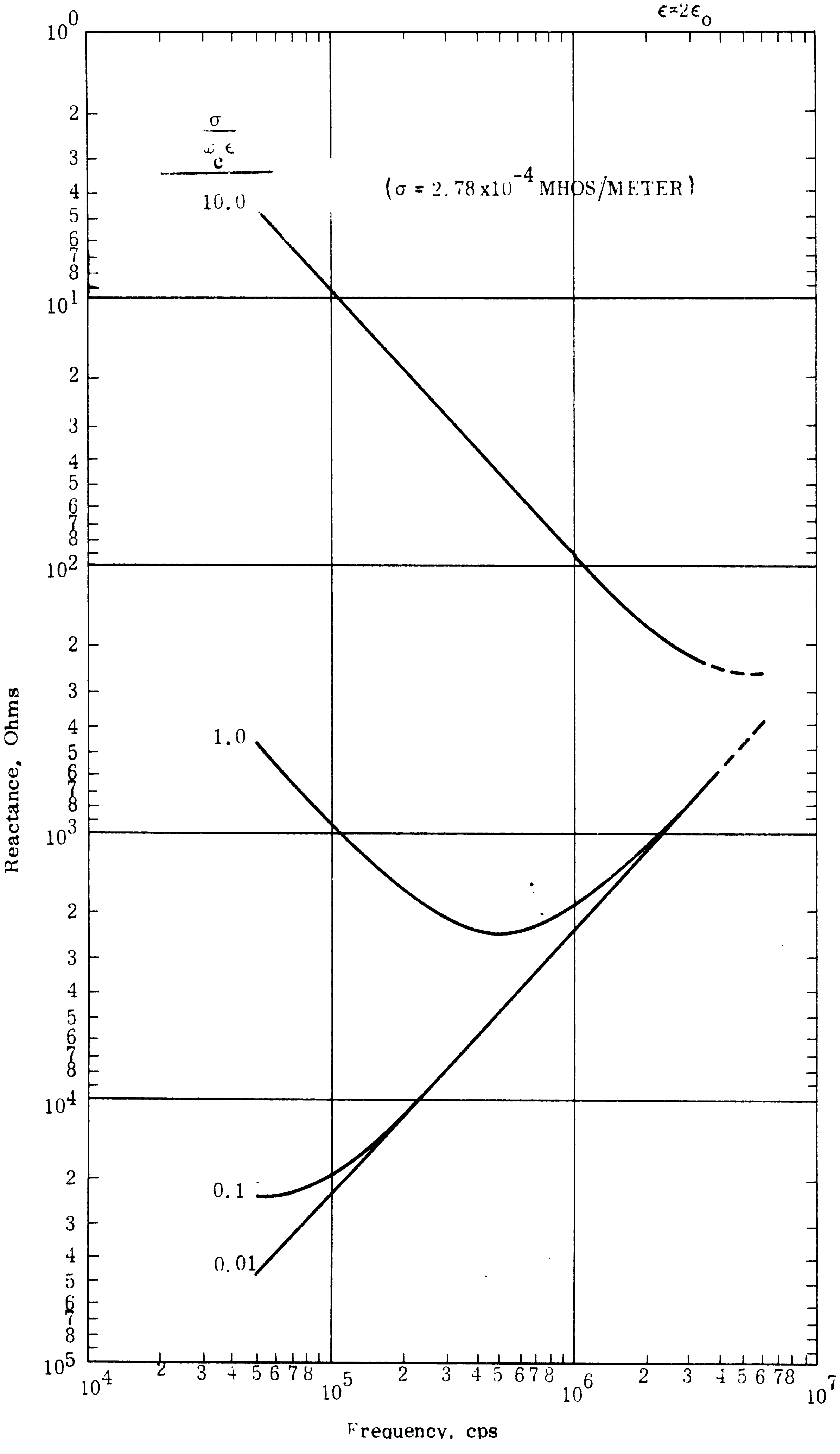


FIG. 2.5. Resistance versus Frequency for Dipole of Length 2h=20 Meters, Radius a=1 mm and $\sigma=0$, $\mu=\mu_0$, $\epsilon = \epsilon' - j\epsilon''$, $\epsilon' = 2\epsilon_0$

