011764-513-M

4 December 1973

MEMO TO: File

FROM: E. F. Knott

SUBJECT: Analysis of resistive sheet with parabolic distribution

This memorandum summarizes the results of a series of radar cross section computations obtained via program RISK from AFAL during November, 1973. The scatterer was an ogival cylinder 3λ wide with a flat resistive sheet attached to the leading edge and lying in the plane of symmetry. The width of the sheet, herein designated as ℓ, was varied from 0.5λ to 1.5λ in steps of 0.05λ, producing 21 distinct cases. The sampling rate over the sheet was fixed at 20 samples per wavelength, hence ranged from as few as 10 to as many as 30 samples.

The resistance distribution was parabolic with R_{max} (the value at the leading edge of the sheet) fixed at 1508 ohms. Unfortunately, because of errors in the input data cards sent to AFAL, three of the runs (for $\ell = 0.5$, 0.55 and 1.0λ) could not be used. However, similar data for two of these widths had been previously obtained using a different sampling rate. By including the data from still another previous run (for $\ell = 0.25\lambda$), the number of cases is restored to 21. The edge-on scattering behavior is listed in Table 1, along with the complex values of P_c, to be discussed in a moment.

The first three columns of Table 1 represent the data read directly from the output of the program. Because of a sign error in program RISK (and thus in its sister program REST), the phase of the scattering printed on the output record is 180 degrees removed from the true phase angle. Hence

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ℓ/λ	10 log σ/λ	ϕ, deg.	Real P_c	Imag. P_c
0.25	-15.83	72.7	-0.38680	0.12048
0.50	-22.35	71.9	0.18178	-0.05942
0.60	-24.66	45.0	0.13061	0.06655
0.65	-26.82	26.9	0.08097	0.08069
0.70	-30.50	4.3	0.03932	0.06367
0.75	-34.88	-28.6	0.02164	0.03968
0.80	-40.13	-95.4	0.02126	0.01257
0.85	-37.40	-170.8	0.03342	-0.00517
0.90	-33.94	149.6	0.04919	-0.01082
0.95	-32.15	119.6	0.06150	-0.00690
1.00	-31.52	90.8	0.06653	0.00093
1.05	-31.58	61.4	0.06553	0.00851
1.10	-32.15	28.9	0.06077	0.01170
1.15	-32.85	-7.4	0.05612	0.01050
1.20	-33.25	-46.9	0.05411	0.00674
1.25	-33.09	-87.0	0.05546	0.00291
1.30	-32.57	-124.7	0.05895	0.00134
1.35	-32.07	-159.6	0.06240	0.00261
1.40	-31.83	167.4	0.06392	0.00604
1.45	-31.94	134.7	0.06267	0.00959
1.50	-32.37	101.0	0.05923	0.01151

Table 1. Raw and corrected edge-on scattering data as obtained from program RISK. The last two columns corrected data.

the values listed in Table 1 should be advanced (or retarded) 180 degrees in order to arrive at the true phase angle. In addition, because of the ambiguity in the true phase (i.e., a phase angle θ cannot be distinguished from $\theta \pm 2m\pi$, where m is an integer), multiples of 2π must be added or subtracted from the data in order to produce smoothly changing phase as a function of sheet width.

The correct multiple of 2π may be estimated knowing the origin of the coordinate system and the suspected location of the dominant scattering center on the body. Figure 1 shows the coordinates and, since it was believed that

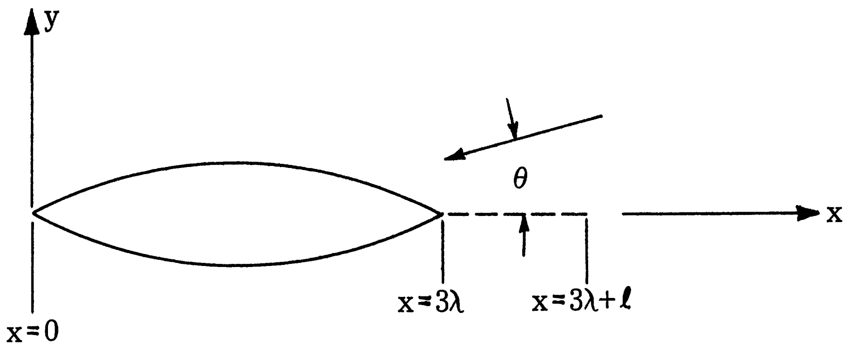

Figure 1. Coordinate system for the digital "measurements".

the leading edge was the dominant source of return, the phase angle for $\ell = 1.5\lambda$ was fixed near -6.5π. With this interpretation, the "measured" amplitude and phase of the edge-on scattering (corresponding to $\theta = 0$ in Figure 1) are plotted in Figure 2.

