011764-515-M

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MEMO TO: File

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SUBJECT: More about the surface field on a constant impedance half plane

In Memorandum 011764-514-M it was suggested that

$$\frac{K_+(k)}{K_+(\xi)} \longrightarrow 1$$

as $|\xi| \to \infty$. This is incorrect. From a more detailed consideration of the split functions (to be reported later), it can be shown that

$$K_+(\xi) = \left\{\frac{\cos\chi\,(1+\cos\theta)}{\cos\chi+\sin\theta}\right\}^{1/2} \left\{\frac{1+\sin(\chi+\theta)}{\cos\chi+\sin\theta}\cdot\frac{1-\sin(\chi-\theta)}{\cos\chi-\sin\theta}\right\}^{1/4}$$

$$\cdot \exp\left\{\frac{1}{2\pi}\left(\int_{\theta+\chi}^{i\infty} + \int_{\theta-\chi}^{i\infty}\right)\frac{v\,dv}{\cos v}\right\} \qquad (1)$$

where $\cos\chi = 1/\eta$ and $\cos\theta = \xi/k$. For large $|\xi|$,

$$\sin\theta \sim \frac{i\xi}{k}, \qquad \theta \sim i\log\frac{2\xi}{k}$$

and, hence, if $\eta \neq 0$,

$$K_+(\xi) \sim \left(\frac{\cos\chi}{i}\right)^{1/2} (i^2)^{1/4} \exp\left\{\frac{1}{2\pi}\left(\int_{\theta+\chi}^{i\infty} + \int_{\theta-\chi}^{i\infty}\right)\frac{v\,dv}{\cos v}\right\}.$$

As $|\xi| \to \infty$, both integrals tend to zero, giving

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$$K_+(\xi) \sim \eta^{-1/2} \quad , \qquad \eta \neq 0; \tag{2}$$

thus

$$\frac{K_+(k)}{K_+(\xi)} \longrightarrow \eta^{1/2} K_+(k) \tag{3}$$

as $|\xi| \to \infty$, from which we obtain

$$E_z(0, \pm 0) = \eta^{1/2} K_+(k) \ . \tag{4}$$

If $|\eta| \gtrsim 1$,

$$K_+(k) \simeq \eta^{-1/2} \exp\left(-\frac{1}{\pi\eta}\right)$$

and in this case we now have

$$E_z(0, \pm 0) \simeq \exp\left(-\frac{1}{\pi\eta}\right) \ . \tag{5}$$

When $\eta = 8$, corresponding to a normalised electrical resistivity of 4, eq. (5) gives

$$E_z(0, \pm 0) = 0.9606$$

which is fantastically close to the value obtained from the computed data for electrically resistive sheets.