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Scattering from Two-Dimensional Bodies with Absorber Sheets

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SCATTERING FROM TWO-DIMENSIONAL BODIES
WITH ABSORBER SHEETS
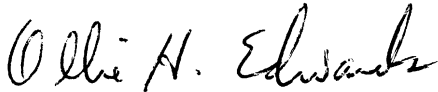
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FOREWORD

This Interim Report describes research performed by The University of Michigan Radiation Laboratory, 2455 Hayward Street, Ann Arbor, Michigan 48105, under USAF Contract F33615-73-C-1174, Project 7633, Task 7633-13, "Non-Specular Radar Cross Section Study." The research was sponsored by the Electromagnetic Division, Air Force Avionics Laboratory, and the Technical Monitor was Dr. Charles H. Kreuger, AFAL/WRP.

The computer program described in the report was developed during the time period 15 October 1973 through 15 March 1974 and was written by Dr. V. V. Liepa.

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(signature)

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ABSTRACT

This report describes a program that computes the far field scattering pattern of a two-dimensional cylindrical body (or bodies) treated with absorbing materials. The body surface is assumed to satisfy an impedance boundary condition and the absorber is modeled by equivalent electric and magnetic sheets. The program reduces the coupled integral equations governing the surface currents to a system of simultaneous linear equations, solves for the unknown currents, and then computes the far field pattern from the solution. The integral equations derived and presented in a previous interim report were used as the basis for preliminary versions of the program, but these equations have been found to be in error. The required correction consists of incorporating terms previously omitted and the corrected equations are presented and discussed. Careful attention is given to a description of the input data necessary to run the program and the results of a sample run are included for illustration.

The program was developed to explore the effects of absorbent materials on the scattering of electromagnetic waves by edges. Programs used previously in similar explorations embodied a surface impedance boundary condition; for lossy materials covering smooth surfaces of large radius of curvature, such an impedance can be estimated from the layer thickness and material properties, but the relationship breaks down near edges. The program described in this report, however, models the effects of actual materials rather than using the nebulous surface impedance boundary condition.

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I

INTRODUCTION

This report describes a computer program (RAMVS) that solves the two-dimensional integral equations for the currents induced upon the surfaces of cylindrical bodies and thin sheets. The fields on the surfaces of solid bodies are assumed to satisfy an impedance boundary condition and the sheets are assumed to be electrically or "magnetically" resistive. Electric and magnetic sheets can be superposed and, if their resistivities are properly chosen, can be used to model the effect of a layer of absorbing material having arbitrary permittivity and permeability. The program reduces the coupled integral equations to a system of simultaneous linear equations, solves for the unknown currents, and from these computes the far field scattering patterns.

RAMVS was developed as a tool to study methods of reducing non-specular scattering and has been used to compute the scattering patterns of simple shapes treated with resistive sheets. As such, it was always a single body that was specified on input. However, the program "understands" only two kinds of surfaces—thin resistive sheets and impedance surfaces—and cannot distinguish between a single body and multiple bodies, as long as all are identified with one class of surface. Thus, although RAMVS has not been used for multiple impedance surface scatterers, it well could have been. Similarly, the program views disjointed resistive sheets as a single surface even though several distinct sheets may be specified on input. These sheets need not be attached to the body or bodies, and in fact the physical separation could be hundreds of wavelengths if, for some reason, such a configuration were of interest.

The program allows the impedances and resistivities to vary from point to point along the profile according to a selection of mathematical distributions provided internally. Metallic bodies and sheets may be modeled by simply specifying a constant surface impedance or resistance of zero. Any combination of sheets and body profiles may be specified on input, but the configuration

must be symmetrical. Only half the profile should be specified because the program generates the other half by imaging about the plane of symmetry ($y = 0$). For running the program on a typical installation ($\sim 60\,K_{10}$ word core), the total perimeter of all surfaces should not exceed ten wavelengths or so. This provides for a sampling rate of 8 or 10 points per wavelength and a maximum of a 150×150 element matrix to be solved. Because of coupling between the integral equations, the maximum number of sampling points must include those on the resistive sheets twice; for example, a body having 100 sampling points, in the presence of a sheet with 25, would represent a total of 150 unknown current elements.

RAMVS is based upon the integral equations derived and presented in a previous Interim Report (Knott and Senior, 1974) and the pertinent equations are repeated and rearranged in a form more appropriate for numerical computation. Late in the development of the program, errors were discovered in the previous integral equation formulation, resulting in the omission of some terms. The necessary correction is discussed in Chapter II, and the corrected equations are given. Both the previous and corrected form of the equations contain higher order singularities which require special numerical treatment, and this treatment has been covered in a previous report.

A description of the code is given in Chapter III and provides the reader with an overview of the functioning of each subprogram. For readers who wish to examine the details of the programming, a source listing of RAMVS is given in the Appendix. A list of FORTRAN symbols may be found in Chapter IV to assist the reader in relating the program variables to the mathematical description of the integral equations.

Detailed instructions for the preparation of input data are presented in Chapter V and it is assumed that the user has some familiarity with FORTRAN data formats. The form of the input deck is illustrated by means of a specific example, and the preliminary calculations and decisions that must be made before the deck can be prepared are discussed. The required content of each

card is described and a sample run is displayed to show the results of the program for the specific geometry chosen for illustration.

RAMVS has been successfully run on the IBM 360/67 computer at The University of Michigan and on the CDC 6600 computer at WPAFB. It was tested for special cases, such as isolated impedance bodies and isolated dielectric strips, by means of comparison with other programs that have been verified, as well as with measured data. However, the program has not been thoroughly tested for general, absorber-clad bodies due to the lack of any data for comparison, and the user should be cautious and critical in surveying the computed results for such cases. The authors would appreciate learning about any comparisons of computed results with other data that either demonstrate the program's capability or that show its weaknesses.

II

MATHEMATICAL FORMULATION

We are here concerned with the problem of a plane electromagnetic wave incident on a structure consisting of a cylinder of arbitrary profile in the presence of a generalized resistive sheet of infinitesimal thickness. The entire structure is independent of the z coordinate of a Cartesian coordinate system (x, y, z) and the plane wave is incident in the xy plane with either the electric or magnetic vector in the z direction, corresponding to E- or H-polarization respectively. The problem is therefore two-dimensional and the geometry is depicted in Fig. 2-1.

The profile of the (closed) cylinder is designated C_2 and here an impedance boundary condition is imposed. The surface impedance η is arbitrary and can vary as a function of position. The resistive sheet profile is designated C_1 and may consist of several disjoint parts, each representing an individual sheet. The boundary conditions at the sheet are specified by the bulk permittivity ϵ_r, the bulk permeability μ_r and the thickness Δ of the material being modelled. Special cases are those of electric or magnetic resistivities alone.

RAMVS is a computer program for the digital solution of the coupled integral equations for the currents induced in the structure, from which the scattered fields are then determined. The integral equations were derived in a previous technical report (Knott and Senior, 1974), but owing to an error, some of the equations presented there are invalid for the type of sheet now considered. We shall therefore begin by detailing the nature of this error.

2.1 Amended Sheet Equations

In the study of the integral equations for thin sheets which are electrically or magnetically resistive, or which have an impedance boundary condition imposed at the surface, an analytical error was made in Knott and Senior (1974) which invalidates the results for simple nonplanar sheets and for multi-sheets which are not co-planar.

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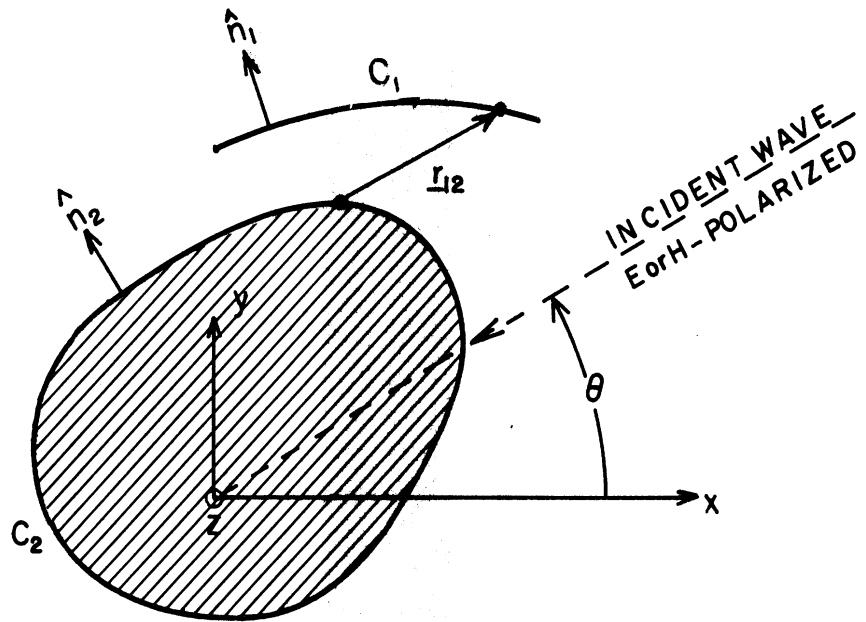
FIG. 2-1: Geometry for the composite scatterer.

C_1: Absorber sheet, described by ϵ_r, μ_r and the thickness of the material

C_2: Impedance body described by arbitrary surface impedance η.

The error first appeared in the reduction of eq. (2.24) for a thin impedance boundary condition sheet with H-polarization. Subtraction of the two limits as the observation point $\underline{\rho}$ approaches the sheet from above and below does, indeed, produce an identity for $J_2(s)$, but on addition we have

$$H_z^i(s) = \frac{1}{2} J_1(s) + \frac{1}{4} \int_{C_+} \eta(s')J_1(s')H_0^{(1)}(kr)d(ks') - \frac{i}{4} \int_{C_+} J_2(s')(\hat{n}' \cdot \hat{r})H_1^{(1)}(kr)d(ks').$$

$$(2.27':2.1)$$

This differs from the equation originally obtained due to the presence of the second integral. As indicated by the slash, this is a Cauchy principal value integral, since the self-cell contributions cancel on summing the limits. Similarly, eq. (2.29) must be replaced by

$$Y_0 E_s^i(s) = \frac{1}{2} \eta(s)J_2(s) + \lim_{\underline{\rho} \to C_+} \frac{1}{4k} \frac{\partial}{\partial n} \int_{C_+} J_2(s')(\hat{n}' \cdot \hat{r})H_1^{(1)}(kr)d(ks')$$

$$- \frac{i}{4} \int_{C_+} \eta(s')J_1(s')(\hat{n} \cdot \hat{r})H_1^{(1)}(kr)d(ks')$$

$$(2.29':2.2)$$

where the normals are all with respect to the upper surface.

The additional terms in (2.27') and (2.29') vanish identically for a single planar sheet since then $\hat{n} \cdot \hat{r} = \hat{n}' \cdot \hat{r} = 0$. In this particular case, eqs. (2.27) and (2.29) are correct as they stand, and are decoupled integral equations for $J_1(s)$ and $J_2(s)$ respectively. In all other cases, however, the equations are coupled and both must be solved to determine even the total electric current, proportional to $J_2(s)$.

The analogous equations for E-polarization can be obtained using duality and are

$$Y_0 E_z^i(s) = \frac{1}{2}\eta(s)J_3(s) + \frac{1}{4}\int_{C_+} J_3(s')H_0^{(1)}(kr)d(ks')$$

$$-\frac{iY_0}{4}\int_{C_+} J_4(s')(\hat{n}'\cdot\hat{r})H_1^{(1)}(kr)d(ks') \qquad (2.32':2.3)$$

and

$$Z_0 H_s^i(s) = -\frac{1}{2\eta(s)}J_4(s) - \lim_{\rho\to C_+}\frac{1}{4k}\frac{1}{\partial n}\int_{C_+} J_4(s')(\hat{n}'\cdot\hat{r})H_1^{(1)}(kr)d(ks')$$

$$+\frac{iZ_0}{4}\int_{C_+} J_3(s')(\hat{n}\cdot\hat{r})H_1^{(1)}(kr)d(ks') \qquad (2.33':2.4)$$

where

$$J_3(s) = \frac{Y_0}{\eta(s)}(E_z^+ + E_z^-) \quad,$$

$$J_4(s) = E_z^+ - E_z^- \quad.$$

The added terms in (2.32') and (2.33') couple the equations, but vanish for a single planar sheet. For this reason, the later equations in Chapter II of Knott and Senior (1974) are correct as they stand.

These same errors recur in Sections 4.2 and 4.3 where generalized (i.e., multiple) resistive sheets are considered, but do not affect any of the equations or discussion through page 33 in Chapter IV. However, eqs. (4.32) and (4.33) for a generalized resistive sheet with E-polarization are in error by the absence of the coupling terms and should read

$$Z_0 H_s^i(s) = Z_0 R^*(s) J_s^*(s) + \lim_{\rho \to C} \frac{1}{4k} \frac{\partial}{\partial n} \int_C J_s^*(s')(\hat{n}' \cdot \hat{r}) H_1^{(1)}(kr) d(ks')$$

$$+ \frac{iZ_0}{4} \oint_C J_z(s')(\hat{n} \cdot \hat{r}) H_1^{(1)}(kr) d(ks') \quad ,$$

$$\text{(4.32': 2.5)}$$

$$Y_0 E_z^i(s) = Y_0 R(s) J_2(s) + \frac{1}{4} \int_C J_z(s') H_0^{(1)}(kr) d(ks')$$

$$+ \frac{iY_0}{4} \oint_C J_s^*(s')(\hat{n}' \cdot \hat{r}) H_1^{(1)}(kr) d(ks') \quad .$$

$$\text{(4.33': 2.6)}$$

These terms must also be introduced into eqs. (4.43) and (4.44) for a generalized resistive sheet in the presence of an imperfectly conducting body, and the equations then become

$$Y_0 E_z^i(s_1) = Y_0 R(s_1) J_z(s_1) + \frac{1}{4} \int_{C_1} J_z(s') H_0^{(1)}(kr_{11}) d(ks')$$

$$+ \frac{iY_0}{4} \oint_{C_1} J_s^*(s')(\hat{n}' \cdot \hat{r}_{11}) H_1^{(1)}(kr) d(ks') + \frac{1}{4} \int_{C_2} K_z(s') H_0^{(1)}(kr_{12}) d(ks')$$

$$- \frac{i}{4} \int_{C_2} \eta(s') K_z(s')(\hat{n}' \cdot \hat{r}_{12}) H_1^{(1)}(kr_{12}) d(ks') \quad , \qquad \text{(4.43': 2.7)}$$

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$$H_s^i(s_1) = R^*(s_1)J_s^*(s_1) + \lim_{\rho \to \rho_1} \frac{Y_0}{4k} \frac{\partial}{\partial n_1} \int_{C_1} J_s^*(s')(\hat{n}' \cdot \hat{r}_{01})H_1^{(1)}(kr_{01})d(ks')$$

$$+ \frac{i}{4} \int_{C_1} J_z(s')(\hat{n}_1 \cdot \hat{r}_{11})H_1^{(1)}(kr_{11})d(ks') + \frac{i}{4} \int_{C_2} K_z(s')(\hat{n}_1 \cdot \hat{r}_{12})H_1^{(1)}(kr_{12})d(ks')$$

$$- \frac{i}{4k} \int_{C_2} \eta(s')K_z(s') \frac{\partial}{\partial n_1} \left\{ (\hat{n}' \cdot \hat{r}_{12})H_1^{(1)}(kr_{12}) \right\} d(ks') . \qquad (4.44' : 2.8)$$

We note in passing that the incorrect eq. (4.42) is the root of all these errors.

The set of three equations necessary for the complete solution of the problem consists of (4.43'), (4.44') and one of (4.45) and (4.46). For reasons stated previously, the more convenient combination is (4.43'), (4.44') and (4.45). The additional terms in (4.43') and (4.44') vis-a-vis (4.43) and (4.44) vanish if

$$\hat{n} \cdot \hat{r}_{11} = \hat{n}' \cdot \hat{r}_{11} = 0$$

i.e., all the sheets are co-planar. Until recently, this was the only case where the equations had been studied in detail and numerical data obtained. For a single electrically resistive sheet ($R^* = \infty$, implying $J_s^* \equiv 0$) and a perfectly conducting body ($\eta = 0$), eq. (4.43') reduces to (4.47), so that the equations (4.47) and (4.43) used in program REST are not affected by the error.