It is immediately apparent from the plot that the character of the scattering changes drastically as the sheet width becomes more than about 1λ. The amplitude decreases rapidly with increasing width and, presumably due to a cancellation effect, drops into a deep null near $\ell = 0.8\lambda$. Beyond this width the amplitude exhibits a gentle undulation characteristic of a small component being added to a much larger one. Aside from a kink in the data between

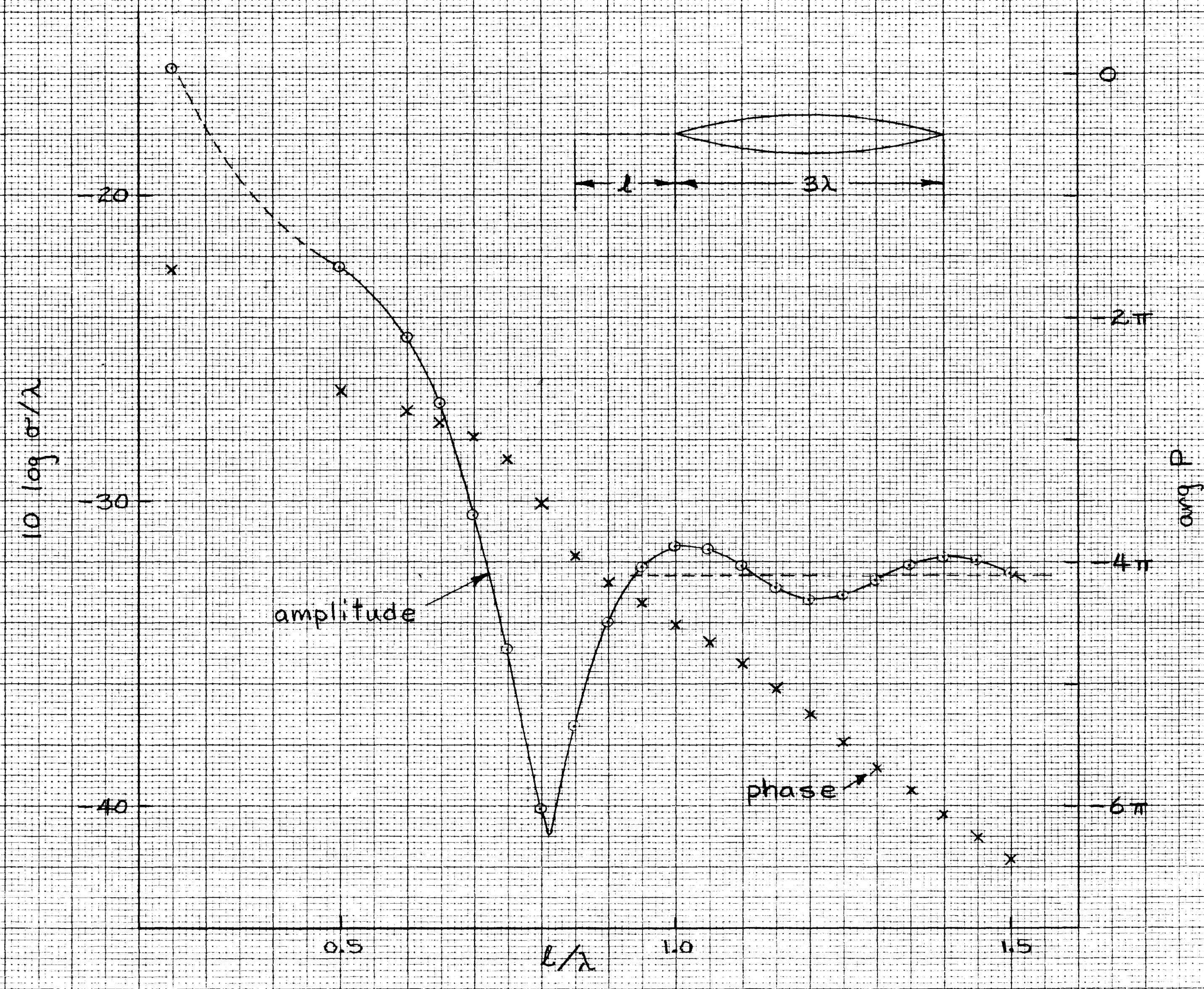

Figure 2. Amplitude and phase of the edge-on return from an ogival cylinder and resistive sheet with parabolic distribution. R = 4.0.

