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DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

Radiation Laboratory

TRANSIENT RADIATION AND RECEPTION OF ELECTROMAGNETIC ENERGY

Final Report (15 December 1972 - 31 May 1975)

Grant GK 36867

by

Professor Chen-To Tai

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Summary

The research completed under this Grant covered the following problems:

1) Transient Radiation from resistively loaded transmission lines and thin biconical antennas,
2) Radiation and reception of transients by linear antennas,
3) Eigenfunction expansion of dyadic Green's functions.

Problem 1) was the Ph.D. thesis topic of Mr. Harold E. Foster, who completed his work in the Spring of 1973. Problem 2) was the subject matter of a chapter, bearing the same title, written jointly by the principal investigator, C.T. Tai and Dr. Dipak Sengupta for a forthcoming book, Transient Electromagnetic Fields, edited by L.B. Felsen of the Polytechnic Institute of New York. The book is scheduled to be published in 1975.

Problem 3) was not a main topic of the original grant. However, as a result of an indaequate treatment of the subject matter in the principal investigator's book, some effort was spent to improve that treatment. The complete work was too long to be published as a journal article, but a technical report was submitted to the National Science Foundation in July 1973. Subsequently, the same report was published as one of the Mathematical Notes by the Weapons System Laboratory of the Kirtland Air Force Base, as requested by Dr. Carl Baum of that laboratory.

The eigenfunction expansion of dyadic Green's functions pertaining to cavities have also been found. Functions pertaining to cylindrical and spherical cavities were part of a Ph.D. thesis written by Pawel Rosenfeld, which was completed in March 1974. Different representations of the dyadic Green's function for a rectangular cavity have been investigated very thoroughly by the principal investigator, and will be presented as an oral paper at the forthcoming meeting of the AP-S/URSI International Symposium in Urbana, Illinois, June 2-5, 1975.

Discussion of Research

For convenience, a copy of the Abstract of Harold E. Foster's Ph.D. thesis is attached to this report. The Abstract gives a detailed synopsis of the work. Of particular significance is the treatment of a resistive loading that varies linearly with position along a transmission line. The problem was solved exactly in terms of Airy functions of complex argument. This model is used to check the numerical analysis for other problems which could not be solved analytically in closed form.

For the problem of radiation and reception of transients by linear antennas, we emphasize the importance of the concept of the transfer function between the transmitting antenna and the receiving antenna. According to our view, the description of the transient field of a transmitting antenna alone is of limited use, because the detection of any electromagnetic signal inevitably requires a receiving antenna. Thus, the characteristics of the receiving antenna play an important role in determining the ultimate response of a transient signal. In our work, we apply the method of Fourier transform to the formulation of the problem and carry out the analysis for some simple interaction problems such as the transfer function analysis between dipole and dipole, dipole and loop.

For resistively loaded antennas, we depend heavily on the numerical method to obtain the final result. By properly loading the antenna, it is possible to create transient fields which exhibit very little oscillatory response to step-voltage excitation. In the work on dyadic Green's functions, we have now amended the missing singular term in all the eigenfunction expansions of the dyadic Green's functions previously studied in the principal investigator's book. This unusual error was not detected for several years in spite of the fact that the manuscript of the book had been reviewed by many experts in this field. The missing singular term only affects the field in a source region. This is presumably the reason why it was not discovered until the whole completeness problem was critically examined by Professor Per-olof Brundell of the University of Lund, Sweden. Since then we have reconstructed all the functions based on a new expansion technique using the inhomogeneous equation for the dyadic Green's function pertaining to the H-field as the starting point, namely,

$$\nabla \times \nabla \times \bar{\bar{G}}_H - k^2 \bar{\bar{G}}_H = \nabla \times \left[\bar{\bar{I}}\delta(\bar{R} - \bar{R}')\right]$$

The solenoidal nature of the function $\bar{\bar{G}}_H$, i.e., $\nabla \cdot \bar{\bar{G}}_H = 0$, enables us to construct the eigenfunction expansion of $\bar{\bar{G}}_H$ using two sets of solenoidal vector wave functions. The corresponding $\bar{\bar{G}}_E$ function is then found by

$$\bar{\bar{G}}_E = \nabla \times \bar{\bar{G}}_H$$

The discontinuous behavior of $\bar{\bar{G}}_H$ across a current source region is responsible for the singular term of $\bar{\bar{G}}_E$ as a result of the differential operation on a discontinuous function.