2.2 The Required Equations

The integral equations used in RAMVS for E-polarization are (2.7) and (2.8) above and eq. (4.45) of Knott and Senior (1974). To facilitate the programming, it is convenient to rearrange the equations and to change the notation slightly. The equations then read

$$Y_0 E_z^i(s_1) = Z_e(s_1)J_z(s_1) + \frac{1}{4} \int_{C_1} J_z(s')H_0^{(1)}(kr)d(ks')$$

$$+ \frac{iY_0}{4} \oint_{C_1} J_s^*(s')(\hat{n}' \cdot \hat{r})H_1^{(1)}(kr)d(ks')$$

$$+ \frac{1}{4} \int_{C_2} K_z(s')H_0^{(1)}(kr)d(ks')$$

$$- \frac{i}{4} \int_{C_2} \eta(s')K_z(s')(\hat{n}' \cdot \hat{r})H_1^{(1)}(kr)d(ks') \quad , \tag{2.7}$$

$$H_s^i(s_1) = \frac{i}{4} \oint_{C_1} J_z(s')(\hat{n} \cdot \hat{r})H_1^{(1)}(kr)d(ks') + Z_m(s_1)J_s^*(s_1)$$

$$+ \lim_{\rho \to \rho_1} \frac{Y_0}{4k} \frac{\partial}{\partial n_1} \int_{C_1} J_s^*(s')(\hat{n}' \cdot \hat{r})H_1^{(1)}(kr)d(ks')$$

$$+ \frac{i}{4} \int_{C_2} K_z(s')(\hat{n} \cdot \hat{r})H_1^{(1)}(kr)d(ks')$$

$$- \frac{1}{4k} \int_{C_2} \eta(s')K_z(s') \frac{\partial}{\partial n_1} \left\{ (\hat{n}' \cdot \hat{r})H_1^{(1)}(kr) \right\} d(ks') \quad , \tag{2.8}$$

$$Y_0 E_z^i(s_2) = \frac{1}{4} \int_{C_1} J_z(s')H_0^{(1)}(kr)d(ks') + \frac{iY_0}{4} \int_{C_1} J_s^*(s')(\hat{n}' \cdot \hat{r})H_1^{(1)}(kr)d(ks')$$

$$+ \frac{1}{2} \eta(s_2)K_z(s_2) + \frac{1}{4} \int_{C_2} K_z(s')H_0^{(1)}(kr)d(ks')$$

$$- \frac{i}{4} \int_{C_2} \eta(s')K_z(s')(\hat{n}' \cdot \hat{r})H_1^{(1)}(kr)d(ks') \ . \tag{2.9}$$

The analogous equations for H-polarization can be obtained using duality. This requires the application of the transformation

$$E_z^i \longrightarrow H_z^i \ , \qquad H_s^i \longrightarrow -E_s^i \ , \qquad Y_0 \longleftrightarrow Z_0 \ ,$$

$$J_z \longrightarrow J_z^* \ , \qquad J_s^* \longrightarrow -J_s \ , \qquad \eta \longrightarrow 1/\eta \ ,$$

$$K_z \longrightarrow K_z^* = \eta Z_0 K_s \ , \qquad Z_e \longleftrightarrow Z_m$$

to the above equations and gives

$$H_z^i(s_1) = Z_m(s_1)Y_0 J_z^*(s_1) + \frac{Y_0}{4} \int_{C_1} J_z^*(s')H_0^{(1)}(kr)d(ks')$$

$$- \frac{i}{4} \int_{C_1} J_s(s')(\hat{n}' \cdot \hat{r})H_1^{(1)}(kr)d(ks') + \frac{1}{4} \int_{C_2} \eta(s')K_s(s')H_0^{(1)}(kr)d(ks')$$

$$- \frac{i}{4} \int_{C_2} K_s(s')(\hat{n}' \cdot \hat{r})H_1^{(1)}(kr)d(ks') \ , \tag{2.10}$$

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$$-Y_0 E_s^i(s_1) = - Z_e(s_1) J_s(s_1) + \frac{iY_0}{4} \oint_{C_1} J_z^*(s')(\hat{n} \cdot \hat{r}) H_1^{(1)}(kr) d(ks')$$

$$- \lim_{\rho \to \rho_1} \frac{1}{4k} \frac{\partial}{\partial n_1} \int_{C_1} J_s(s')(\hat{n}' \cdot \hat{r}) H_1^{(1)}(kr) d(ks')$$

$$+ \frac{i}{4} \int_{C_2} \eta(s') K_s(s')(\hat{n} \cdot \hat{r}) H_1^{(1)}(kr) d(ks')$$

$$- \frac{1}{4k} \int_{C_2} K_s(s') \frac{\partial}{\partial n_1} \left\{ (\hat{n}' \cdot \hat{r}) H_1^{(1)}(kr) \right\} d(ks') \quad , \tag{2.11}$$

$$H_z^i(s_2) = \frac{Y_0}{4} \int_{C_1} J_z^*(s') H_0^{(1)}(kr) d(ks') - \frac{i}{4} \int_{C_1} J_s(s')(\hat{n}' \cdot \hat{r}) H_1^{(1)}(kr) d(ks')$$

$$+ \frac{1}{2} K_s(s_2) + \frac{1}{4} \int_{C_2} \eta(s') K_s(s') H_0^{(1)}(kr) d(ks')$$

$$- \frac{i}{4} \oint_{C_2} K_s(s')(\hat{n}' \cdot \hat{r}) H_1^{(1)}(kr) d(ks') \quad . \tag{2.12}$$

The three equations in each set are coupled by the unknown currents: $J_z(s)$ or $J_s(s)$, and $J_s^*(s)$ or $J_z^*(s)$, representing the total electric or magnetic currents borne by the resistive sheet, and $K_z(s)$ or $K_s(s)$ representing the surface electric current density on the cylinder. The normalized electric and magnetic sheet impedances are defined as

$$Z_e = \frac{i}{2\pi(\epsilon_r - 1)\Delta/\lambda} \quad , \tag{2.13}$$

$$Z_m = \frac{i}{2\pi(\mu_r - 1)\Delta/\lambda} \quad , \tag{2.14}$$

where Δ is the thickness of the sheet and λ is the free space wavelength. These parameters are used in preference to the resistivities R and R^* of Knott and Senior (1974) since they are of order unity for values of ϵ_r, μ_r and Δ/λ of practical interest. In addition, they are dimensionless and in this respect are compatible with the surface impedance η of the cylinder.

Since the two sets of equations are related via the duality transformation, it is not surprising that the corresponding equations are mathematically similar. We can therefore describe the manner in which the various integrals are handled by using the equations for E-polarization as an example.

2.3 Numerical Procedures

To develop the computer code, and especially the sub-programs for generating the matrix elements, it is convenient to write the E-polarized equations as

$$Y_0 E_z^i(s_1) = A_1 J_z(s_1) + A_2 J_s^*(s_1) + A_3 K_z(s_2) \quad , \tag{2.15}$$

$$H_s(s_1) = A_4 J_z(s_1) + A_5 J_s^*(s_1) + A_6 K_z(s_2) \quad , \tag{2.16}$$

$$Y_0 E_z^i(s_2) = A_7 J_z(s_1) + A_8 J_s^*(s_1) + A_9 K_z(s_2) \quad , \tag{2.17}$$

where the A's are integral operators whose specific forms are evident from eqs. (2.7) through (2.9). Thus, for example,

$$A_1 J_z(s_1) = Z_e(s_1) J_z(s_1) + \frac{1}{4} \int_{C_1} J_z(s') H_0^{(1)}(kr) d(ks') \quad ,$$

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$$A_2 J_s^*(s_1) = \frac{iY_0}{4} \oint_{C_1} J_s^*(s')(\hat{n}' \cdot \hat{r}) H_1^{(1)}(kr)d(ks') \quad ,$$

and so on.

The standard practice for evaluating the integrals is to divide each range of integration into segments (or cells) and compute their contributions assuming a specific type of variation of the unknown current over the cells. In prior programs we have, for simplicity, used the elementary "flat-top" approach in which the current is assumed constant over a cell. The contribution from each cell is then proportional to the sampled value of the current at its mid-point. However, several of the integral operators have singular kernels and the "self cell" containing the point of observation then requires individual treatment. For all except one of the singularities, the method that we employ is relatively straightforward, but in the exceptional case, the higher order singularity involved necessitates a special treatment for adjacent cells in addition to the self cell. The reasons for this are described in Knott and Senior (1974; Appendix A) where the details of the treatment can be found.

We now examine in turn each of the integral operators A_1 through A_9 and show how the numerical computations are performed. The kernel of the operator A_1 has a logarithmic singularity, and by approximating the zero order Hankel function, the self cell contribution can be evaluated analytically. The result is

$$A_1 J_z(s_1) = \left\{ Z_e(s_1) + \frac{\Delta(s_1)}{\lambda} \left[\frac{\pi}{2} + i \left(\ell n \ \frac{\Delta(s_1)}{\lambda} + 0.02879837 \right) \right] \right\}_\Delta J_z(s_1)$$

$$+ \int_{\pm 2\Delta} J_z(s')H_0^{(1)}(kr)d(ks') + \int_{C_1 - (\Delta \mp 2\Delta)} J_z(s_1')H_0^{(1)}(kr)d(ks') \quad , \quad (2.18)$$

where the bracketed terms represent contributions by the self cell. The first integral on the right side is evaluated using a three-point Simpson's rule

$$\int_a^b f(x)dx = \frac{b-a}{6}\left\{ f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right\}$$

and the integral over $C_1 - (\Delta \overset{+}{-} 2\Delta)$ represents the contribution from the remainder of the surface C_1. The integral is actually coded as a discrete summation over all the cells $i \neq j$, but we have found it just as convenient (and less confusing) to refer to it as an integral in the coding process.

The integral in A_2 is a Cauchy principal value, and has no self cell. Since

$$H_1^{(1)}(kr) = -\frac{\partial}{\partial(kr)} H_0^{(1)}(kr) \qquad \text{and} \qquad \frac{\partial}{\partial(kr)} = (\hat{s}' \cdot \hat{r})\frac{\partial}{\partial(ks')} \quad ,$$

$$A_2 J_s^*(s_1) = \frac{iY_0}{4} \int_{C_1 - \Delta} J_s^*(s')(\hat{n}' \cdot \hat{r})H_1^{(1)}(kr)d(ks')$$

can be reduced to the form

$$A_2 J_s^*(s_1) = -\frac{iY_0}{4} \int_{\overset{+}{-}2\Delta} \left| J_s^*(s')(\hat{n}' \cdot \hat{r})(\hat{s}' \cdot \hat{r}) \frac{\partial}{\partial(ks')}\left\{ H_0^{(1)}(kr) \right\} d(ks') \right.$$

$$+ \frac{iY_0}{4} \int_{C_1 - (\Delta \overset{-}{+} 2\Delta)} J_s^*(s')(\hat{n}' \cdot \hat{r})H_1^{(1)}(kr)d(ks') \quad .$$

$$(2.19)$$

The first integral is evaluated over each cell as

$$-\frac{iY_0}{4} J_s^*(s_j)(\hat{n}_j \cdot \hat{r}_{ij})(\hat{s}_j \cdot \hat{r}_{ij})H_0^{(1)}(kr) \Bigg|_{a(\Delta_j)}^{b(\Delta_j)} \quad ,$$

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where $a(\Delta_j)$ and $b(\Delta_j)$ are the endpoints of the source cell j, and the second integral is evaluated as in (2.18).

In A_3 the integration is over the surface C_2 with the observation point on C_1. There is again no self cell and the integral

$$A_3 K_z(s_2) = \frac{1}{4} \int_{C_2} K_z(s') \left\{ H_0^{(1)}(kr) - i\eta(s')(\hat{n}' \cdot \hat{r}) H_1^{(1)}(kr) \right\} d(ks') \qquad (2.20)$$

can be evaluated directly. The operator

$$A_4 J_z(s_1) = \frac{i}{4} \int_{C_1 - \Delta} J_z(s')(\hat{n} \cdot \hat{r}) H_1^{(1)}(kr) d(ks')$$

is treated similarly to A_2, giving

$$A_4 J_z(s_1) = -\frac{i}{4} \int_{\pm 2\Delta} J_z(s')(\hat{n} \cdot \hat{r})(\hat{s}' \cdot \hat{r}) \frac{\partial}{\partial(ks')} \left\{ H_0^{(1)}(kr) \right\} d(ks')$$

$$+ \frac{i}{4} \int_{C_1 - (\Delta \mp 2\Delta)} J_z(s')(\hat{n} \cdot \hat{r}) H_1^{(1)}(kr) d(ks') . \qquad (2.21)$$

The singularity in

$$A_5 J_s^*(s_1) = Z_m(s_1) J_s^*(s_1) + \lim_{\rho \to \rho_1} \frac{Y_0}{4k} \frac{1}{\partial n_1} \int_{C_1} J_s^*(s_1)(\hat{n}' \cdot \hat{r}) H_1^{(1)}(kr) d(ks') ,$$

however, is the most difficult one. The differentiation with respect to the normal effectively increases (by $1/r$) the order of the singularity of the kernel, but since the same type of operator also occurs in the integral equation for an electrically resistive sheet with H-polarization, we can use the representation previously developed (Knott and Senior, 1974) for that equation. We then have

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$$A_5 J_s^*(s_1) = \left\{ Z_m(s_1) + \frac{iY_0}{\pi^2 \Delta/\lambda} + Y_0 \frac{\Delta(s_1)}{\lambda} \left[\frac{\pi}{2} + i\left(\ell n \frac{\Delta(s_1)}{\lambda} + 0.02879837 \right) \right] \right\} J_s^*(s_1) \Big|_\Delta$$

$$+ \frac{Y_0}{4} \int_{\pm 2\Delta} J_s^*(s') \left\{ (\hat{n}' \cdot \hat{n}) H_0^{(1)}(kr) + \frac{\partial}{\partial(ks')} \left[(\hat{s} \cdot \hat{r}) H_1^{(1)}(kr) \right] \right\} d(ks')$$

$$+ \frac{Y_0}{4} \int_{C_1 - (\Delta \mp 2\Delta)} J_s^*(s') \left\{ (\hat{n}' \cdot \hat{r})(\hat{n} \cdot \hat{r}) H_0^{(1)}(kr) \right.$$

$$\left. + \left[(\hat{s}' \cdot \hat{r})(\hat{s} \cdot \hat{r}) - (\hat{n}' \cdot \hat{r})(\hat{n} \cdot \hat{r}) \right] \frac{H_1^{(1)}(kr)}{kr} \right\} d(ks') \,, \quad (2.22)$$

where the terms in brackets are produced by the self cell, the next integral is the contribution from the two adjoining cells on either side of the self cell and the last integral is the contribution from the remainder of C_1. In the computer code the first and third parts on the right side of (2.22) are programmed as shown, but the second part is expanded further. The contribution of a single source cell Δ_j to an observation cell Δ_i, $i \neq j$, can be written as

$$\frac{Y_0}{4} J_s^*(s_j)(\hat{n}_j \cdot \hat{n}_i) \int_{\Delta_j} H_0^{(1)}(kr) d(ks') + \frac{Y_0}{4} J_s^*(s_j)(\hat{s}_i \cdot \hat{r}_{ij}) H_1^{(1)}(kr) \Big|_{a(\Delta_j)}^{b(\Delta_j)} ,$$

$$(2.23)$$

where $a(\Delta_j)$ and $b(\Delta_j)$ are the endpoints of the jth cell. The integral over Δ_j is evaluated by a three-point Simpson's rule.