$\ell = 0.5\lambda$ and 0.8λ, the deduced phase angle decreases almost linearly at a rate commensurate with the position of the leading edge of the sheet.

A preliminary analysis focussed on determining the two components producing the interference pattern for the longer lengths. The mean level of the pattern, as indicated by the dashed line in Figure 2, implies a normalized scattering amplitude of 0.0600, which is some 3.6 percent greater than the theoretical value of $A = 0.05793$ given by the resistive half plane result for $R_{max} = 4.0$. The mean level of the other component, call it B, is obtainable from the peak to peak excursion of the undulation. From a graphical magnification of the pattern, the excursion is found to be 1.61 dB, implying that $B = 0.0055$.

There is a peculiarity in the pattern, however. The period is not 0.5λ as expected, but more nearly 0.39λ, and this can occur only if the relative phase between the two contributors changes at a rate different from that inferred from their spatial positions. We believe that A arises from the leading edge of the sheet and that B is the scattering from the trailing edge. It is natural to postulate that, since the rear edge is excited, in part, by a wave that propagates along the resistive strip, the relative phase of the trailing edge return go as $e^{i2nk\ell}$, where $n > 1$ accounts for the increased phase delay in passing over the sheet. Based on a period of 0.39λ, $n = 1.282$.

The conceptual picture of the net scattering is thus

$$\frac{P}{|P_0|} = -i\,(A' + B\,e^{i2nk\ell})\,e^{-i2k\ell} \quad , \qquad (1)$$

where $A' = 0.0600$, (i.e., slightly greater than the theoretical value), $P_0 = -\frac{i}{2}$, the metallic half plane value and B is a complex function which is essentially a small constant for $\ell \geq 1.0$, and a rapidly changing function of ℓ for $\ell < 1.0$. Since P is directly obtainable from the output of program RISK in amplitude and phase, and since all indications suggest that A' is fixed, equation (1) can be solved for B.

The results were dismal disappointments. A series of calculations were performed in which A' and n were specified as above, and given slightly different values as well, produced oscillatory values for $|B|$ for $\ell \geq \lambda$. Instead of an expected value near 0.0055, B inevitably traced out a sinusoidal curve with increasing ℓ, rising to peaks greater than 0.01 and attaining nulls as low as 0.0015. Evidently there was a flaw in the concept.

Returning to equation (1), I found it convenient to adjust the phase of all terms such that the leading edge would be the phase reference, and to multiply by i. The result, call it P_c, is

$$P_c = -i \frac{P}{|P_0|} e^{-i2k\ell} = A' + B e^{i2kn\ell} , \qquad (2)$$

and P_c is in every respect as valid a measurement as is P. The real and imaginary parts of P_c are listed in Table 1 and are plotted on the complex plane in Figure 3.

Figure 3 shows that the net return rapidly spirals inward to a complex value of A' with increasing sheet width and that the only flaw in the conceptual picture was that A' was previously assumed to be purely real. A few exploratory calculations showed that the optimum values of A' and n for sheet widths between 1.15 and 1.50 wavelengths are

$$A' = 0.059 + i\,0.00641,$$
$$n = 1.290.$$

Using these values, I solved equation (2) for B and obtained the numbers listed in Table 2. The amplitude $|B|$ is plotted in Figure 4.

The mean variation in $|B|$ has been indicated by a smooth solid line, but careful examination of the figure shows that the datum points may lie slightly off the smooth curve. The amplitude $|A'|$ has been indicated as a dashed line running horizontally across the graph, since A' was assumed to

