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SOME FACETS OF SURFACE WAVE BEHAVIOR

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FOREWORD

This report (BSD-TR-67-142) was prepared by the Radiation Laboratory of the Department of Electrical Engineering of The University of Michigan under the direction of Dr. Raymond F. Goodrich, Principal Investigator and Burton A. Harrison, Contract Manager. The work was performed under Contract F 04694-67-C-0055, "Investigation of Re-entry Vehicle Surface Fields (SURF)". The work was administered under the direction of the Air Force Ballistic Systems Division, Norton Air Force Base, California 92409, by Lieutenant J. Wheatley, BSYDF and was monitored by Mr. H. J. Katzman of the Aerospace Corporation.

The publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

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ABSTRACT

The surface waves that can be supported by various simple geometrical shapes are examined, and some of the properties that they have in common are pointed out.

TABLE OF CONTENTS

I
INTRODUCTION

During the last half century a considerable number of 'surface waves' have been discovered, and in spite of the many properties which they have in common there appears to have been little attempt to relate them to one another. Recent years in particular have seen a proliferation of such waves and in order to analyze the scattering behavior of relatively general bodies it is now customary to introduce such terms as Goubau waves, traveling waves, creeping waves, etc. Each of these originated in an analysis of the solution for a particular body, and whenever the general surface has the appropriate features (as regards curvature, for example) it is assumed that the corresponding wave will be excited, leading ultimately (or perhaps hopefully) to a complete description of the surface field. It is obvious that an approach which is as piecemeal as this is at best only an approximation, but equally important is the fact that since the relation between the various constituents is not known, the underlying unity of the physical field is lost. Under these circumstances it is difficult to estimate the extent to which a change in the radii of curvature at a point will lead to the reflection of one wave from the discontinuity, or to the excitation of another wave beyond the point.

It is the purpose of this report to examine anew the properties of these surface waves and to consider the general wave of which each can be regarded as a particular case. The general wave is of relatively simple form in spite of its all-enbracing character, and should prove of value in studying the scattering produced by highly conducting bodies. For the present, however, we shall consider only the qualitative behavior of the fields, and the solution of actual boundary value problems is reserved for a future report.

II
SURFACE WAVES

The term 'surface wave' is used here in the sense of a wave which is guided by a surface. It is usually characterized by an exponential attenuation in the direction normal to the surface which serves to confine the energy to the immediate vicinity of the body, and although in some cases the attenuation may only become apparent at large distances from the surface (so that the guiding is of a somewhat tenuous character), it is desirable to include such waves within the general category. Along the surface, however, the wave may propagate with or without attenuation depending upon the shape of the surface and the material of which it is composed.

As long ago as 1899 Sommerfeld pointed out that a straight cylindrical body of finite conductivity can act as a guide for electromagnetic waves. The idea attracted little attention for practical purposes until 1950 when Goubau investigated the problem anew and showed that a wave can be launched along the cylinder by taking the conductor to be the extension of a coaxial line whose outer surface is terminated in a flared horn. The launching efficiency is reasonably good and if the surface of the conductor is modified so as to appear reactive to the field, Goubau (1950, 1952) found that the attenuation could be reduced to an almost insignificant amount, leading to a relatively loss-free method of guiding electromagnetic energy. When the radius of the cylinder is so small that the conductor is merely a wire (radius $\ll$ wavelength) the resulting field has been called a Goubau wave, and we shall see later that this is equivalent to the 'traveling wave' which has been employed in aerial theory for some time, and which has recently been used (Peters, 1958) as a basis for calculating the radar scattering properties of long thin bodies at near nose-on incidence. When the radius is very large compared with the wavelength, the wave is directly analogous to the one which can be supported by a flat surface of non-zero impedance.

The propagation of electromagnetic waves over a flat surface has been studied for many years and because of the intrinsic importance of his problem, together with the fact that a flat surface is more amenable to rigorous analysis, the bulk of the literature on surface waves has originated in connection with this problem. The first significant contribution was provided by Zenneck in 1907, who showed that a wave which is a solution of Maxwell's equations could propagate over a surface of non-zero impedance without change of form, and if the surface is dissipative the phase velocity exceeds that of light. Such a wave is produced by the incidence of a plane wave on the surface at the Brewster angle (corresponding to zero reflection), and suffers only slight attenuation in the direction of propagation, but is rapidly attenuated in the direction normal to the surface. The wave was originally thought to be the vehicle by means of which a field is propagated over the earth's surface at large distances from the transmitter, and that this is not so only became apparent at a much later date.

The more practical problem of a vertical dipole over an homogeneous ground was solved by Sommerfeld in 1909, and to bring out the physical significance of the solution he divided the expression for the field into a 'space wave' and a 'surface wave'. The latter varies inversely as the square root of the range and was immediately identified as the radial counterpart of the Zenneck wave. Although both parts of the solution were necessary to satisfy the conditions of the problem, the surface wave was the dominant contribution near to the surface of the ground.

A further twenty-five years were to elapse before it was discovered that Sommerfeld's solution was incorrect. As a result of the increasingly obvious discrepancy between the measured values of the field and those calculated using Sommerfeld's formula, Norton (1935) was led to re-examine the derivation, and found an error in sign arising from a choice of square root incompatible with the branch cuts used in the analysis (see Niessen (1937)). The correction served to remove the surface wave which was present in the solution, and it is now accepted that the Sommerfeld surface wave (or the radial Zenneck wave) does not appear explicitly in the field of a vertical dipole over a conducting earth. On the other hand, the true field near to the surface can in fact be interpreted as a Sommerfeld wave diffracted under a screen extending down to the image of the transmitter in the earth (see Booker and Clemmow (1950)), so that the surface wave does play a part, albeit a concealed one.

With all the above surface waves an important factor is the (complex) surface impedance, and if this is zero the waves cannot, in general, be supported. In addition, the properties of the waves which are present are governed by the phase of the impedance, and a change from a dielectric to a conductive surface (or a conductive to an inductive one) will vitally affect the nature of the waves. A full discussion of this in relation to the waves over a flat surface has been given by Wait (1957).

When analysing the radar scattering properties of metallic bodies it is commonly supposed that the surface can be treated as perfectly conducting, and thus for a long thin body it is assumed that the traveling wave will be excited even if the surface impedance is zero. This is clearly not so if the traveling wave is indeed a Goubau wave, and it is therefore of interest to examine the role of the surface impedance in this case.

There is also the problem posed by creeping waves, since these would appear to be supported by perfectly conducting structures and to have at least some of the characteristics of surface waves. If the radii of curvature are everywhere infinite, so that the body is merely a flat surface of infinite extent, a surface wave is possible only if the surface impedance η is non-zero, and the wave is then generated by the incidence of a plane wave at the Brewster angle*. The properties of such waves are reviewed in Chapter III, and it is found that they can be supported no matter how small η is providing it is not zero. However, as the impedance approaches zero, so does the Brewster angle, hereby increasing the difficulty of launching.

If one of the radii of curvature is infinite, but the other finite, a surface wave is again possible if $\eta \neq 0$. The wave can be launched by a plane wave incident at the Brewster angle, and travels along the generators of the cylinder with a small amount of attenuation in this direction, but no decay in the circumferential direction. On the other hand, if a field is incident in the plane perpendicular to the axis of the cylinder (or on a body both of whose radii are finite), it would seem that even with a perfectly conducting surface a wave is launched at the shadow boundary and progresses around the body with a form of decay (both radially and circumferentially) which is similar to that for a surface wave. In the absence of absorption it is apparent that the loss of energy must occur due to radiation (rather than dissipation, as in the normal surface wave), but this apart, is it possible that the curvature in the direction of propagation is mathematically equivalent to a surface impedance? To answer this question is one of the aims of the report.

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*If the Brewster angle is complex, a complete spectrum of plane waves may be necessary for the launching in practice.

III
PLANE SURFACE WAVES

It is convenient to begin by considering the surface waves which can be supported by a flat surface of infinite extent the impedance of which is not zero. This is one of the more basic problems in electromagnetic theory and various aspects of it have received quite extensive treatment in the literature. The present discussion, therefore, is only in the nature of a survey.

To achieve the generality which we shall later require, the surface is assumed to be anisotropic and to have constant impedances η_1 and η_2 in the x and y directions respectively, where x and y are Cartesian coordinates in the plane of the sheet. The physical interpretation to be attached to the two impedances will be made clear later, but for the moment we remark only that at the surface $z = 0$ the boundary conditions which are assumed are

$$E_x = -\eta_1 Z H_y \tag{3.1}$$

$$E_y = \eta_2 Z H_x \tag{3.2}$$

(Senior, (1960)), where Z is the intrinsic impedance of free space.

To determine the type of surface waves which can exist it is sufficient to consider a plane wave incident upon the sheet and deduce the surface waves from the condition that the reflected field be zero. Since any three-dimensional plane wave can be broken up into two quasi three dimensional fields each of which is obtainable from a single-component Hertz vector, it is natural to carry out the analysis in terms of these, and in view of the boundary conditions which have to be satisfied, it is convenient to take

$$\underline{E}^{(1)} = (0, \gamma, -\beta)\, e^{ik(\alpha x + \beta y + \gamma z)} \, , \tag{3.2a}$$

$$Z\,\underline{H}^{(1)} = (\alpha^2 - 1, \alpha\beta, \alpha\gamma)\, e^{ik(\alpha x + \beta y + \gamma z)} \, , \tag{3.2b}$$

$$\underline{E}^{(2)} = (-\gamma, 0, \alpha)\, e^{ik(\alpha x + \beta y + \gamma z)} \, , \tag{3.3a}$$

$$Z\,\underline{H}^{(2)} = (\alpha\beta, \beta^2 - 1, \beta\gamma)\, e^{ik(\alpha x + \beta y + \gamma z)} \, , \tag{3.3b}$$

where $\alpha^2 + \beta^2 + \gamma^2 = 1$ and the time dependence is $e^{-i\omega t}$. Because of the interpretation of the above as incident fields, γ will be regarded as having negative real part.