The operator A_6 is relatively simple since the observation point is on C_1 and the source point on C_2. Only the derivative factor needs expansion and this gives

$$A_6 K_z(s_2) = -\frac{1}{4k} \int_{C_2} \eta(s') K_z(s') \left\{ (\hat{n}' \cdot \hat{r})(\hat{n} \cdot \hat{r}) H_0^{(1)}(kr) \right.$$

$$\left. + \left[(\hat{s}' \cdot \hat{r})(\hat{s} \cdot \hat{r}) - (\hat{n}' \cdot \hat{r})(\hat{n} \cdot \hat{r}) \right] \frac{H_1^{(1)}(kr)}{kr} \right\} d(ks') \,. \quad (2.24)$$

The operators A_7 and A_8 are programmed without modification:

$$A_7 J_z(s_2) = \frac{1}{4} \int_{C_1} J_z(s')H_0^{(1)}(kr)d(ks') \,, \tag{2.25}$$

$$A_8 J_s^*(s_2) = \frac{iY_0}{4} \int_{C_1} J_s^*(s')(\hat{n}' \cdot \hat{r})H_1^{(1)}(kr)d(ks') \,, \tag{2.26}$$

and with the final operator A_9, the self cell contribution must be removed from the first of the two integrals. The procedure is directly analogous to that for A_1 and yields

$$A_9 K_z(s_2) = \left\{ \frac{1}{2}\eta(s_2) + \frac{\Delta(s_2)}{\lambda}\left[\frac{\pi}{2} + i\left(\ell n \ \frac{\Delta(s_2)}{\lambda} + 0.02879837\right)\right] \right\} K_z(s_2)\bigg|_\Delta$$

$$+ \frac{1}{4} \int_{C_2-\Delta} K_z(s')\left\{H_0^{(1)}(kr) - i\eta(s')(\hat{n}' \cdot \hat{r})H_1^{(1)}(kr)\right\}d(ks') \,. \tag{2.27}$$

The expressions for the far field scattering can be deduced from eqs. (2.7) and (2.9) for E-polarization and (2.10) and (2.12) for H-polarization by replacing the Hankel functions by their asymptotic expansions for large argument. Each of these four equations has the form

$$F_z^i = F_z^t - F_z^s$$

where F represents either $Y_0 E_z$ or H_z depending on the polarization and the superscript t denotes the total field. Since F_z^s is simply the negative of the integral terms in the equations, a far field value can be obtained by selecting those integrals representing C_2 contributions observed on C_1 or C_1 contributions observed on C_2. It then follows that for E-polarization

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$$Y_0 E_z^s(\rho, \theta) \sim \sqrt{\frac{2}{\pi k \rho}} \, e^{ik\rho - i\frac{\pi}{4}} \frac{1}{4} \left\{ - \int_{C_1} J_z(s') \, e^{-ik(x'\cos\theta + y'\sin\theta)} d(ks') \right.$$

$$- Y_0 \int_{C_1} J_s^*(s')(\hat{n}' \cdot \hat{r}) e^{-ik(x'\cos\theta + y'\sin\theta)} d(ks')$$

$$\left. - \int_{C_2} K_z(s') \left\{ 1 - \eta(s')(\hat{n}' \cdot \hat{r}) \right\} e^{-ik(x'\cos\theta + y'\sin\theta)} d(ks') \right\} \quad , \qquad (2.28)$$

and for H-polarization

$$H_z^s(\rho, \theta) \sim \sqrt{\frac{2}{\pi k \rho}} \, e^{ik\rho - i\frac{\pi}{4}} \frac{1}{4} \left\{ -Y_0 \int_{C_1} J_z^*(s') \, e^{-ik(x'\cos\theta + y'\sin\theta)} d(ks') \right.$$

$$+ \int_{C_1} J_s(s')(\hat{n}' \cdot \hat{r}) e^{-ik(x'\cos\theta + y'\sin\theta)} d(ks')$$

$$\left. - \int_{C_2} K_s(s') \left\{ \eta(s') - (\hat{n}' \cdot \hat{r}) \right\} e^{-ik(x'\cos\theta + y'\sin\theta)} d(ks') \right\} \quad , \qquad (2.29)$$

where θ is the scattering direction measured anticlockwise from the positive x axis and $\hat{r}$ is a unit vector to the observation point.

The two-dimensional scattering cross sections are

$$\frac{\sigma(\theta, \phi)}{\lambda} = \lim_{\rho \to \infty} \frac{2\pi\rho}{\lambda} \left| \frac{Y_0 E_z^s(\theta, \phi)}{Y_0 E_z^i(\phi)} \right|^2 \qquad (2.30)$$

and

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$$\frac{\sigma(\theta,\phi)}{\lambda} = \lim_{\rho \to \infty} \frac{2\pi\rho}{\lambda} \left| \frac{H_z^s(\theta,\phi)}{H_z^i(\phi)} \right|^2 \qquad (2.31)$$

where ϕ is the angle of incidence. These show why it is convenient to choose

$$\left| Y_0 E_z^i(\theta) \right| = \left| H_z^i(\theta) \right| = 1 \quad .$$

PROGRAM DESCRIPTION

RAMVS is based in eqs. (2.7) through (2.12) and uses the particular
mathematical expansions given by eqs. (2.18) through (2.31). There are three
variables to be determined: the total electric and magnetic currents $\underline{J}$ and $\underline{J}^*$,
respectively, carried by the absorber sheet, and a surface current $\underline{K}$ on the
impedance surface. If M is the number of cells used for the absorber sheet,
and K is the number of cells used on the impedance surface, the total number
of unknowns, and hence the number of linear simultaneous equations to be solved,
is $2M + K$. This is (approximately) thrice the unknowns encountered in cross
section computations for impedance cylinders (program RAMD) or resistive
sheets (program REST). (RAMD is discussed by Knott and Senior (1974) and
REST by Knott et al. (1973).) It is therefore important that in developing pro-
gram RAMVS consideration is given to writing a program with the least abuse
of computer core space.

Dimensioning has been simplified by structuring the program in modules,
with MAIN used only for starting the program and allocating core space. The
user of the program must provide appropriate dimensioning only in MAIN.*

The FORTRAN source program consists of 11 modules (a MAIN program
and 10 subprograms) and contains about 850 statements, including comment
cards. The structure of the program is shown in Fig. 3-1, where each block
represents a particular module. The arrows joining these blocks refer to a
typical execution sequence, discussed in the latter part of this section. The

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* When linear vectors are dimensioned in the MAIN program and are passed as
arguments in the subroutine calling sequence, the dimension of the vector in the
subroutine is arbitrary. For two-dimensional vectors, the size of the first
dimension entry is passed in the calling sequence along with the vector, while
the second entry is arbitrary. Thus in program RAMVS the dimension statements
occurring in subroutines are of the form A(1) or B(M, 1).
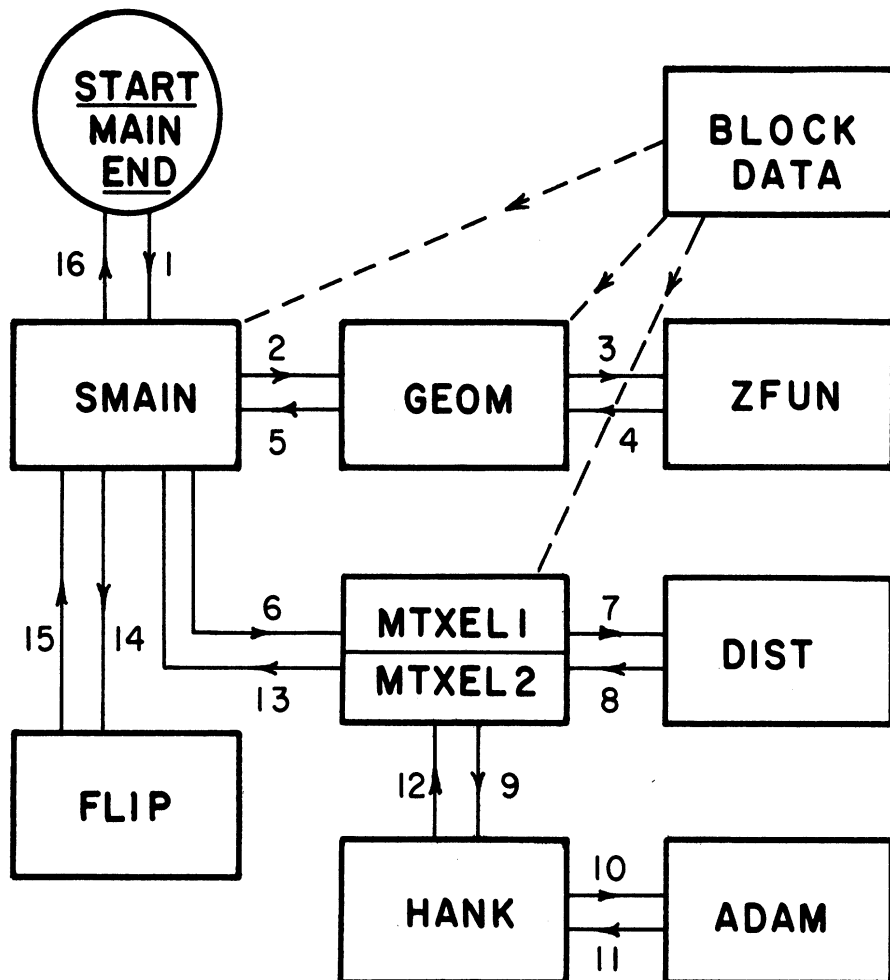
FIG. 3-1: Block diagram and flow graph for the program.

purpose of the modules is as follows.

MAIN   This is a short program (7 FORTRAN statements) and its sole purpose
is to start the program and allocate the core space for the dominant vectors
(dimensioning of arrays). Whenever vectors must be redimensioned, only the
MAIN program need be changed.

SUBROUTINE SMAIN   MAIN calls subroutine SMAIN, a control program that
also performs other operations. These include the reading of control statements,
the setting of appropriate controls for the type of computation required, and the
calling of various subroutines. SMAIN calls a subroutine GEOM for computation
of geometrical and electrical parameters of the scatterer, then calls subroutine
MTXEL to generate the matrix and calls subroutine FLIP to invert the matrix
and compute the unknown (current) vector. Computations of scattering cross
sections and the printing of most of the output is also done in SMAIN.

SUBROUTINE GEOM   GEOM, as the name implies, reads the input data
specifying the geometry and the electrical characteristics of the scatterer and
from these generates point-by-point descriptions of the body required for solving
the integral equations. The computations include the position of each cell, the
components of the surface normal, the cell width, and the electrical parameters
for the cell.

The technique used for reading in the data is the same as used in our
other two-dimensional scattering programs, such as RAMD and REST, developed
under this contract. It is assumed that the profile of a two-dimensional scatterer
can be described by an assembly of circular arcs (a line segment is an arc of very
large radius) and that the body is symmetric about the plane $y = 0$. Thus, it is
sufficient to specify only half of the body and let the computer generate the image.
For the special case of a horizontal line (strip) lying in the $y = 0$ plane, no image
is generated.

In GEOM a segment of circular arc is specified by the coordinates of the
endpoints and the angle subtended by the arc as measured at the center of the
generating circle. While cell coordinates and other parameters are being found,
a running distance along the surface is also computed for each line segment.

This distance is used as an independent variable in establishing the surface impedance or absorber sheet impedance distributions along the surface.

COMPLEX FUNCTION ZFUN   This subprogram computes a surface impedance distribution according to one of the following mathematical relations:

$$Z = (a - bs)^c , \quad *$$

$$Z = a + bs^c ,$$

$$Z = a + be^{-cs} ,$$

where a and b are complex constants and c is a real constant. The distance along the surface of the line segment is zero at the first (input) endpoint and increases toward the other. Thus, if the left-most endpoint of a line segment is read in first, the distance is computed from left-to-right or, if the right-most endpoint is read in first, the distance is computed from right-to-left along the particular segment. This adds additional flexibility to the possible impedance variations available from the three expressions given above.

SUBROUTINE MTXEL1 AND SUBROUTINE MTXEL2   Once the geometrical and electrical parameters have been furnished by GEOM and ZFUN for each cell position, matrix elements are generated either by MTXEL1 or MTXEL2, depend- ing on whether the incident wave is E- or H-polarized, respectively. The two subroutines are quite similar structurally, and in the earlier development of the code a single subroutine was used with numerous IF statements to differentiate between the two polarizations. The resultant program was elegant but confusing, and for the sake of simplicity and computational efficiency, the subroutine was replaced by two separate codes.

In typical numerical solutions of electromagnetic scattering and antenna problems, the matrix possesses certain symmetries that can be exploited in the matrix generation, and even in solving for the unknown vector. In the process of development of RAMVS, various computational possibilities were examined that

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* In the FORTRAN code this equation is written as $Z = \exp \left[c \cdot \log_e (a - bs) \right]$.

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would reduce the core storage and the computational time. Because of the coupled integral equations which are involved here, the matrix defined by eqs. (2.15) through (2.17) does not, in general, have explicit symmetries associated with typical scatterers that could be readily exploited to save appreciable machine time. To keep the code relatively simple and to reduce programming errors, the matrix elements are computed in a relatively straightforward fashion, with the following modification. Since the electric and the magnetic sheets are physically superimposed and therefore have the same geometrical parameters, the computation of radii between the cells, the Hankel functions, and the dot products need not be duplicated. If A is the entire matrix and $A_1, \ldots, A_9$ the submatrices as defined in Chapter II, the matrix elements are computed simultaneously in A_1, A_2, A_4 and A_5, in A_3 and A_6, and in A_7 and A_8.

SUBROUTINE DIST   Subroutine DIST is called by MTXEL1(2) to compute the distance between the source and observation points (cells) and five kinds of dot products, such as $(\hat{n} \cdot \hat{r})$. To save unnecessary computations, one of the arguments in the subroutine calling sequence specifies the particular things to be computed. Thus if the computation of a particular matrix element requires the distance r between two cells and $(\hat{n} \cdot \hat{r})$, only the two are computed by the subroutine.

SUBROUTINE HANKC   The generation of matrix elements requires Hankel functions of the first kind, and these are computed in HANKC, complemented by subroutine ADAM. For computations of $H_0^{(1)}(x)$ and $H_1^{(1)}(x)$, HANKC uses polynomial approximations for $J_0(x)$ and $Y_0(x)$, $J_1(x)$ and $Y_1(x)$ (Abramowitz and Stegun, 1964). For the sake of efficiency, one of the arguments in the calling sequence specifies whether $H_0^{(1)}(x)$ or $H_1^{(1)}(x)$ is to be computed, or both.

SUBROUTINE ADAM   The subroutine ADAM complements HANKC and is used to simplify the code of HANKC.

SUBROUTINE FLIP   As the name indicates, this is a matrix inversion subroutine and is an outgrowth of subroutine ZVO8 used in program RAM1A originally supplied to us by AFAL. The technique used is the standard Gauss-Jordan method (IBM, 1966) and the procedure has been explained in detail by

Oshiro (1963). During the inversion process the original matrix A is replaced by its inverse A^{-1} and subsequently the solution vector (current vector) is obtained by multiplying A^{-1} by the excitation field. When a new excitation field is given, matrix inversion is by-passed and the solution vector obtained by multiplying A^{-1} by the new field.

BLOCK DATA   Numerical constants commonly used in RAMVS are stored in COMMON BLOCK. These constants include Y_0, π, degree-to-radian conversion factor, and others.

A typical computation cycle in RAMVS is as follows.

MAIN starts the cycle by allocating core space and calling subroutine SMAIN (1)[*]. SMAIN, in the first go-around, reads the first three data cards containing a title card and control information, such as incident polarization, bistatic or backscattering cross section to be computed, and others (cards A, B, C, as described in Chapter V). Then GEOM (2) is called, and it reads data specifying geometrical and electrical parameters of the scatterer (cards D through H) which include information such as type of surface, number of cells per segment and type of impedance variation along the surfaces. Using these data, GEOM generates all of the required geometrical parameters and, with help from ZFUN (3,4), the electrical parameters for each cell. Matrix elements are generated next (5,6), either in MTXEL1 or MTXEL2, depending upon the polarization desired. MTXEL first calls DIST (7,8) to compute the distances and various dot products associated with each observation and source point, and then calls on HANKC plus ADAM (9, 10, 11, 12) for the related Hankel function.

Control returns to SMAIN where the incident field vector is computed (13), whereupon FLIP is called (14) to invert the matrix and return (15) the solution vector. SMAIN prints out the geometrical and electrical parameters associated with each cell, along with the current. It then computes either the bistatic or backscattering cross section, and prints out the results. In the case of back-

[*] Numbers in parentheses indicate the position on the flow-graph in Fig. 3-1.

scattering, FLIP is called (14) to calculate the new current for each new incident direction, and FLIP in turn acknowledges (15) by multiplying the previously computed matrix A^{-1} by the incident vector.

