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THE UNIVERSITY OF MICHIGAN

COLLEGE OF ENGINEERING
DEPARTMENT OF ELECTRICAL ENGINEERING
Radiation Laboratory

Investigation of Re-Entry Vehicle Surface Fields (U)

Quarterly Report No. 1

18 December 1966 - 18 March 1967

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By

R. F. GOODRICH, B. A. HARRISON, R. E. KLEINMAN
E. F. KNOTT, T. M. SMITH and V. H. WESTON

March 1967

Contract F 04694-67-C-0055

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Norton Air Force Base, California

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R. F. Goodrich, B. A. Harrison, R. E. Kleinman
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March 1967

Prepared for

BALLISTIC SYSTEMS DIVISION
DEPUTY FOR BALLISTIC MISSILE RE-ENTRY SYSTEMS
AIR FORCE SYSTEMS COMMAND
NORTON AFB, CALIFORNIA

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FOREWORD

(U) This report, BSD-TR-67-141, was prepared by the Radiation Laboratory of the Department of Electrical Engineering of The University of Michigan under the direction of Dr. Raymond F. Goodrich, Principal Investigator and Burton A. Harrison, Contract Manager. The work was performed under Contract F 04694-67-C-0055, "Investigation of Re-entry Vehicle Surface Fields (SURF)". The work was administered under the direction of the Air Force Ballistic Systems Division, Norton Air Force Base, California 92409, by Lt. J. Wheatley, BSYDP, and was monitored by Mr. H. J. Katzman of the Aerospace Corporation.

(U) The studies presented herein cover the period 18 December 1966 through 18 March 1967.

(U) In addition to security requirements which must be met, this document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of Ballistic Systems Division (BSOMS), Norton AFB, California 92409.

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ABSTRACT

(Secret)

This is the First Quarterly Report on Contract F 04694-67-C-0055 and covers the period 18 December 1966 to 18 March 1967. The report discusses work in progress on Project SURF and on related short pulse investigations. The SURF program is a continuing determination of the radar cross section of metallic cone-sphere shaped re-entry bodies and the effect on radar cross section of absorber coatings, antenna and rocket nozzle perturbations, changing the shape of the rear spherical cap, and of the re-entry plasma environment. The objective of the short pulse investigation is the determination of methods of modifying the short pulse signature of cone-sphere shaped re-entry bodies and decoys.

I

INTRODUCTION

(S) This is the First Quarterly Report on Contract F 04694-67-C-0055, "Investigation of Re-entry Vehicle Surface Fields (Backscatter) (SURF)". It covers the period 18 December 1966 to 18 March 1967. Work under this program includes a continuation of the SURF investigation which is a study of methods to compute the radar cross section of cone-sphere shaped re-entry vehicles and the initiation in this Quarter of a short pulse study. The short pulse study has as its objective a determination of methods for modifying the short pulse signature of cone-sphere shaped re-entry vehicles and decoys so as to prevent discrimination. These studies are monitored by Lt. J. Wheatley for the Ballistic Systems Division and by Mr. H. J. Katzman for the Aerospace Corporation.

(S) The approach adopted in the SURF investigation makes use of experimental measurements of the surface fields induced on various scale models of re-entry bodies and related shapes to aid in the construction of a theory to explain radar scattering behavior and in the formulation of mathematical expressions for the computation of radar cross section. In addition to the surface field measurements, backscatter measurements are relied on to furnish substantiation of the theory being developed or to guide the investigation in areas wherein surface field measurements alone do not provide adequate data. A digital computer program is being developed to aid in the study of cases of oblique incidence on the target and to provide supplementary data in cases where the very low backscatter from the target is difficult to measure accurately.

(S) The basic metallic cone-sphere with tip and termination modifications was studied during the first phase of the SURF program. The second phase dealt with the effect on radar cross section of a) modifications of the basic metallic cone-sphere due to the addition of antennas and rocket nozzles, changes in the sphericity of the rear termination of the cone, and the addition of absorbing materials, and b) the re-entry plasma environment. Some aspects of this second phase are continuing while, at the same time, the third phase dealing with the effect on radar cross section of combinations of perturbations and coatings, has been initiated.

(S) Short pulse discrimination methods permit one to distinguish between a warhead and accompanying decoys by a simple numerical count of the number of pulses returned by each body. The short pulse investigation has been undertaken to determine methods for countering this discrimination. The investigation in its early stages will be principally mathematical so that the basic theory of short pulse scattering can be set forth. Its application to re-entry shapes will follow. Experimental data made available by Lincoln Laboratory to the Radiation Laboratory via the Aerospace Corporation will be used as part of this analysis.

(S) During the first quarter of the present phase of the program, a Program Plan was written and submitted for approval. The experimental surface field measurement facility was modified to automate some of the measurement procedures so that a greater number of measurements might be made in a given period and to simplify some of the steps in analysis of the measurement data. The theoretical work continued with a study of diffraction by a parabolic cylinder for its application to modifications to the rear of the re-entry vehicle and a study of perturbations to the coated cone-sphere. In response to requests by Aerospace Corporation, the draft of the Final Report for the work on SURF under Contract AF 04(694)-834 was expanded to include, a) a comparison of cross section computations with model measurements of the Mark -12 re-entry vehicle, and b) presentation of cross section formulas in the form of a handbook. The development of a computer program for the radar cross section of rotationally symmetric metallic re-entry vehicles continued. Analytical difficulties were identified and methods for overcoming them were devised.

(S) The study of the effect on radar cross section of the re-entry plasma environment continued with a study of the conditions under which the impedance boundary condition holds. Work was begun on a determination of temperature effects. The experimental program was begun with a study of plasma sheath simulation on a coated flat plate. A cone-sphere model for plasma sheath studies was obtained from Aerospace Corporation. In addition, modifications of the Program Plan section dealing with this work were requested by Aerospace Corporation at the March 1967 Technical Discussion Meeting and these were incorporated in a revised version of the report.

(S) In order to determine methods for masking effects leading to short pulse discrimination between re-entry body and decoy, it is necessary to be able to calculate the pulse response of an object as a function of pulse width and pulse shape and of target shape and constituent material. The first concern is to develop a method for carrying out such calculations for targets of practical interest. This investigation is being carried out along two main lines. Since the pulse response may always be written in terms of the Fourier transform of the time harmonic scattered field, one line of attack involves calculating the pulse scattering characteristics of interest by taking the transform of the time harmonic response. The second line of investigation involves a direct attack on the time-dependent scattering problem. Work on both of these approaches was begun during this quarter.

(U) A description of the investigations outlined above is given in subsequent sections. An analysis of measurement data is being carried out but the results are not at a stage at which conclusions can be drawn. Instead of presenting fragmentary results in this Quarterly, it was decided to report more fully on the work in the next Quarterly at which time the analysis could be meaningfully related to additional measurements.

TASK 2.0 EXPERIMENTAL INVESTIGATIONS

2.1 Introduction

(S) During this reporting period, the experimental group prepared a Program Plan describing their plans to modify their facility, construct models, and obtain experimental data. They have designed and constructed a semi-automatic surface field intensity recording system. A series of backscatter measurements has also been completed, at frequencies from 2.43 to 3.83 GHz, upon model D-14, a 15^{o} total angle cone-sphere coated with Eccosorb casting resin absorber. Several items of major equipment have been ordered, but not received. They will augment our measurement capability. The re-entry plasma experimental program was begun.

2.2 Measurement Facility Modification

(U) During the two previous years of the SURF program, considerable effort has been expended in the acquisition, reduction and analysis of data; all three processes requiring human judgement and involving human responses. All three require the investment of manpower, but the time expended in the first two can be materially reduced by mechanization. The processing time for data analysis probably cannot be reduced other than by investing more manpower. A method to speed up the processes of data collection and data reduction, so that more experimental conditions may be investigated than might otherwise be possible, has been conceived and implemented.

(U) The rate of data acquisition is being increased by a self-recording system much like that presently used in antenna pattern work. Until now, surface field data were obtained on a point-by-point basis by a human operator who advanced the probe to predetermined locations on the target surface. The operator does this by manually turning on a dirve motor which moves the probe and by turning the motor off when the probe reaches the desired point. The operator then reads a meter which indicates the surface field intensity for this probe setting and he records the observed readings of both position and intensity on a data sheet.

(U) This is precisely the way many antenna patterns were measured in the days when few commercially manufactured recorders and positioners were available and experimenters soon bought or built antenna pattern recording equipment to speed up the data collection process. We would like to follow the same course in our surface field measurements, but our situation is slightly different because of the nature of the probing process.

(U) The test object whose surface is to be probed should ideally be isolated from all other objects in order to simulate immersion in free space, yet a disturbance in the form of the probe leads and the probe itself must be introduced in order to measure the fields. Therefore, any structure supporting the probe should be kept as far away from the target as possible so as not to compound the disturbances already present. To do so, we depend on a few inches of the probe lead itself to hold the probe on the target and the very end of the support structure is never less than 8" from the target surface. Since the probe lead is very slender, this arrangement is flexible enough to permit the probe to 'dance' as much as 0.3" during traversing because of vibrations in the drive system. However, when the traversing motor is turned off, the vibrations die out and the probe comes to rest at very nearly the intended location.

(U) Because of these vibrations and because the probe tends to lag, the motion of the traversing system (due to friction against the surface the probe rests on), position information sampled from the traversing mechanism would not accurately correspond with the true probe position during the traverse. On the other hand, if the traversing system is turned off after some finite probe advancement, the probe will stop vibrating and when it has come to rest, the position of the traversing mechanism corresponds quite closely to the true probe position. In this case, position information sampled from the traversing mechanism is reasonably accurate, and therefore useful. The recording system which has been designed permits the probe to come to rest at the desired surface location before a datum point is recorded; no information is recorded <u>during</u> the traverse. As a result, the recorded data are a selection of equally spaced points of the continuous surface field distribution, but the points lie close enough together that a smooth curve may be drawn through them.

(U) The system which accomplishes this is basically the same as that which is used to obtain antenna patterns with the exception that a timing unit alternately interrupts the probe traverse and energizes the solenoid of a recording pen. The timing unit inserts a time delay of about 3 seconds between the traverse interruption and the datum recording function, which is adequate to insure the probe has stopped vibrating. The pen solenoid is actived by the discharge of a capacitor so that when a datum point is to be recorded, the pen is momentarily driven onto the chart paper, then is immediately restored to its normal (non-recording) position by a restraining spring.

(U) The traversing mechanism has been fitted with selsyns from which probe position may be extracted; the selsyn signals are fed to the recorder for both azimuthal (circular) probe motion as well as for linear motion. Conventional chart paper is used and the probe halts at points which are 4° apart along circular trajectories or points 1/6" apart along trajectories. When the system is in operation an observer may watch a distribution being plotted before him. First he hears the whirring of the drive motor as it advances the probe and at the same time sees that the recorder chart is slowly moving under the pen. At the same time, the pen is responding, but not recording, to the intensity distribution sensed by the probe. Abruptly a relay snaps its contacts together, having been triggered by a limit switch sensing the traverse interval, and for a time the operator hears nothing and sees no changes taking place in the chart motion. Suddenly another relay claps shut and the pen rapidly strikes the chart paper and recovers, leaving a dot on the paper in its wake. At the same time, the drive motor circuit is closed and the traversing begins anew, starting another cycle in the process.

(U) The intervals between points along the probe trajectory have been chosen as a matter of convenience and are determined by the characteristics of the traversing mechanism. The screws which drive the mechanism along a straight line are six pitch lathe screws. A cam has been mounted on one of these screws and since it nudges a limit switch every revolution, the interval between datum points is 1/6". For circular trajectories, a row of rivets around the outer circumference of the turret ring+ is used to activate the limit switch. The switch arm has a roller which rides over each rivet head, since there are 90 equally spaced rivets, the angular interval between adjacent rivets is 4°. Depending on the traverse mode being used, each horizontal division of chart paper corresponds to 1/6" or 4°.

+The entire mechanism is mounted upon a surplus gun turret ring which provides circular (azimuthal) probe trajectories; this is convenient for probing around the base of a cone-sphere or other circular paths.

(U) The square root of the probe signal is plotted so that the surface field intensity, and not its logarithm, is recorded at each point. It is our plan that the measurements be calibrated before a run is made in order that we may pre-adjust the gain settings of the system. This will permit an observer to read intensities directly from the chart since the chart scale itself will be calibrated. This may not be altogether successful, for we have experienced small drifts in amplitude that occur during a recording run.