If both η_1 and η_2 are zero, the only possible expressions for the total electric field are

$$\underline{E}^{(1)}(\alpha, \beta, \gamma) + \underline{E}^{(1)}(\alpha, \beta, -\gamma) \tag{3.4a}$$

and

$$\underline{E}^{(2)}(\alpha, \beta, \gamma) + \underline{E}^{(2)}(\alpha, \beta, -\gamma) \quad . \tag{3.4b}$$

Each of these independently satisfies the conditions of the problem, and in each of them the second term represents a reflected wave. Since this is present whatever the values of α, β and γ, it follows immediately that a perfectly conducting sheet cannot support a surface wave.

If $\eta_1 = 0$ but $\eta_2 \neq 0$ the possible solutions are

$$\underline{E}^{(1)}(\alpha, \beta, \gamma) + \frac{\gamma + \eta_2(1-\alpha^2)}{\gamma - \eta_2(1-\alpha^2)} \underline{E}^{(1)}(\alpha, \beta, -\gamma) \quad , \tag{3.5a}$$

$$\underline{E}^{(2)}(0, \beta, \gamma) + \underline{E}^{(2)}(0, \beta, -\gamma) \quad , \tag{3.5b}$$

$$\underline{E}^{(2)}(\alpha, 0, \gamma) + \underline{E}^{(2)}(\alpha, 0, -\gamma) \quad , \tag{3.5c}$$

together with the 'coupled' solutions

$$\underline{E}^{(1)}(\alpha, \beta, 0) + \frac{1-\alpha^2}{\alpha\beta} \underline{E}^{(2)}(\alpha, \beta, 0) \tag{3.5d}$$

$$\underline{E}^{(1)}(\alpha, \beta, \gamma) + \frac{\gamma + \eta_2(1-\alpha^2)}{2\alpha\beta\eta_2} \left\{ \underline{E}^{(2)}(\alpha, \beta, \gamma) + \underline{E}^{(2)}(\alpha, \beta, -\gamma) \right\} . \tag{3.5e}$$

Of these, (3.5b) and (3.5c) are analogous to (3.4a) and (3.4b) in that the surface appears perfectly conducting, and (3.5d) reduces to the null field when the reflection coefficient is put equal to zero. This leaves only (3.5a) and (3.5e) and here the reflected fields can be removed by taking

$$\gamma = -\eta_2 (1 - \alpha^2) \quad .$$

The resulting field is simply

$$\underline{E}^{(1)}(\alpha, \beta, -\eta_2[1-\alpha^2])$$

which of its own accord satisfies all the conditions of the problem. Since $\alpha^2 + \beta^2 + \gamma^2 = 1$, we have

$$\beta = \pm (1-\alpha^2)\sqrt{\frac{1}{1-\alpha^2} - \eta_2^2}$$

and the field components* are

$$\underline{E} = (0, \eta_2, \mp\sqrt{\frac{1}{1-\alpha^2} - \eta_2^2}) \exp\left[ik\left\{\alpha x \pm (1-\alpha^2)\, y\sqrt{\frac{1}{1-\alpha^2} - \eta_2^2} - \eta_2(1-\alpha^2)\, z\right\}\right] \tag{3.6a}$$

$$Z\underline{H} = (1, \mp \alpha\sqrt{\frac{1}{1-\alpha^2} - \eta_2^2}, \alpha\,\eta_2) \exp\left[ik\left\{\alpha x \pm (1-\alpha^2)\, y\sqrt{\frac{1}{1-\alpha^2} - \eta_2^2} - \eta_2(1-\alpha^2)\, z\right\}\right] . \tag{3.6b}$$

This field is entirely self supporting and providing $\mathrm{Im}\ \eta_2 < 0$ it has all the characteristics of a surface wave.

If $\eta_2 = 0$ but $\eta_1 \neq 0$ the solutions of the boundary value problem are similarly

$$\underline{E}^{(2)}(\alpha, \beta, \gamma) + \frac{\gamma + \eta_1(1-\beta^2)}{\gamma - \eta_1(1-\beta^2)}\, \underline{E}^{(2)}(\alpha, \beta, -\gamma) \tag{3.7a}$$

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* For convenience an amplitude factor $\alpha^2 - 1$ is omitted

$$\underline{E}^{(1)}(0, \beta, \gamma) + \underline{E}^{(1)}(0, \beta, -\gamma) \tag{3.7b}$$

$$\underline{E}^{(1)}(\alpha, 0, \gamma) + \underline{E}^{(1)}(\alpha, 0, -\gamma) \tag{3.7c}$$

$$\underline{E}^{(2)}(\alpha, \beta, 0) + \frac{1-\beta^2}{\alpha\beta}\,\underline{E}^{(1)}(\alpha, \beta, 0) \tag{3.7d}$$

$$\underline{E}^{(2)}(\alpha, \beta, \gamma) + \frac{\gamma + \eta_1(1-\beta^2)}{2\alpha\,\beta\,\eta_1}\left\{\underline{E}^{(1)}(\alpha, \beta, \gamma) + \underline{E}^{(1)}(\alpha, \beta, -\gamma)\right\} \tag{3.7e}$$

(cf Eqs. (3.5a) through (3.5e)), and the only case in which a surface wave can be excited is that in which

$$\gamma = -\eta_1(1-\beta^2) \quad ,$$

giving

$$\alpha = \pm (1-\beta^2)\sqrt{\frac{1}{1-\beta^2} - \eta_1^2} \quad .$$

The resulting field (of type '2') is

$$\underline{E} = \left(\eta_1, 0, \pm\sqrt{\frac{1}{1-\beta^2} - \eta_1^2}\right)\exp\left[ik\left\{\pm(1-\beta^2)\,x\sqrt{\frac{1}{1-\beta^2} - \eta_1^2} + \beta y - \eta_1(1-\beta^2)z\right\}\right] \tag{3.8a}$$

$$Z\underline{H} = \left(\pm\beta\sqrt{\frac{1}{1-\beta^2} - \eta_1^2}, -1, -\beta\eta_1\right)\exp\left[ik\left\{\pm(1-\beta^2)\,x\sqrt{\frac{1}{1-\beta^2} - \eta_1^2} + \beta y - \eta_1(1-\beta^2)z\right\}\right]. \tag{3.8b}$$

This is also self supporting and has the characteristics of a surface wave providing Im $\eta_1 < 0$. Observe that whereas the field (3.6) propagates in the x direction unaffected by the impedance but is attenuated in the y direction, the reverse is true of the field (3.8), and this is in accordance with the interpretation of η_1 and η_2 as impedances associated with the x and y directions respectively.

For the general case in which both η_1 and η_2 are non-zero, the solutions are

$$\underline{E}^{(1)}(0,\beta,\gamma) + \frac{\gamma+\eta_2}{\gamma-\eta_2}\,\underline{E}^{(1)}(0,\beta,-\gamma) \tag{3.9a}$$

$$\underline{E}^{(1)}(\alpha,0,\gamma) + \frac{1+\gamma\eta_2}{1-\gamma\eta_2}\,\underline{E}^{(1)}(\alpha,0,-\gamma) \tag{3.9b}$$

$$\underline{E}^{(2)}(0,\beta,\gamma) + \frac{1+\gamma\eta_1}{1-\gamma\eta_1}\,\underline{E}^{(2)}(0,\beta,-\gamma) \tag{3.9c}$$

$$\underline{E}^{(2)}(\alpha,0,\gamma) + \frac{\gamma+\eta_1}{\gamma-\eta_1}\,\underline{E}^{(2)}(\alpha,0,-\gamma) \tag{3.9d}$$

$$\underline{E}^{(1)}(\alpha,\beta,\gamma) + \frac{\gamma+\eta_2(1-\alpha^2)}{2\alpha\beta\eta_2}\left\{\underline{E}^{(2)}(\alpha,\beta,\gamma) + \underline{E}^{(2)}(\alpha,\beta,-\gamma)\right\}$$

$$- \frac{\eta_1(\eta_2\gamma+1-\beta^2)}{2\alpha\beta\eta_2}\left\{\underline{E}^{(2)}(\alpha,\beta,\gamma) - \underline{E}^{(2)}(\alpha,\beta,-\gamma)\right\} \tag{3.9e}$$

$$\underline{E}^{(2)}(\alpha, \beta, \gamma) + \frac{\gamma + \eta_1(1-\beta^2)}{2\alpha\beta\eta_1}\left\{\underline{E}^{(1)}(\alpha, \beta, \gamma) + \underline{E}^{(1)}(\alpha, \beta, -\gamma)\right\}$$

$$- \frac{\eta_2(\eta_1\gamma + 1-\alpha^2)}{2\alpha\beta\eta_1}\left\{\underline{E}^{(1)}(\alpha, \beta, \gamma) - \underline{E}^{(1)}(\alpha, \beta, -\gamma)\right\} . \qquad (3.9f)$$

The last two are valid if $\alpha\beta\eta_2 \neq 0$ and $\alpha\beta\eta_1 \neq 0$, but if either angle is zero the coupling disappears and the solutions revert to the forms shown in (3.9a) through (3.9d), whilst if $\eta_1 = 0$ or $\eta_2 = 0$ the solutions become identical to those listed previously.