As a sequel to this work, we have completed the derivation of the dyadic Green's functions pertaining to cavities. In the thesis of Pawel Rosenfeld, the function pertaining to rectangular, cylindrical and spherical cavities were derived. The Abstract of his thesis attached. Alternative representations of the rectangular cavity functions have since been thoroughly examined. This work can be considered as a supplement to the work of Morse and Feshbach [Methods of Theoretical Physics, Vol. II, McGraw-Hill Book Company, 1953] who derived one complete expression of $\bar{\bar{G}}_A$, the function pertaining to the vector potential, but gave one incomplete expression in another place. We have now found all the alternative expressions for $\bar{\bar{G}}_E$. The relations between $\bar{\bar{G}}_E$, $\bar{\bar{G}}_H$ and $\bar{\bar{G}}_A$ are also discussed in detail. This work will be presented in the forthcoming meeting of AP-S/URSI in Illinois.

Publications under Grant GK-36867

1. C.T. Tai, "On the eigenfunction expansion of dyadic Green's functions", Proc. IEEE, 61, p. 480, April 1973.

2. C.T. Tai, "Eigenfunction Expansion of Dyadic Green's Functions", technical report submitted to NSF in July, 1973; the same report was reissued by the Weapons System Laboratory, Kirtland Air Force Base, as Mathematics Note No. 28, dated September 1973.

3. P. Rosenfeld, "The Electromagnetic Theory of Three-dimensional Inhomogeneous Lenses and the Dyadic Green's Functions for Cavities", Ph.D. Thesis, Department of Electrical and Computer Engineering, The University of Michigan, 1974.

4. H. E. Foster, "Transient Radiation from Resistively Loaded Transmission Lines and Thin Biconical Antennas", Ph.D. Thesis, Department of Electrical and Computer Engineering, The University of Michigan, 1973.

5. H.E. Foster and C.T. Tai, "Inhomogeneous Resistive Loading of Transmission Media", paper presented at Fall URSI Meeting, Boulder, Colorado, October 14-17, 1974

Personnel

NSF Grant 36867 was awarded to The University of Michigan and was administered in the Radiation Laboratory under the direction of Professor Chen-To Tai. He was assisted by Harold E. Foster and Pawel Rosenfeld, both graduate students in The University of Michigan before they completed their work in 1973 and 1974 respectively. Yu-Ping Lu, another graduate student, assisted the project mainly in computer programming and computation.

frequency shift is proportional to the cosine of the angle of arrival of the incoming wave referred to the direction of vehicle travel, the frequency axis can also be scaled in angle of arrival. The angle specifies a cone only so it is impossible to differentiate between paths arriving from the right or left (or up or down). The figures vividly illustrate that most of the scattered radio energy arriving at the mobile vehicle at the bottom of the 10-m-wide canyon formed by the surrounding buildings was traveling generally up and down the street (nearly maximum Doppler shift plus or minus). Only a small part of the scattered energy arrives from a path nearly perpendicular to the street. If the buildings were not present, the path between transmitter and receiver would be at an angle of 42° to the direction of vehicle travel. This 42° angle corresponds to a Doppler shift of +2.2 cycles/m. For small excess delays there is some energy arriving at this angle. Reference [3] includes some description of the scattering environment that is applicable to the major features on this scattering function. Briefly, for excess delays up to about 1.5 μs, the scattered energy comes from buildings in the vicinity of the vehicle. Up to about 3 μs, scattering is from other buildings on lower Manhattan Island. The general lack of scattered power at excess delays of from about 2 to 8 μs is due to the East River and Hudson River open areas surrounding the island. From about 8 μs to 10 μs, the relatively strong peaks that are Doppler shifted by the negative maximum amount $1/\lambda$ are reflections from Brooklyn across the East River. The negatively shifted small peak at 6 and the positively shifted small peak between 8 and 9 μs do not seem to correspond to any single reflecting location and thus are probably due to multiple reflection.