If additional sets of data are provided, the cycle $(2,3,\dots,15)$ is repeated, otherwise the program returns to MAIN (16) and execution is terminated.

IV

LIST OF FORTRAN SYMBOLS

This list contains symbols that have been assigned a specific meaning in the program. Words marked by an asterisk are used as prefixes (suffixes) to indicate a particular modification to another variable. Input parameters are also included in this list.

FORTRAN Entry	Description
A(I, J)	Matrix A or its inverse A^{-1}
AA	Dummy variable (complex)
AMP*	Usually indicates amplitude
ANG	Angle subtended by an arc segment
BJ	Bessel function $J_{0,1}(x)$
BY	Bessel function $Y_{0,1}(x)$
DIG	Radians-to-degrees conversion factor
DSQ(J)	Cell size
FAC*	Multiplication factor
FIRST	Initial angle of incidence
FORM	Control selects type of impedance variation desired
HONE, H1	Hankel function, $H_1^{(1)}(x)$
HZERO, HZ	Hankel function, $H_0^{(1)}(x)$
I	Index, normally assigned to observation point
ID	Title, "A" format
INK	Angular increment for back/bistatic scattering angles
IAT	Control in subroutine FLIP If IAT = 1, invert matrix If IAT = 2, bypass inversion
II	Index, associated with I

J	Index, usually assigned to source point
JJ	Index, associated with J
K	Number of cells on impedance surface
KODE	Bistatic/backscattering control
L	Index assigned to geometry segment
LAST	Final (last) angle of incidence
LL(I)	A vector in FLIP subroutine
LUMP(2, I)	Array used for cell identification
M	Number of cells on absorber surface
MI	Dimension integer passed to FLIP
MK	M + K
M1	M + 1
MM1	M + M + 1
MMK	M + M + K
MORE	Control integer in input data
NDNP	$\hat{n} \cdot \hat{n}'$
NDR	$\hat{n} \cdot \hat{r}$
NPDR	$\hat{n}' \cdot \hat{r}$
PHASE*	Phase of a complex number
PHI(I)	Current vector to be determined
PI	π
PINK(I)	Excitation vector
PIPI	$\pi \cdot \pi$
PIT	$\pi/2$
P(WAVE)*	P refers to previous value (of wave)
R	$R = |\bar{r}_i - \bar{r}_j|$
RED	Degrees-to-radians conversion factor
RK	$2\pi R$
S	Distance along the surface

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SDR	$\hat{s} \cdot \hat{r}$
SPDR	$\hat{s}' \cdot \hat{r}$
SUM*	Sum, as the name indicates (complex)
TPI	2π
WAVE	Incident wavelength
X(I)	x-coordinate of the cell
XA	x-coordinate for first endpoint of an input arc segment
XB	x-coordinate for second endpoint of an input arc segment
XN(I)	x-component of normal vector, $\hat{n}$
Y(I)	y-coordinate of the cell
YA	y-coordinate for first endpoint of an input arc segment
YB	y-coordinate for second endpoint of an input arc segment
YN(I)	y-component of normal vector, $\hat{n}$
YZ	Y_0
ZE(I)	Electric impedance vector; ZS(I) is stored in part of ZE(I)
ZEA ZEB ZEX	Coefficients used in ZFUN for computation of ZE(I)
ZM(I)	Magnetic impedance vector
ZMA ZMB ZMX	Coefficients used in ZFUN for computation of ZM(I)
ZSA ZSB ZSX	Coefficients used in ZFUN for computation of surface impedance and stored in vector ZE(I)
ZS	Surface impedance constant

V

USER INSTRUCTIONS AND SPECIAL CONSIDERATIONS

The program RAMVS solves the integral equations for currents and bistatic or monostatic scattering from a general two-dimensional body consisting of a closed cylindrical surface with specified surface impedance ZS (stored in ZE) and absorber sheets that are specified by their electric and magnetic sheet resistances (impedances). Mathematical modeling requires that these sheets be thin, and comparison of computed and measured backscattering has shown that for a dielectric sheet reasonable results are obtained when the sheet is as thick as 0.1λ, where λ is the free-space wavelength. If thicker sheets are desired, they can be simulated by two or more equivalent thin sheets, requiring, of course, more sampling points.

The size of the body or the number of sampling points that can be used depends on the core size of the machine and the funds or computer time available. If K is the number of sampling points on the impedance surface and M is the number of sampling points on the absorber sheet, the total number of unknowns to be determined is $2M+K$.

To assist a potential user intending to run RAMVS, we illustrate the preparation of input data for a hypothetical computation. Partial output for the same data is also presented.

5.1 IBM-CDC Compatibility

The program has been compiled and run on the University of Michigan IBM 360/67 computer and the AFAL CDC 6600 machine at WPAFB. Only one discrepancy occurs, and this is in statement 13 (IBM version) in subroutine SMAIN. Depending upon the machine used to run the program, the following (or equivalent) forms should be used:

IBM	CDC
5 READ(5, 100, END=998)ID	5 READ(5, 100)ID
	IF(EOF(5)) 999,998
	998 CONTINUE

5.2 Sample Run

Consider a relatively thin perfectly conducting ogival cylinder to which are attached an absorber fin at the front and two fins at the back. We choose the body to be one wavelength long with its surface generating arc encompassing 25°. The fins are made of material characterized by (complex) permittivity $\epsilon_r = 5.0 + i3.0$ and permeability $\mu_r = 2.0 + i1.0$, and the thickness of the material is 0.05 wavelength. The details of the geometry are shown in Fig. 5-1.
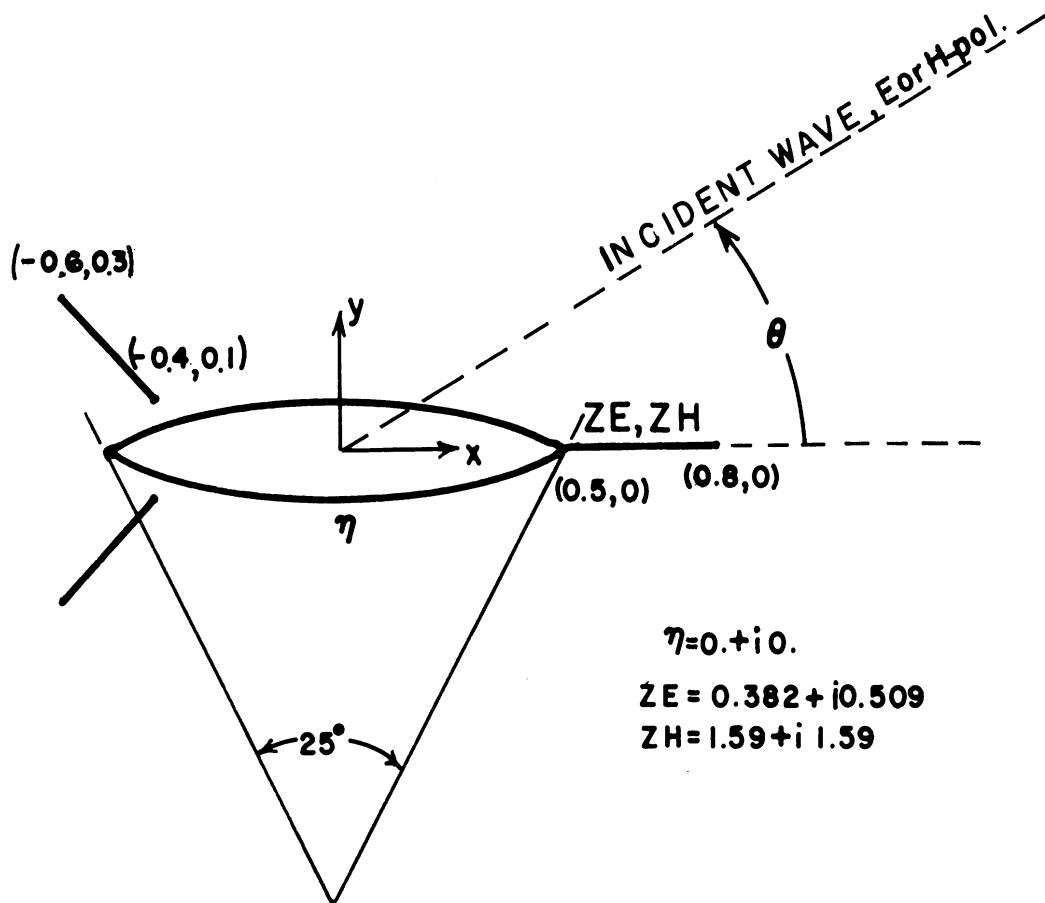
FIG. 5-1: Geometry used in the sample computation.

It is required to compute the backscattering cross section pattern at aspect angles spaced 10 degrees apart for both incident polarizations.

Preliminary computation:

$$ZE = \frac{i}{2\pi(\epsilon_r - 1)\Delta/\lambda} = \frac{i}{2\pi(4.0 + i3.0)(.05)} \approx 0.382 + i0.509$$

$$ZM = \frac{i}{2\pi(\mu_r - 1)\Delta/\lambda} = \frac{i}{2\pi(1.0 + i1.0)(.05)} = 1.59 + i1.59$$

$$ZS = 0. + i0. \quad \text{(perfectly conducting)} .$$

Number of sampling points:

Using the criterion of approximately 10 sampling points per free space wavelength, we choose:

10 points on each of the upper and lower halves of the ogival cylinder,

3 points on the front fin, and

3 points on each of the rear fins.

Dimensioning:

If the program is used as listed in the Appendix, the array dimensions are more than adequate. On the other hand, if larger or smaller arrays are desired, the following guidelines should be applied:

(a) Dimensioning need be changed only in the MAIN program.

(b) Estimate the number of sampling points that will be used on impedance and absorber surfaces.

(c) Determine the dimensions of arrays from the following formula:

If $\qquad$ K = number of points on the impedance surface

and $\qquad$ M = number of points on the absorber sheet,

then the pertinent values MK and MMK that must be entered in the MAIN program are

$$MK = M + K \qquad \text{and} \qquad MMK = M + M + K .$$

For particular details, see the comments at the beginning of the program listing (Appendix).

Input data formats:

Below are the input data formats for the RAMVS program. The letters assigned to each format do not necessarily represent the order of the data cards. Depending upon the geometry used and the computational requirements, some formats may be repeated and others not used at all.

```
                        *** INPUT DATA FORMAT ***
_____
A    FORMAT (18A4)              TITLE CARD: USE UP TO 72 COLUMNS
_____
B    FORMAT (I2,I3,4F10.5)    MORE,IPOL,WAVE,ZSFAC,ZEFAC,ZMFAC
     MORE=0             THIS WILL BE THE LAST RUN FOR THIS DATA SET
     MORE=1             THERE ARE MORE DATA TO BE READ AFTER THIS SET
     IPOL=1             E-POLARIZATION
     IPOL=2             H-POLARIZATION
     WAVE               WAVELENGTH
     ZSFAC              MULTIPLYING FACTOR(REAL) FOR ALL ZS
     ZEFAC              MULTIPLYING FACTOR(REAL) FOR ALL ZE
     ZMFAC              MULTIPLYING FACTOR(REAL) FOR ALL ZM
_____
C    FORMAT (I2,3X,3F10.5)   KODE,FIRST,LAST,INK
     KODE=0             COMPUTES BISTATIC SCATTERING PATTERN
     KODE=1             COMPUTES BACKSCATTERING PATTERN
     FIRST              INITIAL SCATTERING AND INCIDENCE ANGLE
     LAST               FINAL ANGLE
     INK                ANGULAR INCREMENT
_____
         (DATA D, E, AND F REQUIRED FOR EACH ABSORBER SEGMENT;
          AT LEAST ONE ABSORBER AND ONE IMPEDANCE SEGMENT IS
          REQUIRED. ABSRORBER SEGMENTS MUST BE READ IN FIRST.)
D    FORMAT (I2,I3,5F10.5)   TYPE,N,XA,YA,XB,YB,ANG
     TYPE=1 ABSORBER SHEET
     TYPE=2 IMPEDANCE SURFACE
     N                  NUMBER OF SAMPLING POINTS ON THIS SEGMENT
     XA,YA,XB,YB        SEGMENT ENDPOINTS
     ANG                ANGLE SUBTENDED BY THE SEGMENT
_____
E    FORMAT (I2,3X,5F10.5)   FORM,ZEA,ZEB,ZEX
     FORM=-1            ZE(I)=(ZEA-ZEB*S(I))**ZEX
     FORM= 0            ZE(I)=ZEA+ZEB*S(I)**ZEX
     FORM= 1            ZE(I)=ZEA+ZEB*EXP(-ZEX*S(I))
     ZEA,ZEB            COMPLEX IMPEDANCE CONSTANTS
     ZEX                REAL IMPEDANCE CONSTANT
```

```
F    FORMAT (I2,3X,5F10.5)  FORM,ZMA,ZMB,ZMX
     FORM=-1     ZM(I)=(ZMA-ZMB*S(I))**ZMX
     FORM= 0     ZM(I)=ZMA+ZMB*S(I)**ZMX
     FORM= 1     ZM(I)=ZMA+ZMB*EXP(-ZMX*S(I))
     ZMA,ZMB     COMPLEX IMPEDANCE CONSTANTS
     ZMX         REAL IMPEDANCE CONSTANT
```

```
         (DATA G AND H REQUIRED FOR EACH IMPEDANCE SEGMENT.)
G    FORMAT (I2,I3,5F10.5)  TYPE,N,XA,YA,XB,YB,ANG
     TYPE=1 ABSORBER SHEET
     TYPE=2 IMPEDANCE SURFACE
     N           NUMBER OF SAMPLING POINTS ON THIS SEGMENT
     XA,YA,XB,YB SEGMENT ENDPOINTS
     ANG         ANGLE SUBTENDED BY THE SEGMENT·
```

```
H    FORMAT (I2,3X,5F10.5)  FORM,ZSA,ZSB,ZSX
     FORM=-1     ZS(I)=(ZSA-ZSB*S(I))**ZSX
     FORM= 0     ZS(I)=ZSA+ZSB*S(I)**ZSX
     FORM= 1     ZS(I)=ZSA+ZSB*EXP(-ZSX*S(I))
     ZSA,ZSB     COMPLEX IMPEDANCE CONSTANTS
     ZSX         REAL IMPEDANCE CONSTANT.
```

```
I    FORMAT (I5)             INTEGER ZERO IN COLUMN 5 SHUTS OFF
                             READING OF SEGMENT PARAMETERS
```

```
         (USE THIS CARD ONLY IF, IN B, MORE=1)
J    FORMAT (I2,I3,4F10.5)   MORE,IPOL,WAVE,ZSFAC,ZEFAC,ZMFAC
```

Input data cards:

The data set may consist of as few as 9 cards but can be many more,
depending upon the number of segments used to specify the geometry, the number
of polarizations and frequencies desired, and the number of constant multiplica-
tion factors for the impedances. The geometry must always consist of two sur-
faces, an impedance surface and an absorber sheet, though either may consist
of only two cells far removed from the main scatterer. The data and explanation
follows.

(card)	2	5	10	20	30	40	50 (column)
1	OGIVAL CYLINDER WITH FINS						
2	1	2	1.	1.	1.	1.	
3	1		0.	180.	10.		
4	1	3	0.5	0.	0.8	0.	0.
5	0		0.382	0.509	0.	0.	0.
6	0		1.59	1.59	0.	0.	0.
7	1	3	-0.4	0.1	-0.6	0.3	0.
8	0		0.582	0.509	0.	0.	0.
9	0		1.59	1.59	0.5	0.	0.
10	2	10	-0.5	0.	0.	0.	25.
11	1		0.	0.	0.	0.	0.
12	0	0					
13	0	1	1.	1.	1.	1.	

Structure of the input deck for the example of Fig. 5-1.

<u>CARD 1</u>(A) FORMAT (18A4) Title card; use up to 72 columns.

This is a title card and is repeated in the output format. The letter (A) after <u>CARD 1</u> refers to the format used as described above in Input data formats.

<u>CARD 2</u>(B) FORMAT (I2, I3, 4F10. 5) MORE, IPOL, WAVE, ZSFAC, ZEFAC, ZMFAC

This is a control card; the same card is repeated at the end of the data set, but with MORE = 0 (last run for this data set) and IPOL = 1 (E-polarization).