In order that a surface wave be generated, it is necessary for the field to be incident at the angle corresponding to the zero of the reflection coefficient, and taking first the coupled solution (3.9e) the requirement is

$$\gamma + \eta_2(1-\alpha^2) = 0 \quad , \qquad (3.10a)$$

$$\eta_2\gamma + 1 - \beta^2 = 0 \quad , \qquad (3.10b)$$

implying

$$(\gamma + \eta_2)(\gamma + 1/\eta_2) = 0 \quad . \qquad (3.10c)$$

If $\gamma = -\eta_2$, then from (3.10a) and (3.10b),

$$\alpha = 0, \ \beta = \pm\sqrt{1-\eta_2^2}$$

and the condition is the same as that imposed by (3.9a). Similarly, if $\gamma = -1/\eta_2$, then $\alpha = \pm\sqrt{1 - 1/\eta_2^2}$, $\beta = 0$, which could otherwise have been obtained by taking (3.9b) and putting the reflection coefficient equal to zero. Consequently, as regards surface wave generation, the coupled solution (3.9e) is equivalent to (3.9a) and (3.9b), and (3.9f) is equivalent to (3.9c) and (3.9d). The basic surface wave solutions are therefore

$$\underline{E}^{(1)}\left(0,\ \pm\sqrt{1-\eta_2^2},\ -\eta_2\right) \tag{3.11a}$$

$$\underline{E}^{(1)}\left(\pm\sqrt{1-1/\eta_2^2},\ 0,\ -1/\eta_2\right) \tag{3.11b}$$

$$\underline{E}^{(2)}\left(0,\ \pm\sqrt{1-1/\eta_1^2},\ -1/\eta_1\right) \tag{3.11c}$$

$$\underline{E}^{(2)}\left(\pm\sqrt{1-\eta_1^2},\ 0,\ -\eta_1\right) \tag{3.11d}$$

and associated with these are the exponential factors

$$\exp\left[ik\left\{\pm y\sqrt{1-\eta_2^2} - \eta_2 z\right\}\right] \tag{3.12a}$$

$$\exp\left[ik\left\{\pm x\sqrt{1-1/\eta_2^2} - z/\eta_2\right\}\right] \tag{3.12b}$$

$$\exp\left[ik\left\{\pm y\sqrt{1-1/\eta_1^2} - z/\eta_1\right\}\right] \tag{3.12c}$$

$$\exp\left[ik\left\{\pm x\sqrt{1-\eta_1^2} - \eta_1 z\right\}\right] \tag{3.12d}$$

respectively.

It is clear that for complex η_1 and η_2 two of these waves are exponentially increasing in the positive z direction, and are therefore inadmissible both as surface waves and as solutions of a scattering problem. If η is the surface impedance for a material of permittivity ϵ, permeability μ and conductivity σ, then

$$\eta = \frac{1}{\sqrt{\frac{\mu_o}{\mu}\left(\frac{\epsilon}{\epsilon_o} + i\,\frac{\sigma}{\omega\epsilon_o}\right)}} \tag{3.13}$$

where ϵ_o and μ_o are the permittivity and permeability respectively of free space, and hence

$$\frac{-\pi}{4} \leq \arg\eta \leq 0$$

with the lower limit corresponding to a perfect conductor and the upper limit to a pure dielectric. If, on the other hand, the impedance is produced not by the material constants of the surface but by some form of loading (for example, by corrugations or by coating a perfect conductor with a dielectric) then

$$\frac{-\pi}{2} \leq \arg\eta \leq \frac{-\pi}{4} \quad ,$$

so that in general we may assume

$$\frac{-\pi}{2} \le \arg \eta_1 \,,\ \arg \eta_2 \le 0 \ . \qquad (3.14)$$

It now follows that of the waves listed in (3.11a) through (3.11d), only the first and fourth are allowable, the field components of which are

$$\underline{E} = (0,\ \eta_2,\ \pm\sqrt{1-\eta_2^2})\ \exp\left[ik\left\{\pm y\sqrt{1-\eta_2^2} - \eta_2 z\right\}\right] \ , \qquad (3.15a)$$

$$Z\underline{H} = (1,\ 0,\ 0)\ \exp\left[ik\left\{\pm y\sqrt{1-\eta_2^2} - \eta_2 z\right\}\right] \ ; \qquad (3.15b)$$

$$\underline{E} = (\eta_1,\ 0,\ \pm\sqrt{1-\eta_1^2})\ \exp\left[ik\left\{\pm x\sqrt{1-\eta_1^2} - \eta_1 z\right\}\right] \ , \qquad (3.16a)$$

$$Z\underline{H} = (0,\ -1,\ 0)\ \exp\left[ik\left\{\pm\sqrt{1-\eta_1^2} - \eta_1 z\right\}\right] \ . \qquad (3.16b)$$

These are the basic surface waves which can be supported by the anisotropic sheet, and it will be observed that (3.15) is the limit of (3.6) as $\alpha \longrightarrow 0$. If, therefore, the impedance η_1 is zero, a surface wave can be excited by a wave incident at any angle to the x axis, but if η_1 is not zero, the field must be incident at the Brewster angle for this impedance, thereby specifying the angle of incidence completely. Similarly, (3.16) is the limit of (3.8) as $\beta \longrightarrow 0$, and the fact that the number of conditions on α and β which must be satisfied for a surface wave to be generated is directly proportional to the number of the non-zero impedances is in accordance with the statement that η_1 and η_2 only affect the electric vectors in the x and y directions respectively, and can be chosen independently of one another.

A final remark is necessary concerning the exceptional case of a dielectric surface. The impedances η_1 and η_2 are then real so that all four of the fields listed in (3.11a) through (3.11d) are now possible. If η_1 and η_2 are less than unity, as will be true in most practical circumstances, the fields corresponding to the exponential factors (3.12b) and (3.12c) are rapidly attenuated in the direction parallel to the surface, but propagate in the normal direction. As such, the fields still do not qualify as surface waves, but even here the association of η_1 and η_2 with E_x and E_y respectively still stands.

IV
TRAVELING AND GOUBAU WAVES

The concept of a traveling wave has played an important role in antenna theory for many years and lies at the root of the standard treatment of radiation from long thin conductors. In practice, however, the wave is used only as a physical description of the means by which a current is produced whose amplitude is constant and whose phase varies linearly with distance along the antenna, and seldom is any consideration given to the field of which the current is merely a consequence. Providing the antenna is sufficiently long it is assumed that it can be represented by a cylindrical conductor whose radius and surface characteristics do not affect the current distribution to any significant extent, and from the agreement between theory and experiment it is apparent that this is a valid approximation for a number of antenna systems. Thus, a single-wire antenna terminated in its characteristic impedance must have essentially this type of current, as may also a terminated rhombic antenna or even a long helix. Nevertheless, in most textbooks on antenna theory (see, for example, Kraus (1950)) the current distribution is merely assumed, with little attempt to justify the choice and no discussion of the actual waves which the antenna can support. In short, a traveling wave is only a term applied to a particular current behavior , and one might conclude from this, for example, that the phase velocity of the 'wave' is determined by the nature of the excitation and not by the properties of the actual conductor.

In view of the similarity between problems in radiation and diffraction it is not surprising to find that the traveling wave picture also proves useful in analysing the scattering properties of long thin bodies illuminated near to nose-on, and perhaps the astonishing thing is that the application was not appreciated for so many years. Prior to 1956 it was believed that for "any pointed object of revolution that has a smooth surface at the geometrical shadow with all appendages well within the shadow, the dominant back scattering comes from the point" (Hansen and Schiff, (1948)), but in 1956 Peters showed that this could only be true when the contribution due to the traveling wave mode was negligible. A corrected version of the latter report was published in 1958.

By likening the body to a traveling wave antenna Peters was led to an expression for the back scattering cross section which is proportional to the squares of the gain, the wavelength, and the current (or voltage) reflection coefficient at the apparent location of the terminals of the antenna. If the traveling wave is assumed to be the same as for a long thin wire, the gain can be calculated in terms of the relative phase velocity, so that there are now three parameters present in the formula for the cross section: the phase velocity p, the reflection coefficient γ and the effective length L of the wire (which will differ from the actual length of the body only when the body departs significantly from a cylindrical shape). Unfortunately, it is not always easy to estimate the values which these should have. Taking first the parameter L there is the question of whether this is determined by the overall length of the body or by its surface length in the plane of the incident propagation vector and the axis, and a small variation can have a marked effect on the location of the nulls in the calculated scattering pattern as a function of aspect. For the relative phase velocity, it is customary to choose a value which will match the time of travel along the equivalent wire to that of the actual wave along the body, so giving an average matching between the corresponding phase planes. But even this depends on a knowledge of the current path (including the point of reflection), and will certainly be a function of the polarization and may well vary with aspect. Finally there is the reflection coefficient γ, and although it produces only a scaling in the overall scattering pattern, the estimation of this parameter is perhaps the most difficult of them all. It will clearly depend on the point at which the current is reflected (which will usually be taken as the 'location' of the antenna terminals), and may therefore vary with aspect angle. It will also be a function of the current path, but probably the most important factor is the curvature of the body at the actual point of reflection. Some idea of the magnitude of γ can be obtained from the experimental results of Peters (1958) and it would appear to range from unity for bodies which are so terminated (for example, in a disc or a baffle) as to prevent radiation from the rear, to a lower limit of about $1/3$ for bodies whose rear is pointed. In reality, however, γ will almost certainly be complex, and the phase will then modify the electrical path length and hence the chosen value for p.