Parameters that describe the multipath delay spread of this path are defined and tabulated in [3].

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REFERENCES

[1] R. S. Kennedy, *Fading Dispersive Communication Channels*. New York: Wiley Interscience, 1969, ch. 3.
[2] D. C. Cox, "Delay Doppler characteristics of multipath propagation at 910 MHz in a suburban mobile radio environment," *IEEE Trans. Antennas Propagat.*, vol. AP-20, pp. 625–635, Sept. 1972.
[3] ——, "910 MHz urban mobile radio propagation: Multipath characteristics in New York City," submitted to *IEEE Trans. Commun.*
[4] W. R. Young, Jr., and L. Y. Lacy, "Echoes in transmission at 450 megacycles from land-to-car radio units," *Proc. IRE*, vol. 38, pp. 255–258, Mar. 1950.
[5] G. L. Turin, F. D. Clapp, T. L. Johnston, S. B. Fine, and D. Lavry, "A statistical model of urban multipath propagation," *IEEE Trans. Veh. Technol.*, vol. VT-21, pp. 1–9, Feb. 1972.
[6] D. C. Cox, "Doppler spectrum measurements at 910 MHz over a suburban mobile radio path," *Proc. IEEE*, vol. 59, pp. 1017–1018, June 1971.
[7] B. B. Barrow, L. G. Abraham, W. M. Cowan, Jr., and R. M. Gallant, "Indirect atmospheric measurements utilizing Rake tropospheric scatter techniques—Part I: The Rake tropospheric scatter technique," *Proc. IEEE*, vol. 57, pp. 537–551, Apr. 1969.
[8] D. C. Cox, "Time and frequency domain characterizations of multipath propagation at 910 MHz in a suburban mobile radio environment," *Radio Sci.*, vol. 7, Dec. 1972.

On the Eigenfunction Expansion of Dyadic Green's Functions

CHEN-TO TAI

Abstract—As a result of an error, the singular behavior of the eigenfunction expansion of the dyadic Green's function is not correctly formulated in my book (C. T. Tai, *Dyadic Green's Functions in Electromagnetic Theory*, Scranton, Pa.: International Textbook, 1971). The correct expressions are given here and an improved method for deriving the residue series is presented.

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Professor Per-Olof Brundell of the University of Lund, Lund, Sweden, has kindly called my attention to an error in the treatment of the eigenfunction expansion of the dyadic Green's functions discussed in my book [1]. In that work, only two sets of solenoidal vector wave functions are used in the expansion of the dyadic delta function $\overline{I}\delta(R-R')$. These two sets are indeed complete within the space of solenoidal vector fields but not within the space of all vector fields. As $\overline{I}\delta(R-R')$ is not solenoidal, having the divergence equal to $\nabla\delta(R-R')$, the two sets of solenoidal vector wave functions employed are not sufficient to represent such a quantity.

As a result of the error, the singular behavior of the eigenfunction expansion of the dyadic Green's function is not correctly formulated in my book. Of the residue series derived in the book, all of them happen to be valid for $R\neq R'$ except the one dealing with a rectangular waveguide with moving medium which is incorrect. However, the method used to derive these residue series is misleading and unsatisfactory. For the spherical case based on the algebraic method, the singular behavior of the dyadic Green's function can be preserved if so desired although only the result for $R\neq R'$ is given in the book.