MORE = 1 Another run will be made for this data set.

IPOL = 2 H-polarization.

WAVE = 1. Wavelength in units used. If geometry is normalized with respect to wavelength, WAVE = 1.

ZSFAC⎫
ZEFAC⎬ = 1. Relative multiplication factors for impedances.
ZMFAC⎭

<u>CARD 3</u>(C) FORMAT (2I, 3X, 3F10. 5) KODE, FIRST, LAST, INK

KODE = 1 Bistatic pattern is computed.

FIRST = 0. Initial incidence angle. Only for this incidence are the surface currents printed out.

LAST = 180. Final angle. Since the body is symmetrical about the y-plane, running as far as 360 degrees would be repetitious. The program requires that LAST > FIRST.

INK = 10. Scattering will be computed at 10-degree increments.

<u>CARD 4</u>(D) FORMAT (I2, I3, 5F10. 5) TYPE, N, XA, YA, XB, YB, ANG

This and the next eight cards prescribe the scatterer geometry and associated electrical properties. The program requires that absorber surfaces be read in first. In this data set the following order of input is used: (1) frontal fin, (2) rear fins, (3) ogival cylinder.

TYPE = 1 Indicates this is an absorber surface.

N = 3 Number of sampling points on this segment.

XA = 0. 5⎫
YA = 0 ⎬ Specifies endpoints for the segment.
XB = 0. 8⎪
YB = 0 ⎭

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ANG = 0. Implies that this is a straight line segment.

In generation of the geometry in subroutine GEOM it has been assumed that a body is symmetric about the y = 0 axis, and only the parameters for the upper profile must be supplied. However, when a segment is a straight line and lies on the y = 0 axis, its image is not generated. Such is the case here for CARD 4.

CARD 5(E)   FORMAT (I2, 3X, 5F10.5) FORM, ZEA, ZEB, ZEX

This card prescribes the electrical sheet impedance parameters for the frontal strip.

FORM = 0   Selects equation for computation of ZE in ZFUN.

ZEA = (0.382, 0.509) Electric impedance constants appropriate for the
ZEB = (0., 0.)   equation selected by FORM = 0. Note that in this
ZEX = 0.   particular case the value of ZEX is arbitrary.

CARD 6(F)   FORMAT (I2, 3X, 5F10.5) FORM, ZMA, ZMB, ZMX

FORM = 0   Selects equation for computation of ZM in ZFUN

ZMA = (1.59, 1.59)   Magnetic impedance constants appropriate for the
ZMB = (0., 0.)   equation selected by FORM = 0.
ZMX = 0.

CARD 7(D)   FORMAT (I2, I3, 5F10.5) TYPE, N, XA, YA, XB, YB, ANG

This card specifies the geometry for the upper tail fin. Its format is the same as that of CARD 4. The lower fin (an image) will be generated automatically by the program.

CARD 8(E)   FORMAT (I2, 3X, 5F10.5) FORM, ZEA, ZEB, ZEX

Since the same material is used in the tail fins as in the frontal fin, this card is identical to CARD 5.

CARD 9(F)   FORMAT (I2, 3X, 5F10.5) FORM, ZMA, ZMB, ZMX

This card is identical to CARD 6.

CARD 10(D) FORMAT (I2,I3,5F10.5) TYPE, N,XA, YA, XY, YB, ANG

This card specifies the geometry for the upper portion of the ogival cylinder. Again, the lower portion will be generated automatically in the program.

TYPE = 2 Indicates this is an impedance surface.

XA = -0.5 ⎫
YA = 0.   ⎪
           ⎬ Specifies endpoints for the segment.
XB = 0.5   ⎪
YB = 0.   ⎭

ANG = 25. Angle encompassed by the circular arc.

CARD 11(G) FORMAT (I2,3X,5F10.5) FORM, ZSA, ZSB, ZSX

This card specifies parameters for computation of normalized surface impedance ZS. Since the body is perfectly conducting, this card is easy to do, viz.

FORM = 1 Selects equation for computing ZS

ZSA = (0.,0.) ⎫
               ⎪ Surface impedance constants appropriate for the
ZSB = (0.,0.) ⎬ equation selected by FORM = 1.
ZEX = 0   ⎭

CARD 12(H) FORMAT (I5)

Integer in column 5 shuts off reading of segment parameters.

CARD 13(I) FORMAT (I2,I3,4F10.5) MORE, IPOL, WAVE, ZSFAC, ZEFAC, ZMFAC

With the use of this card (in addition to MORE = 1 in CARD 2), computations for the same geometry are repeated, but for different electrical parameters. Here, only the polarization is changed (IPOL = 1), and since this is the last run for this data set, MORE = 0. Other entries are the same as in CARD 2.

MORE = 0 Last run for this data set.

IPOL = 1 E-polarization.

WAVE = 1. Wavelength.

ZSFAC ⎫
ZEFAC ⎬ = 1. Relative multiplication factor for impedances.
ZMFAC ⎭

Output data:

The sample data set was run on the University of Michigan IBM 360/67 computer and the H-polarization results are shown in the following pages.

For the shape computed, there are a total of 29 sampling points on the body: 20 on the ogival cylinder and 9 on the absorber fins. The resultant matrix is 38 x 38 (MMK = 9+9+20), and 44.2 seconds of CPU time were required for both the E- and H-polarization computations.

The output consists of three major blocks and in this case each group requires a page of this report. On the first page (p. 41), geometrical and electrical input parameters are given , along with other useful data such as the radius and length of arcs used in generating the profile. Such output is helpful in verifying that all input parameters have been correctly punched and read into the machine.

The second block (p. 42) consists of geometrical and electrical parameters for all cells on the body, and includes the computed currents. When the incident wave direction is specified for a range of incident angles, the currents are printed out only for the first angle; if currents are required for other angles of incidence, the input data set must be repeated for the new incidence angle.

The third block (p. 43) lists an array of computed cross sections. Since there are three types of current contributing to the scattering, the individual contribution of each type is also given. These include electric current (ELECTRIC) and magnetic current (MAGNETIC) from the absorber sheets, and the surface current (IMPEDANCE) from the impedance surface. The (ABSORBER) column lists the net contributions of the absorber sheet (electric plus magnetic current), and the (TOTAL) column is the sum of the three contributions. Such a breakdown of the cross section into its components is often helpful in determining the sources of scattering (the dominant scatterer), analyzing the performance of the absorber, and studying other scattering effects.

The expressions used for computing the bistatic and backscattering cross sections are

OGIVAL-CYLINDER WITH FINS

SEG NUM	SEG TYP	NUM CELLS	XA	YA	XB	YB	ANGLE	RADIUS	LENGTH	FORM	ZEA(ZMA)	ZEB(ZMB)	ZEX(ZMX)
1	ABS	3	0.5000	0.0	0.8000	0.0	0.0	999.00	0.300	0	0.382 (0.509)	0.0 (0.0)	0.0 (0.0)
2	ABS	3	-0.4000	0.1000	-0.6000	0.3000	0.0	999.00	0.283	0	1.590 (1.590)	0.0 (0.0)	0.0 (0.0)
3	ABS	3	-0.4000	-0.1000	-0.6000	-0.3000	0.0	999.00	0.283	0	0.382 (0.509)	0.0 (0.0)	0.0 (0.0)
4	IMP	10	-0.5000	0.0	0.5000	0.0	25.00	2.31	1.008	1	1.590 (1.590)	0.0 (0.0)	0.0 (0.0)
5	IMP	10	-0.5000	0.0	0.5000	0.0	25.00	2.31	1.008	1	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)

KEY PARAMETERS

INCIDENT POLARIZATION	H
SURFACE IMPEDANCE FACTOR	1.000
ELECTRIC IMPEDANCE FACTOR	1.000
MAGNETIC IMPEDANCE FACTOR	1.000
TOTAL NUMBER OF POINTS ON THE BODY	29
NUMBER OF SEGMENTS USED	5
NUMBER OF INCIDENT FIELD DIRECTIONS	19
NUMBER OF BISTATIC DIRECTIONS	0
WAVELENGTH	1.000

OGIVAL-CYLINDER WITH FINS

ABSORBER SURFACE

I SEG	X	Y	S	DSQ	--- ZE ---	MOD(JE)	ARG(JE)	MOD(JM)	ARG(JM)	--- ZM ---
1 1	0.5500	0.0	0.0500	0.1000	0.382 0.500	0.0015	107.274	175.9988	130.740	1.590
2 1	0.6500	0.0	0.1500	0.1000	0.382 0.500	0.0030	81.665	154.2007	95.055	1.590
3 1	0.7500	0.0	0.2500	0.1000	0.382 0.500	0.0057	104.946	151.1338	46.245	1.590
4 2	-0.4333	0.1333	0.1471	0.0943	0.382 0.500	0.6425	-56.877	127.6986	112.808	1.590
5 2	-0.2000	0.2000	0.2357	0.0943	0.382 0.500	0.8052	-38.139	143.1452	141.711	1.590
6 2	-0.5667	0.2667	0.0471	0.0943	0.382 0.500	0.7464	-26.004	154.7719	168.516	1.590
7 3	-0.4333	-0.1333	0.1471	0.0943	0.382 0.500	0.6464	-59.428	128.3813	120.774	1.590
8 3	-0.5000	-0.2000	0.0471	0.0943	0.382 0.500	0.8324	-38.695	132.1762	143.525	1.590
9 3	-0.5667	-0.2667	0.2357	0.0943	0.382 0.500	0.7670	-24.537	142.6163	162.476	1.590

IMPEDANCE SURFACE

I SEG	X	Y	S	DSQ	--- ZS ---	MOD(JS)	ARG(JS)
10 4	-0.4507	0.0104	0.0504	0.1008	0.0	0.7305	174.281
11 4	-0.3514	0.0279	0.1512	0.1008	0.0	0.8100	131.881
12 4	-0.2515	0.0410	0.2520	0.1008	0.0	0.0088	81.928
13 4	-0.1511	0.0498	0.3528	0.1008	0.0	0.2416	53.786
14 4	-0.0504	0.0542	0.4536	0.1008	0.0	0.1703	28.453
15 4	0.0504	0.0542	0.5544	0.1008	0.0	0.9292	-9.181
16 4	0.1511	0.0498	0.6552	0.1008	0.0	0.9239	-61.240
17 4	0.2515	0.0410	0.7560	0.1008	0.0	1.1662	-99.740
18 4	0.3514	0.0279	0.8568	0.1008	0.0	1.2643	-125.755
19 4	0.4507	0.0104	0.9576	0.1008	0.0	0.8253	-149.047
20 5	-0.4507	-0.0104	0.0504	0.1008	0.0	0.7342	174.316
21 5	-0.3514	-0.0279	0.1512	0.1008	0.0	0.8016	131.735
22 5	-0.2515	-0.0410	0.2520	0.1008	0.0	0.0070	81.447
23 5	-0.1511	-0.0498	0.3528	0.1008	0.0	0.2464	53.523
24 5	-0.0504	-0.0542	0.4536	0.1008	0.0	1.1759	28.475
25 5	0.0504	-0.0542	0.5544	0.1008	0.0	0.9294	-8.918
26 5	0.1511	-0.0498	0.6552	0.1008	0.0	0.9192	-61.276
27 5	0.2515	-0.0410	0.7560	0.1008	0.0	1.1644	-100.???
28 5	0.3514	-0.0279	0.8568	0.1008	0.0	1.2661	-125.961
29 5	0.4507	-0.0104	0.9576	0.1008	0.0	0.8240	-148.968

BACKSCATTERING CROSS SECTION
10*LOG(SIGMA/LAMBDA)
(H-POLARIZATION)

THETA	(ELECTRIC) DB	PHASE	(MAGNETIC) DB	PHASE	(ABSORBER) DB	PHASE	(IMPEDANCE) DB	PHASE	(TOTAL) DB	PHASE
0.0	-1C.02	-38.07	-13.84	123.39	-17.16	-16.23	-21.36	1C2.8C	-17.64	26.51
10.0	-10.69	-58.16	-13.87	111.68	-2C.C5	-37.C9	-16.19	1C2.89	-19.87	63.86
20.0	-9.47	-159.19	-14.01	83.86	-16.55	-125.81	-9.32	99.91	-11.59	124.17
30.0	-7.11	-151.76	-16.71	49.02	-16.21	-161.4C	-4.C1	93.99	-4.82	125.39
40.0	-8.85	162.19	-19.2C	-71.33	-16.21	-168.78	-3.6C	84.41	-2.55	1C8.8C
50.0	-8.22	81.68	-9.56	-113.31	-19.52	132.56	-3.9C	64.53	-3.29	72.77
60.0	-7.98	57.61	-9.98	-121.8C	-21.72	55.32	-4.5C	13.23	-3.7C	18.C5
70.0	-7.53	-168.2C	-31.8C	64.27	-14.69	-174.35	-C.8C	-26.66	-3.49	-32.71
80.0	-13.06	-137.74	-12.47	64.16	-9.87	-153.79	4.C1	-35.3C	-4.25	-45.26
90.0	-2C.51	-52.73	-29.24	173.19	-22.55	-72.15	6.36	-34.22	6.61	-35.44
100.0	-18.96	45.89	-14.1C	-113.48	-12.46	18.35	5.17	-29.22	5.94	-24.13
110.0	-5.61	35.44	-14.13	-98.87	-16.9C	-64.59	-C.73	-18.C1	1.52	-23.CC
120.0	-7.67	-117.94	-18.73	52.35	-7.74	-115.22	-7.47	27.11	-11.39	-41.46
130.0	-13.02	-134.81	-16.32	69.72	-12.94	-146.59	-6.C8	1C2.13	-6.55	137.48
140.0	-8.71	119.42	-25.C9	-166.57	-13.61	85.51	-6.87	117.17	-1.96	121.63
150.0	-9.88	6C.C1	-14.1C	-145.22	-2C.C2	20.39	-11.87	122.45	-11.C5	11C.92
160.0	-1C.55	2C.53	-13.13	-159.41	-17.17	-56.72	-18.9C	123.81	-11.99	1C1.1C
170.0	-11.13	-22.98	-14.18	179.97	-15.25	-71.71	-23.7C	12C.11	-31.69	-42.78
180.0		-39.66	-14.47	160.48				112.67	-19.34	-74.36

$$\sigma/\lambda = \lim_{\rho \to \infty} \frac{2\pi}{\lambda} \rho \left| Y_0 E_z^S \right|^2 \quad , \qquad \text{E-polarization}$$

$$\sigma/\lambda = \lim_{\rho \to \infty} \frac{2\pi}{\lambda} \rho \left| H_z^S \right|^2 \quad , \qquad \text{H-polarization}$$

and the associated phases are $\text{Arg}(E^S)$ and $\text{Arg}(H^S)$ for E- or H-polarization, respectively.

VI

CONCLUSIONS

In the previous sections a description of a computer program RAMVS for computation of scattering from a general two-dimensional body consisting of an arbitrary impedance cylinder and absorber sheets specified by equivalent electric and magnetic sheet impedances is given. The sheet impedances must be determined from the permittivity, permeability and thickness of the material, and then read in as input data. Consistent with the core limitations of the particular installation used to run the program, the program has the potential for solving scattering from many two-dimensional problems of interest. These include perfectly conducting bodies, bodies with prescribed surface impedance with or without absorber sheets present, and absorber-clad bodies. The RAM- series programs, such as RAMD (impedance body, E-polarization), REST (resistive sheets, E-polarization), or RASP (electric impedance sheets, E-polarization) developed and used under this contract are special cases of RAMVS. RAMVS was tested against these programs and, for the cases computed, gave the same results.

Many other aspects of RAMVS, however, have not been tested. To determine the capabilities as well as the shortcomings of the program, computed patterns should be compared against experimental measurements for structures such as isolated absorber sheets, sheets attached to metallic bodies, and thick absorber layers, either isolated or when coated on metallic cylinders. Of particular importance would be the determination of optimum cell size in absorber layers. Since there are two unknowns associated with each cell, it would be desirable to use cell sizes as large as possible, thereby minimizing the size of the matrix. On the other hand, cells that are too large may lead to a degradation of accuracy.