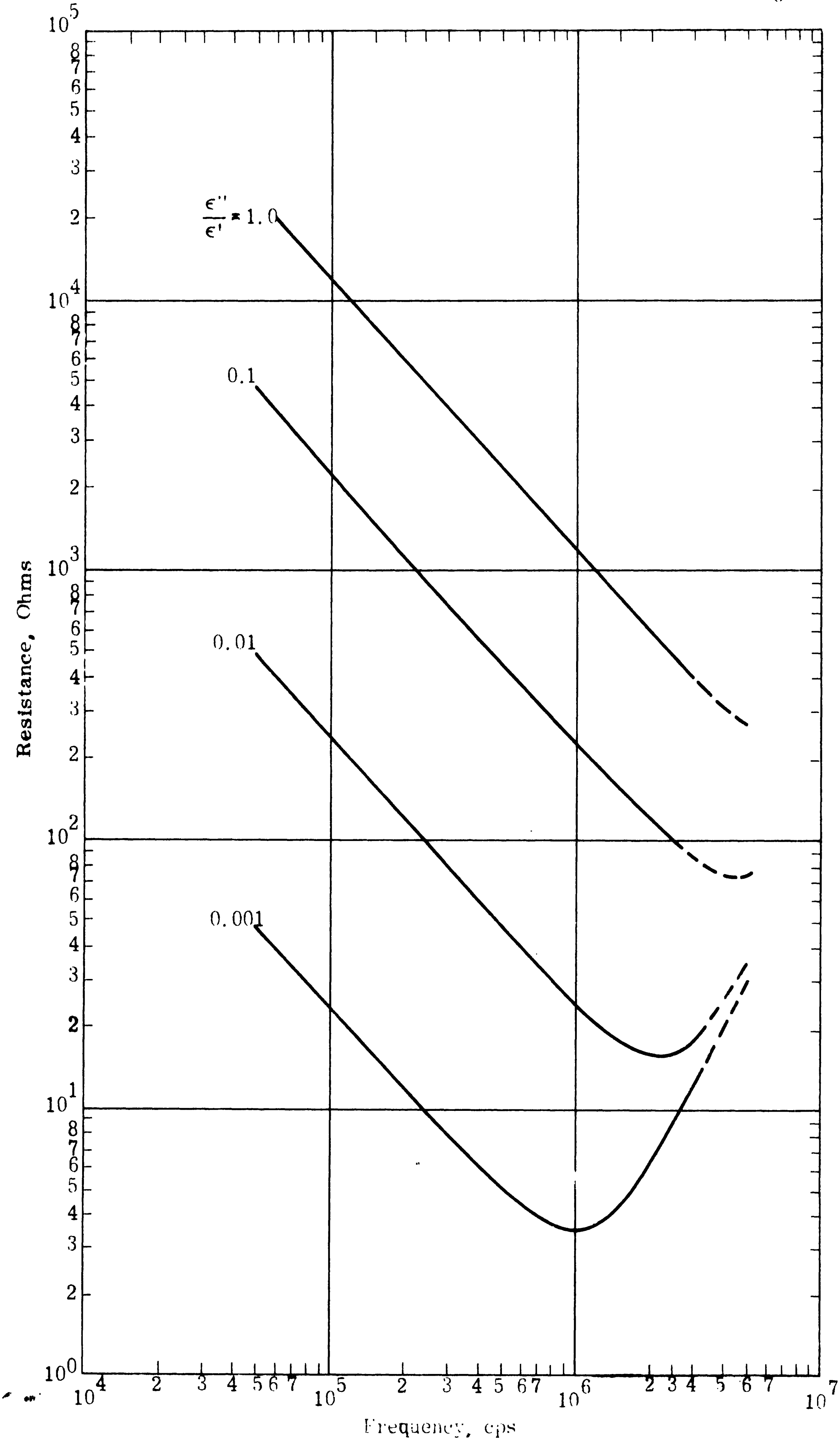


FIG. 2.6. Reactance versus Frequency for Dipole Antenna of Length 2h = 20 Meters. Radius a = 1 mm and $\sigma = 0$, $\mu = \mu_0$. $\epsilon = \epsilon' - j\epsilon''$, $\epsilon' = 2\epsilon_0$.