In this letter we outline a method whereby only two sets of solenoidal vector wave functions are sufficient to derive the correct solution which is valid everywhere including the source region. We consider, for example, the dyadic Green's function of the first kind pertaining to a rectangular waveguide which satisfies the equation:

$$\nabla\times\nabla\times\overline{G}_1(R\,|\,R') - k^2\overline{G}_1(R\,|\,R') = \overline{I}\delta(R-R'). \quad (1)$$

By taking the curl of (1) we obtain

$$\nabla\times\nabla\times\nabla\times\overline{G}_1(R\,|\,R') - k^2\nabla\times\overline{G}_1(R\,|\,R') = \nabla\times[\overline{I}\delta(R-R')]. \quad (2)$$

The function $\nabla\times\delta\overline{I}(R-R')$ is solenoidal so it can be expanded in terms of the vector eigenfunctions M and N defined by

$$M_{omn}(h) = \nabla\times\left[\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{ihz}\hat{z}\right] \quad (3)$$

$$N_{emn}(h) = \frac{1}{\kappa}\nabla\times\nabla\times\left[\cos\frac{m\pi x}{a}\cos\frac{m\pi y}{b}e^{ihz}\hat{z}\right] \quad (4)$$

where

$$\kappa^2 = h^2 + k_c^2$$
$$k_c^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2.$$

These vector wave functions satisfy the boundary condition at the walls of the waveguide corresponding to $x=0$, $x=a$, $y=0$, and $y=b$:

$$\hat{n}\times\nabla\times M_{omn}(h) = 0$$
$$\hat{n}\times\nabla\times N_{emn}(h) = 0.$$

They are compatible with the requirement that $\hat{n}\times\overline{G}_1(R\,|\,R')=0$ on the boundary of the waveguide.

We now let

$$\nabla\times[\overline{I}\delta(R-R')] = \int_{-\infty}^{\infty}\sum_m\sum_n [M_{omn}(h)A_{omn}(h) + N_{emn}(h)B_{emn}(h)]\,dh \quad (5)$$

where A and B are two unknown sets of vector coefficients to be determined. The integral in (5) is to be interpreted in the generalized sense as the Fourier integral representation of the distribution $\nabla\times[\overline{I}\delta(R-R')]$. By means of the orthogonal relations of the functions $M_{omn}(h)$ and $N_{emn}(h)$ we find

$$A_{omn}(h) = \frac{2-\delta_0}{\pi abk_c^2}\nabla'\times M_{omn}'(-h)$$

$$B_{emn}(h) = \frac{2-\delta_0}{\pi abk_c^2}\nabla'\times N_{emn}'(-h)$$

where the primed functions are defined with respect to the primed variables (x', y', z') pertaining to R'. Applying now the Ohm-Rayleigh method to (2), we can express $\nabla\times\overline{G}_1(R\,|\,R')$, which represents a solenoidal vector field, in a similar form. The expression for $\nabla\times\overline{G}_1(R\,|\,R')$ is found to be

$$\nabla\times\overline{G}_1(R\,|\,R') = \int_{-\infty}^{\infty}\sum_m\sum_n \frac{2-\delta_0}{\pi abk_c^2(h^2+k_c^2-k^2)} \cdot [M_{omn}(h)\nabla'\times M_{omn}'(-h) + N_{emn}(h)\nabla'\times N_{emn}'(-h)]\,dh$$

In view of (1), we have

$$\overline{G}_1(R\,|\,R') = \frac{1}{k^2}\left[\nabla\times\nabla\times\overline{G}_1(R\,|\,R') - \overline{I}\delta(R-R')\right]$$
$$= -\frac{\overline{I}\delta(R-R')}{k^2} + \frac{1}{k^2}\int_{-\infty}^{\infty}\sum_m\sum_n \frac{2-\delta_0}{\pi ab k_c^2(h^2+k_c^2-k^2)}$$
$$\cdot\left[\nabla\times M_{omn}(h)\nabla'\times M_{omn}'(-h) + \nabla\times N_{emn}(h)\nabla'\times N_{emn}'(-h)\right]dh. \quad (7)$$