(U) Since this modified system of data recording will provide the field intensities directly, and not in decibels as was previously done, a step in data reduction will have been avoided. It may be desirable to record the data on tape or punched cards so that a computer may organize, compute, and present the data in any form desired by the data analyst. In its present form, this will have to be done manually by a person or persons reading values from the raw pattern and punching the information on cards. Automatic key punching equipment is available, but the cost is high and this approach is not being considered.

2.3 Re-entry Plasma Experiments

(S) As part of the plasma experiment to assist in the derivation of theoretical results for the cross section of plasma coated cone-spheres, the surface fields generated on a cone covered with a simulated plasma sheath will be measured. As a preliminary, experiment, and to assist in the measuring procedures, the surface fields will be measured on a coated flat plate. The plasma sheaths under consideration will be simulated by a plane of wires. Details of the simulation are given below.

2.3.1 Conditions for Simulating a Thin Plasma Sheath with a Plane of Wires

(S) A recent paper by R. L. Fante (1967) gives additional support for the usefullness of the thin sheath approximation for re-entry type plasmas. Our experimental efforts are based upon the thin sheath assumption.

a) Reflection Coefficient for a Plane of Wires.

$$R = \frac{-1}{1 + 2\,\dfrac{Z_{sw}}{N(\perp, /\!/)}}$$

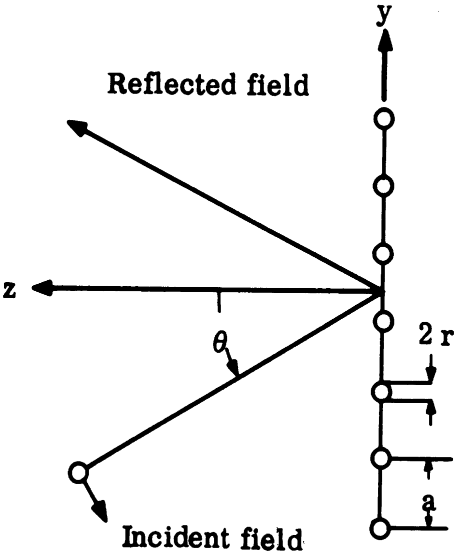
where $N(\perp) = Z_0/\cos\theta$, $N(//) = Z_0 \cos\theta$

$Z_{sw} = R_s + jZ_0 \frac{a}{\lambda} \ln_\epsilon \left(\frac{a}{2\pi r}\right)$ surface impedance for a plane of wires

$R_s = R_o a$

R_o = ohm/meter

$Z_o = \sqrt{\frac{\mu_o}{\epsilon_o}}$ impedance of free space



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b) Reflection Coefficient for a Thin Current Sheath.

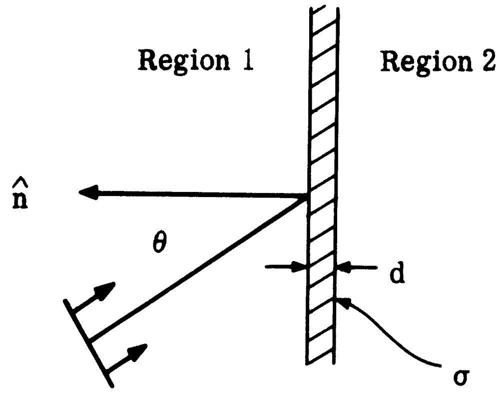
$$R = \frac{-1}{1+2\frac{Z_{sp}}{N(\perp, //)}}$$

where $N(\perp) = Z_o/\cos\theta$, $N(||) = Z_o \cos\theta$

$Z_{sp} = \frac{1}{\sigma d}$ surface impedance of a plasma sheath

$\sigma = \frac{\omega_p^2 \epsilon_o}{\nu_c + j\omega}$ $Z_{sp} = \frac{Z_o}{W^2 D}(V+1)$ $V = \frac{\nu_c}{\omega}$ collision ratio

$Z_o = \sqrt{\mu_o/\epsilon_o}$, $W = \frac{\omega_p}{\omega}$ plasma ratio $D = k_o d$ thickness ratio



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These results are obtained by applying the Wait-Poeverline jump conditions to Maxwell's equations

$$\hat{n} \times (\bar{E}_1 - \bar{E}_2) = 0$$

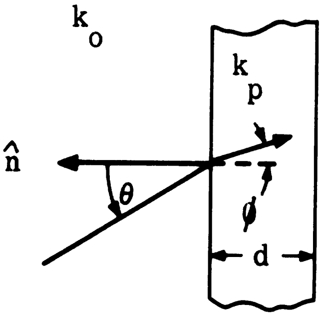
$$\hat{n} \times (\bar{H}_1 - \bar{H}_2) = \bar{J}_s = \bar{E}_t / Z_s$$

$\therefore$ From (a) and (b) the simulation conditions are:

$$\frac{R_s}{Z_o} = \frac{V}{W^2 D} \qquad \frac{a}{\lambda} \ell n_\epsilon \left(\frac{a}{2\pi r}\right) = \frac{1}{W^2 D}$$

c) Reflection Coefficient for a Finite Cold Plasma Slab of Arbitrary Thickness d.

$$R(\perp, ||) = \frac{\frac{j}{2}\left[\frac{k_o}{k_p}\left(\frac{\cos\theta}{\cos\phi}\right)^{\pm 1} - \frac{k_p}{k_o}\left(\frac{\cos\phi}{\cos\theta}\right)^{\pm 1}\right]\sin(k_p d\cos\phi)}{\cos\left[k_p d\cos\phi\right] + \frac{j}{2}\left[\frac{k_o}{k_p}\left(\frac{\cos\theta}{\cos\phi}\right)^{\pm 1} + \frac{k_p}{k_o}\left(\frac{\cos\phi}{\cos\theta}\right)^{\pm 1}\right]\sin(k_p d\cos\phi)}$$



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For Cold Plasma $\quad k_p = k_o\left[1 - \frac{jW^2}{v+j}\right]^{1/2}$

Snell's Law: $\quad \cos\phi = \left[1 - \left(\frac{k_o}{k_p}\right)^2 \sin^2\theta\right]^{1/2}$.

1st Condition. $\quad |k_p|^2 >> k_o^2 \Longrightarrow \cos\phi \approx 1 \qquad \therefore \phi \approx 0^{\circ}$ (see Fig. 1).

This reduces the above equation to

$$R(\perp, ||) \cong \frac{\frac{j}{2}\left[\frac{k_o}{k_p'}(\cos\theta)^{\pm 1} - \frac{k_p'}{k_o}(\cos\theta)^{\pm 1}\right]\sin\left[k_p' d\right]}{\cos\left[k_p' d\right] + \frac{j}{2}\left[\frac{k_o}{k_p'}(\cos\theta)^{\pm 1} + \frac{k_p'}{k_o}(\cos\theta)^{\pm 1}\right]\sin\left[k_p' d\right]}$$

where

$$k_p' \approx k_o\left[\frac{-jW^2}{V+j}\right]^{1/2}$$

2nd Condition

$$k'_p d \ll 1 \Longrightarrow \begin{cases} \cos\left[k'_p d\right] \approx 1 \\ \sin\left[k'_p d\right] \approx k_p d \end{cases}$$

$$\therefore R(\perp, ||) \cong \frac{\frac{j}{2}\frac{k'^2_p d}{k_o}(\cos\theta)^{\mp 1}\left[\left(\frac{k_o}{k'_p}\right)^2(\cos\theta)^{\mp 2} - 1\right]}{1 + \frac{j}{2}\frac{k'^2_p d}{k_o}(\cos\theta)^{\mp 1}\left[\left(\frac{k_o}{k'_p}\right)^2(\cos\theta)^{\mp 2} + 1\right]}$$

and this reduces to

$$R(\perp, ||) \cong \frac{-1}{1 + \frac{2}{j}\left(\frac{k_o}{k'_p}\right)^2 \frac{(\cos\theta)^{\pm 1}}{k_o d}} = \frac{-1}{1 + 2\frac{(V+j)}{W^2 D}(\cos\theta)^{\pm 1}}$$

(S) If we allow $\frac{V+j}{W^2 D} = Z_s$ and $(\cos\theta)^{\pm 1} = N(\perp, ||)$, it is seen that the above equation for $R(\perp, ||)$ is the same as those given in parts (a) and (b). Condition (1) and (2) above are the restrictions which are necessary to reduce the reflection coefficient for a slab of arbitrary thickness to that for a thin current sheath. They are also the conditions which are necessary in order to apply the Wait-Poeverline jump condition mentioned at the end of part (b).

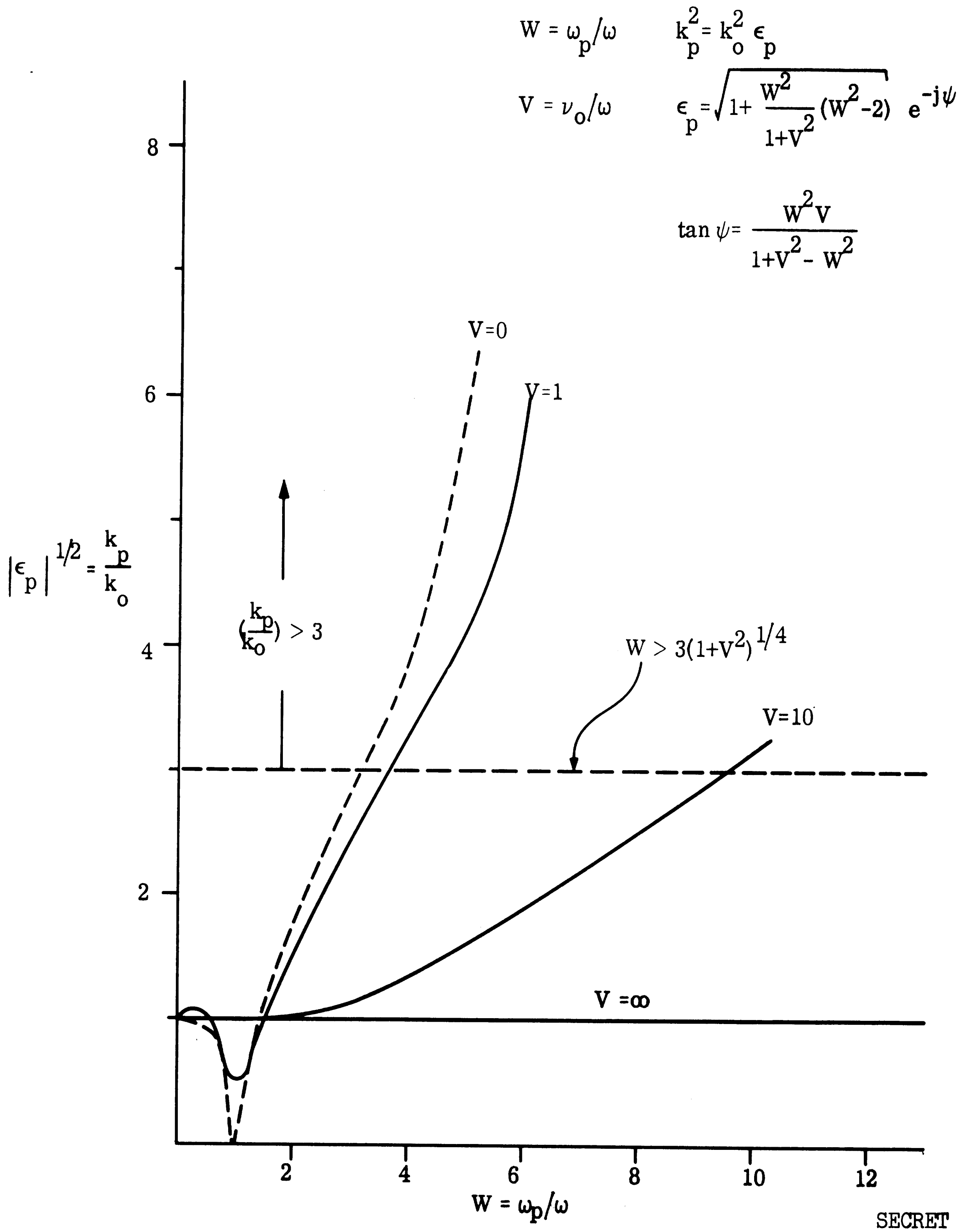

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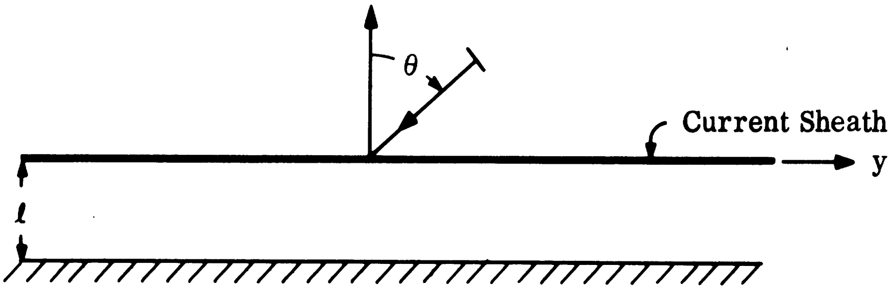
FIG. 1: Plasma Dielectric Constant $|\epsilon_p|$ as a Function of $W = \frac{\omega_p}{\omega}$ with $V = \frac{\nu_c}{\omega}$ as the Parameter. If $|\epsilon_p|^{1/2} > 3$, the condition $|k_p|^2 >> k_o^2$ is Approximately Satisfied.