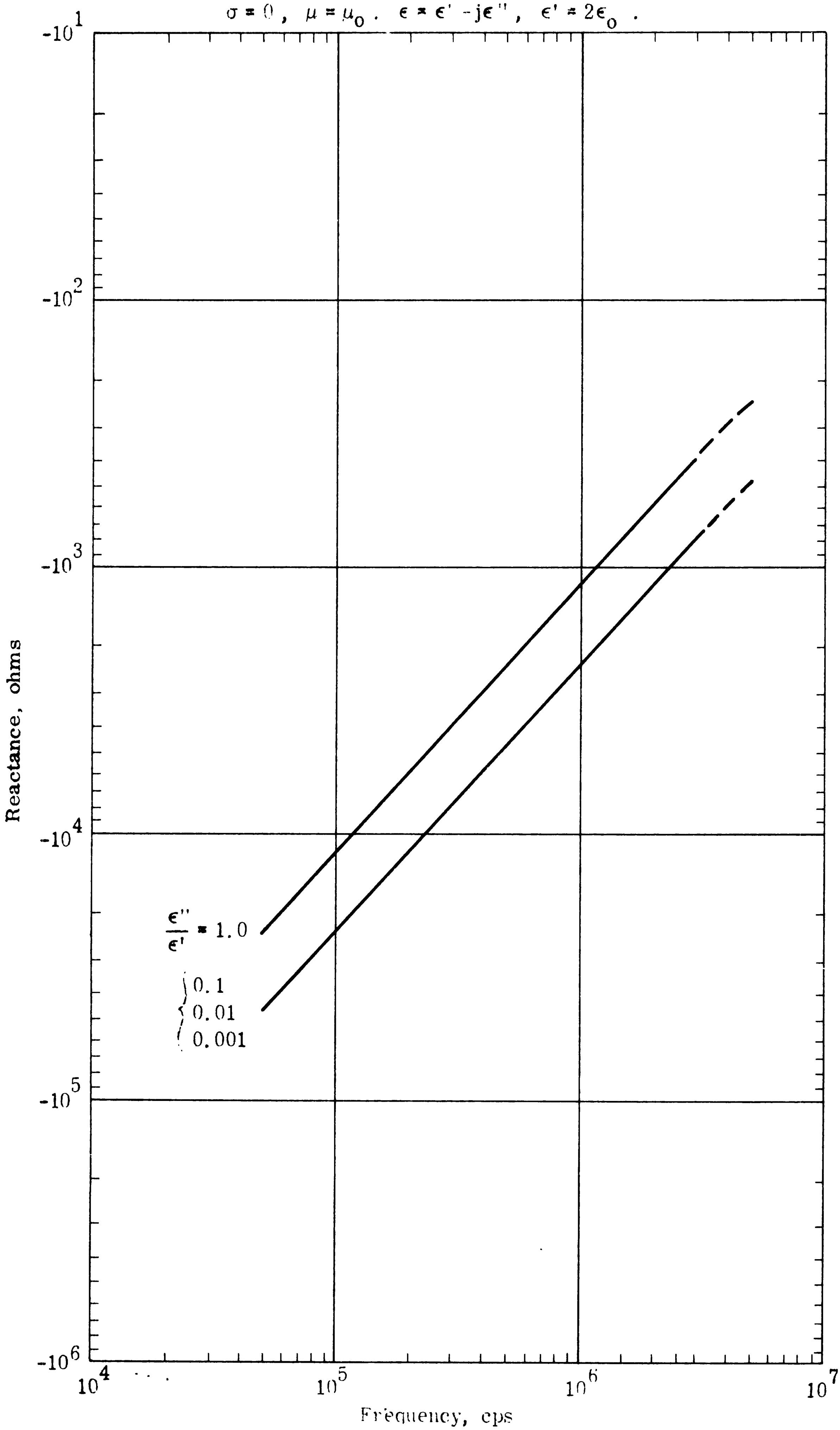


FIG. 2.7. Resistance versus Frequency for Dipole Antenna of Length 2h = 20 meters. Radius a = 1 mm and $\sigma = 0$, $\epsilon = 2\epsilon_0$, $\mu = \mu' - j\mu''$, $\mu' = 2\mu_0$.