The integral in (7), representing $\nabla\times\nabla\times\overline{G}_1(R\,|\,R')$, contains a generalized singular function which can be extracted from the integral. The remaining part can be evaluated in a closed form by the residue theorem. The final expression for $\overline{G}_1(R\,|\,R')$ is given by

$$\overline{G}_1(R\,|\,R') = -\frac{\hat{z}\hat{z}\delta(R-R')}{k^2} + \frac{i}{ab}\sum_m\sum_n \frac{2-\delta_0}{k_g k_c^2} \cdot \begin{cases} M_{emn}(k_g)M_{emn}'(-k_g) + N_{omn}(k_g)N_{omn}'(-k_g) \\ M_{emn}(-k_g)M_{emn}'(k_g) + N_{omn}(-k_g)N_{omn}'(k_g) \end{cases} \quad (8)$$

where k_g denotes the guided wave number $(k^2-k_c^2)^{1/2}$. The series contained in (8) converges to a distribution as R approaches R'. For $R\neq R'$, the top series applies to $z>z'$ and the bottom one to $z<z'$. The series contained in (8) is the same as that found in the author's book [1, p. 79, eq. (8)]. However, the expression for $\overline{G}_1(R\,|\,R')$ derived here has an extra term $-\hat{z}\hat{z}\delta(R-R')/k^2$ which together with the series gives the complete representation of $\overline{G}_1(R\,|\,R')$ in the entire spatial domain of the waveguide including the source region. The method described here is applicable to all eigenfunction expansions of the dyadic functions which satisfy an equation of the form represented by (1). The details of evaluating the integral representing $\nabla\times\nabla\times\overline{G}(R\,|\,R')$ are rather intricate. They will be described elsewhere together with other cases.

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REFERENCES

[1] C. T. Tai, *Dyadic Green's Functions in Electromagnetic Theory.* Scranton, Pa.: International Textbook, 1971.

A Note on the Modified Kalman Filter for Channel Equalization

JON W. MARK

Abstract—An application of the Kalman filter to equalization of a ligital communication channel is described. The resultant modified Kalman equalizer (KE) is a nonlinear system in which the channel ap gains are estimated via a decision feedback approach and the iitial state variable is estimated by a prediction process.

I. Introduction

In the design of a digital communication system a tapped-delay ne model has often been used to represent the dispersive channel. the digital message is encoded such that it is characteristically milar to white Gaussian noise, the channel model may be formuted using a state variable representation similar to the filtering odel advanced by Kalman [1]. Here the state variables are the ccessive message symbols. Under a known channel condition the alman filter, which is a dual to the channel model, represents the timum linear equalizer in that the number of equalizer taps needs ly be the same as the number of channel taps. For a conventional ear equalizer (CLE) the performance is a direct function of the gree of freedom associated with the CLE [6], [7]. Optimum linear equalization is attained by the CLE only when the number of taps is infinite. Also, the tap gains further away from the main or central tap are smaller and are difficult to adjust. Thus a Kalman equalizer (KE) which needs only a finite number of taps to attain optimality is definitely superior to the CLE from the implementation point of view. In the KE the estimates of the state variables constitute the estimates of the transmitted message symbols. A utilization of the Kalman filter for channel equalization under a known channel condition has been described recently [2].