(S)

2.3 2 Some Electric Properties of a Flat Plate Covered by a Thin Plasma Sheath

a) Reflection Coefficient



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$$R(\perp/\parallel) = \left[\frac{1-A(\perp/\parallel)}{1+A(\perp/\parallel)}\right]^{1/2} e^{j\left[\pi - \xi(\perp/\parallel)\right]}$$

$$A(\perp/\parallel) = \frac{2\,\frac{W^2D}{V^2+1}\,V(\cos\theta)^{\mp 1}}{1+\left[\frac{W^2D}{V^2+1}(\cos\theta)^{\mp 1}+\cot(k_o\ell\cos\theta)\right]^2+\left[\frac{W^2D}{V^2+1}V(\cos\theta)^{\mp 1}\right]^2}$$

$$\xi(\perp/\parallel) = \tan^{-1}\left\{\frac{2\left[\frac{W^2D}{V^2+1}(\cos\theta)^{\mp 1}+\cot(k_o\ell\cos\theta)\right]}{-1+\left[\frac{W^2D}{V^2+1}(\cos\theta)^{\mp 1}+\cot(k_o\ell\cos\theta)\right]^2+\left[\frac{W^2D}{V^2+1}V\cos\theta^{\mp 1}\right]^2}\right\}$$

if $V=0$ $\quad A(\perp/\parallel) \equiv 0$

$$\xi(\perp/\parallel) = \tan^{-1}\left\{\frac{2\left[W^2D(\cos\theta)^{\mp 1}+\cot(k_o\ell\cos\theta)\right]}{W^2D(\cos\theta)^{\mp 1}+\cot(k_o\ell\cos\theta)-1}\right\}$$

and $\quad R(\perp/\parallel) = e^{j\left(\pi-\xi(\perp/\parallel)\right)}$

(S) Therefore when $V=0$, $|R|=1$ and has a phase dependent on W^2D and $k_o\ell=L$ (see Fig. 2).

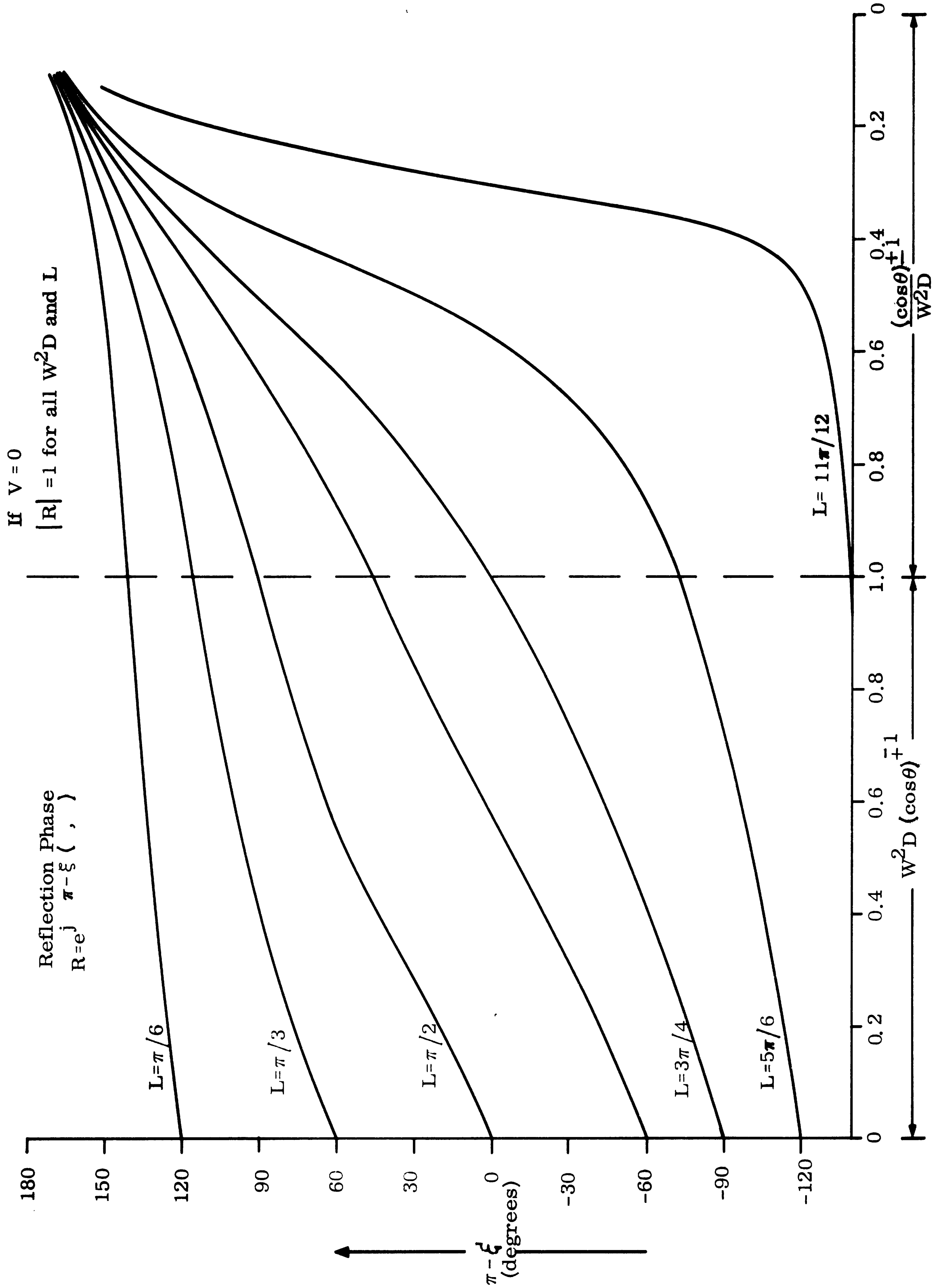

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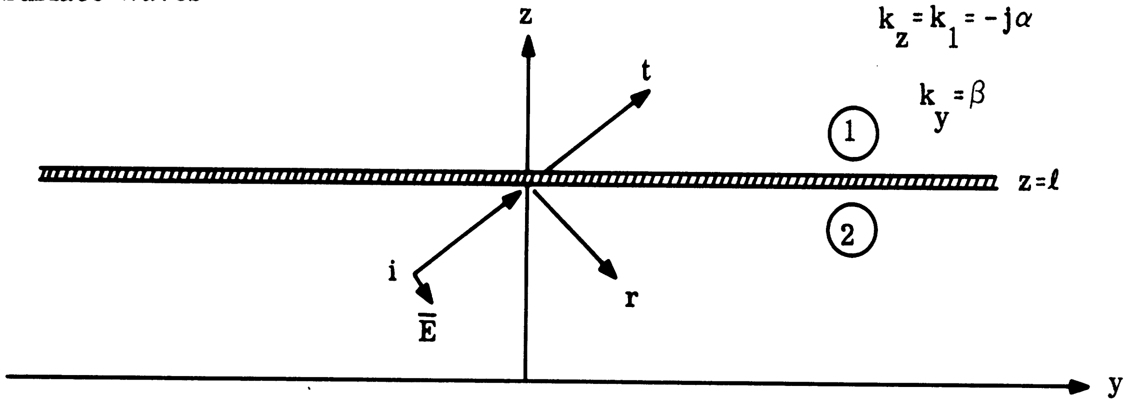
FIG. 2: Reflection Phase π-ξ as a Function W^2D with L as the Parameter.

b) Surface Waves



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An inductive-reactive surface will support surface waves for only the TM (parallel) mode.

The boundary conditions give the following set of equations

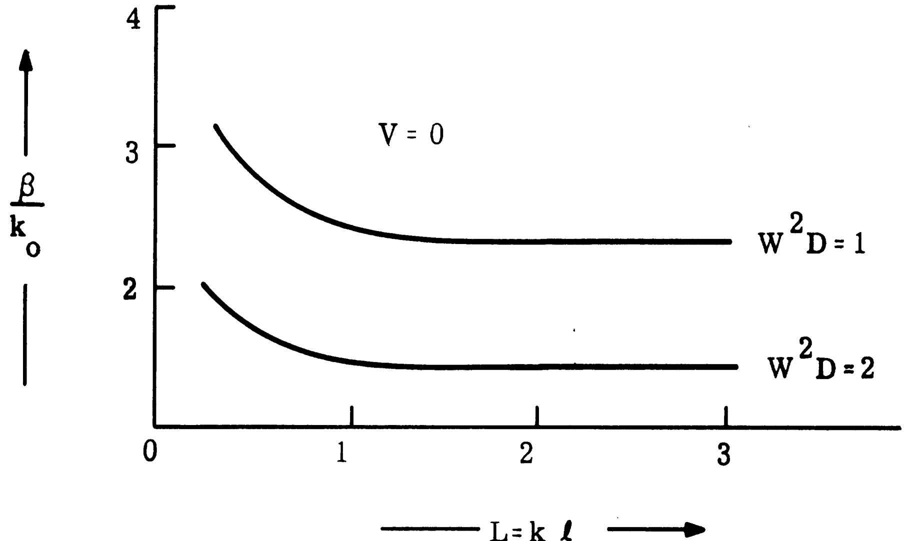
$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ 1 & -e^{j2k_1\ell} & -1 \\ 1 & e^{j2k_1\ell} & -\left(1+\frac{k_1}{\omega\epsilon_o Z_s}\right) \end{pmatrix} \begin{pmatrix} H^i \\ H^r \\ H^t \end{pmatrix}$$

If $k_1 = -j\alpha$, and $Z_s = j(Z_o/W^2D)$

$$W^2D = \frac{k_o}{\alpha}(1+\coth 2\ell)$$

From Helmholtz' equation $\beta^2 = k_o^2 + \alpha^2$

$\frac{\beta}{k_o} = \sqrt{1+(A/L)^2}$ $A = \alpha\ell$, $L = k_o\ell$, for $L > 2$, $\beta \approx k_o\sqrt{1+(2/W^2D)^2}$.