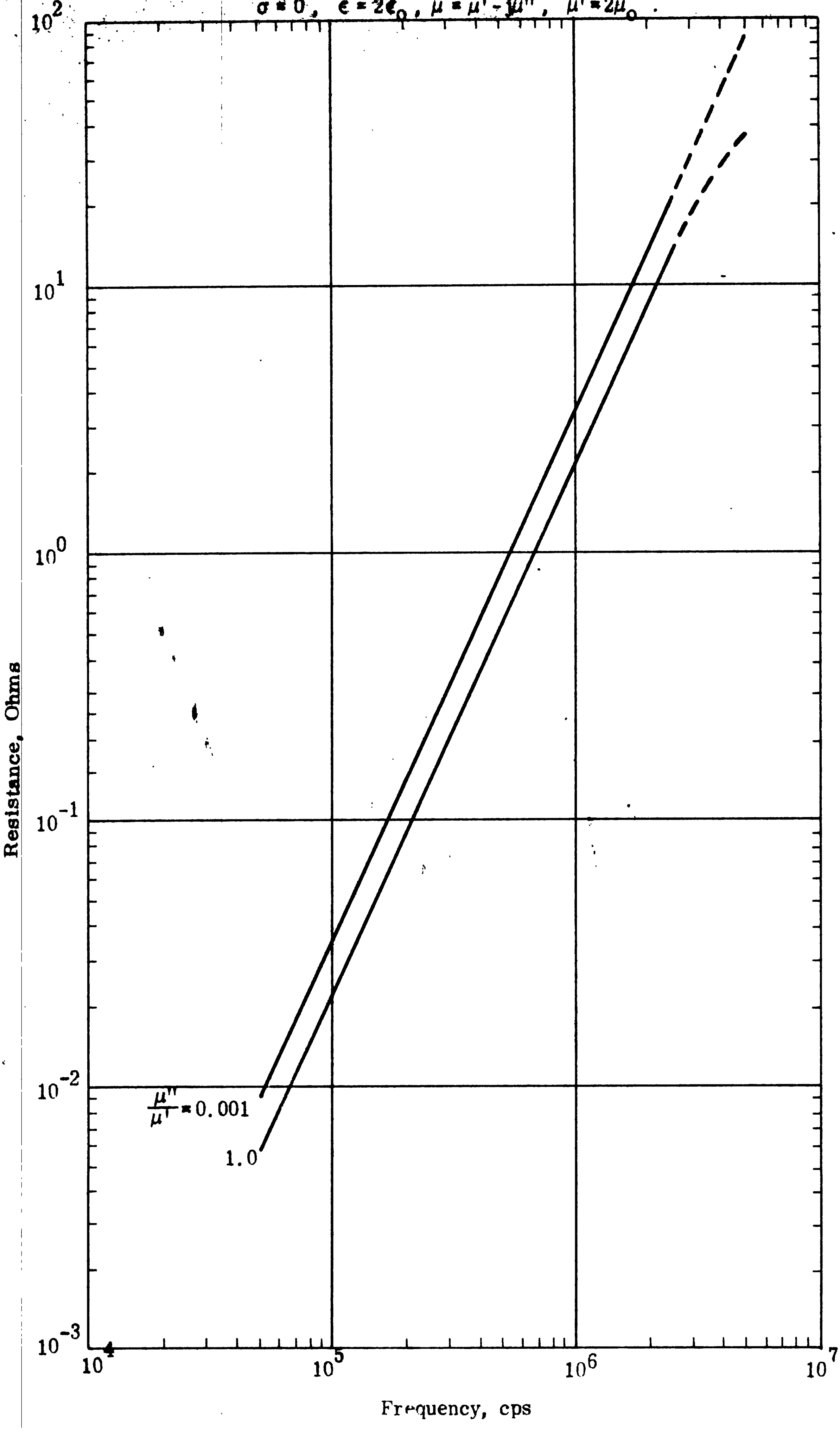


FIG. 2.8. Reactance versus Frequency for Dipole Antenna of Length 2h = 20 Meters, Radius a = 1 mm and $\sigma = 