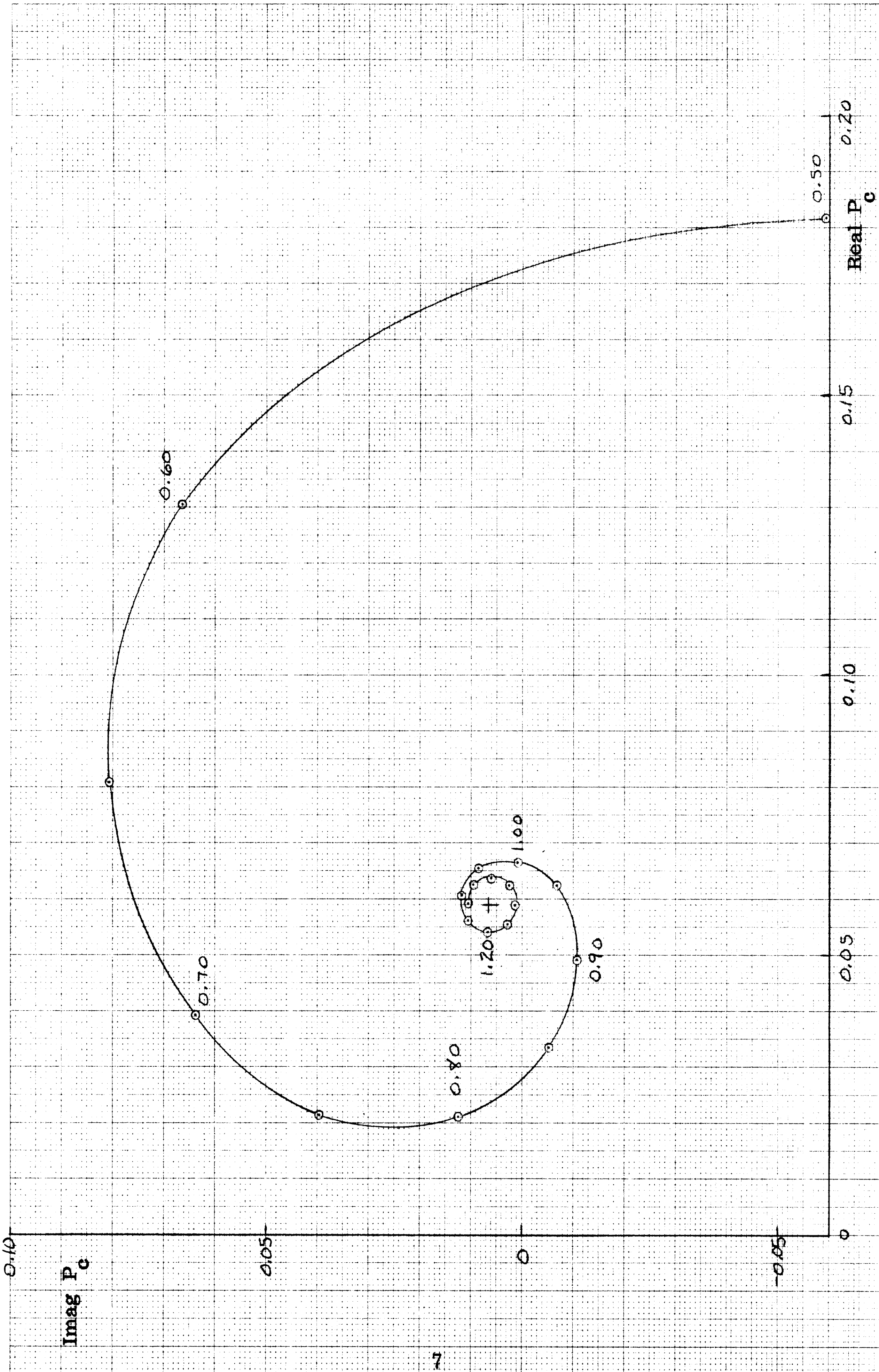

Figure 3. A plot of P_c on the complex plane.

ℓ/λ	mod B	arg B (degrees)
0.25	0.46016	-66.55
0.50	0.13931	-132.60
0.60	0.09351	-157.26
0.65	0.07746	-170.20
0.70	0.06055	178.81
0.75	0.05003	161.72
0.80	0.03824	147.69
0.85	0.02808	134.88
0.90	0.01982	124.41
0.95	0.01354	118.28
1.00	0.00932	115.16
1.05	0.00686	122.58
1.10	0.00558	129.83
1.15	0.00500	137.02
1.20	0.00490	141.60
1.25	0.00498	143.72
1.30	0.00508	142.00
1.35	0.00510	138.01
1.40	0.00494	135.39
1.45	0.00486	134.13
1.50	0.00511	134.22

Table 2. Deduced value of B for $A' = 0.059 + 0.00641$ and $n = 1.290$.

be independent of ℓ. Note that although the two intersect near $\ell = 0.70\lambda$, the null in Figure 2 occurs near $\ell = 0.80\lambda$ because the relative phase between them is not 180°. Note that the decay of B appears to be exponential and bottoms out at a threshold value of $|B| \approx 0.005$. The curve has no inclination to fall below this level.