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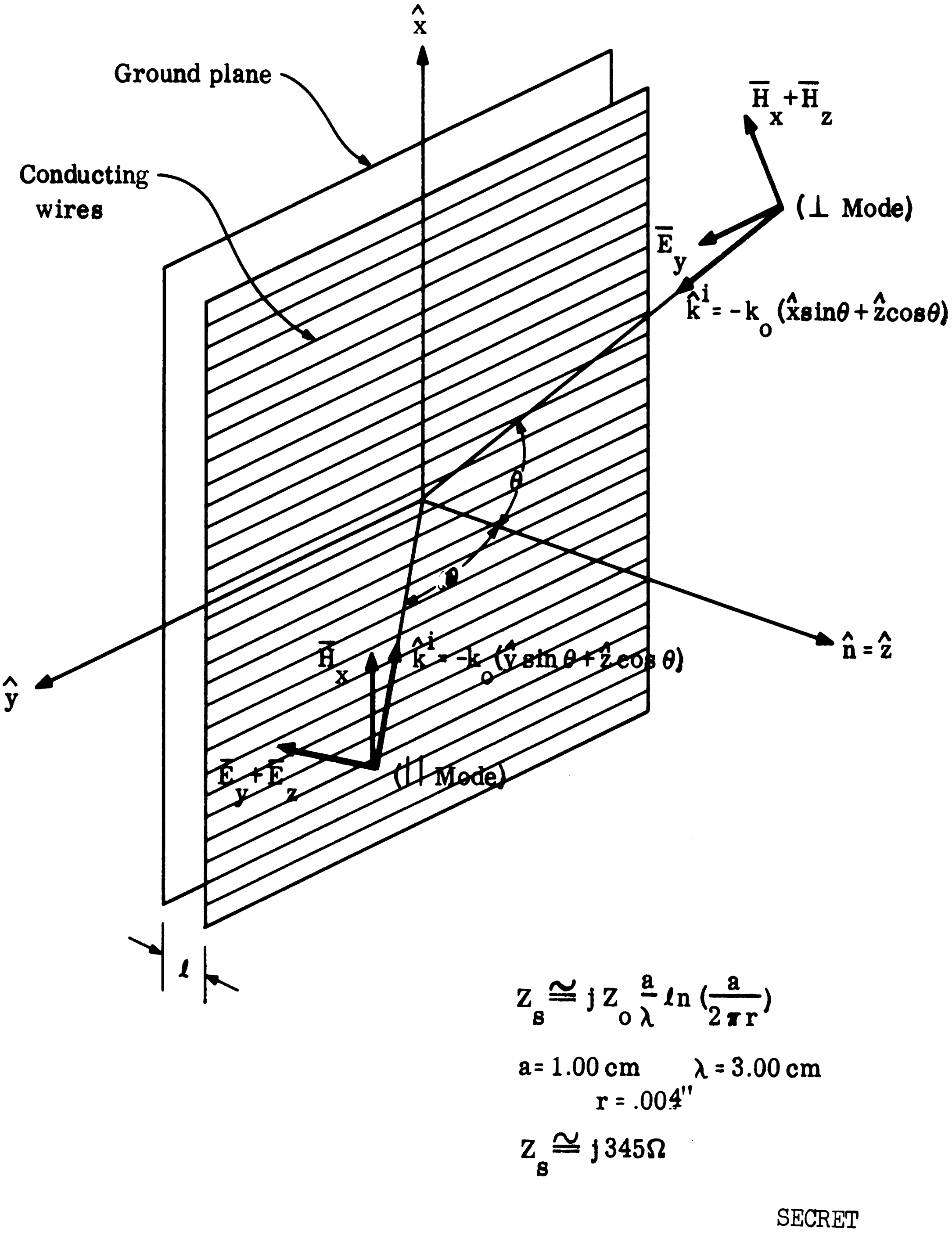


FIG. 3: Experimental Geometry for a Flat Plate Covered by a Current Sheath.

TASK 3.0 THEORETICAL SURF INVESTIGATIONS

3.1 Generalized Computer Program

(S) Under Task 3.1.1 and as an adjunct to other tasks both experimental and theoretical, computer programs have been devised to provide data for comparison with experimentally obtained data or to develop the theory leading to the computation of radar cross sections of cone-sphere shaped re-entry bodies. For example, there is an operating program for the computation of field components on cylindrical surfaces for specified surface impedance conditions and as a function of frequency of incident radiation. This program is used to study the surface fields on the sides of coated cones since incremental sections through the cone can be compared to incremental sections through a cylinder of the same radius. Similarly, there is a program to compute the surface fields on the surface of a metallic sphere. These values are used to compare with experimental values measured by probes on the surface of calibration or test spheres. The comparison allows one to judge the accuracy with which the probe would measure the current along the sides of a cone sphere.

(S) A computer program which is similar in its application but very much more complex than the programs of the type illustrated above, is the program to compute the surface fields and radar cross section of rotationally symmetric metallic bodies with specific application to the cone-sphere. Because of its complexity and the analytic problems encountered, this program has taken longer to make operational than others of the type described. In addition, because it could be a powerful analytical tool in the SURF investigation, a great deal of interest in the details of the analysis has been shown by Aerospace monitors. To illustrate some of the details of the work, a description is given here of the methods which were used at the onset and the subsequent refinements which were required.

(S) The equations defining the matrix elements are as follows:

$$T_{2i-1,\ 2j-1} = \int_a^b dt'\, f_1(t')K_j(t')f_4(t') \int_c^d dt\, f_1(t)K_i(t)f_4(t)\, G_m(t,t')$$

$$+ \int_a^b dt'\, f_1(t')K_j(t')f_3(t') \int_c^d dt\, f_1(t)K_i(t)f_3(t)\, \frac{G_{m-1}(t,t')+G_{m+1}(t,t')}{2}$$

$$- \int_a^b dt'\, f_1(t')Y_j(t') \int_c^d dt\, f_1(t)Y_i(t)\, G_m(t,t')$$

$$T_{2i-1,\ 2j} = \int_a^b dt'\, f_1(t')K_j(t') \int_c^d dt\, f_1(t)\, K_i(t)\, f_3(t)\, \frac{G_{m-1}(t,t')-G_{m+1}(t,t')}{2}$$

$$+ \int_a^b dt'\, K_j(t')\frac{m}{C} \int_c^d dt\, f_1(t)\, Y_i(t)\, G_m(t,t')$$

$$T_{2i,\ 2j-1} = \int_a^b dt'\, f_p(t')K_j(t')f_3(t') \int_c^d dt\, f_1(t)K_i(t)\, \frac{G_{m-1}(t,t')-G_{m+1}(t,t')}{2}$$

$$+ \int_a^b dt'\, f_1(t')Y_j(t') \int_c^d dt\, K_i(t)\, \frac{m}{C}\, G_m(t,t')$$

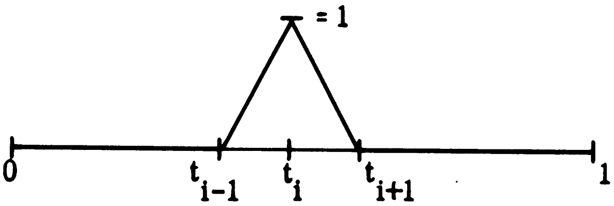
$$T_{2i,\ 2j} = \int_a^b dt'\, f_1(t')\, K_j(t') \int_c^d dt\, f_1(t)\, K_i(t) \frac{G_{m-1}(t,t') + G_{m+1}(t,t')}{2}$$

$$- \int_a^b dt'\, K_j(t') \frac{m^2}{C^2} \int_c^d dt\, K_i(t)\, G_m(t,t')$$

(S) In the equations above, the indices i and j run from 1 to N, the number of sampling points. The limits of integration in all cases are 0 to 1 but due to characteristics of the K functions (trial functions), these limits can be narrowed. The symbol m refers to the mode number and C is half the circumference of the body.

(S) The expansion function for the surface currents is given by:

$K_i(t) =$



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$$Y_i(t) = \frac{\left(K_i'(t) + f_5(t)\, K_i(t)\right)}{C} \quad .$$

The m^{th} Fourier component of the free space Green's function is:

$$G_m(t,t') = \int_0^{\pi} \frac{e^{iK_oR}}{K_oR} \cos m\phi\, d\phi$$

$$K_oR = \sqrt{\left(f_2(t) - f_2(t')\right)^2 + f_1^2(t) + f_1^2(t') - 2 f_1(t) f_1(t') \cos m\phi}$$

for $t = t'$ and $\phi = 0$

$$K_o R = 0.$$

At $t = t'$, G_m is singular but the singularity is integrable.

(S) The matrix elements definition of the geometry of the shape being programmed is given by the f functions. The f functions for the test shape being considered during the checkout of the program and to be replaced by the cone-sphere definition when the computer program is operational are:

$$f_1(t)=\pi\sin\pi t \qquad f_2(t)=\pi(1-\cos\pi t) \qquad f_3(t)=\cos\pi t \qquad f_4(t)=\sin\pi t$$

$$f_5(t)=\pi\frac{\cos\pi t}{\sin\pi t} .$$

Each integral may be written in the form

$$\int_a^b dt' F_1(t') \int_c^d dt\ F_2(t) G_m(t,t') .$$

The method originally followed and since discarded was to make a Taylor series expansion about t' throwing out all terms of second order or above

$$F_2(t)=F_2(t')+F_2'(t')(t-t') .$$

Substitution gives the sum of two integrals;

$$\int_a^b dt' F_1(t')F_2(t') \int_c^d dt\ G_m(t,t') + \int_a^b F_1(t')F_2'(t') \int_c^d dt\ (t-t')G_m(t,t') .$$

A new function I_m^n is defined and the indicated integration acorss the singularity is done analytically.

$$I_m^n(t')= \int_c^d dt\ (t-t')^n\ G_m(t,t') .$$

By substitution, one obtains the sum of two single integrals which may be done by numerical quadratures:

$$\int_a^b dt' \, F_1(t')F_2(t')I_m^0(t't) + \int_a^b dt' F_1(t')F_2'(t')I_m^1(t') \quad .$$

One problem which was encountered was that the analysis assumed that the singularity be approached with the outer limits of integration greater than the inner limits.

$$\int_4^5 dt' F_1(t') \int_3^4 dt \; F_2(t) \, G_m(t, t') \quad .$$

(S) This sometimes made necessary an interchange in the order of integration and led to ambiguity as to which function was to be expanded in the Taylor series.

$$\int_3^4 dt' \, X(t') \int_4^5 dt \; Z(t) \; G_m(t, t') \qquad \text{(incorrect) } F_1 = X, \; F_2 = Z$$

$$\int_4^5 dt' Z(t') \int_3^4 dt \; X(t) \; G_m(t, t') \qquad \text{(correct) } F_1 = Z, \; F_2 = X \quad .$$

(S) The discontinuity in the derivative of the K function made this an important consideration. If

$K_5'(t) = 1/\Delta$ then $K_4' = -1/\Delta$

$X'(t) \neq Z'(t)$

in the general case. This showed the contradiction in the method being followed. The existence of a sort of symmetry for the test shape was then observed.

$f_1(1-t) = f_1(t)$ $\qquad$ $f_3(1-t) = -f_3(t)$

$f_4(1-t) = f_4(t)$ $\qquad$ $f_5(1-t) = -f_5(t)$

$K_{N+1-i}(1-t) = K_i(t)$ $\qquad$ $K'_{N+1-i}(1-t) = -K_i'(t)$

$$Y_{N+1-i}(1-t) = Y_i(t)$$

$I = N+1-i \qquad\qquad J = N+1-j$

$$T_{2i-1,\,2j-1} = T_{2I-1,\,2J-1}$$
$$T_{2i,\,2j} = T_{2I,\,2J}$$
$$T_{2i-1,\,2j} = -T_{2I-1,\,2J}$$
$$T_{2i,\,2j-1} = -T_{2I,\,2J-1} \quad .$$

(S) Then the test results obtained by running the program on the computer did not show the expected symmetry, it was decided that the approximation due to ignoring all terms in the Taylor series of second order or higher was not accurate enough and this method was discarded. Following a new approach, G_m was rewritten as follows:

$$G = e^{iK_oR} / K_oR \qquad\qquad S = t-t'$$

$$G_m(t,t') = \int_0^{\pi} (\cos m\phi) G d\phi = \left\{ \int_{n_m}^{\pi} (\cos m\phi) G d\phi + \int_0^{n_m} i\, e^{i\frac{K_oR}{2}} \frac{\sin\frac{K_oR}{2}}{\frac{K_oR}{2}} \cos m\phi\, d\phi + \left(\phi_o + \frac{\ln|S|}{R_1}\right) - \frac{m^2}{2}\phi_2 \right\} - \frac{\ln|S|}{R_1}$$

where

$$\phi_o = \frac{1}{R_1} \ln \frac{R_1N_m + \sqrt{R_0^2 + R_1^2N_m^2}}{R_0}$$

$$\phi_2 = \frac{1}{2R_1^3}\left[R_1N_m\sqrt{R_0^2+R_1^2N_m^2} - R_0^2 \ln \frac{R_1N_m + \sqrt{R_0^2+R_1^2N_m^2}}{R_0}\right]$$

$$R_1^2 = f_1(t)f_1(t')$$

$$R_0^2 = \left(f_2(t)-f_2(t')\right)^2 + \left(f_1(t)-f_1(t')\right)^2$$

$$\left(\phi_o + \frac{\ln|S|}{R_1}\right) \xrightarrow[S\to 0]{} \frac{1}{R_1} \ln \frac{2R_1N_m}{C}$$

and with the result that the terms inside $\{\ \}$ are not singular when $t = t'$.