0$, $\epsilon = 2\epsilon_0$, $\mu = \mu' - j\mu''$, $\mu' = 2\mu_0$.

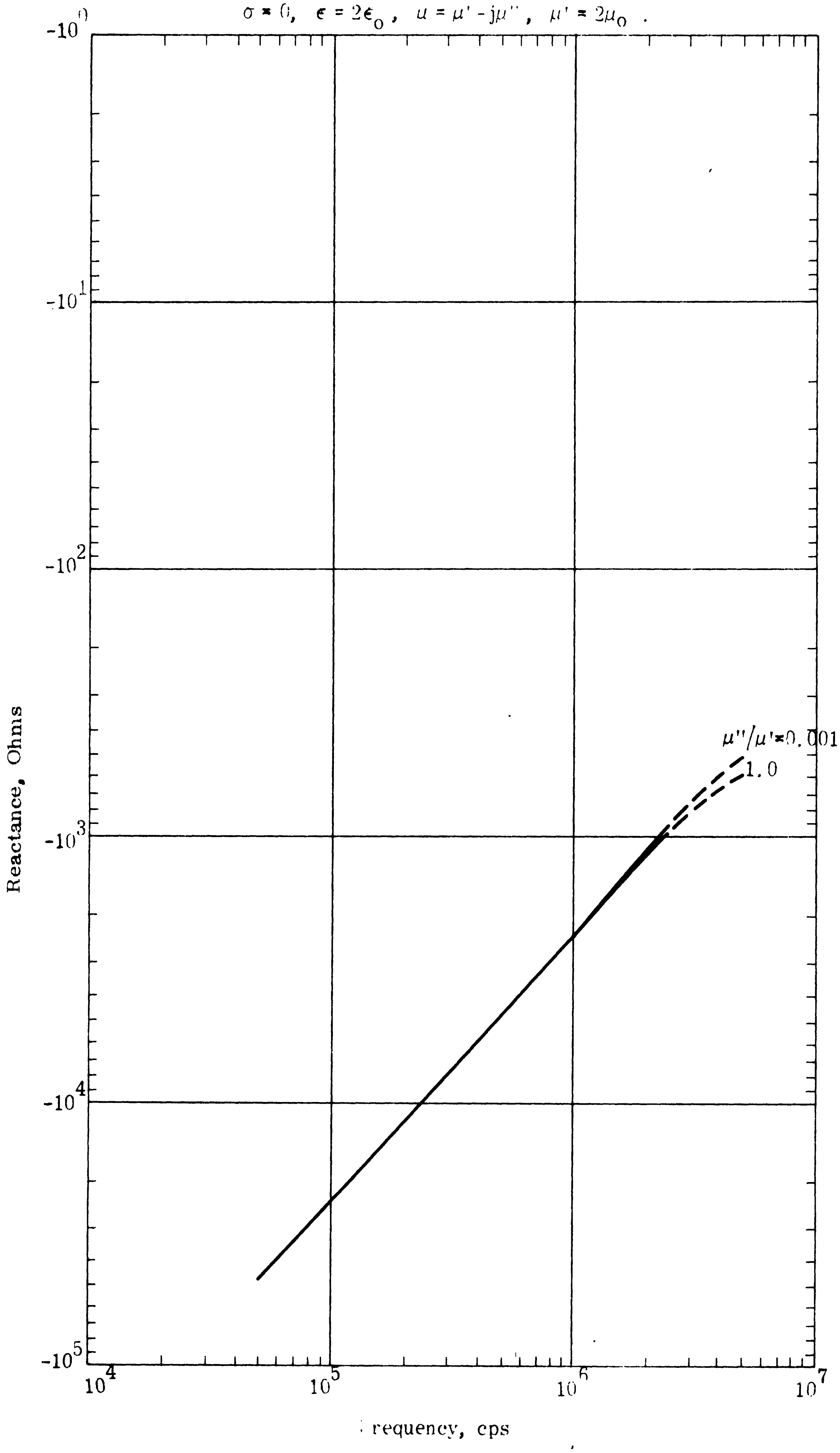
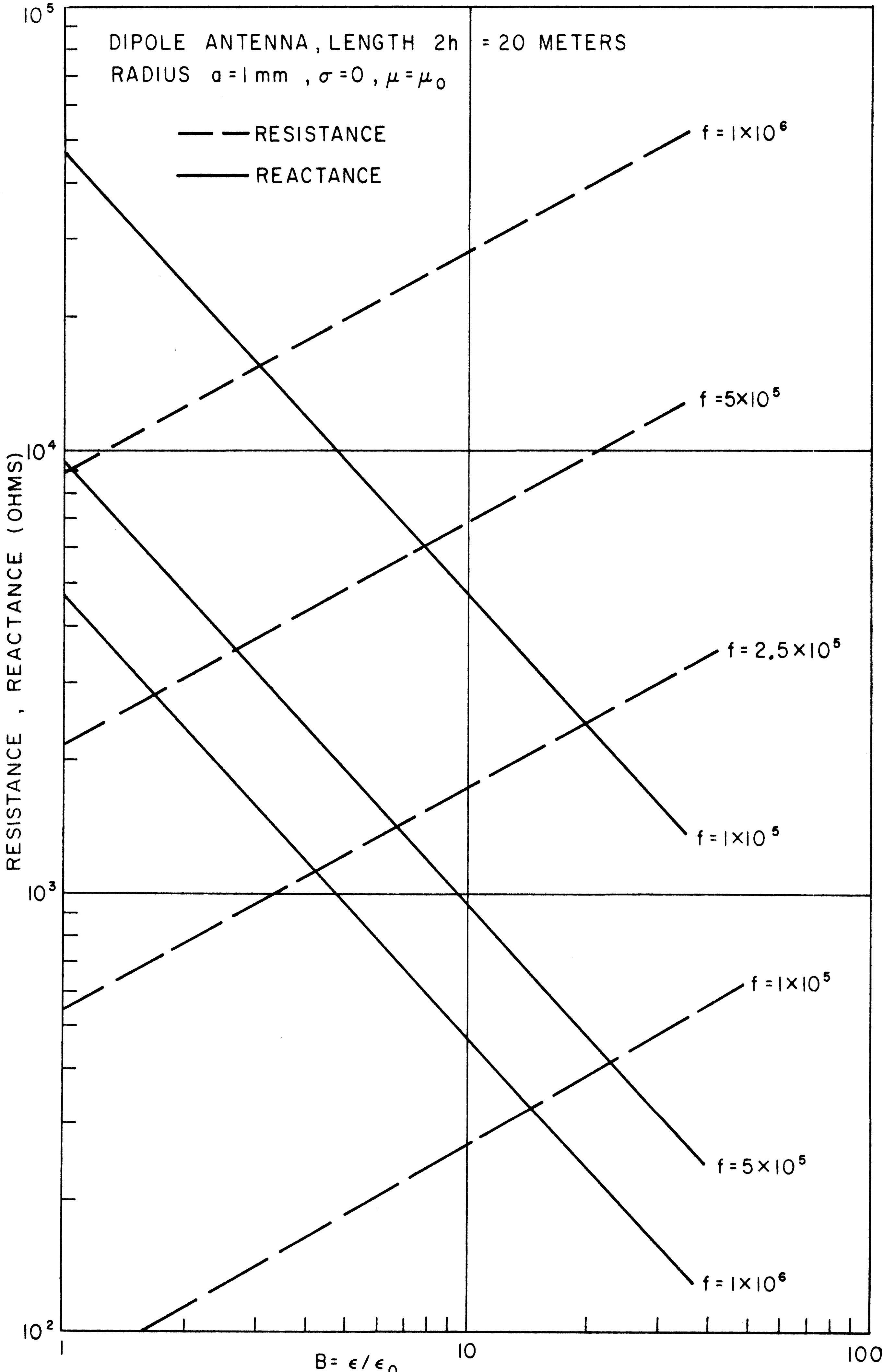