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Application of the Kalman filter to channel equalization requires 1) a knowledge of the initial state variable estimate and the initial covariance matrix, and 2) a knowledge of the channel impulse response. If the initial estimates are outside the feasible region to admit convergence, the Kalman filter will diverge. In the equalization context, a divergent situation will not necessarily mean an unstable condition. Rather, the system error will run away. Unless the initial state variable and the initial covariance estimates are chosen judiciously, the stringent requirement of initial conditions may obviate the optimality of the Kalman filter as an equalizer. Also, for unknown channels, the gains of the tapped-delay line model (which correspond to the components of the output vector in the Kalman filter model) need to be measured or estimated. Taking into account the optimality of the Kalman filter, this letter is concerned with the estimation of an initial state variable estimate and the adaptive computation of the channel tap gains $\{c_j\}$.

II. Problem Statement

Let the discrete message sequence $\{a(n)\}$ be sample-to-sample independent. Let the digital communication channel be characterized by a tapped-delay line model with a finite number of nonzero tap gains $\{c_j\}_{j=-L}^{M}$, where c_0 is the main tap with L precursors and M delay echoes. The channel may be represented by the following state vector equation:

$$m(n) = Fm(n-1) + ga(n) \quad (1)$$

with the channel output given by

$$y(n) = c^t(n)m(n) + \eta(n)$$
$$= x(n) + \eta(n) \quad (2)$$

where F is an $(M+L+1)\times(M+L+1)$ system matrix which characterizes the tapped-delay line model, g is an $(M+L+1)\times 1$ input vector, and $c(n)$ is an $(M+L+1)\times 1$ output vector (channel tap gain vector). F and g are given by

$$F = \begin{bmatrix} 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & \cdot & \cdot & \cdot & 0 \\ \cdot & & & & & \\ \cdot & & & & & \\ 0 & 0 & \cdot & \cdot & 1 & 0 \end{bmatrix} \quad \text{and} \quad g = \begin{bmatrix} 1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix}. \quad (3)$$

In (1) and (2), $m^t(n) \triangleq (m_{-L}(n), \cdots, m_0(n), \cdots, m_M(n))$ is a state vector, $x(n)$ is the channel output scalar, and $\eta(n)$ is a sample of an additive white Gaussian noise sequence. The superscript t denotes matrix transpose. In the KE the final state variable estimate represents the estimate of the message symbol, i.e., $\hat{a}(n-M) = \hat{m}_M(n\,|\,n)$. Our problem is to devise an adaptive technique for computing the conditional estimate

$$\hat{a}(n-M) = E\{a(n-M)\,|\,y(n), y(n-1), \cdots, y(1)\}$$
$$= E\{a(n-M)\,|\,y(n)\}, \qquad n \geq M \quad (4)$$

where the last equality holds due to the implicit Gauss–Markov properties of the $\{y(n)\}$ sequence. Also, it is assumed that at the start, the precursors of the KE have been filled with data.

III. Adaptive Modified Kalman Equalizer (KE)

If the transmitted sequence $\{a(n)\}$ is assumed to be white Gaussian noise, the state vector $m(n)$ is a Gauss–Markov vector. Application of the conditional mean (4) while taking into account the implicit Gauss–Markov property of $\{y(n)\}$ leads to the Kalman filtering equations

$$\hat{m}(n\,|\,n) = \hat{m}(n\,|\,n-1) + k(n)[y(n) - \hat{x}(n\,|\,n-1)] \quad (5)$$
$$\hat{m}(n\,|\,n-1) = F\hat{m}(n-1\,|\,n-1) + g\hat{m}_{-L}(n\,|\,n-1) \quad (6)$$
$$\hat{x}(n\,|\,n-1) = c^t(n)\hat{m}(n\,|\,n-1). \quad (7)$$

The unknown parameters in (5) to (7) are $\hat{m}_{-L}(n\,|\,n-1)$, $k(n)$, and

ABSTRACT

THE ELECTROMAGNETIC THEORY OF THREE-DIMENSIONAL INHOMOGENEOUS LENSES AND THE DYADIC GREEN'S FUNCTIONS FOR CAVITIES

by
Pawel Rozenfeld

Co-Chairmen: Chen-To Tai, Chiao-Min Chu

In this thesis the dyadic Green's functions of a number of cavities have been derived and the characteristics of some inhomogeneous lenses have been investigated.