In retrospect, there are two areas in which the program could be modified. First, a substantial saving in computation time could be achieved by dispensing with the actual inversion of the matrix and solving the system of linear algebraic

equations by a direct technique such as Gaussian elimination (Southworth and Deleeuw, 1965). This would require developing, or obtaining if possible, an appropriate code to replace the subroutine FLIP. Additional changes might be required in MAIN and SMAIN to accommodate new variables, if such are needed. Before making such changes in the program, however, an accurate assessment of the CPU time should be made for a typical computation based on the matrix inversion approach and the direct solution technique. A "rule of thumb" is that it requires almost three times as many operations to invert a matrix as it does to solve the system of equations directly, but this applies only for a single incident vector. When a number of incident vectors are present, as is the case in computing backscattering patterns, it may be more efficient to invert the matrix and perform a simple multiplication with each incident vector, rather than to repeat a portion (back substitution) of a direct solution procedure for each.

The other possible modification of the program would be to add a capability to compute the tangential electric and magnetic fields along a prescribed surface and form the ratio of the two to determine the impedance. This would be helpful in relating the material properties of the coatings to surface impedances, especially near edges or other surface or electrical discontinuities, where no known analytical formulas exist. To change the present version to RAMVS to include this capability would require an additional subroutine for computing the electric and magnetic fields along a presecribed surface representing, for example, the outer surface of the absorber. Subroutines GEOM and SMAIN would have to be expanded to accommodate such a surface.

The next test for the program RAMVS will be to compute the scattering from the geometries the program was intended to handle. This will encompass scattering by ogival cylinders with magnetic fins ($\epsilon_r = 1$), isolated absorber strips, and perhaps absorber-clad bodies. When possible, experimental data will be used for comparison. The results will be included in the Final Report under this contract.

APPENDIX
PROGRAM LISTING

The following is an IBM version of the FORTRAN source listing of the program RAMVS. To run this program on a machine such as the CDC 6600 at WPAFB (AFAL), two cards should be changed in subroutine SMAIN as described in Section 5.1 of this report.

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                        C
C                         *** RAMVS ***                                  C
C                       (03-10-74 VERSION)                        VVL    C
C                                                                        C
C     THIS PROGRAM COMPUTES NEAR-FIELD AND FAR-FIELD SCATTERING FROM     C
C     A GENELAL IMPEDANCE BODY IN PRESENCE OF ABSORBER SHEETS;           C
C     TWO-DIMENSIONAL GEOMETRY, EXP(-IWT) TIME CONVENTION.               C
C                                                                        C
C     RE: V.V.LIEPA,E.F.KNOTT,AND T.B.A.SENIOR,"COMPUTER PROGRAM FOR     C
C     SCATTERING FROM TWO-DIMENSIONAL BODIES WITH ABSORBER SHEETS",      C
C     THE UNIVERSITY OF MICHIGAN RADIATION LABORATORY REPORT             C
C     NO.011764-2-T (AFAL-TR-74   ),       1974.                         C
C                                                                        C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                        C
C                    *** INPUT DATA FORMAT ***                           C
C------------------------------------------------------------------------C
C     A     FORMAT (18A4)             TITLE CARD; USE UP TO 72 COLUMNS    C
C------------------------------------------------------------------------C
C     B     FORMAT (I2,I3,4F10.5)   MORE,IPOL,WAVE,ZSFAC,ZEFAC,ZMFAC     C
C           MORE=0         THIS WILL BE THE LAST RUN FOR THIS DATA SET    C
C           MORE=1         THERE ARE MORE DATA TO BE READ AFTER THIS SET  C
C           IPOL=1         E-POLARIZATION                                 C
C           IPOL=2         H-POLARIZATION                                 C
C           WAVE           WAVELENGTH                                     C
C           ZSFAC          MULTIPLYING FACTOR(REAL) FOR ALL ZS            C
C           ZEFAC          MULTIPLYING FACTOR(REAL) FOR ALL ZE            C
C           ZMFAC          MULTIPLYING FACTOR(REAL) FOR ALL ZM            C
C------------------------------------------------------------------------C
C     C     FORMAT (I2,3X,3F10.5)   KODE,FIRST,LAST,INK                  C
C           KODE=0         COMPUTES BISTATIC SCATTERING PATTERN           C
C           KODE=1         COMPUTES BACKSCATTERING PATTERN                C
C           FIRST          INITIAL SCATTERING AND INCIDENCE ANGLE         C
C           LAST           FINAL ANGLE                                    C
C           INK            ANGULAR INCREMENT                              C
C------------------------------------------------------------------------C
C                 (DATA D, E, AND F REQUIRED FOR EACH ABSORBER SEGMENT;   C
C                  AT LEAST ONE ABSORBER AND ONE IMPEDANCE SEGMENT IS     C
C                  REQUIRED. ABSRORBER SEGMENTS MUST BE READ IN FIRST.)   C
C     D     FORMAT (I2,I3,5F10.5)   TYPE,N,XA,YA,XB,YB,ANG               C
C           TYPE=1 ABSORBER SHEET                                         C
C           TYPE=2 IMPEDANCE SURFACE                                      C
C           N              NUMBER OF SAMPLING POINTS ON THIS SEGMENT      C
C           XA,YA,XB,YB    SEGMENT ENDPOINTS                             C
C           ANG            ANGLE SUBTENDED BY THE SEGMENT                 C
C------------------------------------------------------------------------C
C     E     FORMAT (I2,3X,5F10.5)   FORM,ZEA,ZEB,ZEX                     C
C           FORM=-1        ZE(I)=(ZEA-ZEB*S(I))**ZEX                      C
C           FORM= 0        ZE(I)=ZEA+ZEB*S(I)**ZEX                        C
C           FORM= 1        ZE(I)=ZEA+ZEB*EXP(-ZEX*S(I))                   C
C           ZEA,ZEB        COMPLEX IMPEDANCE CONSTANTS                    C
C           ZEX            REAL IMPEDANCE CONSTANT                        C
```

```
C------------------------------------------------------------------C
C      F     FORMAT (I2,3X,5F10.5)   FORM,ZMA,ZMB,ZMX             C
C            FORM=-1        ZM(I)=(ZMA-ZMB*S(I))**ZMX             C
C            FORM= 0        ZM(I)=ZMA+ZMB*S(I)**ZMX               C
C            FORM= 1        ZM(I)=ZMA+ZMB*EXP(-ZMX*S(I))          C
C            ZMA,ZMB        COMPLEX IMPEDANCE CONSTANTS           C
C            ZMX            REAL IMPEDANCE CONSTANT               C
C------------------------------------------------------------------C
C               (DATA G AND H REQUIRED FOR EACH IMPEDANCE SEGMENT.) C
C      G     FORMAT (I2,I3,5F10.5)   TYPE,N,XA,YA,XB,YB,ANG       C
C            TYPE=1 ABSORBER SHEET                                C
C            TYPE=2 IMPEDANCE SURFACE                             C
C            N              NUMBER OF SAMPLING POINTS ON THIS SEGMENT C
C            XA,YA,XB,YB    SEGMENT ENDPOINTS                     C
C            ANG            ANGLE SUBTENDED BY THE SEGMENT        C
C------------------------------------------------------------------C
C      H     FORMAT (I2,3X,5F10.5)   FORM,ZSA,ZSB,ZSX             C
C            FORM=-1        ZS(I)=(ZSA-ZSB*S(I))**ZSX             C
C            FORM= 0        ZS(I)=ZSA+ZSB*S(I)**ZSX               C
C            FORM= 1        ZS(I)=ZSA+ZSB*EXP(-ZSX*S(I))          C
C            ZSA,ZSB        COMPLEX IMPEDANCE CONSTANTS           C
C            ZSX            REAL IMPEDANCE CONSTANT               C
C------------------------------------------------------------------C
C      I     FORMAT (I5)                 INTEGER ZERO IN COLUMN 5 SHUTS OFF C
C                                        READING OF SEGMENT PARAMETERS C
C------------------------------------------------------------------C
C               (USE THIS CARD ONLY IF, IN B, MORE=1)            C
C      J     FORMAT (I2,I3,4F10.5)   MORE,IPOL,WAVE,ZSFAC,ZEFAC,ZMFAC C
C                                                                 C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                 C
C               *** DIMENSIONING FORMAT ***                      C
C------------------------------------------------------------------C
C   VECTORS ARE DIMENSIONED ONLY IN THE MAIN PROGRAM.            C
C   IF K=NO. POINTS(CELLS) ON THE IMPEDANCE SURFACE AND          C
C   M=NO. OF POINTS(CELLS) ON THE ABSORBER SHEET, THEN           C
C               MK=M+K AND MMK=M+M+K                             C
C                                                                 C
C.....MAIN PROGRAM-----A STARTER PROGRAM                         C
C   *   COMPLEX A(MMK,MMK+1),PHI(MMK),PINK(MMK),LL(MMK),MM(MMK)  C
C   *   COMPLEX ZE(MK),ZM(MK)                                    C
C   *   DIMENSION X(MK),Y(MK),XN(MK),YN(MK),S(MK),DSQ(MK)        C
C   *   DIMENSION LUMP(2,MK)                                     C
C   *   DATA MI/MMK/                                             C
C       CALL SMAIN(MI,A,PHI,PINK,ZE,ZM,X,Y,XN,YN,S,DSQ,LL,MM,LUMP) C
C       END                                                      C
C                       * CARDS TO BE CHANGED WHEN REDIMENSIONING C
C                                                                 C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
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C.....MAIN PROGRAM-----A STARTER PROGRAM
C**** RAMVS VERSION
      COMPLEX A(150,151),PHI(150),PINK(150),LL(150),MM(150)
      COMPLEX ZE(100),ZM(100)
      DIMENSION X(100),Y(100),XN(100),YN(100),S(100),DSQ(100)
      DIMENSION LUMP(2,100)
      DATA MI/150/
      CALL SMAIN(MI,A,PHI,PINK,ZE,ZM,X,Y,XN,YN,S,DSQ,LL,MM,LUMP)
      END
```

```fortran
      SUBROUTINE SMAIN(MI,A,PHI,PINK,ZE,ZM,X,Y,XN,YN,
     &S,DSQ,LL,MM,LUMP)
C**** RAMVS VERSION F-&H-POLARIZATIONS
      DIMENSION LUMP(2,1)
      COMPLEX A(MI,1)
      COMPLEX PHI(1),PINK(1),ZE(1),ZM(1)
      COMPLEX SUM,DEL,SUME,SUMM,SUMK
      DIMENSION X(1),Y(1),XN(1),YN(1),S(1),DSQ(1),LL(1),MM(1)
      DIMENSION ID(18),IPP(2)
      COMMON/PIES/PI,TPI,PIT,PIPI,YZ,RFD,DIG
      REAL LAST,INK
      DATA IPP/4HEEEE,4HHHHH/
C.....READ INPUT DATA AND GENERATE BODY PROFILE
    5 READ(5,100,END=999) ID
      READ (5,200) MORE,IPOL,WAVE,ZSFAC,ZEFAC,ZMFAC
      READ (5,210) KODE,FIRST,LAST,INK
      WRITE (6,150) ID
      CALL GEOM(LUMP,X,Y,XN,YN,S,DSQ,ZE,ZM,K,M)
      IF (KODE.NE.0) GO TO 25
      NINC=1
      NBIT=1+IFIX((LAST-FIRST)/INK)
      GO TO 28
   25 NBIT=0
      NINC=1+IFIX((LAST-FIRST)/INK)
   28 CONTINUE
C.....CONSTRUCT MATRIX ELEMENTS
      M1=M+1
      MK=M+K
      MMK=MK+M
      MM1=M1+M
      DO 35 I=M1,MK
   35 ZE(I)=ZE(I)*ZSFAC
      FACE=ZEFAC*WAVE
      FACM=ZMFAC*WAVE
      XK=TPI/WAVE
      DO 37 I=1,M
      ZE(I)=ZE(I)*FACE
   37 ZM(I)=ZM(I)*FACM
      DO 39 I=1,MK
      S(I)=S(I)/WAVE
   39 DSQ(I)=DSQ(I)/WAVE
      GO TO 50
   40 FAC=ZSFAC/PZSFAC
      DO 45 I=M1,MK
   45 ZE(I)=ZE(I)*FAC
      FAC=WAVE/PWAVE
      FACE=ZEFAC/PZEFAC*FAC
      FACM=ZMFAC/PZMFAC*FAC
      DO 47 I=1,M
      ZE(I)=ZE(I)*FACE
   47 ZM(I)=ZM(I)*FACM
      FAC=PWAVE/WAVE
```

```
      XK=TPI/WAVE
      DO 49 I=1,MK
      S(I)=S(I)*FAC
   49 DSQ(I)=DSQ(I)*FAC
   50 CONTINUE
      IF (IPOL.EQ.1) CALL MTXEL1(MI,M,K,X,Y,XN,YN,DSQ,ZE,ZM,A)
      IF (IPOL.EQ.2) CALL MTXEL2(MI,M,K,X,Y,XN,YN,DSQ,ZE,ZM,A)
      I=LUMP(2,MK)
      WRITE(6,400) IPP(IPOL),ZSFAC,ZEFAC,ZMFAC,MK,I,NINC,NBIT,WAVE
C.....COMPUTE INCIDENT FIELD AND INVERT MATRIX
      TETA=RED*FIRST
      CT=COS(TETA)
      ST=SIN(TETA)
      DO 60 I=1,M
      HOLD=-XK*(CT*X(I)+ST*Y(I))
      DEL=CMPLX(COS(HOLD),SIN(HOLD))
      PINK(I)=DEL
   60 PINK(I+M)=-DEL*(XN(I)*CT+YN(I)*ST)
      DO 63 I=M1,MK
      HOLD=-XK*(CT*X(I)+ST*Y(I))
   63 PINK(I+M)=CMPLX(COS(HOLD),SIN(HOLD))
      CALL FLIP(A,MMK,MI,LL,MM,PINK,PHI,1)
      WRITE (6,150) ID
C.....PRINT OUT STUFF FOR THE ABSORBER SURFACE (FIRST ANGLE ONLY)
      WRITE (6,350)
      WRITE (6,375)
      DO 67 I=1,M
      DEL=PHI(I)+(1.E-50,0.)
      AMPE=CABS(DEL)
      PHASEE=DIG*ATAN2(AIMAG(DEL),REAL(DEL))
      DEL=PHI(I+M)+(1.E-50,0.)
      AMPH=CABS(DEL)
      PHASEH=DIG*ATAN2(AIMAG(DEL),REAL(DEL))
      IF (IPOL.EQ.1) WRITE(6,395) (LUMP(J,I),J=1,2),X(I),Y(I),S(I),
     &DSQ(I),ZE(I),ZM(I),AMPE,PHASEE,AMPH,PHASEH
   67 IF (IPOL.EQ.2) WRITE(6,395) (LUMP(J,I),J=1,2),X(I),Y(I),S(I),
     &DSQ(I),ZE(I),ZM(I),AMPH,PHASEH,AMPE,PHASEE
C.....PRINT OUT STUFF FOR THE IMPEDANCE SURFACE (FIRST ANGLE ONLY)
      WRITE (6,300)
      WRITE (6,325)
      DO 65 I=M1,MK
      IM=I+M
      DEL=PHI(IM)+(1.E-50,0.)
      AMP=CABS(DEL)
      PHASE=DIG*ATAN2(AIMAG(DEL),REAL(DEL))
      DEL=ZE(I)
   65 WRITE (6,250) (LUMP(J,I),J=1,2),X(I),Y(I),S(I),DSQ(I),
     &DEL,AMP,PHASE
```