(S) We make the following definitions:

$$G_m^{(0)} = \left\{ \quad \right\} \qquad G_m^{(1)} = \frac{\ln|S|}{R_1}$$

$$G_m(t,t') = G_m^{(0)} + G_m^{(1)}$$

$$ME^{(0)} = \int dt'\ F_1(t') \int dt\ F_2(t) G_m^{(0)}$$

$$ME^{(1)} = \int dt'\ F_1(t') \int dt\ F_2(t)\ G_m^{(1)}$$

$$\int dt' F_1(t') \int dt\, F_2(t) G_m(t,t') = ME^{(0)} + ME^{(1)}$$

which again gives the sum of two integrals, one of which can be done by numerical quadratures.

(S) An analytic treatment of the singular part produces two integrals each of which can be done by numerical quadratures.

$$\mathcal{F}_1(t) = \frac{F_1(t)}{\sqrt{f_1(t)}} \qquad \mathcal{F}_2(t) = \frac{F_2(t)}{\sqrt{f_2(t)}}$$

$$ME^{(1)} = -\int dt'\ \mathcal{F}_1(t') \int_{\ell}^{u} dt\ \mathcal{F}_2(t)\ \ln|S|$$

$$\beta_u = u - t' \qquad \beta_{\ell} = \ell - t'$$

$$ME^{(1)} = \int dt'\ \mathcal{F}_1(t')\ \mathcal{F}_2(t') \Big[\beta_u \ln|\beta_u| - \beta_u - \beta_{\ell} \ln|\beta_{\ell}| + \beta_{\ell} \Big]$$
$$- \int dt'\ \mathcal{F}_1(t') \int dt \left(\mathcal{F}_2(t) - \mathcal{F}_2(t') \right) \ln|S|.$$

$$\left(\mathcal{F}_2(t) - \mathcal{F}_2(t') \right) \ln|S| \xrightarrow[S \to 0]{} 0$$

(S) The integral across the singularity is now expressed as the sum of two integrals all of which can be done numerically.

(S) The program is in the debugging stage. Successive runs are giving increased accuracy when compared with the results for the exact solution for the current on the sphere used as a test object. When sufficient accuracy is achieved, the cone-sphere input function will replace the sphere input function and the program will be run for the cone-sphere.

3.2 Perturbations on Coated Shapes

(S) To obtain the effect of perturbations near the tip of a coated cone we have first considered the field induced on a semi-infinite cone in order to obtain a better expression for the exciting field. The approach we use to this problem of the coated cone is as follows.

(S) The electromagnetic fields of a right-circular cone whose surface is characterized by an impedance were considered. To simplify the problem, a delta current source (either magnetic or electric) is assumed on the extension of the conical axis and oriented in that direction. Using Stratton and Chu's integral representation of electromagnetic fields in a closed region, a perturbational scheme for the electromagnetic fields of the impedance cone is formulated. In that scheme, the perturbed fields are represented by an ascending series of the surface impedance where the first term is the unperturbed field, i.e., electromagnetic fields of a perfectly conducting cone. The coefficients of two successive terms of the series are related by an integral equation. Thus the perturbational scheme breaks the original mixed boundary value problem into a set of infinitely many integral equations Fortunately, all these integral equations have the same kernel. The integral equations of this type are being studied.

3.3 Effect of Small Base Radius

(U) Using the parabolic cylinder as the canonical problem we are studying the transition between the edge diffraction region, the cylinder very near a half-plane, and the creeping wave region, the minimum radius of curvature R_0 such that $kR_0 > 1$.

(U) We start with the analog of the Watson transform in its integral form and examine the poles of the integrand. For $kR_0 << 1$ the poles are such that the residue is the Fresnel integral representation of the half plane field; for $kR_0 >> 1$ the poles are such that we are led to the Fock function representation of the field. Our problem then is to find the transition between these two. We suspect that the vestiges of the half plane poles remain even in the high frequency region; hence we are examining various representations of the parabolic cylinder functions in order to prove or disprove this conjecture.

(U) In particular the propagation of the creeping wave is investigated by varying the focal length of the parabolic cylinder. An exact series solution in terms of Hermite function is transformed to a complex integral by Watson's method. In the shadow region the contour is deformed and the asymptotic solution is defined by the poles of the integrand enclosed by the contour. The location of the pole is determined by e-valuating the root of a Hermite function which may be calculated approximately by the method of steepest descent. The result expresses the field in terms of residues which are recognized as creeping waves.

(U) The focal length of interest is assumed to be comparable with the wavelength. The behavior of the creeping wave is then to be observed between two extreme cases, i.e. the half plane and the large cylinder.

3.4 Plasma Re-entry Environment

3.4.1 Derivation of Generalized Impedance Boundary Condition

(S) In order to compute the radar cross section of a plasma-sheathed conical re-entry body, it would be useful to show that thin lossy sheaths (with large gradients) can be represented by some type of general impedance boundary condition, and at the same time find the conditions on the geometrical and electrical parameters for which the boundary condition holds. In particular it is hoped that under certain restrictions, that the electromagnetic properties of the sheath can be represented in terms of an impedance boundary of the form

$$\underline{E} - (\underline{E}\cdot\underline{n})\underline{n} = \eta\sqrt{\mu_0/\epsilon_0}\ \underline{n}\times\underline{H}$$

is imposed on the outer surface, where η is a parameter which is a function of the electrical properties of the sheath and the geometrical properties of the surface. It is well known that uniform overdense sheaths can be represented by such a condition. It would be useful to see if and when the results can be extended to non-uniform sheaths.

(U) A general approach to the problem is to first formulate an integral equation and from this extrapolate the results using such physical assumptions as the principle of local analysis. The development of an integral equation is given below.

(U) The following definitions will be made

S_0 the surface of the vehicle

S_1 the outer surface of the sheath

V_s the volume of the sheath

V_∞ the volume of free space exterior to the sheath

ϵ' the relative dielectric constant of the sheath (non-homogeneous)

$k_0 = 2\pi/\lambda$ free space wave number

$\kappa = k_0\sqrt{\epsilon'}$.

(U) The development of the integral equation follows from the relation given in Stratton involving the two vector fields $\underline{P}$ and $\underline{Q}$

$$\int_V \left\{\underline{Q}\cdot\underline{\nabla}\wedge\underline{\nabla}\wedge\underline{P} - \underline{P}\cdot\underline{\nabla}\times\underline{\nabla}\times\underline{Q}\right\} dv = \int_S \underline{n}\cdot\left\{\underline{P}\times\underline{\nabla}\times\underline{Q} - \underline{Q}\wedge\underline{\nabla}\wedge\underline{P}\right\} ds \qquad (3.1)$$

where $\underline{n}$ is the unit normal vector pointing out of the volume. The vector fields $\underline{P}$ and $\underline{Q}$ will be chosen as follows.

(1) $\underline{P} = \underline{E}$ (electric intensity)

(2) $\underline{Q} = F\,\underline{a}$

where $\underline{a}$ is an arbitrary unit vector, and the generalized Green's function F is given by

$$F = -e^{is}/4\pi\rho$$

with

$$\rho = |\underline{r}-\underline{r}'| ,$$

$$s(\underline{r}, \underline{r}') = \int_0^\rho \kappa(\underline{r}'+\underline{e}\, s)ds,$$

$$\underline{e} = \underline{\nabla}\rho \quad .$$

It is easily seen that the vector $\underline{e}$ is a unit vector directed from $\underline{r}'$ to $\underline{r}$. To employ the above integral equation, the following quantities are required.

$$\underline{\nabla}\wedge\underline{P} = i\omega\mu_0 \underline{H}$$

$$\underline{\nabla}\wedge\underline{\nabla}\wedge\underline{P} = \kappa^2 \underline{E}$$

$$\underline{\nabla}\wedge\underline{Q} = \underline{\nabla}F \times \underline{a}$$

$$\underline{\nabla}\wedge\underline{\nabla}\wedge\underline{Q} = -\underline{a}\nabla^2 F + \underline{\nabla}(\underline{a}\cdot\underline{\nabla}F) = \underline{a}\left\{\kappa^2 F - \delta(\underline{r}-\underline{r}') - iF\underline{e}\cdot\underline{\nabla}\kappa\right\} + \underline{\nabla}(\underline{a}\cdot\underline{\nabla}F) \quad .$$

It can be shown the surface integral in Eq. (3.1) can be expressed in the form

$$\underline{a}\cdot\int_s \left\{i\omega\mu_0\, \underline{n}\wedge\underline{H}\,F + (\underline{n}\wedge\underline{E})\wedge\underline{\nabla}F\right\} ds$$

and the volume integral reduces to

$$\underline{a}\cdot\left\{\underline{E}(\underline{r}') + \int_v \left\{i\underline{E}(\underline{e}\cdot\underline{\nabla}\kappa)F + \underline{\nabla}F(\underline{\nabla}\cdot\underline{E})\right\} - \int_s \underline{\nabla}F(\underline{E}\cdot\underline{n})\, ds\right\}.$$

Since $\underline{a}$ is an arbitrary unit vector, the following integral equation is obtained provided that $\underline{r}'$ is contained in v.

$$\underline{E}(\underline{r}') = \int_s \left\{i\omega\mu_0\,\underline{n}\wedge\underline{H}\,F + (\underline{n}\wedge\underline{E})\wedge\underline{\nabla}F + \underline{\nabla}F(\underline{E}\cdot\underline{n})\right\} ds$$
$$- \int_v \left\{i\underline{E}(\underline{e}\cdot\underline{\nabla}\kappa)F + \underline{\nabla}F(\underline{\nabla}\cdot\underline{E})\right\} dv \quad . \qquad (3.2)$$

(U) In order to apply representation (3.1) for a volume v, it is necessary that the fields are continuous in v implying that ϵ' is continuous in v. The above integral equation can be applied to the sheath itself where $v = v_0$ and s is the combined surface s_1 and s_0. However in this formalization the incident field is not contained explicitly. To take into account the incident field, the volume v_∞ exterior to the sheath must be included. In this case the surface s will be s_0 and s_∞ a sphere with infinite radius. In the region exterior to s_0, the total fields $\underline{E}$ and $\underline{H}$ can be decomposed into the incident ($\underline{E}_0$, $\underline{H}_0$) and scattered fields. The integral over s_∞ containing the scattered field will vanish, whereas the integral

$$\int_{s_\infty} \left\{ i\omega\mu_0 \, \underline{n}\wedge\underline{H}_0 F + (\underline{n}\wedge \underline{E}_0)\wedge \underline{\nabla} F + \underline{\nabla} F \, (\underline{E}_0 \cdot \underline{n}) \right\} ds = \underline{f}_0 (\underline{r}') \tag{3.3}$$

does not vanish. For overdense sheaths of sufficient thickness, such that negligible energy penetrates to the surface of the vehicle, it can be shown that the above integral can be represented in the form

$$\underline{f}_0(\underline{r}') = \int_{v_0} \left\{ (k_0^2 - k^2)\, \underline{E}_0 F + i\underline{E}_0 (\underline{e} \cdot \underline{\nabla} \kappa)F \right\} dv + \underline{E}_0(\underline{r}') \quad . \tag{3.4}$$

There exist other representations of the above integral (3.3) but these will not be given here. The final form of the integral Eq. (3.2) is given in the form

$$\underline{E}(\underline{r}') = \underline{f}_0(\underline{r}') - \int_{v_0} \left[i\underline{E}(\underline{e} \cdot \underline{\nabla} \kappa) F + \underline{\nabla} F (\underline{\nabla} \cdot \underline{E}) \right] dv$$
$$+ \int_{s_0} \left\{ i\omega\mu_0 (\underline{n} \wedge \underline{H}) F + (\underline{n} \wedge \underline{E}) \wedge \underline{\nabla} F + \underline{\nabla} F (\underline{n} \cdot \underline{E}) \right\} ds \quad . \tag{3.5}$$

One advantage of using the above typed integral equation for either overdense or lossy sheaths is that the kernel of the integral equation contains a decaying exponential in the form

$$\exp\left\{-\mathrm{Im}\int_0^{\rho}\kappa\ (\underline{r}'+\underline{e}\ s)ds\right\}$$

which allows in general the principle of local analysis to be applied.