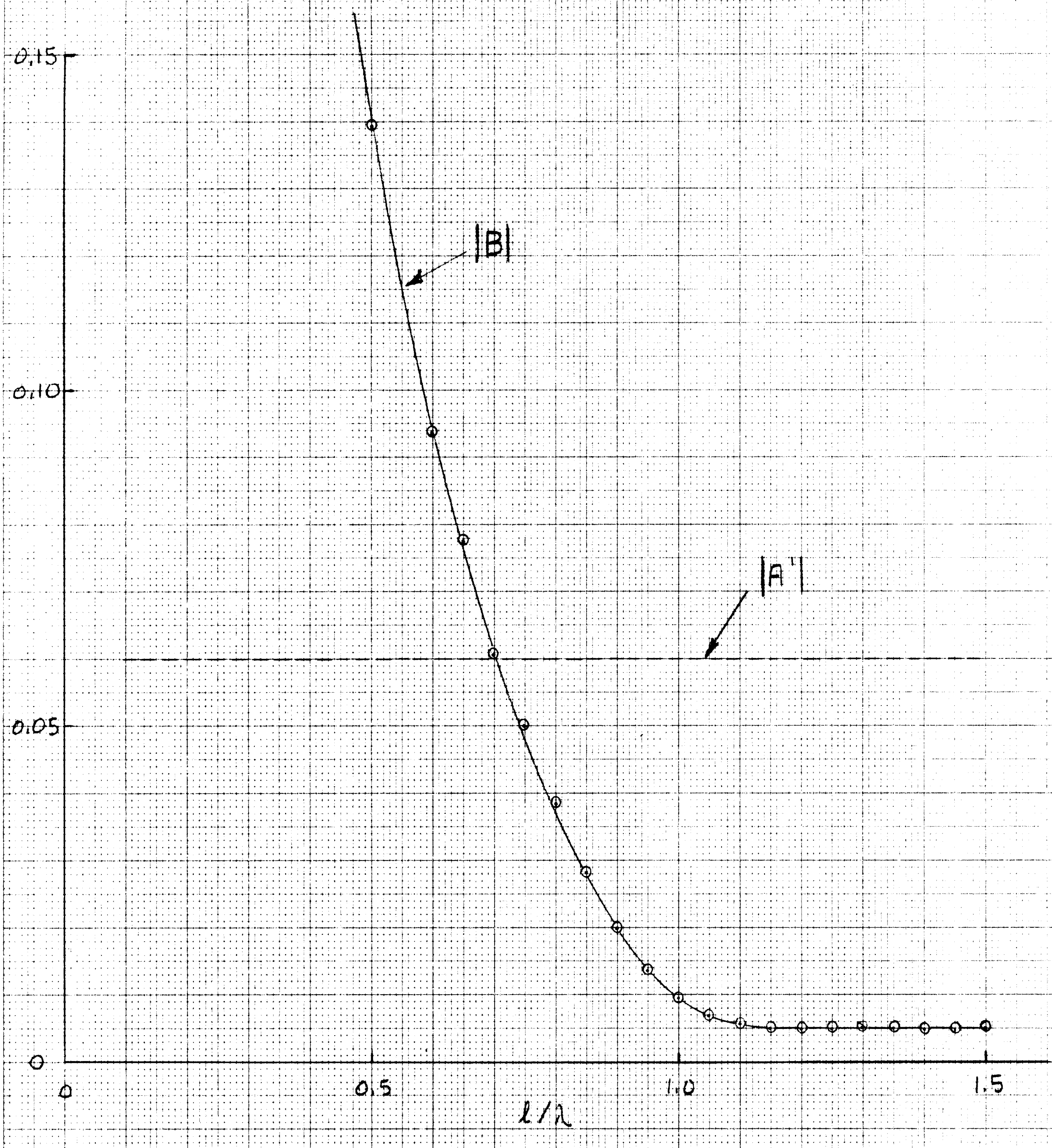

Figure 4. Dependence of |B| on ℓ/λ.

The radar cross section of the trailing edge component is plotted in Figure 5, along with that of the (constant) leading edge term for reference. The decay is not quite exponential and seems to have a sinusoidal perturbation; this may be because the factor n in equation (2) is not a constant but is, instead, a function of ℓ. The mean rate of decay is about 45 dB per wavelength, suggesting that, at least for the narrower sheet widths,

$$|B| \propto e^{-2.25\ell/\lambda} ,$$

a quite rapid decay.