In spite of these ambiguities (or perhaps because of them), traveling wave theory has proved remarkably effective in explaining the enhanced scattering from slender bodies at aspects near* to nose-on. Peters (1958) found that for a polyrod antenna of dimensions 6λ by (approximately) $\lambda/4$ terminated in a disc (so as to give $\gamma = 1$) the formula is in excellent agreement with experiment out to angles of about 30^o off nose-on. Similar agreement was obtained for a metallic rod 39λ by $\lambda/4$ with $\gamma = 0.32$ and $p = 1$, and here the value for the reflection coefficient was determined experimentally by comparing scattering patterns with and without a disc termination on the body. Even for an ogive of 30^o angle and length 39λ the agreement was quite good out to about 20^o if the phase velocity was taken as 0.99, although the measured reflection coefficient of 0.7 produced a calculated scattering cross section whose maxima were some 2 db higher than the experimental ones, and the theoretical nulls appeared only as shallow minima in practice. A further application of the theory (to a 10 to 1 prolate spheroid) is given in Siegel (1959).

*It should be noted that one of the shortcomings of the theory is its prediction of a null at nose-on incidence even for bodies of asymmetrical shape. This is a direct consequence of the assumption of a thin-wire current distribution, and rules out any enhancement of the scattering from the nose at this aspect.

Notwithstanding the uncertainties which are associated with Peters' theory, it has served to pinpoint a physical mechanism for producing a scattered field from long thin objects at near nose-on incidence. It has also focussed attention on the importance of guided waves in a common type of radar scattering problem, but it would seem that a basic study of such waves should throw some light on the true value of the phase velocity and the manner in which the wave (or the current) is reflected at places where the radius of curvature of the body changes sharply. To this end we shall now briefly consider the properties of waves which are guided along a cylindrical surface, and in the course of this show the identity of traveling and Goubau waves.

In terms of the cylindrical polar coordinates (r, θ, z) the cylinder is defined by the equation $r = a$ and at its surface the boundary conditions are taken as

$$E_\theta = -\eta Z H_z \tag{4.1a}$$

$$E_z = \eta Z H_\theta \tag{4.1b}$$

(see, for example, Senior (1960)), where η is the surface impedance. In seeking to satisfy the conditions it is convenient to consider two basic fields each of which is derived from a single-component Hertz vector. The first of these is obtained from the electric Hertz vector

$$\underline{\pi} = (0, 0, \pi_z) ,$$

with

$$\pi_z = H_m(hr) \exp\left\{ i\left(\pm z\sqrt{k^2 - h^2} + m\theta\right)\right\} , \tag{4.2}$$

and to provide the necessary continuity throughout the free space region m must be an integer ($m = 0, \pm 1, \pm 2, \ldots$). $H_m(hr)$ is the cylindrical Hankel function of the first or second kind depending on whether h has positive or negative imaginary part respectively, and the resulting field components are

$$\underline{E}^{(1)} = \left(\pm ih\sqrt{k^2-h^2}\,H'_m,\ \mp\frac{m}{r}\sqrt{k^2-h^2}\,H_m,\ h^2H_m\right)\exp\left\{i\left(\pm z\sqrt{k^2-h^2}+m\theta\right)\right\} \tag{4.3a}$$

$$Z\underline{H}^{(1)} = \left(\frac{mk}{r}H_m,\ ikhH'_m,\ 0\right)\exp\left\{i\left(\pm z\sqrt{k^2-h^2}+m\theta\right)\right\} \tag{4.3b}$$

where the prime denotes differentiation with respect to the argument hr. Similarly, from a magnetic Hertz vector whose only non-zero component is (4.2), we have

$$\underline{E}^{(2)} = \left(-\frac{mk}{r}H_m,\ -ikhH'_m,\ 0\right)\ \exp\left\{i\left(\pm z\sqrt{k^2-h^2}+m\theta\right)\right\} \tag{4.4a}$$

$$Z\underline{H}^{(2)} = \left(\pm ih\sqrt{k^2-h^2}\,H'_m,\ \mp\frac{m}{r}\sqrt{k^2-h^2}\,H_m,\ h^2H_m\right)\exp\left\{i\left(\pm z\sqrt{k^2-h^2}+m\theta\right)\right\} \tag{4.4b}$$

and the most general field can be expressed as a superposition of the above

If $m \neq 0$ and $\eta \neq 0$ a combination of $(\underline{E}^{(1)}, \underline{H}^{(1)})$ and $(\underline{E}^{(2)}, \underline{H}^{(2)})$ is necessary* to satisfy the boundary conditions, and the admissible values of h are then given by the roots of the transcendental equation

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*We are here ignoring the discrete frequency spectra obtained from the solutions of the equations $H_m(ka) - i\eta H'_m(ka) = 0$, $H_m(ka) - \frac{i}{\eta}H'_m(ka) = 0$ as functions of k.

$$\left\{H_m(ha) - \frac{ik}{h}\frac{1}{\eta}H'_m(ha)\right\}\left\{H_m(ha) - \frac{ik}{h}\eta H'_m(ha)\right\} + (k^2 - h^2)\left\{\frac{m}{h^2 a}H_m(ha)\right\}^2 = 0 . \tag{4.5}$$

Unfortunately no general method for solving this equation is available, but it is nevertheless apparent that the roots are discrete and form a two-fold infinity. The corresponding fields are the natural modes of the system and are quasi-longitudinal (in the sense that neither the electric nor magnetic vector is transverse) with cyclical periodicity around the cylinder. For most practical purposes these modes are not important since the energy is mainly confined to the interior of the cylinder, leading to an extremely high rate of attenuation.

In the particular case of a perfectly conducting cylinder ($\eta = 0$) Eq. (4.5) reduces to

$$H_m(ha)\,H'_m(ha) = 0$$

and modes of the transverse electric or transverse magnetic type are now possible. The appropriate values of ha are given by the zeros of the Hankel function or its derivative (which requires $m > 1$ for such zeros to exist), and since these are complex the resulting modes are attenuated in the z direction.

When $\eta \neq 0$ transverse modes can only exist if $m = 0$ and this is a situation of some interest in surface wave theory. Taking first the transverse electric mode, substitution of the field components (4.4) into the boundary conditions (4.1) gives

$$H_o(ha) - \frac{ik}{h\eta}H'_o(ha) = 0 \tag{4.6}$$

as the equation from which to determine the admissible values of h. In the practical case to be considered, η is a small impedance produced either by a finite conductivity or by a small amount of surface loading accidentally or intentionally introduced into the structure, and as such its argument is restricted to the range $[-\pi/2, 0]$. A typical value for $|\eta|$ can be taken as 10^{-6}, corresponding to the conductivity of copper at a frequency of 1 Mc, and unless ka is of the same order of magnitude (or smaller) the only solution of Eq. (4.6) occurs for $ha >> 1$. The precise root can now be found by inserting the asymptotic expansion of $H_0(ha)$ for large ha, from which we obtain

$$h = \mp k/\eta \tag{4.7}$$

with the upper sign if the Hankel function is of the first kind and the lower if it is of the second. Because of the restriction on $\arg \eta$, each of these implies a field which builds up exponentially away from the cylinder through a factor $e^{-ikr/\eta}$, and which therefore violates the radiation condition. Moreover, since the propagation constant is

$$\pm \frac{ik}{\eta} \sqrt{1-\eta^2} \tag{4.8}$$

it follows that if the mode could be excited, it would be damped out almost immediately in the z direction.

Although this result is well known, it is of interest to note that the behavior of the field in the normal direction is precisely the same as that of the plane surface wave (3.11b) and indeed the dependence on the mutually perpendicular coordinates r, θ and z is identical with the dependence on the coordinates z, y and x

respectively which the field (3.11b) exhibits. For a large cylinder such a correspondence would come as no surprise, but in the present instance the solution is valid even for cylinders whose radii are as small as $\lambda|\eta|$, where λ is the free space wavelength.

Considering now the transverse magnetic mode, the equation from which to determine h is

$$H_o(ha) - \frac{ik}{h}\eta H'_o(ha) = 0 \tag{4.9}$$

which differs from (4.6) only in having η replaced by $1/\eta$. If ka is very large compared with unity (it is sufficient if $ka > 5/|\eta|$), a solution can again be found by inserting the asymptotic expansion of the Hankel function, and this gives

$$h = \mp k\eta \tag{4.10}$$

where the two signs apply in the same manner as those in Eq. (4.7). Both solutions correspond to a wave which is exponentially attenuated in the radial direction through a factor

$$e^{-ikr\eta} \tag{4.11}$$

and since the propagation constant in the z direction is

$$\pm k\sqrt{1-\eta^2} \tag{4.12}$$

the field travels along the cylinder with relatively little attenuation and with a phase velocity which is infinitesimally less than the velocity of light. It almost goes without saying that the field is directly analogous to the surface wave which can be supported by a plane structure (c.f. (3.11a)).