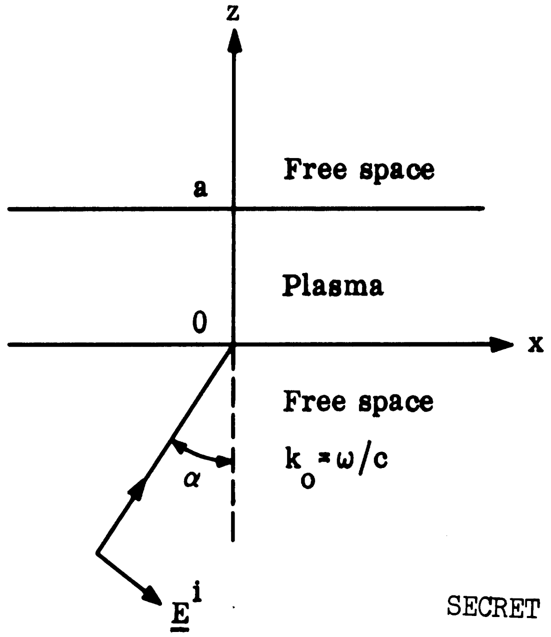
(U) In investigating approximate solutions in the hope of deriving a general impedance boundary condition, the two dimensional problem will be considered first. The simplification of the integral equation to the two-dimensional case will be given in future reports.

3.4.2 Temperature Effects

(U) In an infinite, homogeneous and isotropic, plasma medium the allowed longitudinal and transverse oscillations are not coupled. In practice, however, boundaries exist and coupling of the various wave modes arises as a consequence of the boundary conditions. We consider, then, the reflection of a plane electromagnetic wave incident obliquely upon a finite plasma layer, where the equilibrium plasma is assumed to be homogeneous and isotropic. The electric vector of the wave is taken in the plane of incidence. This case is of special interest because a longitudinal wave, which cannot be excited in the other polarization case, can now penetrate into the plasma. Treating only the electrons dynamically, we have derived an exact solution of the coupled Maxwell-Vlasov equations under the assumption that the electrons are specularly reflected at the boundaries. The solution differs from those given in previous investigations (notably in the Soviet literature) in that the coupling between the longitudinal and transverse modes has been taken into account. The previous investigators have obtained their solution by employing the permittivity tensor derived for an infinite plasma, and this, in general, is not valid in the case of a bounded plasma. Some preliminary asymptotic results for low temperatures have been obtained. A brief outline is given as follows.

(U) The fundamental equations to be solved are

$$\underline{j} = \frac{1}{i\omega\mu_o}\left(\underline{\nabla}\wedge\underline{\nabla}\wedge\underline{E} - k_0^2\,\underline{E}\right) \quad \text{(Maxwell equations)} \qquad (3.6)$$



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and the linearized Vlasov equation

$$-i\omega f + \underline{v}\cdot\underline{\nabla} f = -\frac{n_o e}{\kappa T}\left(\underline{E}\cdot\underline{v}\right) f_o \qquad (3.7)$$

where

$$f_o = \frac{1}{\pi^{3/2}\, v_T^3}\, e^{-v^2/v_T^2}\ , \qquad v_T = \sqrt{\frac{2\kappa T}{m}}\ . \qquad (3.8)$$

Equations (3.6) and (3.7) are coupled self-consistently by

$$\underline{j} = -e\ \ \underline{v}\, f\, d\underline{v}\ . \qquad (3.9)$$

For polarization in the plane of incidence, solutions of the form

$$\underline{E} = e^{ik_x x}\left[E_x(z),\, 0,\, E_z(z)\right]\ , \quad H = e^{ik_x x}\left[0,\, H_y(z),\, 0\right]\ , \quad k_x = k_o \sin\alpha \qquad (3.10)$$

will be sought. The following boundary condition on $f(\underline{x},\underline{v})$ (specular reflection)

$$f(v_z > 0) = f(v_z < 0) \text{ at } z = 0,\ a, \tag{3.11}$$

will be employed.

With the assumption, $\underline{E}$, $\underline{H}$, $f \sim e^{i\underline{k}\cdot\underline{x}}$, the Maxwell-Vlasov equations become

$$-i\omega\epsilon_o(\underline{k}\cdot \underline{E})+(\underline{k}\cdot \underline{j})=0, \quad (k_o^2-k^2)(\underline{k}\ \underline{E})+i\omega\mu_o(\underline{k}\wedge\underline{j})=0, \quad (\omega-\underline{k}\cdot\underline{v})f= -\frac{in_o e}{\kappa T}(\underline{E}\cdot\underline{v})f_o. \tag{3.12}$$

The decomposition of $\underline{E}$ and $\underline{v}$ into longitudinal and transverse components, e.g.

$$\underline{E} = \underline{E}^{\ell}+\underline{E}^{t} \text{ where } \begin{cases} \underline{E}^{\ell} = \dfrac{(\underline{k}\cdot\underline{E})}{k^2}\ \underline{k} \\ \underline{E}^{t} = \dfrac{1}{k^2}\underline{k}\wedge(\underline{E}\wedge\underline{k}) \end{cases} \tag{3.13}$$

yields the following equations expressed in terms of the components E^{ℓ} and E^{t}

$$\epsilon^{\ell}(\omega,\underline{k})E^{\ell} - \frac{E^t}{\lambda_D^2 k}\int \frac{v^t f_o}{\omega-\underline{k}\cdot\underline{v}}\,d\underline{v} = 0\ , \qquad \lambda_D = \sqrt{\frac{\epsilon_o\kappa T}{e^2 n_o}}$$

$$-k_o^2\ \frac{E^{\ell}}{\lambda_D^2 k}\int \frac{v^t f_o}{\omega-\underline{k}\cdot\underline{v}}\,d\underline{v} + \left[k_o^2\ \epsilon^t(\omega,\underline{k})-k^2\right] E^t = 0\ . \tag{3.14}$$

(U) For complex k, the integrals are non-zero, and non-trivial solutions exist only if the determinant vanishes, i.e.

$$\epsilon^{\ell}(\omega,\underline{k})\left[k_o^2\ \epsilon^t(\omega,\underline{k})-k^2\right] - \left[\frac{k_o}{\lambda_D^2 k}\int\frac{v^t f_o}{\omega-\underline{k}\cdot\underline{v}}\,d\underline{v}\right]^2 = 0 \tag{3.15}$$

This gives rise to a discrete set of coupled modes. The surface impedance is given by

$$S(0) = \sqrt{\frac{\epsilon_o}{\mu_o}}\left(\frac{E_x}{H_y}\right)_{z=0} = S_2 - \frac{S_1^2}{S_2+\cos\alpha} \tag{3.16}$$

where

$$S_1 = \frac{2i}{k_o a} \sum_{n=o}^{\infty}{}' \frac{(-)^n}{\alpha_n^2+k_x^2} \frac{k_x^2(k_o^2\epsilon^t-\alpha_n^2-k_x^2)+k_o^2\alpha_n^2\epsilon^\ell-2i\pi\beta k_o^2\frac{\omega^2 k_x\alpha_n}{\alpha_n^2+k_x^2}}{\epsilon^\ell(k_o^2\epsilon^t-\alpha_n^2-k_x^2)+\pi^2\beta^2 k_o^2\frac{\omega^4}{(\alpha_n^2+k_x^2)^2}} , \qquad (3.17)$$

S_2= (same thing without the $(-)^n$) ,

and

$$\alpha_n = n\pi/a, \ \beta = -2\omega_p^2/\pi v_T^2\omega^2 \ \exp\left(-\frac{c^2}{v_T^2}\frac{1}{\sin^2\alpha}\right), \ \omega_p = \sqrt{e^2 n_o/\epsilon_o m} \ .$$

For $(v_T/c) \ll 1$ and $\omega \simeq \omega_p$,

$$S_1 \sim \frac{i}{1-\frac{\omega_p^2}{\omega^2}} \left\{ \frac{k_o \sin^2\alpha}{k^\ell \sin(k^\ell a)} + \frac{k^t}{k_o \sin(k^t a)} \right\}$$

$$S_2 \sim \frac{i}{1-\frac{\omega_p^2}{\omega^2}} \left\{ \frac{k_o \sin^2\alpha}{k^\ell} \cot(k^\ell a) + \frac{k^t}{k_o} \cot(k^t a) \right\} + \frac{2\omega_p^2}{\sqrt{\pi}\,\omega^2} \frac{v_T}{c} \sin^2\alpha \qquad (3.18)$$

where

$$k^\ell = \frac{\omega}{v_T}\left[\frac{2}{3}\left(\frac{\omega^2}{\omega_p^2}-1\right) - \frac{v_T^2}{c^2}\sin^2\alpha\right]^{1/2}$$

$$k^t = \frac{\omega}{c}\left[\left(1-\frac{\omega_p^2}{\omega^2}\right)\left(1-\frac{1}{2}\frac{\omega_p^2}{\omega^2}\frac{v_T^2}{c^2}\right) - \sin^2\alpha\right]^{1/2} \qquad (3.19)$$

The critical frequency occurs when $\omega^2/\omega_p^2 = 1 + \frac{3}{2}\frac{v_T^2}{c^2}\sin^2\alpha$, in which case S(0) is infinite and the plasma layer is opaque.

(U) In future work, the results should be extended to the case of a plasma slab backed by a conducting plane, separated from the plasma by a very thin veneer of ablative material. It may be important to consider the effect of temperature on surface waves. Later on effects of the velocity and temperature gradient in the sheath may have to be included.

TASK 4.0 SHORT PULSE INVESTIGATIONS

4.1 Introduction

(S) The overall aim of the program is to determine the conditions under which back scattering measurements of short incident pulses can be used effectively to discriminate between two objects whose cw back scattered radar cross sections are indistinquishable in significant frequency ranges. The basis for the investigation is the conjecture that while two different objects may have the same effective cw back scattered radar cross section, short incident pulses will give rise to different secondary scattered pulses which though small compared with the primary scattered pulse (hence completely lost in a cw return) are not only observable, but will be different in shape, spacing and/or number for the different objects. Thus a basis for discrimination will be provided. On the other hand, the basis for discrimination may be removable if the secondary pulses can be suppressed or if spurious secondary pulses can be generated.

(S) In order to answer the two basic questions: can the decomposition of the scattered field into separate pulses provide a means of practical discrimination between objects whose time harmonic scattered fields or radar cross sections are indistinguishable? and can these effects be masked by suitable changes in shape and constituent material? it is necessary to be able to calculate the pulse response of an object as a function of both pulse parameters (width and shape) and target parameters (shape and constituent material).

(S) The first concern is to develop a method for carrying out such calculations for targets of practical interest. This investigation is being carried out along two main lines. Since the pulse response may always be written in terms of the Fourier transform of the time harmonic scattered field, one line of attack involves calculating the pulse scattering characteristics of interest by taking the transform of the time harmonic response. The second line of investigation involves a direct attack on the time-dependent scattering problem. This, in the long run, may prove to be the more fruitful approach and will be discussed first.

4.2 Time Dependent Scattering

(U) There is a theory of propagation of discontinuities in EM fields which was developed by R.K. Luneberg. This is available in roughly the same form in books by Luneberg (1964) and Kline and Kay (1965). The subsequent development in Kline and Kay however, is along the lines that should be applicable to diffraction of pulses.

(U) The theory concerns weak solutions of Maxwell's equations (6 coupled hyperbolic differential equations). It is assumed that on a hypersurface $\phi(x, y, z, t) = 0$ the field vectors are discontinuous. The discontinuity may be due to an interface between media, or it may be the boundary of the domain in which a nonzero field exists when sources within a bounded domain are turned on at $t = 0$. When $\theta(x, y, z, t) = \psi(x, y, z) - ct = 0$ the ψ = constant are called wave fronts: their normal trajectories are called rays. In uniform media the rays are straight lines.

(U) The magnitude of the discontinuities in E and H and their derivative in the ray direction, say $\left[E^{(n)}\right]$ and $\left[H^{(x)}\right]$ are governed by ordinary differential equations along the rays, the so-called transport equations. These are recursive equations, the solution of $\left[E^{(n)}\right]$ requiring knowledge of $\left[E^{(n-1)}\right]$ for example. They are quite simple in homogeneous media. It is also simple to treat the effect of reflection of rays at an interface. The initial values are determined by the source behavior, and, for reflection, by Fresnel laws.