Fig. 2.9 Resistance & Reactance versus Permittivity



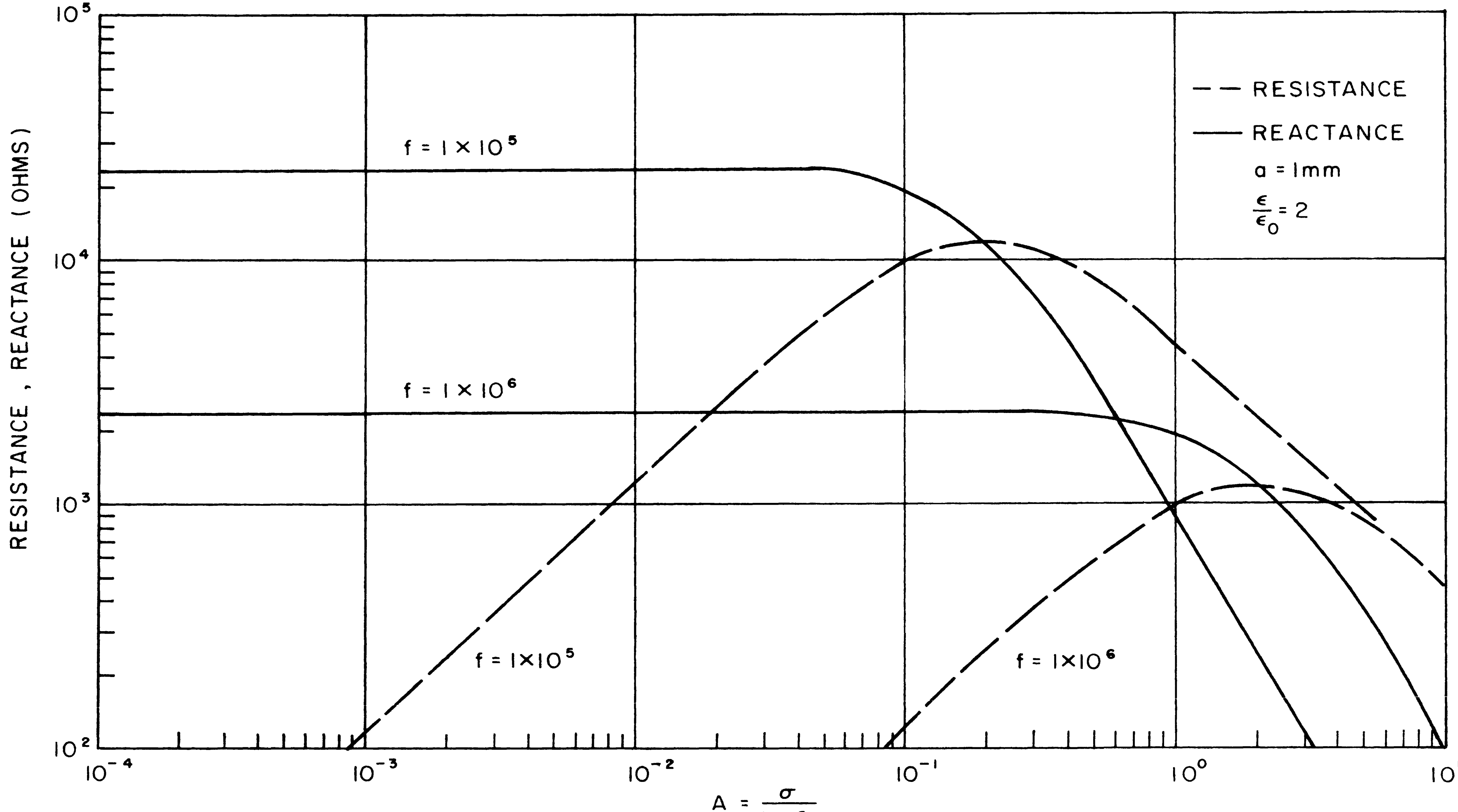


Fig. 2.10 Resistance & Reactance versus Conductivity, Dipole Antenna, Length 2h= 20 meters

FIG. 3.1. Resistance of Dipole Antenna of Length 2h = 20 Meters and radius a = 1 mm in Free Space Near Material Medium Half Space.