The above analysis is only valid if $ka|\eta| >> 1$ and in many cases of practical interest (particularly those to which traveling wave theory might be applied) the transverse radius of curvature is not large enough to satisfy this condition. If $ka|\eta| << 1$, however, an alternative method of solving Eq. (4.9) is available. This is based on the expansion of the Hankel function for small argument and taking, for example, the Hankel function of the first kind, we have

$$H_o(ha) = 1 + \frac{2i}{\pi}(\log ha - \gamma) + O(\overline{ha}^2 \log ha)$$

which can be substituted into Eq. (4.9) to give

$$\xi \left\{1 + O(\xi)\right\} \log \xi + \frac{ika}{2}\eta e^{2\gamma}\left\{1 + O(\xi \log \xi)\right\} = 0 \qquad (4.13)$$

Here,

$$\xi = \left(\frac{ha}{2} e^{\gamma - i\pi/2}\right)^2 \qquad (4.14)$$

and γ is Euler's constant (= 0.5772157. . . .), and providing terms in $\xi \log \xi$ are negligible compared with unity, (4.13) can be written as

$$\xi \log \xi + \frac{ika}{2}\eta e^{2\gamma} = 0 \quad . \qquad (4.15)$$

For simplicity let us further assume that $\arg \eta = -\pi/4$, corresponding to the impedance of a highly conducting metallic surface. Equation (4.15) then becomes

$$\xi \log \xi = -\frac{ka}{2}|\eta| e^{i\pi/4} \tag{4.16}$$

and the solution of this has been discussed in some detail by Goubau (1950).

If b and β are real quantities defined by the relation

$$\xi = b e^{i(\pi/4 - \beta)}, \tag{4.17}$$

Eq. (4.16) can be broken up into the two real equations

$$b \log b = -\frac{ka}{2}|\eta| \cos\beta \tag{4.18}$$

$$\tan\beta = \frac{\pi/4 - \beta}{\log b}, \tag{4.19}$$

and for $ka|\eta| < 10^{-3}$ (say), β is so small that (4.18) and (4.19) can be approximated as

$$b \log b = -\frac{ka}{2}|\eta|, \tag{4.20}$$

$$\beta = \frac{\pi/4}{1 + \log b}. \tag{4.21}$$

The first of these can be solved by a simple graphical approach, and β can then be obtained by substituting the resulting values of b into (4.21).

Equation (4.16), on the other hand, is actually valid for a somewhat wider range of ξ, and bearing in mind that the only restriction is for $\xi \log \xi$ to be small compared with unity (say, less than 0.1 in magnitude), Eqs. (4.18) and (4.19) can be used even for $ka|\eta|$ as large as 0.1. When $ka|\eta|$ exceeds 10^{-3} the approximations leading to (4.20) and (4.21) are no longer justifiable, but by working with the complete Eqs. (4.18) and (4.19) it is again possible to determine b and β by graphical means. In Figures 4-1 and 4-2 the values of $\frac{b}{ka|\eta|}$ and β are plotted as functions of $ka|\eta|$, and from these it is seen that the condition on $\xi \log \xi$ is indeed satisfied if $ka|\eta| < 0.1$ approximately.

To calculate the phase velocity v and the attenuation coefficient X it is necessary to specify the value of $|\eta|$, and to illustrate the dependence of these quantities on the parameter ka we shall choose $|\eta| = 10^{-6}$. Since v and X are given by the formulae

$$v = c\left\{1 - \frac{2b}{(ka)^2} e^{-2\gamma} \cos\left(\frac{\pi}{4} - \beta\right)\right\}, \tag{4.22}$$

$$X = k\frac{2b}{(ka)^2} e^{-2\gamma} \sin\left(\frac{\pi}{4} - \beta\right), \tag{4.23}$$

both of these can now be obtained from Figures 4-1 and 4-2, and in Figure 4-3 $\log_{10}(1 - \frac{v}{c})$ and $\log_{10}\frac{X}{k}$ are shown for $0.2 < ka < 10^5$. Both $1 - \frac{v}{c}$ and $\frac{X}{k}$ decrease with increasing ka, and throughout this range the attenuation is insignificant, with the phase velocity differing infinitesimally from that of free space. In fact, it is necessary to go down to values of ka of order 10^{-6} before either becomes as large as 10^{-2}.

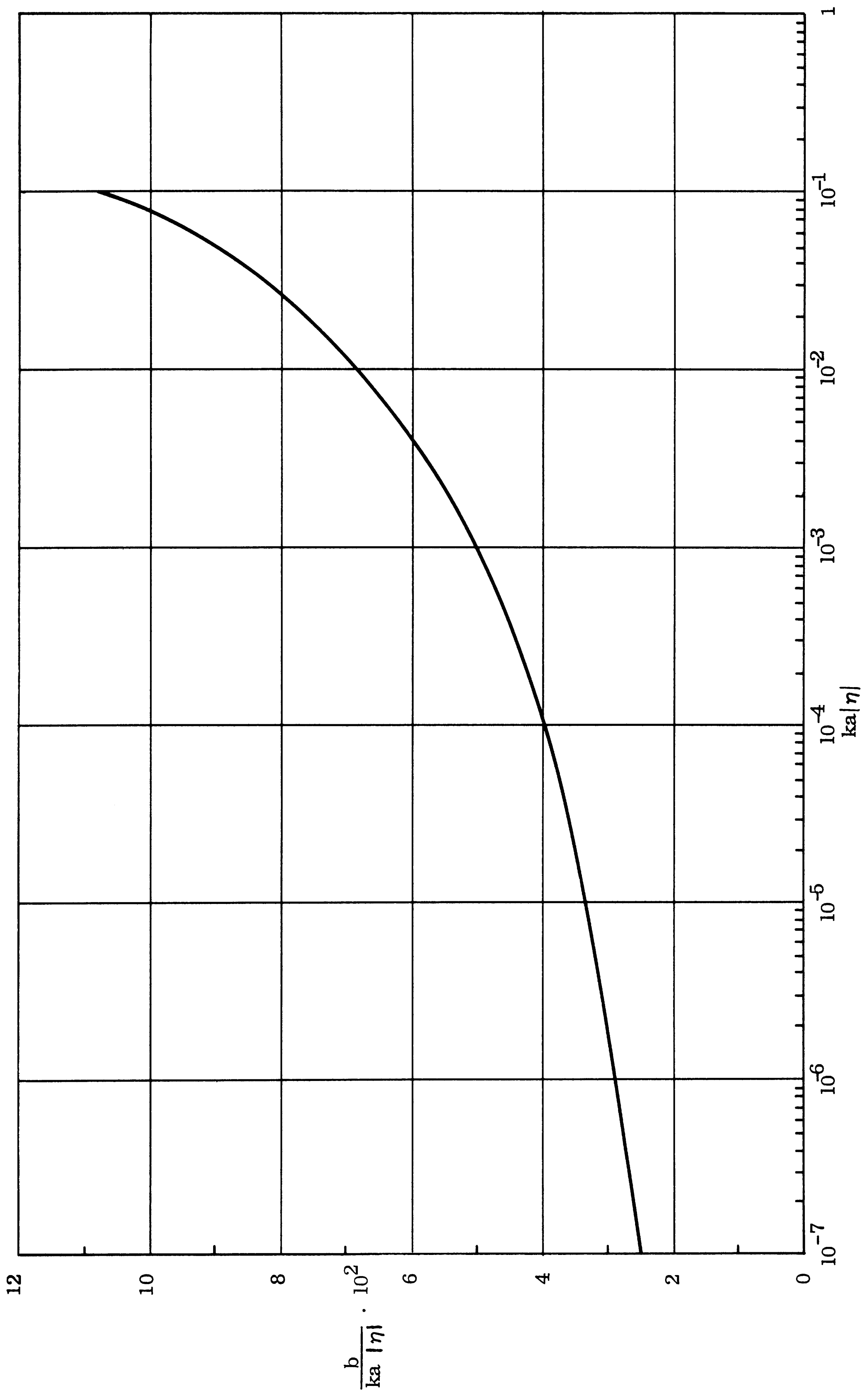

FIG. 4-1: SOLUTION OF EQUATION (4.20).