```
C.....DOPE OUT THE APPROPRIATE FIELD FACTORS
      WRITE (6,150) ID
      THE=FIRST-INK
      IF (KODE.EQ.1) GO TO 70
      WRITE (6,800) FIRST,IPP(IPOL)
      GO TO 75
70    WRITE (6,600) IPP(IPOL)
75    THE=THE+INK
      IF (THE.GT.LAST) GO TO 105
      IF (THE.EQ.FIRST) GO TO 85
      TETA=RED*THE
      CT=COS(TETA)
      ST=SIN(TETA)
C.....IN THE FOLLOWING LOOP COMPUTE THE NEW INCIDENT FIELD
      DO 80 I=1,M
      HOLD=-XK*(CT*X(I)+ST*Y(I))
      DEL=CMPLX(COS(HOLD),SIN(HOLD))
      PINK(I)=DEL
80    PINK(I+M)=-DEL*(XN(I)*CT+YN(I)*ST)
      DO 83 I=M1,MK
      HOLD=-XK*(CT*X(I)+ST*Y(I))
83    PINK(I+M)=CMPLX(COS(HOLD),SIN(HOLD))
      IF (KODE.EQ.0) GO TO 85
      CALL FLIP(A,MMK,MI,LL,MM,PINK,PHI,2)
85    CONTINUE
C.....ADD UP THE CURRENTS FOR FAR FIELD
      SUM=(1.E-25,0.)
      SUME=SUM
      SUMM=SUM
      SUMK=SUM
      DO 93 I=1,M
      IM=I+M
      DS=DSQ(I)
      SUME=SUME-PHI(I)*PINK(I)*DS
93    SUMM=SUMM+PHI(IM)*PINK(IM)*DS
      SUMM=SUMM*YZ
      IF (IPOL.EQ.2) GO TO 90
      DO 95 I=M1,MK
      IM=I+M
95    SUMK=SUMK+(-1.+ZE(I)*(XN(I)*CT+YN(I)*ST))*PINK(IM)*DSQ(I)*PHI(IM)
      GO TO 99
90    DEL=SUME
      SUME=-SUMM/YZ
      SUMM=DEL*YZ
      DO 97 I=M1,MK
      IM=I+M
97    SUMK=SUMK+(-ZE(I)+(XN(I)*CT+YN(I)*ST))*PINK(IM)*DSQ(I)*PHI(IM)
99    DEL=SUME+SUMM
      SUM=DEL+SUMK
      AMPR=REAL(SUME)
      AMPI=AIMAG(SUME)
      PHASEF=DIG*ATAN2(AMPI,AMPR)
      AMPR=PIT*(AMPR*AMPR+AMPI*AMPI)
```

```
        SCATE=10.*ALOG10(AMPR)
        AMPR=REAL(SUMM)
        AMPI=AIMAG(SUMM)
        PHASEM=DIG*ATAN2(AMPI,AMPR)
        AMPR=PIT*(AMPR*AMPR+AMPI*AMPI)
        SCATM=10.*ALOG10(AMPR)
        AMPR=REAL(DEL)
        AMPI=AIMAG(DEL)
        PHASED=DIG*ATAN2(AMPI,AMPR)
        AMPR=PIT*(AMPR*AMPR+AMPI*AMPI)
        SCATD=10.*ALOG10(AMPR)
        AMPR=REAL(SUMK)
        AMPI=AIMAG(SUMK)
        PHASEK=DIG*ATAN2(AMPI,AMPR)
        AMPR=PIT*(AMPR*AMPR+AMPI*AMPI)
        SCATK=10.*ALOG10(AMPR)
        AMPR=REAL(SUM)
        AMPI=AIMAG(SUM)
        PHASET=DIG*ATAN2(AMPI,AMPR)
        AMPR=PIT*(AMPR*AMPR+AMPI*AMPI)
        SCATT=10.*ALOG10(AMPR)
        WRITE(6,900) THE,SCATE,PHASEE,SCATM,PHASEM,SCATD,PHASED,
       &SCATK,PHASEK,SCATT,PHASET
        GO TO 75
105     PZSFAC=ZSFAC
        PWAVE=WAVE
        PZEFAC=ZEFAC
        PZMFAC=ZMFAC
        IF (MORE.EQ.0) GO TO 5
        READ (5,200) MORE,IPOL,WAVE,ZSFAC,ZEFAC,ZMFAC
        WRITE (6,150) ID
        GO TO 40
100     FORMAT (18A4)
150     FORMAT (1H1,18A4)
200     FORMAT (I2,I3,4F10.5)
210     FORMAT (I2,3X,3F10.5)
250     FORMAT(1H ,2I3,4F8.4,1X,2F11.3,F9.4,F9.3)
300     FORMAT(////18HOIMPEDANCE SURFACE)
350     FORMAT(17HOABSORBER SURFACE)
325     FORMAT(8HO  I SEG,4X,1HX,7X,1HY,7X,1HS,6X,3HDSQ,
       &11X,10H--- ZS ---,5X,7HMOD(JS),2X,7HARG(JS)/)
375     FORMAT(8HO  I SEG,4X,1HX,7X,1HY,7X,1HS,6X,3HDSQ,
       &11X,10H--- ZE ---,13X,10H--- ZM ---,5X,7HMOD(JE),2X,7HARG(JE),2X,
       &7HMOD(JM),2X,7HARG(JM)/)
395     FORMAT(1H ,2I3,4F8.4,2(1X,2F11.3),2(F9.4,F9.3))
400     FORMAT (////55X,14HKEY PARAMETERS//
       &40X,21HINCIDENT POLARIZATION,22X,1A1//
       &40X,24HSURFACE IMPEDANCE FACTOR,F20.3//
       &40X,25HELECTRIC IMPEDANCE FACTOR,F19.3//
       &40X,25HMAGNETIC IMPEDANCE FACTOR,F19.3//
       &40X,34HTOTAL NUMBER OF POINTS ON THE BODY,I10//
       &40X,23HNUMBER OF SEGMENTS USED,I21//
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      &40X,35HNUMBER OF INCIDENT FIELD DIRECTIONS,I9//
      &40X,29HNUMBER OF BISTATIC DIRECTIONS,I15//
      &40X,10HWAVELENGTH,F34.3)
600   FORMAT(///,51X,28HBACKSCATTERING CROSS SECTION,/,
      &55X,20H10*LOG(SIGMA/LAMBDA),/,
      &57X,1H(,1A1,14H-POLARIZATION),///,
      &31X,10H(ELECTRIC),7X,10H(MAGNETIC),7X,10H(ABSORBER),7X,
      &11H(IMPEDANCE),8X,7H(TOTAL),/,21X,5HTHETA,2X,
      &5(17H    DB      PHASE  )/)
800   FORMAT(///,49X,33HBISTATIC SCATTERING CROSS SECTION,/,
      &55X,20H10*LOG(SIGMA/LAMBDA),/,
      &48X,29HFOR INCIDENT FIELD DIRECTION=,F6.1,/,
      &57X,1H(,1A1,14H-POLARIZATION),///,
      &31X,10H(ELECTRIC),7X,10H(MAGNETIC),7X,10H(ABSORBER),7X,
      &11H(IMPEDANCE),8X,7H(TOTAL),/,21X,5HTHETA,2X,
      &5(17H    DB      PHASE  )/)
900   FORMAT(19X,F7.2,5(1X,2F8.2))
999   RETURN
      END
```