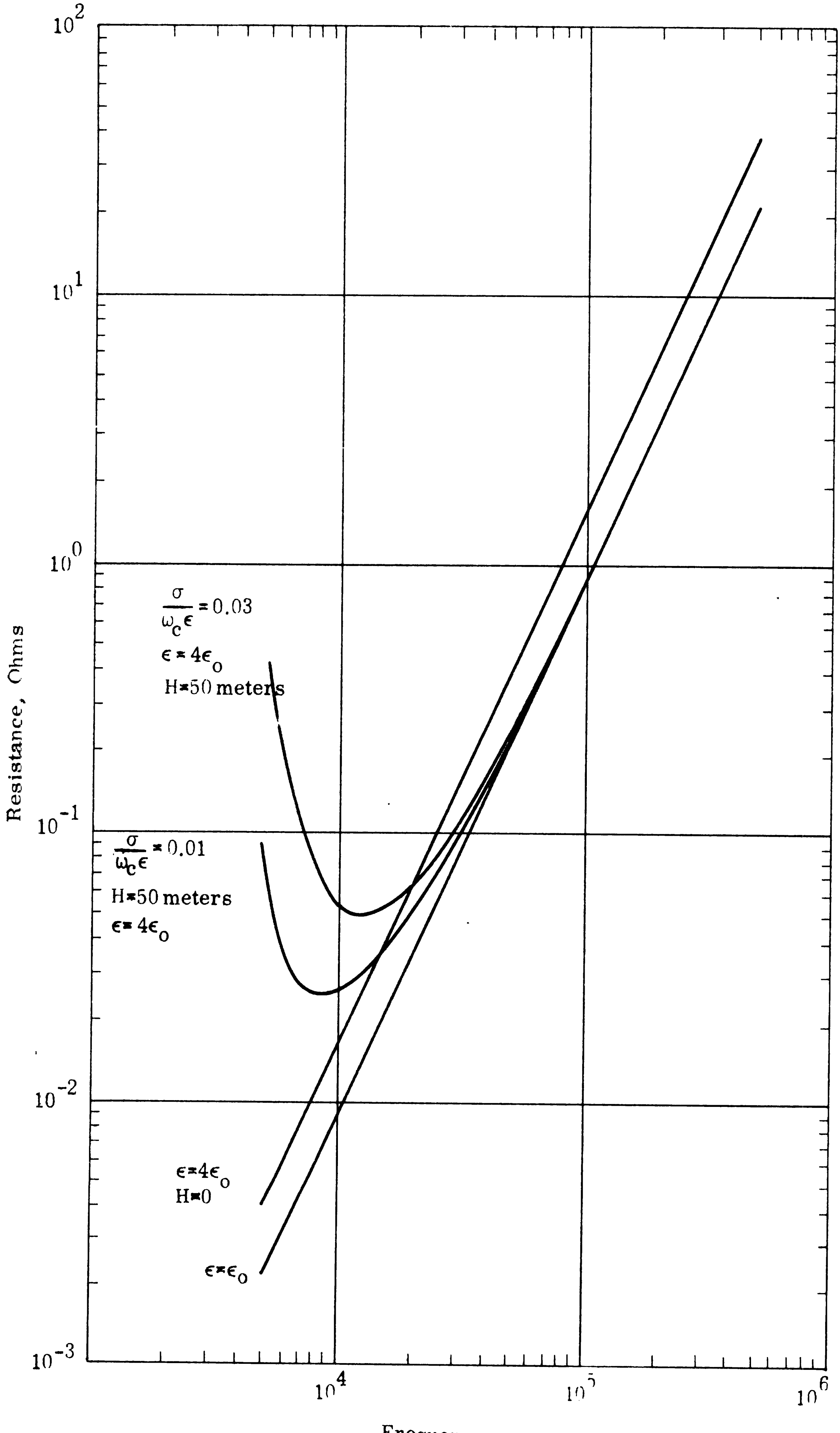


FIG. 3.2. Resistance of Dipole Antenna of Length 2h=20 Meters, Radius a = 1 mm in Free Space as Function of Height H Above Material Medium with $\epsilon = 4\epsilon_0$, $\mu = \mu_0$, $\sigma = 0$ and Frequency = 5×10^5 cps.

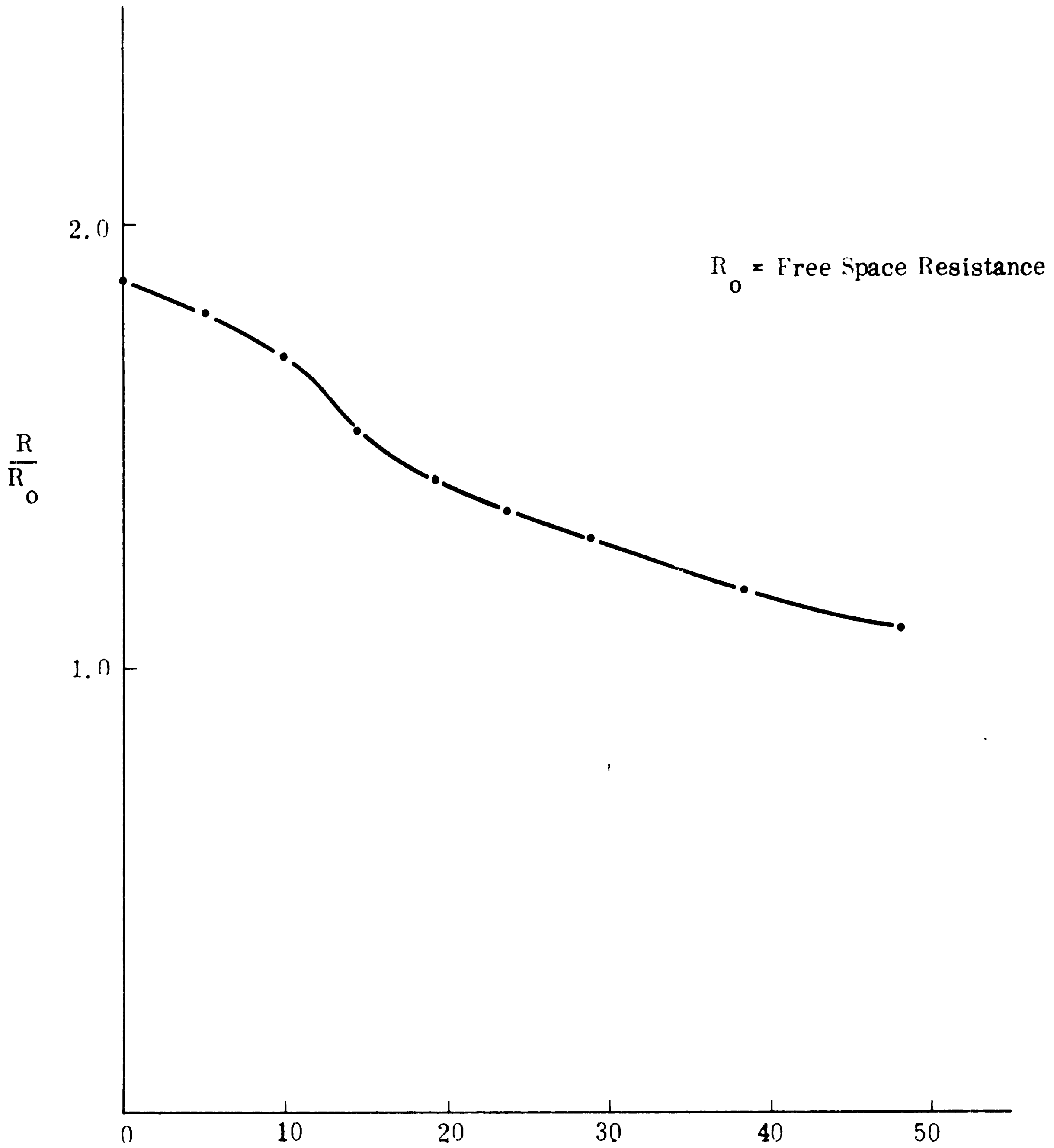
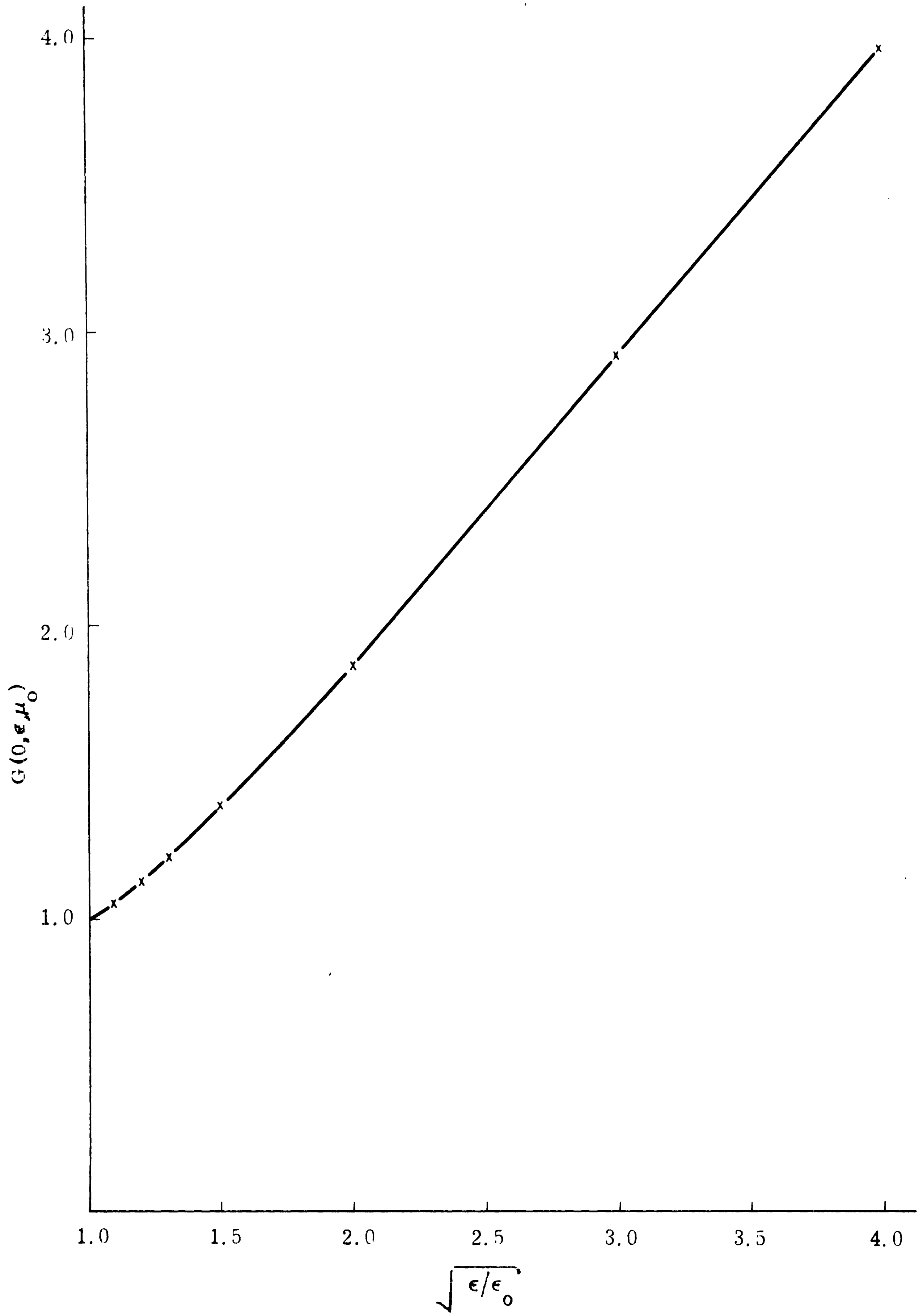


