THE UNIVERSITY OF MICHIGAN
COLLEGE OF ENGINEERING
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING
Radiation Laboratory

APPLICATION OF THE LARGE GRADIENT VOR ANTENNA

By

Dipak L. Sengupta and Philip Chan

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FEDERAL AVIATION ADMINISTRATION
800 Independence Avenue, S.W.
Washington, D.C. 20591

2216 Space Research Building
2455 Hayward Street
Ann Arbor, Michigan 48105
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I

INTRODUCTION

This is the second Interim Report on Contract No. DOT-FA72WA-2882, "Application of the Large Gradient VOR Antenna," and covers the period 1 September to 30 November, 1972.

The present report investigates theoretically the radiation pattern of Scanwell antennas located in free space and above a perfectly conducting infinite planar ground. The organization of the report and some of the notations used are similar to our Interim Engineering Report No. 1 (Sengupta and Chan, 1972).
II

THE SCANWELL ANTENNA

The Scanwell antenna is a large gradient VOR antenna developed by the Scanwell Laboratories. It consists of a linear array of five elements or bays. Each bay consists of a standard 4-loop VOR array (Anderson et al, 1953). Figure 1 shows schematically the array geometry. The direction of the \( z \)-axis will be referred to as the vertical direction. The numbering of the array elements is as shown in Fig. 1. The position of the \( n \)-th element with respect to the origin is denoted by \( d_n \). Note that \( d_n \) is positive for elements above the \( x \)-axis and negative for elements below the \( x \)-axis. As can be seen from Figure 1, the array is non-uniformly spaced and excited. However, the spacing of the antenna elements are symmetrical with respect to the \( x \)-axis.

![Schematic representation of the Scanwell antenna.](image)

FIGURE 1: Schematic representation of the Scanwell antenna.
The Scanwell laboratories optimized the antenna parameters $I_n = |I_n| e^{i\alpha_n}$, $d_n$ such that the free space pattern of the antenna in the x-z plane produces maximum field gradient ($\alpha_g$) at the horizon. The optimum parameters of the antenna under such conditions are such that the following conditions hold:

\[
\begin{align*}
|I_n| &= |I_{-n}| \\
\alpha_n &= -\alpha_{-n}, \\
\text{where } \alpha_n &\text{ is the angle of } I_n, \\
d_n &= |d_{-n}|
\end{align*}
\]

(1)

It is now assumed that each element or bay has a $\sin \theta$ - type of pattern in the z-x plane. Thus, the free space vertical plane complex pattern of the antenna may be written as follows:

\[
S(\theta) = \sin \theta \left[ |I_0| + 2 \sum_{n=1}^{2} |I_n| \cos (kd_n \cos \theta - \alpha_n) \right]
\]

(2)

The fundamental approximation made in Equation (2) is that the element pattern, i.e., the pattern of the 4-loop VOR array is $\sin \theta$ - type in the x-z plane and that it is the same in both carrier and side-band modes. Because of this Equation (2) indicates that the antenna has the same pattern in the carrier
and side band mode of operation. As discussed earlier (Sengupta and Chan, 1972), the slightly different nature of the side-band and carrier mode patterns can be taken into account by using the proper element pattern in Equation (2).
FREE SPACE PATTERNS

In this section we give the free space vertical plane radiation patterns of the Scanwell antenna. Figure 2 shows the free space pattern of the optimum Scanwell antenna. The excitation coefficients of the optimum antenna, as obtained by the Scanwell Laboratories, are as follows:

\[
\begin{align*}
|I_0| &= 1, \quad \alpha_0 = 0 \\
|I_1| &= 0.62, \quad \alpha_1 = 96^\circ.3 \\
|I_2| &= 0.19, \quad \alpha_2 = 108^\circ.9.
\end{align*}
\]

The element spacings in the optimum antenna are \(d_1 = \lambda/2, \quad d_2 = 3\lambda/2\) where \(\lambda\) is the operating wavelength. The field gradient obtained for the optimum antenna is \(\alpha_g \simeq 16 \text{ dB/}^\circ\).