(U) The value of these $\left[E^{(n)}\right]$ for pulse solutions occurs as follows. If one considers a spatial point (x_o, y_o, z_o) at the time t_o at which it lies on a discontinuity front (wave front), then at a later time $t > t_o$ this point lies behind the wave front. If the field $E(x_o, y_o, x_o, t)$ is expanded in a Taylor's series about (x_o, y_o, z_o, t):

$$E(x_o, y_o, z_o, t) = E(x_o, y_o, z_o, t) + \left.\frac{\partial E(x_o, y_o, z_o, t)}{\partial t}\right]_{t=t_o^+} (t - t_o) + \ldots \qquad (4.1)$$

the coefficients are just $\frac{1}{n!}\left[E^{(n)}\right]$ where the $\left[E^{(n)}\right]$ are the jumps of the field and its derivatives on the wave front. One thus has the field expressed for a time after the passage of the wave front when these discontinuities are known as solutions of the transport equations. For a sufficiently short pulse of duration τ less than the radius of convergence of the series we then have a representation of the field.

(U) For the initial conditions for the $\left[E^{(n)}\right]$ one needs the corresponding derivatives at the source at $t=0$. This gives a Taylor series for the emitted pulse about $t=0$ and (4.1) probably does not converge for greater $(t-t_0)$ than the radius of convergence of this initial series which puts a limitation on the original pulse form we can treat in this fashion. We can, however, ameliorate this by using the linearity of the equations to build up more complicated pulses by superposition.

(U) So far we have mentioned only direct rays and reflected rays. The rays are determined by the requirement that their path $\int n ds$ be an extremum, i.e. Fermat's principle (n is refractive index). For diffraction by edges or smooth bodies (as for rays tangent to smooth bodies), suitable extensions of Fermat's principle are given by Keller's geometric theory of diffraction (Keller, 1958). For the smooth bodies, one has, corresponding to the creeping waves, both a "surface diffracted ray" which follows the surface and "shed diffracted" rays corresponding to energy shed into the shadow region.

(U) Just as in the time harmonic surface field investigations, by considering known surfaces as canonical problems which can be solved by other means, suitable generalization of the transport equation for the field discontinuities for such rays may be found. One major problem is to determine the initial values of $[E]$, $[H]$ at the points of origin of these rays. The justification for using canonical surfaces is that the initial values determined by purely local effects and similar factors will hold for other shaped bodies as found by fitting the canonical surface to the actual surface at the ray-launching point. The extent to which this program has been carried out and whether one has also transport equations for $\left[E^{(n)}\right]$ and $\left[H^{(n)}\right]$ $n > 0$ are being investigated. If it is found that one can use this Taylor series method for creeping wave contributions to pulses, with the theory as it now stands, it will be used to calculate the pulse response from a cone-sphere.

(U) This is essentially an analytic approach; however, the applicability of numerical methods to the solution of the transport equation will also be examined.

4.3 Time Harmonic Scattering

(U) In contrast to this attack on the pulse scattering problem, there is a method for proceeding to the time dependent scattering problem from the time harmonic case via a Fourier transform (see Appendix A). Actually, there are two ways of constructing the scattered pulse due to an arbitrary incident pulse. We may write

$$\mathcal{E}^s = \int_{-\infty}^{\infty} E^s(\omega)F(\omega)e^{-i\omega t}\,d\omega \qquad (4\text{-}2)$$

where $\mathcal{E}^s$ denotes back scattered pulse, $F(\omega)$ is the spectral function of the incident pulse and $E^s(\omega)$ is the back scattered field for an incident plane wave. Clearly if $F(\omega)$ is effectively band limited, it is only necessary to know $E^s(\omega)$ over a limited frequency range in order to calculate $\mathcal{E}^s$. If, in addition, this frequency range is one in which the time harmonic field may be found asymptotically, the exact field need never be determined. This equation will be used to determine the scattered pulse, first for a sphere then for a cone-sphere and other shapes of interest where the requisite time harmonic response is available, either from the SURF program or elsewhere. The sphere calculation will be carried out using the exact, approximate and experimental time harmonic field to check both the validity of the use of the approximate form of the cw scattered field and the extent to which cw experimental data may be used to obtain the short pulse response. (The transforms of the amplitude and phase of the scattered field will also be computed separately as a test of their information content in the transient case.)

(U) The pulse response may also be calculated using the expression

$$\mathcal{E}^s = \int_0^T \mathcal{E}(t-\tau)\,\mathfrak{F}^i(\tau)\,d\tau \qquad (4\text{-}3)$$

where $\mathcal{E}$ represents the response to a delta function source, i.e., the impulse response, and $\mathfrak{F}^i(\tau)$ is the incident pulse form. In general the impulse response is a difficult quantitiy to calculate, since it requires a knowledge of the time harmonic response at all frequencies viz.

$$\mathcal{E} = \frac{1}{2\pi} \int_{-\infty}^{\infty} E^{s}(\omega) e^{-i\omega t} d\omega \quad . \tag{4.4}$$

However, in the case of the sphere $E^{s}(\omega)$ is known for all frequencies, thus the impulse response may be found and, with equation (4.3), the response for any incident pulse may be found. Such a calculation not only provides a check of the equivalent result found with (4.2), but also provides a means of determining the variation in scattered pulse with changes in incident pulse. The impulse response is therefore being calculated and will then be used to calculate the response from a limited but realistic range of incident pulse shapes. Barring computational difficulties (it is not intended that this calculation become a major task), the response from square, triangular, cosine, cosine squared, and cosine cubed pulses, each for a range of two to eight carrier cycles, will be found for center frequencies in X, C, S and L-bands for spheres of radius .2 to 2 meters. The relevance to this task of the impulse approximation developed, by Kennaugh and co-workers (e.g. Kennaugh and Cosgriff, 1958) is being investigated.

(U) Understood in all these investigations is the fact that we are dealing with far zone fields. In calculating the impulse response, however, the range of integration includes zero frequency where any far field approximation is invalid. That this introduces no serious error is indicated, though admittedly not proven, in Appendix B.

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APPENDIX A

ON THE TRANSFORMATION FROM TIME HARMONIC TO TRANSIENT SCATTERING

(U) In this Appendix we review the procedure where by the time harmonic field scattered by a general body when illuminated by a plane wave may be transformed into the response of the same body to an arbitrary incident pulse.

(U) We denote functions in the frequency domain by capital letters and functions in the time domain with script.

(U) Fourier transform pairs are then

$$\mathcal{E}(t) = \int_{-\infty}^{\infty} E(\omega)\, e^{-i\omega t}\, d\omega \tag{A.1}$$

$$E(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathcal{E}(t)\, e^{+i\omega t}\, dt \tag{A.2}$$

and the spectral representation of the delta function is:

$$\delta(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t} d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\omega t}\, d\omega . \tag{A.3}$$

Let $\underline{E} = \underline{E}^{i}(\overline{r}, \omega) + \underline{E}^{s}(\overline{r}, \omega)$, $\omega \geqslant 0$, be the total electric field when an incident field, $\underline{E}^{i}$, is scattered by some body with harmonic time dependence $e^{-i\omega t}$. Extend the definition of $\underline{E}$ over all ω with the equation

$$\underline{E}(\overline{r}, -\omega) = \underline{E}^{*}(\overline{r}, \omega)$$

where * denotes complex conjugate.

If $\underline{E}^i(\bar{r},\omega)$ is a plane wave

$$\underline{E}^i(\bar{r},\omega) = \hat{a}\, e^{ik\hat{\alpha}\cdot\bar{r}} = \hat{a}\, e^{i\omega\frac{\hat{\alpha}\cdot\bar{r}}{c}} \qquad (A.4)$$

then

$$\mathcal{E}^i(\bar{r},t) = \hat{a}\int_{-\infty}^{\infty} e^{i\omega(\frac{\hat{\alpha}\cdot\bar{r}}{c}-t)}\, d\omega = 2\pi\hat{a}\,\delta\left(\frac{\hat{\alpha}\cdot\bar{r}}{c}-t\right) \quad . \qquad (A.5)$$

The scattered pulse, due to this δ function pulse is then

$$\mathcal{E}^s(\bar{r},t) = \int_{-\infty}^{\infty} \underline{E}^s(\bar{r},\omega)\, e^{-i\omega t}\, d\omega \quad . \qquad (A.6)$$

Let $\underline{\mathcal{F}}^i(t-\frac{\hat{\alpha}\cdot\bar{r}}{c})$ be a plane pulse of arbitrary shape (along rays normal to the plane of propagation) and width T ($\bar{r}$ is a radius vector to a field point, t is time, $\hat{\alpha}$ is the direction of propagation, and c is the velocity of propagation)

$$\underline{\mathcal{F}}^i(x) = 0 \qquad x > x_2, \qquad x < x_1, \qquad x_2 - x_1 = T \quad .$$

We have the identities

$$\underline{\mathcal{F}}^i(x) = \int_{-\infty}^{\infty} \underline{\mathcal{F}}^i(y)\,\delta(y-x)\,dy = \int_{-\infty}^{\infty} \underline{\mathcal{F}}^i(y)\,\delta(x-y)\,dy\,, \qquad (A.7)$$

then

$$\underline{\mathcal{F}}^i\left(t-\frac{\hat{\alpha}\cdot\bar{r}}{c}\right) = \int_{-\infty}^{\infty} \underline{\mathcal{F}}^i(y)\,\delta\left(y-t+\frac{\hat{\alpha}\cdot\bar{r}}{c}\right)dy \quad . \qquad (A.8)$$

(U) If $\underline{\mathcal{J}}^{i}(x) = \hat{a}\, \mathcal{J}^{i}(x)$, $\hat{a}$ constant and $\mathcal{J}$ a scalar function, then

$$\hat{a}\ \mathcal{J}^{i}\ (t-\frac{\hat{\alpha}\cdot\overline{r}}{c}) = \int_{-\infty}^{\infty} {}^{i}(y)\ \hat{a}\,\delta\ (y-t+\frac{\hat{\alpha}\cdot\overline{r}}{c})\, dy \qquad (A.9)$$

or, with (3)

$$\hat{a}\ \mathcal{J}^{i}\ (t-\frac{\hat{\alpha}\cdot\overline{r}}{c}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathcal{J}^{i}(y)\hat{a}\int_{-\infty}^{\infty} e^{i\omega(y-t+\frac{\hat{\alpha}\cdot\overline{r}}{c})}\, d\omega\, dy\ . \qquad (A.10)$$

If interchange of integration may be justified, then

$$\hat{a}\ \mathcal{J}^{i}\ (t-\frac{\hat{\alpha}\cdot\overline{r}}{c}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{a}\, e^{i\omega(\frac{\hat{\alpha}\cdot\overline{r}}{c}-t)}\int_{-\infty}^{\infty} \mathcal{J}^{i}(y)\, e^{i\omega y}\, dy\, d\omega. \qquad (A.11)$$

If we denote by F^{i} the inverse Fourier transform of $\mathcal{J}^{i}$ (in keeping with our convention)

$$F^{i}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathcal{J}^{i}(y)\, e^{i\omega y}\, dy\ . \qquad (A.12)$$

(F^{i} is the spectral function of $\mathcal{J}^{i}$).

then

$$\hat{a}\ \mathcal{J}^{i}(t-\frac{\hat{\alpha}\cdot\overline{r}}{c}) = \int_{-\infty}^{\infty} \hat{a}\, e^{i\omega(\frac{\hat{\alpha}\cdot\overline{r}}{c}+t)} F^{i}(\omega)\, d\omega = \int_{-\infty}^{\infty} \underline{E}^{i}(\overline{r},\omega)\, F^{i}(\omega)\, e^{-i\omega t}\, d\omega. \qquad (A.13)$$

The scattered pulse due to an incident pulse $\hat{a}\ \mathcal{J}^{i}(t-\frac{\hat{\alpha}\cdot\overline{r}}{c})$ is given similarly as

$$\underline{\mathcal{J}}^{s}(\overline{r},t) = \int_{-\infty}^{\infty} \underline{E}^{s}(\overline{r},\omega)\, F^{i}(\omega)\, e^{-i\omega t}\, d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \underline{E}^{s}(\overline{r},\omega)\, \mathcal{J}^{i}(y)\, e^{i\omega(y-t)}\, dy\, d\omega \qquad (A.14)$$

$$\mathfrak{J}^{s}(\overline{r},t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \underline{E}^{s}(\overline{r},\omega)\ \mathfrak{J}^{i}(-y)\,e^{-i\omega(y+t)}\,dy\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \underline{E}^{s}(\overline{r},\omega)\ \mathfrak{J}^{i}(t-y)\ e^{-i\omega y}\,dy\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \underline{\mathcal{E}}^{s}(\overline{r},y)\ \mathfrak{J}^{i}(t-y)\,dy \qquad \text{(A.15)}$$

where $\underline{\mathcal{E}}^{s}(\overline{r},y)$ is the scattered pulse due to an incident pulse $2\pi\, a\delta\left(\frac{\hat{\alpha}\cdot\overline{r}}{c} - y\right)$.