FIG. 3.3. The Function $G(H, \epsilon, \mu)$ for $\mu=\mu_0$, $H=0$ as a Function of $\sqrt{\epsilon/\epsilon_0}$



						Calculated Free Space		Measured	
Source	Frequency	H/λ	ϵ/ϵ_o	$\sigma/\omega\epsilon$	$k_o h$	R_o	jX_o	R	jX
Iizuka (Monopole)	114 Mc	2.97×10^{-3}	78	0.088	0.178	0.192	- 726	68.0	- 611.0
	"	"	77	1.06	"	"	"	295.0	-1,120.0
	"	1.43×10^{-3}	73	0.088	"	"	"	6.1	- 9.9
	"	"	77	1.06	"	"	"	13.6	- 1.0
Seeley et al (Dipole)	3 Kc	1.52×10^{-5}	*	1.88×10^{6}	0.164	0.54	-8,010	325.0	-2800.0
	"	4.4×10^{-8}		1.88×10^{6}	0.164	0.54	-8,010	100.0	- 970.0
	10 Kc	5.07×10^{-5}		$5.64-10^{5}$	0.548	6.25	-2,400	100.0	- 700.0
	"	1.37×10^{-7}		5.64×10^{5}	"	"	"	60.0	- 140.0
	20 Kc	1.01×10^{-4}		2.82×10^{5}	1.096	27.7	-1,090	10[illegible].0	40.0
	"	2.74×10^{-7}		2.82×10^{5}	1.096	27.7	"	310.0	425.0
Proctor (Dipole)	53.4 Mc	6.3×10^{-4}	6.0	0	$\pi/2$	80.0	0	160.0	0
	"	$.10^{-1}$	"	"	"	"	0	70.0	0

* No dielectric constant given. Island consisted of sandy soil

TABLE I

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