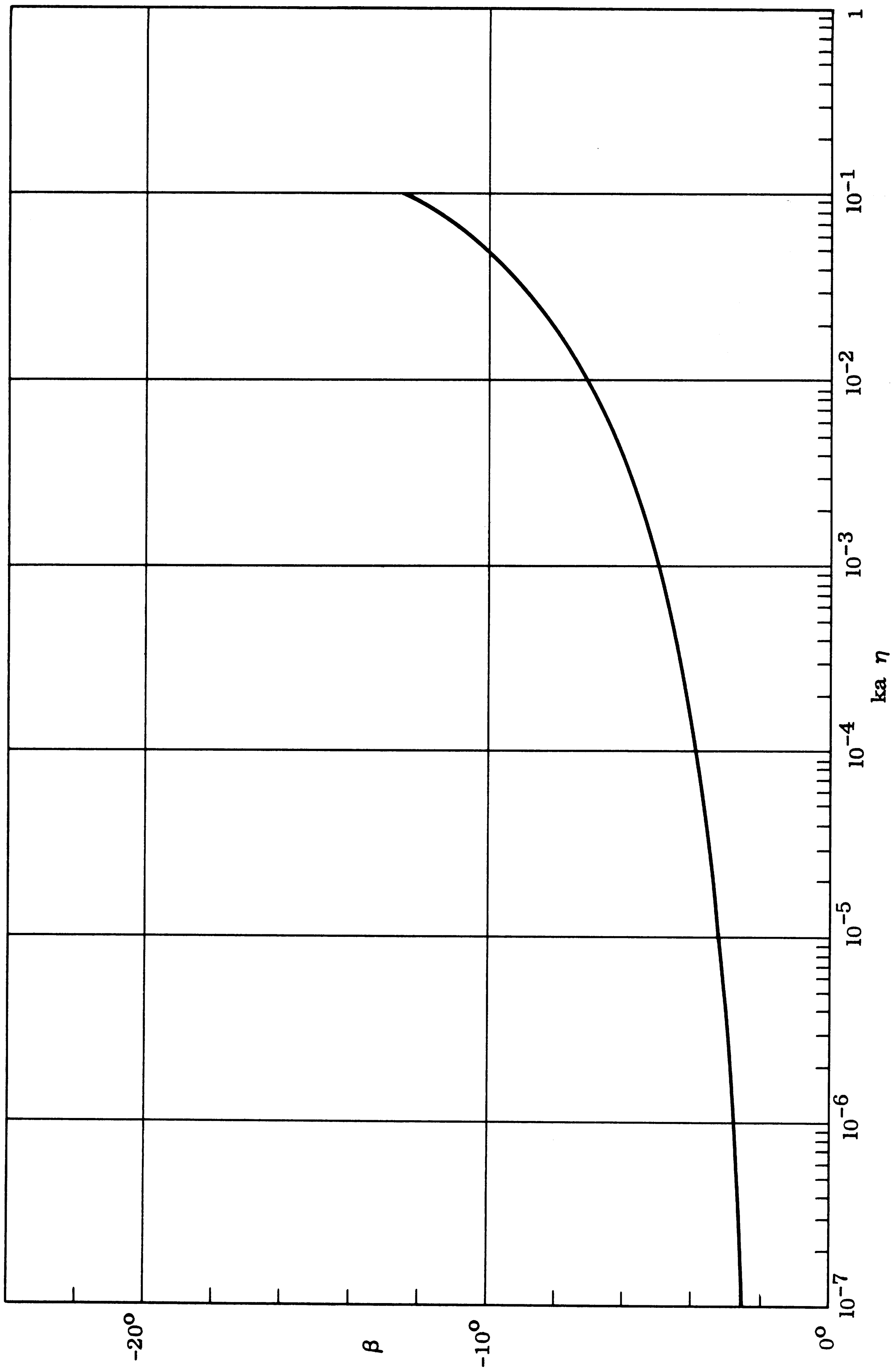

FIG. 4-2: SOLUTION OF EQUATION (4.21)

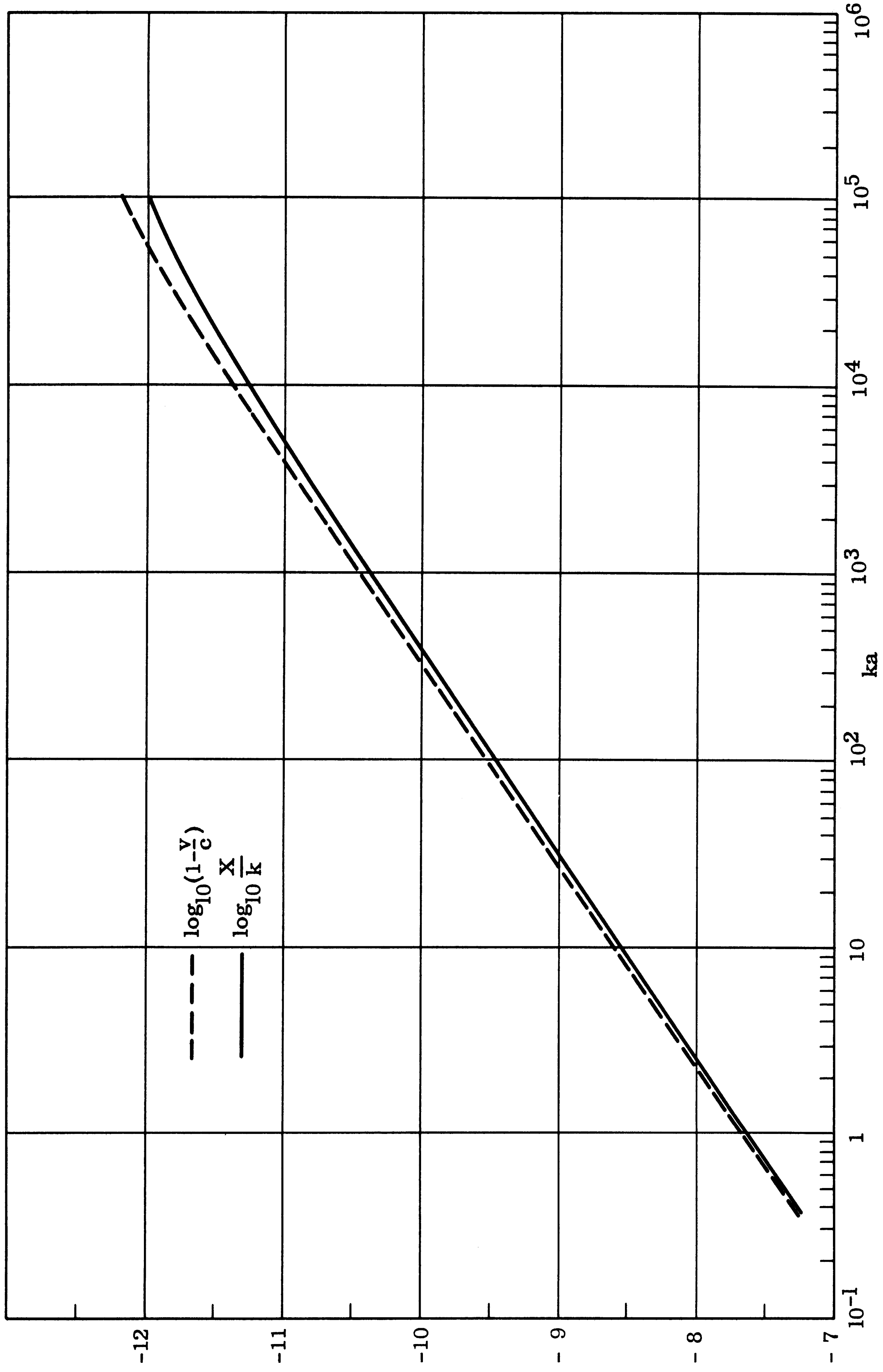

FIG. 4-3: PHASE VELOCITY AND ATTENUATION COEFFICIENT FOR $|\eta| = 10^{-6}$.

An interesting feature of the results is the relative smallness of the interval in ka separating the ranges for which Eqs. (4.10) and (4.15) are applicable. For $ka >> 1/|\eta|$,

$$v = c\left\{1 - \frac{1}{2}\,\mathrm{Re}\,\eta^2\right\}$$

$$X = -k\frac{1}{2}\,\mathrm{Im}\,\eta^2$$

and if $\eta = 10^{-6}\, e^{-i\pi/4}$ the above formulae are valid for $ka > 5.10^6$. The corresponding values of $1 - \frac{v}{c}$ and $\frac{X}{k}$ are zero and 5×10^{-13}, which are in excellent agreement with the trend indicated in Figure 4-3.

It is now apparent that in any application of traveling waves to radar scattering problems in which the transverse radius of curvature is more than $10^{-5}\lambda$ (say) there is little justification for taking the phase velocity to be other than that of light. In addition, the attenuation can be ignored. If ka is very large (greater than, perhaps, $5/|\eta|$) the surface wave has the same characteristics as the wave which can be supported by a flat sheet, including the same type of radial dependence, but for ka less than this the behavior of the field in the radial direction does differ from that of the plane surface wave. In particular, the exponential decay does not set in until the radial distance r is such that $kr > 5/|\eta|$ approximately, and at distances less than this the field amplitude oscillates or may even increase with r.

V

CREEPING WAVES

The term 'creeping wave' was first introduced by Deppermann and Franz (1954) to describe the type of wave which can be supported by a circular cylinder or sphere of large radius. Once launched, the wave travels around the body indefinitely, spilling off energy in the tangential direction as it goes, and thereby contributing to the scattering cross section of the body. As a consequence of the leakage, the wave is exponentially attenuated along the surface, and the form of the attenuation (as regards its dependence on radius) is, perhaps, the most characteristic feature of a creeping wave.

To obtain an expression for the wave it is only necessary to apply a Watson transformation to the standard solution for the current distribution on a cylinder or sphere. After a deformation of contour the solution then appears as a residue series each term of which can be interpreted as a wave originating at the shadow boundary and traveling around the body. The attenuation is determined by the curvature of the surface, and no surface impedance per se is necessary to support the wave. In this respect traveling* and creeping waves are quite distinct, but in both cases the wave is merely a term used to describe a mode in a representation of the current distribution, which representation proves convenient for mathematical purposes. Little attention is paid to the field behavior of this wave, and notwithstanding the fact that it is a valid solution of Maxwell's equations, there are those who doubt whether it has any physical reality. To them at least the resolution into creeping waves is no more than a mathematical device for obtaining a more rapidly convergent expansion for the current.

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*Assuming that the mathematical analogue of a traveling wave is a Goubau wave (Goubau, 1950, 1952).