```
          SUBROUTINE GEOM(LUMP,X,Y,XN,YN,S,DSQ,ZF,ZM,K,M)
C**** RAMVS VERSION
          COMPLEX ZEA,ZEB,ZMA,ZMB,ZFUN,ZF(1),ZM(1)
          DIMENSION X(1),Y(1),XN(1),YN(1),DSQ(1),S(1)
          DIMENSION LUMP(2,1)
          COMMON/PIES/PI,TPI,PIT,PIPI,YZ,RED,DIG
          I=0
          K=0
          L=0
          M=0
          WRITE (6,500)
C......READ INPUT PARAMETERS AND PREPARE TO GENERATE SAMPLING POINTS
C......IF TYPE=1 ABSORBER SHEET, M CELLS TOTAL
C......IF TYPE=2 IMPEDANCE SURFACE,  K CELLS TOTAL
C......TYPE=1 SURFACE MUST BE READ IN FIRST
   10     READ (5,200) ITYPE,N,XA,YA,XB,YB,ANG
          IF (N.EQ.0) GO TO 120
          LIM=2*N-1
          READ (5,250) IZEFRM,ZEA,ZEB,ZEX
          IF (ITYPE.EQ.1) READ (5,250) IZMFRM,ZMA,ZMB,ZMX
          TX=XB-XA
          TY=YB-YA
          D=SQRT(TX*TX+TY*TY)
          IF (ANG.EQ.0.0) GO TO 20
          T=0.5*RED*ANG
          TRX=TX+TY/TAN(T)
          TRY=TY-TX/TAN(T)
          RAD=0.5*D/SIN(T)
          ARC=2.0*RAD*T
          ALF=T/N
          DID=2.0*RAD*ALF
          GO TO 30
   20     RAD=999.
          ARC=D
          DID=D/N
C......START GENERATING
   30     LAST=2
          IF(YA.EQ.0.0.AND.YB.EQ.0.0.AND.ANG.EQ.0.0) LAST=1
          DO 110 JIM=1,LAST
          L=L+1
          DO 100 J=1,LIM,2
          I=I+1
          LUMP(2,I)=L
          LUMP(1,I)=I
          IF (I.EQ.1000) WRITE (6,400)
          IF (JIM.EQ.2) GO TO 90
          IF (ANG.EQ.0.0) GO TO 40
          SINQ=SIN(J*ALF)
          COSQ=COS(J*ALF)
          X(I)=XA+0.5*(TRX*(1.0-COSQ)-TRY*SINQ)
```

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```
         Y(I)=YA+0.5*(TRX*SINQ+TRY*(1.0-COSQ))
         XN(I)=-0.5*(TRX*COSQ+TRY*SINQ)/RAD
         YN(I)= 0.5*(TRX*SINQ-TRY*COSQ)/RAD
         GO TO 50
  40     X(I)=XA+0.5*J*TX/N
         Y(I)=YA+0.5*J*TY/N
         XN(I)=-TY/D
         YN(I)= TX/D
  50     ST=0.5*J*DID
         S(I)=ST
C.....COMPUTE THE ELECTRIC PARAMETERS
         IF(ITYPE.EQ.1) GO TO 60
C.....ZS IS STORED IN THE ZE VECTOR
         ZE(I)=ZFUN(IZEFRM,ZEA,ZEB,ZEX,ST)
         GO TO 100
  60     ZM(I)=ZFUN(IZMFRM,ZMA,ZMB,ZMX,ST)
         ZE(I)=ZFUN(IZEFRM,ZEA,ZEB,ZEX,ST)
         GO TO 100
C.....FROM HERE TO 100 WE CREATE THE SEGMENT IMAGE
  90     K=I-N
         X(I)=X(K)
         Y(I)=-Y(K)
         XN(I)=XN(K)
         YN(I)=-YN(K)
         S(I)=S(K)
         ZE(I)=ZE(K)
         ZM(I)=ZM(K)
 100     DSQ(I)=DID
         IF (JIM.EQ.1) GO TO 102
         YA=-YA
         YB=-YB
 102     IF(ITYPE.EQ.1) GO TO 105
         WRITE(6,300) L,N,XA,YA,XB,YB,ANG,RAD,ARC,IZEFRM,ZEA,ZEB,ZEX
         K=I-M
         GO TO 110
 105     WRITE(6,350) L,N,XA,YA,XB,YB,ANG,RAD,ARC,IZEFRM,ZEA,ZEB,ZEX
         WRITE(6,351) IZMFRM,ZMA,ZMB,ZMX
         M=I
 110     CONTINUE
         GO TO 10
 200     FORMAT (I2,I3,5F10.5)
 250     FORMAT(I2,3X,5F10.5)
 300     FORMAT(1H ,I2,5H   IMP,I4,1X,4F9.4,1X,2F7.2,F7.3,I4,1X,2F9.3,2X,
        &2F9.3,F11.3)
 350     FORMAT(1H ,I2,5H   ABS,I4,1X,4F9.4,1X,2F7.2,F7.3,I4,1X,2F9.3,2X,
        &2F9.3,F11.3)
 351     FORMAT(71X,I4,2H (,F8.3,1X,F8.3,3H) (,F8.3,1X,F8.3,3H) (,
        &F8.3,1H))
 400     FORMAT(37HOWARNING: WE'VE GENERATED 1000 POINTS/)
```

```
500    FORMAT (13HOSEG SEG   NUM,3X,6H      -- ,21HENDPOINTS OF SEGMENTS,6H -
      &--   ,5X,27H -------------------------- ,18HSEGMENT PARAMETERS,27H -
      &-------------------------- /            TYP CELLS,4X,2HXA,7X,2HYA,7X,
      &2HXB,7X,2HYB,6X,24HANGLE RADIUS LENGTH FORM,6X,8HZEA(ZMA),12X,
      &8HZEB(ZMB),8X,8HZEX(ZMX)/)
120    RETURN
       END
```

```
       COMPLEX FUNCTION ZFUN(IFORM,ZA,ZB,ZEX,ST)
C****  RAMVS VERSION
       COMPLEX ZA,ZB
       IF(IFORM) 10,15,20
  10   ZFUN=CEXP(ZEX*CLOG(ZA-ZB*ST))
       RETURN
  15   ZFUN=ZA+ZB*ST**ZEX
       RETURN
  20   ZFUN=ZA+ZB*EXP(-ZEX*ST)
       RETURN
       END
```

```
      SUBROUTINE MTXFL1(MI,M,K,X,Y,XN,YN,DSQ,ZE,ZM,A)
C**** RAMVS VERSION  E-POLARIZATION
      DIMENSION X(1),Y(1),XN(1),YN(1),DSQ(1)
      COMPLEX ZE(1),ZM(1),A(MI,1)
      COMPLEX AA,HZ,HZA,HZB,H1,H1A,H1B
      REAL NPDR,NDR,NDNP
      COMMON/PIES/PI,TPI,PIT,PIPI,YZ,RED,DIG
      M1=M+1
      MK=M+K
      DO 300 II=1,M
      I=II
      IM=II+M
      XI=X(I)
      YI=Y(I)
      XNI=XN(I)
      YNI=YN(I)
C.....GENERATE ELEMENTS IN 1,2,4, AND 5
      DO 100 JJ=1,M
      J=JJ
      JM=J+M
      DS=DSQ(J)
      PDS=1./PIPI/DS
      DDS=0.25*DS*DS
      TPIDS=TPI*DS
      TDS=TPIDS/24.
      PITDS=PIT*DS
      IF (I.EQ.J) GO TO 120
      IJ=IABS(I-J)
      IF (IJ.LE.2) GO TO 110
      CALL DIST(XI,YI,X(J),Y(J),XNI,YNI,XN(J),YN(J),4,
     &R,NPDR,NDR,NDNP,SDR,SPDR)
      RK=R*TPI
      CALL HANKC(RK,2,HZ,H1)
      A(I,J)=HZ*PITDS
      AA=H1*PITDS
      A(I,JM)=YZ*CMPLX(0.,NPDR)*AA
      A(IM,J)=CMPLX(0.,NDR)*AA
      B=NPDR*NDR
      AA=B*HZ+(SPDR*SDR-B)*H1/RK
      A(IM,JM)= AA*PITDS*YZ
      GO TO 100
110   CALL DIST(XI,YI,X(J),Y(J),XNI,YNI,XN(J),YN(J),5,
     &R,NPDR,NDR,NDNP,SDR,SPDR)
      B=R*R+DDS
      C=DS*R*SPDR
      RAK=TPI*SQRT(B+C)
      IF (RAK.NE.RBK) GO TO 103
      HZA=HZB
      H1A=H1B
      GO TO 105
```

```
103      CALL HANKC(RAK,2,HZA,H1A)
105      RBK=TPI*SQRT(B-C)
         RK=R*TPI
         CALL HANKC(RK,0,HZ,H1)
         CALL HANKC(RBK,2,HZB,H1B)
         AA=TDS*(HZA+4.*HZ+HZB)
         A(I,J)=AA
         A(IM,JM)=(0.25*SDR*(H1B-H1A)+NDNP*AA)*YZ
         AA=CMPLX(0.,-0.25)*SPDR*(HZB-HZA)
         A(I,JM)=AA*NPDR*YZ
         A(IM,J)=AA*NDR
         GO TO 100
120      AA=CMPLX(PIT,ALOG(DS)+0.02879837)*DS
         A(I,J)= ZE(J)+AA
         A(I,JM)=(0.,0.)
         A(IM,J)=(0.,0.)
         A(IM,JM)=(ZM(J)+CMPLX(0.,PDS)+AA)*YZ
100      CONTINUE
C.....GENERATE ELEMENTS IN 3 AND 6
         DO 300 JJ=M1,MK
         J=JJ
         JM=J+M
         PITDS=PIT*DSQ(J)
         CALL DIST(XI,YI,X(J),Y(J),XNI,YNI,XN(J),YN(J),4,
        &R,NPDR,NDR,NDNP,SDR,SPDR)
         RK=R*TPI
         CALL HANKC(RK,2,HZ,H1)
C        ***REMEMBER ZS IS STORED IN ZE***
         HZA=ZE(J)
         AA=HZ-CMPLX(0.,NPDR)*H1*HZA
         A(I,JM)=AA*PITDS
         B=NPDR*NDR
         AA=B*HZ+(SPDR*SDR-B)*H1/RK
         AA=CMPLX(0.,NDR)*H1-AA*HZA
300      A(IM,JM)=AA*PITDS
         DO 500 II=M1,MK
         I=II
         IM=I+M
         XI=X(I)
         YI=Y(I)
         XNI=XN(I)
         YNI=YN(I)
C.....GENERATE ELEMENTS IN 7 AND 8
         DO 400 JJ=1,M
         J=JJ
         PITDS=PIT*DSQ(J)
         CALL DIST(XI,YI,X(J),Y(J),XNI,YNI,XN(J),YN(J),0,
        &R,NPDR,NDR,NDNP,SDR,SPDR)
         RK=R*TPI
         CALL HANKC(RK,2,HZ,H1)
         A(IM,J)=HZ*PITDS
400      A(IM,J+M)=CMPLX(0.,NPDR)*H1*PITDS*YZ
```

```
C.....GENERATE ELEMENTS IN 9
      DO 500 JJ=M1,MK
      J=JJ
      IF (I.EQ.J) GO TO 510
      CALL DIST(XI,YI,X(J),Y(J),XNI,YNI,XN(J),YN(J),O,
     &R,NPDR,NDR,NDNP,SDR,SPDR)
      RK=R*TPI
      CALL HANKC(RK,2,HZ,H1)
      AA=HZ-CMPLX(O.,NPDR)*H1*ZE(J)
      A(IM,J+M)=PIT*AA*DSQ(J)
      GO TO 500
510   DS=DSQ(J)
      A(IM,J+M)=0.5*ZE(J)+DS*CMPLX(PIT,ALOG(DS)+0.02879837)
500   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MTXEL2(MI,M,K,X,Y,XN,YN,DSQ,ZE,ZM,A)
C**** RAMVS VERSION  H-POLARIZATION
C**** RAMVS VERSION
      DIMENSION X(1),Y(1),XN(1),YN(1),DSQ(1)
      COMPLEX ZE(1),ZM(1),A(MI,1)
      COMPLEX AA,HZ,HZA,HZB,H1,H1A,H1B
      REAL NPDR,NDR,NDNP
      COMMON/PIES/PI,TPI,PIT,PIPI,YZ,RED,DIG
      M1=M+1
      MK=M+K
      DO 300 II=1,M
      I=II
      IM=II+M
      XI=X(I)
      YI=Y(I)
      XNI=XN(I)
      YNI=YN(I)
C.....GENERATE ELEMENTS IN 1,2,4, AND 5
      DO 100 JJ=1,M
      J=JJ
      JM=J+M
      DS=DSQ(J)
      PDS=1./PIPI/DS
      DDS=0.25*DS*DS
      TPIDS=TPI*DS
      TDS=TPIDS/24.
      PITDS=PIT*DS
      IF (I.EQ.J) GO TO 120
      IJ=IABS(I-J)
      IF (IJ.LE.2) GO TO 110
      CALL DIST(XI,YI,X(J),Y(J),XNI,YNI,XN(J),YN(J),4,
     &R,NPDR,NDR,NDNP,SDR,SPDR)
      RK=R*TPI
      CALL HANKC(RK,2,HZ,H1)
      A(I,J)=HZ*PITDS*YZ
      AA=H1*PITDS
      A(I,JM)=CMPLX(0.,-NPDR)*AA
      A(IM,J)=CMPLX(0.,NDR)*AA*YZ
      B=NPDR*NDR
      AA=B*HZ+(SPDR*SDR-B)*H1/RK
      A(IM,JM)=-AA*PITDS
      GO TO 100
  110 CALL DIST(XI,YI,X(J),Y(J),XNI,YNI,XN(J),YN(J),5,
     &R,NPDR,NDR,NDNP,SDR,SPDR)
      B=R*R+DDS
      C=DS*R*SPDR
      RAK=TPI*SQRT(B+C)
      IF (RAK.NE.RBK) GO TO 103
      HZA=HZB
      H1A=H1B
      GO TO 105
```

```
103     CALL HANKC(RAK,2,HZA,H1A)
105     RBK=TPI*SQRT(B-C)
        RK=R*TPI
        CALL HANKC(RK,0,HZ,H1)
        CALL HANKC(RBK,2,HZB,H1B)
        AA=TDS*(HZA+4.*HZ+HZB)
        A(I,J)=AA*YZ
        A(IM,JM)=-0.25*SDR*(H1B-H1A)-NDNP*AA
        AA=CMPLX(0.,0.25)*SPDR*(HZB-HZA)
        A(I,JM)=AA*NPDR
        A(IM,J)=-AA*NDR*YZ
        GO TO 100
120     AA=CMPLX(PIT,ALOG(DS)+0.02879837)*DS
        A(I,J)=(ZM(J)+AA)*YZ
        A(I,JM)=(0.,0.)
        A(IM,J)=(0.,0.)
        A(IM,JM)=-ZE(J)-CMPLX(0.,PDS)-AA
100     CONTINUE
C.....GENERATE ELEMENTS IN 3 AND 6
        DO 300 JJ=M1,MK
        J=JJ
        JM=J+M
        PITDS=PIT*DSQ(J)
        CALL DIST(XI,YI,X(J),Y(J),XNI,YNI,XN(J),YN(J),4,
       &R,NPDR,NDR,NDNP,SDR,SPDR)
        RK=R*TPI
        CALL HANKC(RK,2,HZ,H1)
C       ***REMEMBER ZS IS STORED IN ZE***
        HZA=ZE(J)
        AA=HZ*HZA-CMPLX(0.,NPDR)*H1
        A(I,JM)=AA*PITDS
        B=NPDR*NDR
        AA=B*HZ+(SPDR*SDR-B)*H1/RK
        AA=CMPLX(0.,NDR)*H1*HZA-AA
300     A(IM,JM)=AA*PITDS
        DO 500 II=M1,MK
        I=II
        IM=I+M
        XI=X(I)
        YI=Y(I)
        XNI=XN(I)
        YNI=YN(I)
C.....GENERATE ELEMENTS IN 7 AND 8
        DO 400 JJ=1,M
        J=JJ
        PITDS=PIT*DSQ(J)
        CALL DIST(XI,YI,X(J),Y(J),XNI,YNI,XN(J),YN(J),0,
       &R,NPDR,NDR,NDNP,SDR,SPDR)
        RK=R*TPI
        CALL HANKC(RK,2,HZ,H1)
        A(IM,J)=HZ*PITDS*YZ
400     A(IM,J+M)=CMPLX(0.,NPDR)*H1*PITDS
```

```
C.....GENERATE ELEMENTS IN 9
      DO 500 JJ=M1,MK
      J=JJ
      IF (I.EQ.J) GO TO 510
      CALL DIST(XI,YI,X(J),Y(J),XNI,YNI,XN(J),YN(J),0,
     &R,NPDR,NDR,NDNP,SDR,SPDR)
      RK=R*TPI
      CALL HANKC(RK,2,HZ,H1)
      AA=HZ*ZE(J)-CMPLX(0.,NPDR)*H1
      A(IM,J+M)=PIT*AA*DSQ(J)
      GO TO 500
510   DS=DSQ(J)
      A(IM,J+M)=0.5+DS*CMPLX(PIT,ALOG(DS)+0.02879837)*ZE(J)
500   CONTINUE
      RETURN
      END




      SUBROUTINE DIST(XI,YI,XJ,YJ,XNI,YNI,XNJ,YNJ,I,
     &R,NPDR,NDR,NDNP,SDR,SPDR)
C.....I=0  R,NPDR
C.....I=1  R,NPDR,NDR
C.....I=2  R,NPDR,NDR,NDNP
C.....I=3  R              NDNP,SDR,SPDR
C.....I=4  R,NPDR,NDR,     SDR,SPDR
C.....I=5  R,NPDR,NDR,NDNP,SDR,SPDR
      REAL NPDR,NDR,NDNP
      IF (I.LT.0.OR.I.GT.5) GO TO 50
      RX=XI-XJ
      RY=YI-YJ
      R=SQRT(RX*RX+RY*RY)
      IF (I.EQ.3) GO TO 10
      NPDR=(RX*XNJ+RY*YNJ)/R
      IF (I.EQ.0) RETURN
      NDR=(RX*XNI+RY*YNI)/R
      IF (I.EQ.1) RETURN
      IF(I.EQ.4) GO TO 15
10    NDNP=XNI*XNJ+YNI*YNJ
15    IF (I.EQ.2) RETURN
      SDR=(RX*YNI-RY*XNI)/R
      SPDR=(RX*YNJ-RY*XNJ)/R
      RETURN
50    WRITE(6,90) I,R
90    FORMAT(31HOSICK DATA IN DIST   *QUIT*   I=,I2,2X,2HR=,E11.3)
      CALL SYSTEM
      END
```

```
      SUBROUTINE HANKC(R,N,HZERO,HONE)
C.....HANKEL FUNCTIONS ARE OF FIRST KIND--J+IY
C.....    N=0 RETURNS HZERO
C.....    N=1 RETURNS HONE
C.....    N=2 RETURNS HZERO AND HONE
C.....SUBROUTINE REQUIRES R>0
C.....SUBROUTINE ADAM MUST BE SUPPLIED BY USER
      DIMENSION A(7),B(7),C(7),D(7),E(7),F(7),G(7),H(7)
      COMPLEX HZERO,HONE
      DATA A,B,C,D,E,F,G,H/1.0,-2.2499997,1.2656208,-0.3163866,
     &0.0444479,-0.0039444,0.00021,0.36746691,0.60559366,-0.74350384,
     &0.25300117,-0.04261214,0.00427916,-0.00024846,0.5,-0.56249985,
     &0.21093573,-0.03954289,0.00443319,-0.00031761,-0.00001109,
     &-0.6366198,0.2212091,2.1682709,-1.3164827,0.3123951,-0.0400976,
     &0.0027873,0.79788456,-0.00000077,-0.0055274,-0.00009512,
     &0.00137237,-0.00072805,0.00014476,-0.78539816,-0.04166397,
     &-0.00003954,0.00262573,-0.00054125,-0.00029333,0.00013558,
     &0.79788456,0.00000156,0.01659667,0.00017105,-0.00249511,
     &0.00113653,-0.00020033,-2.35619449,0.12499612,0.0000565,
     &-0.00637879,0.00074348,0.00079824,-0.00029166/
      IF (R.LE.0.0) GO TO 50
      IF (N.LT.0.OR.N.GT.2) GO TO 50
      IF (R.GT.3.0) GO TO 20
      X=R*R/9.0
      IF (N.EQ.1) GO TO 10
      CALL ADAM(A,X,BJ)
      CALL ADAM(B,X,Y)
      BY=0.6366198*ALOG(0.5*R)*BJ+Y
      HZERO=CMPLX(BJ,BY)
      IF (N.EQ.0) RETURN
   10 CALL ADAM(C,X,Y)
      BJ=R*Y
      CALL ADAM(D,X,Y)
      BY=0.6366198*ALOG(0.5*R)*BJ+Y/R
      HONE=CMPLX(BJ,BY)
      RETURN
   20 X=3.0/R
      IF (N.EQ.1) GO TO 30
      CALL ADAM(E,X,Y)
      FOOL=Y/SQRT(R)
      CALL ADAM(F,X,Y)
      T=R+Y
      BJ=FOOL*COS(T)
      BY=FOOL*SIN(T)
      HZERO=CMPLX(BJ,BY)
      IF (N.EQ.0) RETURN
```

```
30    CALL ADAM(G,X,Y)
      FOOL=Y/SQRT(R)
      CALL ADAM(H,X,Y)
      T=R+Y
      BJ=FOOL*COS(T)
      BY=FOOL*SIN(T)
      HONE=CMPLX(BJ,BY)
      RETURN
50    WRITE(6,90) N,R
90    FORMAT(31HOSICK DATA IN HANKC   *QUIT*   N=,I2,2X,2HR=,E11.3)
      CALL SYSTEM
      END




      SUBROUTINE ADAM(C,X,Y)
      DIMENSION C(7)
      Y=X*C(7)
      DO 10 I=1,5
10    Y=X*(C(7-I)+Y)
      Y=Y+C(1)
      RETURN
      END
```

```
      SUBROUTINE FLIP(A,N,MI,L,M,X,Y,IAT)
      COMPLEX A(MI,1),X(1),Y(1),D,BIGA,HOLD
      DIMENSION L(1),M(1)
      IF (IAT.GT.1) GO TO 150
      D=CMPLX(1.0,0.0)
      DO 80 K=1,N
      L(K)=K
      M(K)=K
      BIGA=A(K,K)
      DO 20 J=K,N
      DO 20 I=K,N
10    IF (CABS(BIGA).GE.CABS(A(I,J))) GO TO 20
      BIGA=A(I,J)
      L(K)=I
      M(K)=J
20    CONTINUE
      J=L(K)
      IF (J.LE.K) GO TO 35
      DO 30 I=1,N
      HOLD=-A(K,I)
      A(K,I)=A(J,I)
30    A(J,I)=HOLD
35    I=M(K)
      IF (I.LE.K) GO TO 45
      DO 40 J=1,N
      HOLD=-A(J,K)
      A(J,K)=A(J,I)
40    A(J,I)=HOLD
45    IF (CABS(BIGA).NE.0.0) GO TO 50
      D=CMPLX(0.0,0.0)
      RETURN
50    DO 55 I=1,N
      IF (I.EQ.K) GO TO 55
      A(I,K)=-A(I,K)/BIGA
55    CONTINUE
      DO 65 I=1,N
      DO 65 J=1,N
      IF (I.EQ.K.OR.J.EQ.K) GO TO 65
      A(I,J)=A(I,K)*A(K,J)+A(I,J)
65    CONTINUE
      DO 75 J=1,N
      IF (J.EQ.K) GO TO 75
      A(K,J)=A(K,J)/BIGA
75    CONTINUE
      D=D*BIGA
80    A(K,K)=1.0/BIGA
      K=N
```

```
100    K=K-1
       IF (K.LE.0) GO TO 150
       I=L(K)
       IF (I.LE.K) GO TO 120
       DO 110 J=1,N
       HOLD=A(J,K)
       A(J,K)=-A(J,I)
110    A(J,I)=HOLD
120    J=M(K)
       IF (J.LE.K) GO TO 100
       DO 130 I=1,N
       HOLD=A(K,I)
       A(K,I)=-A(J,I)
130    A(J,I)=HOLD
       GO TO 100
150    DO 200 I=1,N
       Y(I)=CMPLX(0.0,0.0)
       DO 200 J=1,N
200    Y(I)=A(I,J)*X(J)+Y(I)
       RETURN
       END
```

```
BLOCK DATA
COMMON/PIES/PI,TPI,PIT,PIPI,YZ,RED,DIG
DATA PI,TPI,PIT,PIPI,YZ,RED,DIG/3.1415927,6.2831853,
&1.5707963,9.8696044,0.0026525824,0.01745329,57.29578/
END
```

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13. ABSTRACT

This report describes a program that computes the far field scattering pattern of a two-dimensional cylindrical body (or bodies) treated with absorbing materials. The body surface is assumed to satisfy an impedance boundary condition and the absorber is modeled by equivalent electric and magnetic sheets. The program reduces the coupled integral equations governing the surface currents to a system of simultaneous linear equations, solves for the unknown currents, and then computes the far field pattern from the solution. The integral equations derived and presented in a previous interim report were used as the basis for preliminary versions of the program, but these equations have been found to be in error. The required correction consists of incorporating terms previously omitted and the corrected equations are presented and discussed. Careful attention is given to a description of the input data necessary to run the program and the results of a sample run are included for illustration.

The program was developed to explore the effects of absorbent materials on the scattering of electromagnetic waves by edges. Programs used previously in similar explorations embodied a surface impedance boundary condition; for lossy materials covering smooth surfaces of large radius of curvature, such an impedance can be estimated from the layer thickness and material properties, but the relationship breaks down near edges. The program described in this report, however, models the effects of actual materials rather than using the nebulous surface impedance boundary condition.

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