We have also obtained free space patterns of the antenna for slightly different excitations but with the same element spacings. The following table gives the excitations and the corresponding field gradients obtained:

<table>
<thead>
<tr>
<th>(I_0)</th>
<th>(I_1)</th>
<th>(I_2)</th>
<th>(\alpha_0)</th>
<th>(\alpha_1)</th>
<th>(\alpha_2)</th>
<th>(\alpha_g)</th>
</tr>
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<tr>
<td>1</td>
<td>0.55</td>
<td>0.15</td>
<td>0</td>
<td>96^\circ.3</td>
<td>108^\circ.9</td>
<td>10.20 dB/^\circ</td>
</tr>
<tr>
<td>1</td>
<td>0.50</td>
<td>0.10</td>
<td>0</td>
<td>96^\circ.3</td>
<td>108.9</td>
<td>6.74 dB/^\circ</td>
</tr>
<tr>
<td>1</td>
<td>0.62</td>
<td>0</td>
<td>0</td>
<td>96.3</td>
<td>0</td>
<td>5.27 dB/^\circ</td>
</tr>
<tr>
<td>1</td>
<td>0.4</td>
<td>0.10</td>
<td>0</td>
<td>96^\circ.3</td>
<td>108.9</td>
<td>4.98 dB/^\circ</td>
</tr>
</tbody>
</table>
\[ d^2 = \frac{3\lambda^2}{\phi} \]

\[ f = 109 \text{ MHz} \]

\[ a_0 = 2, a_1 = 3.96, a_2 = 1089, b_0 = 0.62, b_1 = 0.10, b_2 = 0.10 \]

**FIG. 2:** Theoretical free space elevation plane radiation pattern of the optimum Scanwell antenna.
Figure 3 shows the complete patterns of the antenna for different excitations. It can be seen from Figure 3 that for the variation of the excitation used here, the main beam of the pattern is not affected appreciably. However, the field gradient as well as the minor lobe details are affected considerably by the change in the excitations. It should be noted that the standard 4-loop array pattern has a field gradient of about $3\text{dB}/6^\circ$ in both carrier and side-band mode of operation.
FIG. 3: Theoretical free space elevation plane radiation patterns of the Scanwell antenna for different excitations.
IV
SCANWELL ANTENNAS ABOVE GROUND

In this section we discuss the patterns of Scanwell antennas located above a perfectly conducting infinite planar ground. The far field patterns are obtained by using the following expression:

\[ S_T(\theta) = e^{-ikz \cos \theta} S(\theta) - e^{ikz \cos \theta} S(\pi - \theta), \quad (3) \]

\[ 0 \leq \theta \leq \pi/2, \]

where \( S(\theta) \) is given by Equation (2) and \( z \) is the height of the center element \( n = 0 \) above ground.

Complete patterns for the antenna have been computed by using Equation (3) for selected values of \( Z_1 \) in the range \( 0 \leq Z_1 \leq 500^\circ \). During this part of the computation the patterns have been calculated in the range \( 0 \leq \theta \leq \pi/2 \) at \( 1^\circ \) intervals. Some of the patterns for selected values of \( Z_1 \) are shown in Figures (4a–4d). Figures (5a–e) give the patterns of the same antenna in the range \( 80^\circ \leq \theta \leq 90^\circ \) and for different values of the heights. These patterns have been computed at \( 0.1^\circ \) intervals for sufficient accuracy. Figure 6 shows the positions of the few minima above horizon \( \theta = \pi/2 \) as functions of the antenna height.

Notice that the position of a minimum is expressed in angles above the horizon, i.e., above \( 90^\circ \). The curves given in Figure 6 are self-explanatory. The index
n in Figure 6 denotes the number of the minimum above the horizon; \( n = 1 \) is the minimum in the pattern closest to the horizon. Figure 7 shows the depths of the first few minima as a function of the antenna height. In general it can be said that the depths of a minimum increases continuously with \( Z_1 \). For a given \( Z_1 \), the minimum nearest to the horizon is deepest and the depth decreases with the order of the minimum. Figure 8 shows the variation of the filling factor versus height for the first few minima in the pattern obtained by using the optimum antenna as compared with those obtained with a standard VOR antenna when the two antennas are located at the same height. Figure 8 indicates that for each minimum initially the filling factor increases rapidly with height and then it approaches a constant value of about 10dB.