In that no method has been found whereby a single creeping wave can be excited in the absence of the others, there is something to be said for this viewpoint, but nevertheless the concept of creeping waves has proved extremely useful in diffraction theory. The essential feature of these waves was discovered as long ago as 1946 by Fock who, in discussing the diffraction of a plane wave by an arbitrary convex cylinder, showed that the field in the neighborhood of the shadow boundary could be approximated in such a way as to give a parabolic equation in a variable related to one of the components. Depending upon the polarization considered, the field on the surface of the cylinder is then given in terms of one or other of two basic functions whose argument is

$$\xi = \left(\frac{ka}{2}\right)^{1/3} \frac{x}{a}$$

where a is the radius of the cylinder at the boundary, and x is measured in the direction of propagation. These functions serve to bridge the gap between the geometrical optics current in the lit region and the exponential attenuation characteristic of the shadow, and both have been tabulated by Logan (1959). Within the shadow each function is essentially the sum of the appropriate creeping wave terms, and can be resolved in this manner, but for a more detailed discussion of the relationship between Fock theory and creeping waves the reader is referred to Goodrich (1959).

One of the most striking demonstrations of the usefulness of the creeping wave concept is provided by the geometrical theory of diffraction (Keller, 1957). This is basically an extension of ray theory, but one of the tenets of the method is that for a body of large radius the diffracted rays follow geodesics on the surface, with a 'birth weight', attenuation and 'radiation strength' determined (to a first order) by the local properties of the surface. It is assumed that these parameters can be obtained by reference to selected canonical problems (for example, diffraction by a circular cylinder). In this way Keller has been able to derive the dominant terms in the high frequency expansion of the diffracted field for a wide variety of bodies, and additional correction terms have also been obtained by using the creeping wave expansions for bodies of varying curvature (Keller and Levy, 1959; Franz and Klante, 1959).

In order to examine the form of a creeping wave, consider a circular cylinder of radius a whose axis coincides with the z axis of a system of cylindrical polar coordinates (r, θ, z). If the propagation is confined entirely to the (r, θ) plane the problem is two-dimensional, and solutions are then possible in which the only non-zero component of either $\underline{E}$ or $\underline{H}$ is in the z direction. In the former case (E polarization)

$$\underline{E} = (0, 0, \phi) \; , \; \underline{H} = \frac{Y}{ik}\left(\frac{1}{r}\frac{\partial \phi}{\partial \theta}, -\frac{\partial \phi}{\partial r}, 0\right) \tag{5.1}$$

and in the latter (H polarization)

$$\underline{E} = \frac{Y}{ik}\left(-\frac{1}{r}\frac{\partial \phi}{\partial \theta}, \frac{\partial \phi}{\partial r}, 0\right) \; , \; \underline{H} = (0, 0, \phi) \tag{5.2}$$

where ϕ satisfies the equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} + k^2 \phi = 0 .$$

The most general solution of this is of the form

$$\phi = H_{\nu}^{(1)}(kr)\, e^{i\nu\theta} , \tag{5.3}$$

where the Hankel function is chosen in accordance with the requirement of outgoing radiation at infinity, and as long as θ is restricted to the 'physical' plane (say, $0 \leq \theta < 2\pi$), continuity demands that ν be an integer. On the other hand, if the θ plane is unrolled, allowing θ to range from $-\infty$ to ∞, and a Riemann surface constructed of which only one sheet is the physical plane, there is no longer any a priori restriction on ν, and this can now be chosen to satisfy the boundary conditions at the surface.

If the cylinder is perfectly conducting the boundary conditions are

$$E_{\theta} = E_{z} = 0$$

and taking the case of E polarization, the equation from which to determine ν is

$$H_{\nu}^{(1)}(ka) = 0 \quad . \tag{5.4}$$

On the assumption that $ka >> 1$, the solution can be found by replacing the Hankel function by its Airy integral approximation, and is (see, for example, Franz, 1954)

$$\nu = ka + \left(\frac{ka}{6}\right)^{1/3} e^{i\pi/3}\, q - \left(\frac{6}{ka}\right)^{1/3} e^{-i\pi/3}\, \frac{q^2}{180} + O(\overline{ka}^{-1}) \tag{5.5}$$

where $q = q_n$ is a zero of the Airy function

$$Ai(q) = \frac{1}{2} \int_{-\infty}^{\infty} e^{i(qr - r^3)} dr . \tag{5.6}$$

The zeros are real and infinite in number, and if they are ordered according to their magnitude, then for large n

$$q_n \sim \frac{3}{4} \left\{ \pi (4n - 1) \right\}^{2/3} . \tag{5.7}$$

For $n = 1, 2, \ldots 5$ the precise values of the q_n are

3.372134
5.895843
7.962025
9.788127
11.457423

but even when $n = 1$ the error in using the asymptotic formula for the q_n is less than 1 per cent.

It is now apparent that an infinite number of solutions of the form (5.3) are possible, and these are in fact the creeping waves. Each satisfies the wave equation, the boundary conditions on the cylinder and the radiation condition, but does violate the requirement of continuity throughout the physical plane $(0 \leq \theta < 2\pi)$. Consequently, if the body is closed a field of the above type can only exist in combination with such others as to remove the discontinuity, and in practice it is almost certain that an infinity of these modes will be excited. Nevertheless, it is still of interest to investigate the properties of a single creeping wave, and this we shall now do.

Continuing with the case of E polarization, the θ dependence of the n th creeping wave is

$$e^{i\nu_n \theta} ,$$

which can be written as

$$e^{ika(\alpha_n + i\beta_n)\theta} \tag{5.8}$$

where $k\alpha_n$ is the propagation constant (so that c/α_n is the phase velocity) and β_n is the attenuation coefficient per unit length. From (5.5) we have

$$\alpha_n = 1 + \tau_n - \frac{1}{15}\tau_n^2 + 0(\tau_n^3) \tag{5.9}$$

$$\beta_n = \tau_n\sqrt{3}\left\{1 + \frac{1}{15}\tau_n + 0(\tau_n^3)\right\} \tag{5.10}$$

where

$$\tau_n = \frac{q_n}{3}\left(\frac{3}{4ka}\right)^{2/3} . \tag{5.11}$$

The phase velocity at the surface of the cylinder is therefore less than the velocity of light and is, in fact,

$$c\left\{1 - \tau_n + \frac{16}{15}\tau_n^2 + 0(\tau_n^3)\right\} , \tag{5.12}$$

which decreases with increasing n. This is also equivalent to the wave having traveled with the velocity of light around a cylinder of larger radius $a\alpha_n$, and hence, as $ka \to \infty$, the velocity approaches c.

The attenuation coefficient is given by (5.10) and its non-zero value is a consequence of the radiation of energy in the direction tangential to the cylinder. As n increases, β_n increases, but for increasing a the attenuation decreases and ultimately becomes zero in the limit of a flat plane. This suggests that some insight into the behavior of the field can be obtained by comparing its properties with those of the surface wave which an impedance sheet can support. For such a sheet the tangential field variation is of the form

$$e^{ikx\sqrt{1-\eta^2}}$$

where x is measured in the direction of propagation and η is the surface impedance* (see Eq. (3.15). Comparison with (5.8) now gives

$$\sqrt{1-\eta^2} = \alpha_n + i\beta_n$$

from which we have

$$\eta = \pm 2\sqrt{\tau_n}\, e^{-i\pi/3} \left\{ 1 - \frac{8}{15}\tau_n e^{i\pi/3} + O(\tau_n^2) \right\}, \qquad (5.13)$$

and if the upper of the two signs is chosen, the impedance is of similar type to that produced by corrugations. It is therefore possible to associate the tangential

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*An alternative is to interpret η as the reciprocal of the impedance, but the resulting field either violates the radiation condition, or corresponds to an impedance which is not realizable in practice.

variation with either the curvature of the body or the loading of a flat surface notwithstanding the fact that the origin of the attenuation is quite different, being radiation in one case and dissipation in the other. For a complete identification, however, it is necessary to examine the radial variation.

As a function of kr for $r \geq a$ the field is given by

$$H_\nu(kr)$$

but since ν is complex any direct computation is out of the question when ka is large. This is similarly true of the Airy function representation

$$H_\nu(kr) \sim \frac{2}{\pi} e^{-i\pi/3} \left(\frac{6}{kr}\right)^{1/3} Ai(q)$$

where $q = \left(\frac{6}{kr}\right)^{1/3} e^{-i\pi/3} (\nu - kr)$, although here the series expansion for small arguments can be used to cater for small values of $r = a$.

In seeking to calculate the radial variation the approach which has been found most convenient is to replace the Hankel function by its tangent approximation

$$H_\nu(kr) \sim 2i\sqrt{\frac{2}{\pi kr \sin\alpha}} \sin\left[kr(\sin\alpha - \alpha\cos\alpha) - \pi/4\right] \qquad (5.14)$$

(see, Sommerfeld, 1949), where

$$\nu = kr\cos\alpha \qquad (5.15)$$

and α has a positive real part whenever it has a negative imaginary. Providing ka is sufficiently large, α is small near to the surface of the cylinder and accordingly

$$\alpha \simeq e^{-i\pi/2}\sqrt{\frac{2}{kr}(\nu - kr)} \; .$$

Since

$$\sin\alpha - \alpha\cos\alpha \simeq \frac{\alpha^3}{3}$$

we now have

$$H_\nu(kr) \sim \frac{2^{5/4}\, e^{-i\pi/4}}{\pi^{1/2}(kr)^{1/4}(\nu - kr)^{1/4}} \sin\left\{\frac{\pi}{4} - \frac{i}{3}\left[2(\nu - kr)(kr)^{-1/3}\right]^{3/2}\right\} \tag{5.16}$$

the zeros of which are given by

$$\frac{\pi}{4} - \frac{i}{3}\left[2(\nu - kr)(kr)^{-1/3}\right]^{3/2} = n\pi \; , \quad n = 1, 2, 3, \ldots$$

When $r = a$ the expression for the zeros is

$$\nu = ka + \frac{1}{2}(ka)^{1/3} e^{i\pi/3}\left[\frac{3}{4}(4n-1)\pi\right]^{2/3}, \tag{5.17}$$

which agrees with the first two terms of (5.5) if q_n is replaced by its asymptotic formula (5.7). In the following it is therefore assumed that ka is so large that terms $O(\overline{ka}^{-4/3})$ can be neglected in comparison with unity.