The above analysis strengthens the conceptual model that the sources of scattering are concentrated near the leading and trailing edges of the resistive sheet. It shows that an unvarying leading edge component pegged at very nearly the theoretical resistive half plane value is almost correct, but that it must be allowed to take on a slightly reactive character. And although the parameters A' and n have been determined for this particular case of a parabolic distribution for $R_{max} = 4.0$, it will obviously be of interest to determine the corresponding values for other R_{max} and resistance distributions.

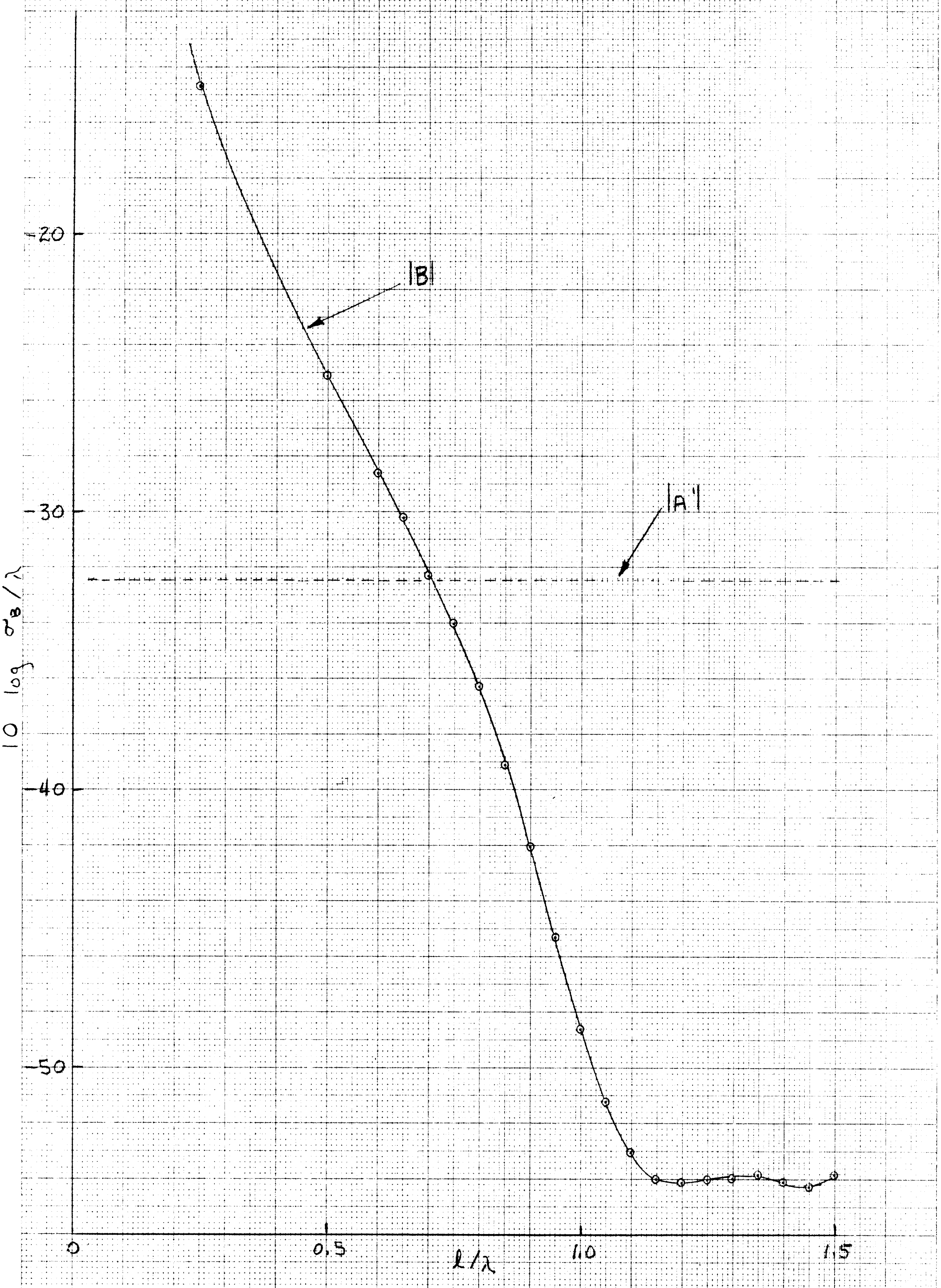

Figure 5. Radar cross section contributions of leading and trailing edges.