Figures 9, 10 and 11 show respectively the position of the minima, depths of the minima and the fillings factors as functions of height for the Scanwell antenna with \( \alpha = 10 \text{ dB/}^\circ \). The corresponding curves for \( \alpha = 6.74 \text{dB/}^\circ \) are given in Figures 12 - 14 and those for \( \alpha = 4.98 \text{dB/}^\circ \) are given in Figures 15 - 17.

In general, for a given \( Z_1 \), the depth of a minimum (say \( n = 1 \)) increases as \( \alpha \) decreases. This is as it should be. The positions of the minima appears to be independent of \( \alpha \) for \( Z_1 \geq 100' \). For \( Z_1 < 100' \), the positions of the minima increase as the gradient decreases. Most importantly, the saturation value of the filling factor decreases as the field gradient value \( \alpha \) decreases.
The depth of the first minimum and the filling factor of the first minimum are shown in Figure 18 as functions of $\alpha_g$ for the particular height $Z_1 = 200'$. 
FIG. 4a: Theoretical elevation plane pattern of the optimum Scanwell antenna above ground. 
\[ Z_1 = 15' \].
FIG. 4b: Theoretical elevation plane pattern of the optimum Scanwell antenna above ground. 
$Z_1 = 25'$. 
FIG. 4c: Theoretical elevation plane pattern of the optimum Scanwell antenna. 
$Z_1 = 50'$. 
FIG. 4d: Theoretical elevation plane pattern of the optimum Scanwell antenna. $Z_1 = 75'$. 
FIG. 5a: Theoretical elevation plane radiation pattern near the horizon for the optimum Scanwell antenna above ground. $Z_1 = 150'$. 
FIG. 5b: Theoretical elevation plane radiation pattern near the horizon for the optimum Scanwell antenna above ground. $Z_1 = 200'$. 
FIG. 5c: Theoretical elevation plane radiation pattern near the horizon for the optimum Scanwell antenna above ground. $Z_1 = 350'$.
FIG. 5d: Theoretical elevation plane radiation pattern near the horizon for the optimum Scanwell antenna above ground. $Z_1 = 400'$. 
FIG. 5e: Theoretical elevation plane radiation pattern near the horizon for the optimum Scanwell antenna above ground. 
$Z_1 = 450'$. 

FIG. 6: Positions of the elevation plane pattern minima versus height of the antenna above ground for the optimum Scanwell antenna (n = 1 is the minimum nearest to the horizon).
FIG. 7: Depths of the pattern minima versus height for the optimum Scanwell antenna.
FIG. 8: Filling factor as a function of height for the optimum Scanwell antenna.
FIG. 9: Positions of the elevation plane pattern minima versus height of the antenna above ground for the Scanwell antenna having $\alpha_g = 10.2\,\text{dB/6}^\circ$. 
FIG. 10: Depths of the pattern minima versus height for the Scanwell antenna having $\alpha_g = 10.2 \text{dB/}^\circ$. 
FIG. 11: Filling factor as a function of height for the Scanwell antenna having $\alpha = 10.2 \text{dB/6°}$. 
FIG. 12: Positions of the elevation plane pattern minima versus height of the antenna above ground for the Scanwell antenna having $\alpha_g = 6.74\text{dB}/6^\circ$. 
FIG. 13: Depths of the pattern minima versus height for the Scanwell antenna having $\alpha = 6.74 \text{dB/6}^\circ$. 

Depth of the minimum in dB

HEIGHT (FEET)
FIG. 14: Filling factor as a function of height for the Scanwell antenna having $\alpha_g = 6.74\text{dB/6}^\circ$. 
FIG. 15: Positions of the elevation plane pattern minima versus height of the antenna above ground for the Scanwell antenna having $\alpha_g = 4.98\text{dB/}^\circ$.
FIG. 16: Depths of the pattern minima versus height for the Scanwell antenna having $\sigma = 4.9805/60$. 

Depth of minimum in db
FIG. 17: Filling factor as a function of height for the Scanwell antenna having \( \alpha_g = 4.98 \text{dB/}^6\text{o} \).
FIG. 18: Depth of the first minimum and the filling factor for the first minimum as functions of the field gradient $\alpha_g$. 

Depth of First Minimum $Z_1 = 200'$
REFERENCES