Using Eq. (5.16) it is a simple matter to compute $H_\nu(kr)$ when $k(r-a)$ is small. On the right hand side of (5.16) r can be replaced by a except in those places where $\nu - kr$ is involved, and if we further define δ by the equation

$$kr = ka + \frac{1}{2}(ka)^{1/3}\left[\frac{3}{4}(4n-1)\pi\right]^{2/3}\delta\ , \qquad (5.18)$$

(5.16) becomes

$$H_\nu(kr) \sim \frac{2\left(\frac{2}{\pi}\right)^{1/2} e^{-i\pi/4}}{(ka)^{1/3}\left[\frac{3}{4}(4n-1)\pi\right]^{1/6}\left(e^{i\pi/3}-\delta\right)^{1/4}} \sin\left[\frac{\pi}{4}\left\{1-i(4n-1)\left(e^{i\pi/3}-\delta\right)^{3/2}\right\}\right]. \qquad (5.19)$$

For small values of δ,

$$\left(e^{i\pi/3}-\delta\right)^{3/2} \sim i\left(1-\frac{3}{2}\delta e^{-i\pi/3}\right), \qquad (5.20)$$

and if indeed δ differs only infinitesimally from zero, the sine can be replaced by its argument to give

$$H_\nu(kr) \sim \frac{\left(\frac{2}{\pi}\right)^{1/2}\left[\frac{3}{4}(4n-1)\pi\right]^{5/6}}{(ka)^{1/3}}(-1)^n\ e^{i\pi/3}\delta, \qquad (5.21)$$

indicating a linear increase in field strength immediately away from the surface. In particular, for $n=1$

$$\left|H_\nu(kr)\right| \sim 3.2^{-1/2}\left(\frac{9\pi}{4ka}\right)^{1/3}\delta \qquad (5.22)$$

and this is valid for (say) $\delta < 0.2$. As δ increases further but remains consistent with the approximation (5.20), the sine becomes dominated by the positive exponential contributed by the δ term, so that

$$H_\nu(kr) \quad \frac{\left(\frac{2}{\pi}\right)^{1/2} e^{i\pi/6}}{(ka)^{1/3}\left[\frac{3}{4}(4n-1)\pi\right]^{1/6}} \, e^{-in\pi + \frac{3i\pi}{8}(4n-1)e^{-i\pi/3}\delta} \tag{5.23}$$

implying

$$\left|H_\nu(kr)\right| \sim 2^{1/2}\left(\frac{2}{3ka\pi^2}\right)^{1/3} e^{(9/16)\pi\sqrt{3}\,\delta} \tag{5.24}$$

for $n = 1$, and the linear increase has now given way to an exponential build-up in the radial direction.

This last result assumes even more significance when expressed in terms of the parameter τ_n. The appropriate expressions for ν and kr are then

$$\nu = ka(1 + 2\tau_n e^{i\pi/3})$$

$$kr = ka(1 + 2\tau_n \delta)$$

(cf (5.17) and (5.18)), which can be substituted into (5.16) to give

$$H_\nu(kr) \sim \frac{2e^{-i\pi/4}}{(\pi ka)^{1/2}\,\tau_n^{1/4}\,(e^{i\pi/3} - \delta)^{1/4}} \sin\left\{\frac{\pi}{4} - \frac{8i}{3}ka\,\tau_n^{3/2}\left(e^{i\pi/3} - \delta\right)^{3/2}\right\}.$$

The formula corresponding to (5.23) is therefore

$$H_{\nu}(kr) \sim \frac{e^{-i\pi/12 - i(8/3)ka\tau_n^{3/2}}}{(\pi ka)^{1/2}\tau_n^{1/4}} \exp\left\{ i\delta\, 4ka\tau_n^{3/2} e^{-i\frac{\pi}{3}} \right\} \tag{5.25}$$

the radial dependence of which is of the form

$$e^{ik(r-a)\eta}$$

with

$$\eta = 2\tau_n^{1/2} e^{-i\pi/3} . \tag{5.26}$$

It will be recalled that this is one of the two values of η compatible with the tangential variation of the field, and consequently, near (but not too near) the surface of the cylinder the field variation is

$$e^{ik(r-a)\eta + ika\theta\sqrt{1-\eta^2}} \tag{5.27}$$

where η is defined in (5.26). This has precisely the same mathematical form as the surface wave which can be supported by a flat sheet, through it should be emphasized that the phase of the impedance is not one which could be realized in practice. In effect the surface appears 'active', leading to an increase in the field in the normal direction and the ultimate violation of the condition at infinity, but if this condition were dispensed with, some information about the creeping wave behavior near to the surface of the cylinder could be obtained by taking a flat but non-perfectly conducting surface as a model.

$$\left| H_\nu(kr) \right| \sim \frac{1}{2} e^{9\sqrt{3}/16\,\pi\,\delta} \quad , \tag{5.30}$$

and these are plotted as functions of δ in Fig. 5-1. Also shown are the 'exact' values obtained by direct computation of the expression in Eq. (5.29), together with the small δ approximation derived from Eq. (5.22). The extent to which the various formulae are applicable is at once apparent.

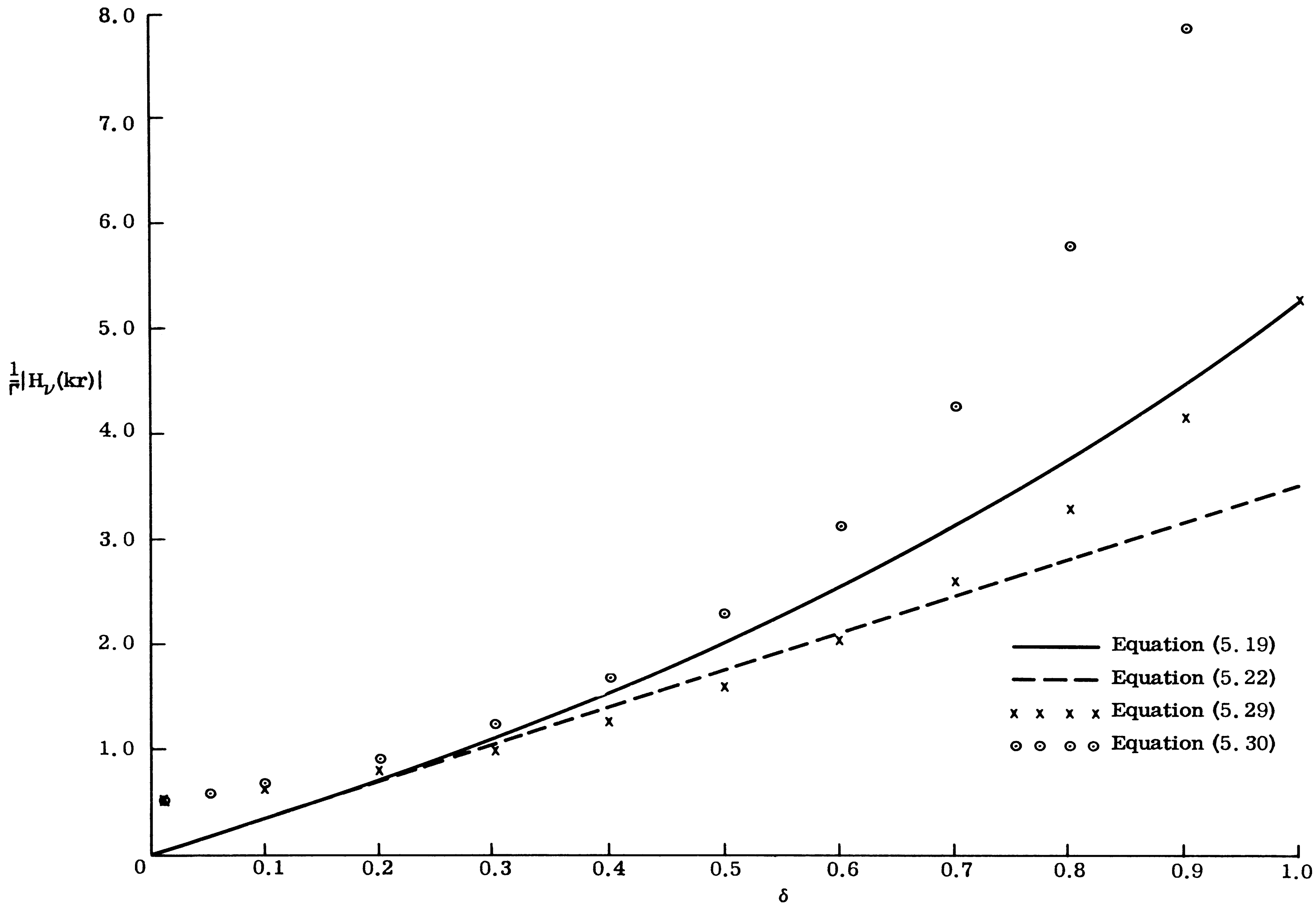

FIG. 5-1: RADIAL VARIATION OF THE FIELD INTENSITY.

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(U) The surface waves that can be supported by various simple geometrical shapes are examined, and some of the properties that they have in common are pointed out.

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