(U) Furthermore

$$\mathfrak{J}^{i}(x) = 0 \qquad x > x_2, \qquad x < x_1 \qquad \text{(A.16)}$$

thus

$$\mathfrak{J}^{i}(t-y) = 0 \quad \text{for } t-y > x_2 \text{ or } y < t - x_2$$
$$\text{and } t-y < x_1 \text{ or } y > t-x_1$$

and

$$\underline{\mathfrak{J}}^{s}(\overline{r},t) = \frac{1}{2\pi}\int_{t-x_2}^{t-x_1} \underline{\mathcal{E}}^{s}(\overline{r},y)\ \mathfrak{J}^{i}(t-y)\,dy\ . \qquad \text{(A.17)}$$

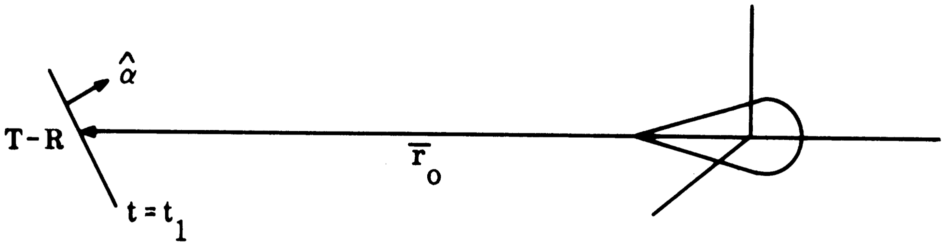
This may be rewritten as

$$\underline{\mathfrak{J}}^{s}(\overline{r},t) = \frac{1}{2\pi}\int_{-x_2}^{-x_1} \underline{\mathcal{E}}^{s}(\overline{r},y+t)\ \mathfrak{J}^{i}(-y)\,dy$$

$$= \frac{1}{2\pi}\int_{x_2}^{x_1} \underline{\mathcal{E}}^{s}(\overline{r},t-y)\,\mathfrak{J}^{i}(y)\,dy$$

$$= \frac{1}{2\pi}\int_{x_1}^{x_2} \underline{\mathcal{E}}^{s}(\overline{r},t-y)\ \mathfrak{J}^{i}(y)\,dy \qquad \text{(A.18)}$$

(U) If we erect a coordinate system in the target let $\bar{r}$ be a radial vector to a general field point, and $\bar{r}_o$ a radial vector to the transmitter-receiver.



Suppose a plane pulse is turned on at time t_1, propagating in the direction $\hat{\alpha}$ (if the transmitter is pointed at the target, $\hat{\alpha} = -\hat{r}_o$). The plane pulse includes the point $\bar{r}_o$ at t_1; the pulse front is the plane $(\bar{r}-\bar{r}_o)\cdot\hat{\alpha} = 0$. At time t_2 the pulse is zero a again in this plane Thus $\mathcal{J}^i(t-\frac{\hat{\alpha}\cdot\bar{r}}{c})$ is nonzero as the argument varies between

$$x_1 = t_1 - \frac{\hat{\alpha}\cdot\bar{r}_o}{c} < t - \frac{\hat{\alpha}\cdot\bar{r}}{c} < t_2 - \frac{\hat{\alpha}\cdot\bar{r}_o}{c} = x_2 \tag{A.19}$$

or

$$t_1 - \frac{\hat{\alpha}\cdot\bar{r}_o}{c} < t - \frac{\hat{\alpha}\cdot\bar{r}}{c} < t_1 + T - \frac{\hat{\alpha}\cdot\bar{r}_o}{c}$$

hence

$$\underline{\mathcal{J}}^s(\bar{r},t) = \frac{1}{2\pi}\int_{t_1-\frac{\hat{\alpha}\cdot\bar{r}_o}{c}}^{t_1-\frac{\hat{\alpha}\cdot\bar{r}_o}{c}+T} \underline{\mathcal{E}}^s(\bar{r},t-y)\,\mathcal{J}^i(y)\,dy \tag{A.20}$$

This can be written

$$\underline{\mathcal{J}}^s(\bar{r},t) \doteq \frac{1}{2\pi}\int_0^T \underline{\mathcal{E}}^s\left(\bar{r}, t-t_1+\frac{\hat{\alpha}\cdot\bar{r}_o}{c}-y\right)\mathcal{J}^i\left(y+t_1-\frac{\hat{\alpha}\cdot\bar{r}_o}{c}\right)dy\,. \tag{A.21}$$

(U) Since t_1 is apparently arbitrary it seems convenient to choose

$$t_1 = \frac{\hat{\alpha} \cdot \bar{r}_o}{c} , \qquad \text{i.e.} \qquad t \geqslant \frac{\hat{\alpha} \cdot \bar{r}_o}{c} .$$

Then

$$\mathcal{J}^s(\bar{r}, t) = \frac{1}{2\pi} \int_0^T \underline{\mathcal{E}}^s(\bar{r}, t-y) \; \mathcal{J}^i(y)\, dy$$

$$= \frac{1}{2\pi} \int_{-t}^{T-t} \underline{\mathcal{E}}^s(\bar{r}, -y) \; \mathcal{J}^i(y+t)\, dy$$

$$= \frac{1}{2\pi} \int_{t-T}^{t} \underline{\mathcal{E}}^s(\bar{r}, y) \; \mathcal{J}^i(t-y)\, dy . \qquad \text{(A.22)}$$

APPENDIX B

ON THE USE OF FAR ZONE APPROXIMATIONS AT LOW FREQUENCIES

(U) The pulse response from an object may be obtained if the time harmonic response is known by integrating over all frequencies (i.e. a Fourier transform), that is

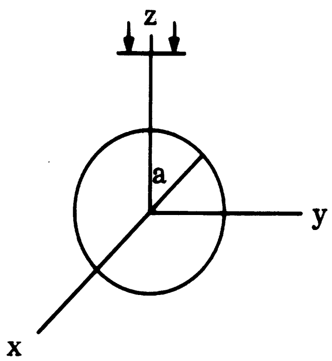
$$\mathcal{E}(\bar{r}, t) = \int_{-\infty}^{\infty} E(\bar{r}, \omega) e^{-i\omega t} d\omega . \tag{B.1}$$

Here $\mathcal{E}$ denotes the pulse response and E the steady state response. If the quantity of interest is the pulse response in the far zone, it is tempting to employ the far zone approximation of E in the Fourier transform. One notices, however, that the range of integration includes $\omega = 0$ at which point any approximation based on large $kr = \omega r/c$ is clearly inapplicable. How significant is the error incurred if one approximates the far zone pulse response with the Fourier transform of the far zone time harmonic response? In an attempt to gain some insight into this question, this appendix is devoted to a specific case, the back scattered far field response from a perfectly conducting sphere.

(U) For a plane wave of the form

$$\bar{E}^i = \hat{x} e^{-ikz-i\omega t} \tag{B.2}$$

incident on a perfectly conducting sphere of radius a (see figure), the exact back



scattered field is*

$$E^s = -\frac{\hat{x}}{2}\sum_{n=1}^{\infty}(-i)^n(2n+1)\left\{\left[\frac{j_n(ka)}{h_n(ka)}h_n(kr)+i\frac{\left[ka\,j_n(ka)\right]'}{\left[ka\,h_n(ka)\right]'}\frac{\left[kr\,h_n(kr)\right]'}{kr}\right]\right\} \tag{B.3}$$

where h_n is a spherical Hankel function of the first kind and primes denote differentiation with respect to the argument (either ka or kr).

(U) Making use of relation

$$E^*(\bar{r},\omega) = E(\bar{r},-\omega)$$

where * denotes complex conjugate and the fact that $\omega = kc$, we may rewrite (B.1) as

$$\mathcal{E}(r,t) = 2c\,\mathrm{Re}\int_0^{\infty} E^s(r,k)\,e^{-ikct}\,dk \tag{B.4}$$

* "Studies in Radar Cross Sections-XLVII: Diffraction and Scattering by Regular Bodies-I: The Sphere," R. F. Goodrich, B. A. Harrison, R. E. Kleinman and T. B. A. Senior, University of Michigan Radiation Laboratory Report 3648-1-T, AD 273006 UNCLASSIFIED December 1961

where $E^s(r,k)$ is given in Eq. (B.3).

(U) The far field form of the time harmonic scattered field is

$$E^s_\infty = \frac{\hat{x}}{2}\frac{i e^{ikr}}{kr}\sum_{n=1}^{\infty}(-1)^n(2n+1)\left\{\frac{j_n(ka)}{h_n(ka)} - \frac{\left[ka\, j_n(ka)\right]'}{\left[ka\, h_n(ka)\right]'}\right\} . \qquad (B.5)$$

This expression is presumably valid for large kr, and is certainly not applicable in a neighborhood of $k=0$. The question at issue is whether an appreciable error is made if the scattered pulse in the "far field"* is approximated by employing Eq. (B.5) (the time harmonic far field) rather than (B.3) (the exact far field) in the integral (B.4) which included $k=0$ in the range of integration.

(U) This question is apparently a difficult one and rather surprisingly unexplored (anything concerning scattering by a sphere that has not yet been done seems surprising). Indeed, to answer the question properly it appears that one must carefully estimate the number of terms required to obtain desired accuracy in the exact expression (B.3) for the desired values of a and r, then show that this accuracy is not impaired if the $h_n(kr)$ are replaced by their far field form. This is not a trivial task and will not be attempted here.

(U) Rather we shall obtain a much weaker result though one which hints at the plausibility of the approximation

$$\mathcal{E}_\infty = 2c\,\mathrm{Re}\int_0^\infty E^s_\infty\, e^{-ikct}\, dk \qquad (B.6)$$

for many cases of interest. [Equation (B.6) defines $\mathcal{E}_\infty$.]

*The far field in the time domain will be characterized by $r \gg a$ (rather than $kr \gg ka$ or $kr \gg 1$) or terms of order $1/r^2$ negligible with respect to terms of order $1/r$.

(U) It may be shown that the limiting forms of the two expressions for the back scattered field, (B.3) and (B.5) are

$$\lim_{k \to 0} E^s_\infty = 0 \qquad \text{(B.7)}$$

and

$$\lim_{k \to 0} E^s = -\hat{x}(a/r)^3 \quad . \qquad \text{(B.8)}$$

(U) If $a/r \leqslant 10^{-4}$ which means that one is looking at a sphere of 1 meter radius or less at a range of 10 kilometers or more (fairly representative of some practical situations) we find

$$\left|E^s - E^s_\infty\right| \leqslant 10^{-12} \left|E^i\right| \quad .$$

Furthermore for $k > .01/a$, which is rather small for spheres of radius in the 1 meter range, kr has already grown to exceed 10^2 and lends confidence to (though does not rigorously justify) the use of the far field approximation over most of the range of integration. Even in the narrow range of least validity, $0 \leqslant k \leqslant .01/a$, we know that at the left hand end point the error is of the order $10^{-12}\left|E^i\right|$.

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13. ABSTRACT

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This is the First Quarterly Report on Contract F 04694-67-C-0055 and covers the period 18 December 1966 to 18 March 1967. The report discusses work in progress on Project SURF and on a related short pulse investigation. The SURF program is a continuing determination of the radar cross section of metallic cone-sphere shaped re-entry bodies and the effect on radar cross section of absorber coatings, antenna and rocket nozzle perturbations, changing the shape of the rear spherical cap, and of the re-entry plasma environment. The objective of the short pulse investigation is the determination of methods of modifying the short pulse signature of cone-sphere shaped re-entry bodies and decoys.

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