To facilitate the treatment of problems involving cavities we have found the expressions for the dyadic Green's functions pertaining to rectangular, cylindrical and spherical cavities. Expressions for the electric and magnetic field involving the Green's functions are presented. An example of the application of the dyadic Green's function technique to the computations of the input admittance of the rectangular cavity is given.

The lenses covered in our work include: the Luneburg, Eaton-Lippmann and Eaton. The dyadic Green's functions for electric and magnetic dipoles in the presence of these lenses are found. The expressions for the electric field of an Huygens source in the presence of an inhomogeneous lens are constructed. Radiation patterns and the bistatic scattering cross sections for the small-diameter lenses and the directivity and the distribution of the energy around the geometrical focus of the Luneburg lenses are examined in detail.

ABSTRACT

TRANSIENT RADIATION FROM RESISTIVELY LOADED TRANSMISSION LINES AND THIN BICONICAL ANTENNAS

by
Harold Edwin Foster

Co-Chairmen: Chen-To Tai, Ralph E. Hiatt

This dissertation presents a theoretical analysis of the radiation and reception of transient electromagnetic signals by resistively loaded transmission lines and thin biconical antennas. The resistively loaded transmission line analysis, in addition to providing an advance in its own right, supplies a basis for the study of transients in antennas. Transmission line theory and mechanisms apply to the modeling of a variety of antennas and to the detailed understanding of their performance.

The transient analysis is attacked by the Fourier transform approach to make use of established concepts such as that of impedance. Investigations are performed first in the frequency domain and then transformed into the time domain for inspection of transient results. The Fast Fourier Transform technique of truncating series of sinusoids provides some economy where numerical computations are needed for the transformations.

General transmission line equations are developed to account for time-dependent and position-dependent transmission line parameters. These equations are then specialized to accommodate the case under investigation which is that of an open-circuited transmission line loaded with series resistance. Several functional distributions of resistance along the transmission line are considered. For a resistive loading that varies linearly with position along the line, a closed form expression for the resulting current on the line is found in terms of Airy functions with complex arguments. For resistance distributions other than linear, the transmission line equation does

not in general possess a closed form functional solution. These problems are solved by a numerical analysis which is implemented digitally on a computer.

An examination is made of the control which the different resistance distributions exert over the transmission line's input impedance, current distribution, radiated transient waveform and received transient waveform. It is shown that an inverse functional form of resistance loading is optimum, based on criteria of maximizing current on the transmission line while minimizing reflections.

Discretely lumped resistance loadings as well as continuous resistance distributions are analyzed. Results of the discrete and continuous analyses are in excellent agreement when the discrete resistances are sufficiently close together.

The concept of a position-dependent characteristic impedance is developed for the resistively loaded transmission line. In addition to varying with position along the line, this quantity also differs in the forward and backward directions. Such characteristic impedances are formulated in general and for the several resistance loading functions that are treated in this dissertation.

An approximate step function voltage source is considered to energize the loaded transmission lines. The resulting current waveforms at positions along the line and the resulting transient radiated fields are computed. The different shapes of the transient waveforms that are radiated in different directions from the loaded transmission line are shown. The radiation patterns which change shape with time are also computed and shown. The maximum amplitude of radiation, over all time, is shown to be in an off-broadside direction that is consistent with a predominantly traveling wave. This occurs for the optimally loaded transmission line in which reflected waves are minimized.

Reception of transient signals by a resistively loaded transmission line is formulated in terms of the vector effective height function. Transient radiation coupling between different loaded transmission lines is also formulated in this way.

The conical transmission line fields associated with a thin biconical antenna are used in an analysis of the transient behavior of this antenna. Determination of transient currents on the antenna takes account of internal complementary fields. Radiation of transient waveforms is analyzed. Transient reception is analyzed via the vector effective height function of the antenna.