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Thomas B.A. Senior

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SCATTERING BY SMALL AEROSOLS

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The investigation was concerned with the electromagnetic scattering characteristics of dielectric crystals such as those occurring in ice crystal clouds and other aerosol-laden atmospheres. Whereas most current theories have modelled the crystals as spheres, our interest is centered on the computation of the scattering behavior of the specific crystal shapes which are known to occur. A portion of the frequency spectrum where this task is feasible is the so-called "low frequency" or Rayleigh region where the wavelength of the radiation is greater by an order of magnitude or more than the maximum linear dimension of the scatterer. The particles of interest consist mainly of plate and columnar crystals whose linear dimensions range from 10 to 500μ.m. For even the largest of these crystals, the Rayleigh region encompasses the entire range of radio frequencies, with wavelengths down to the millimeter range, and for the smaller particles extends into the far infrared as well.

Our studies have been directed entirely at the scattering from a single homogeneous isotropic dielectric particle at these frequencies. A general mathematical formulation has been derived to specify the scattering behavior of such a particle and has been applied to the special cases of rotationally symmetric bodies and rectangular parallelepipeds. Techniques resulting from recent applications of digital methods in potential theory to electromagnetic problems have been utilized to obtain efficient numerical methods for computing the dipole moments which characterize the scattering, and data obtained for selected geometries.

Formulation

In our general formulation we have adopted an approach based on the electric and magnetic polarizability tensors $\overline{\overline{P}}(\epsilon_r)$ and $\overline{\overline{M}}(\mu_r)$ respectively, in terms of which the electric and magnetic dipole moments which characterize the far zone

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scattered fields are simply the dot products of the polarizability tensors with the incident field polarization vector. A particularly convenient aspect of this approach is that the tensors are functions only of the material and geometry of the scatterer and are independent of the direction and polarization vectors of the fields. This explicit separation of the direction and polarization from the intrinsic properties of the particle makes the formulation highly advantageous for any subsequent development of a multiple scattering theory by a cloud of particles. The formulation is described in detail in attachment A where it is shown that both tensors are special cases of a general polarizability tensor $\overline{\overline{X}}(\tau)$, where τ is a material parameter representing either the relative permittivity ϵ_r or relative permeability μ_r of the particle. The tensor elements X_{ij} $(i,j = 1,2,3)$ are expressed as weighted surface integrals of certain potential functions or alternatively as integrals of the normal derivatives of the potentials. To compute the tensor elements it is therefore sufficient to determine either the potentials or their normal derivatives at the surface of the body, and integral equations have been derived for both of these quantities. Examination of these integral equations shows that unique solutions exist for $\tau > 0$, so that the tensor $\overline{\overline{X}}(\tau)$ can be uniquely determined for the values of τ which are of practical interest, and requires the solution of at most three integral equations.

Properties of $X(\tau)$

Since the essential ingredient in the solution of the scattering problem is now the determination of the general polarizability tensor, we have explored the mathematical properties of $\overline{\overline{X}}(\tau)$. For real τ, the tensor is real and symmetric; and for a body which is rotationally symmetric about the x_3 axis, or indeed for any body having symmetry about the two perpendicular planes $x_1 = 0$, $x_2 = 0$ and the plane $x_1 = x_2$, the tensor is diagonal with $X_{11} = X_{22}$, having therefore at most two independent elements and requiring the solution of (at most) two corresponding integral equations. In the case of a spheroidal body we have proved the relation

$$X_{11}(\tau) = - 2 X_{33}\left(\frac{2}{1 + \tau}\right) \tag{1}$$

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holding for all spheroids, both oblate and prolate. This implies that the computation of X_{33} for $0 < \tau \leqslant \infty$ yields the values of $X_{11} (= X_{22})$ for $-1 < \tau \leqslant \infty$ and obviates the solution of a second integral equation. For the general case of a body of arbitrary shape, it is possible to obtain bounds of a geometrical nature on the diagonal elements of the tensor. A lower bound has been established as (c.f. eq. (28) attachment A)

$$X_{ii} \geqslant \frac{\tau - 1}{\tau} V \tag{2}$$

where V is the volume of the body, and we have shown as well that an upper bound is given by

$$X_{ii} \leqslant (\tau - 1) V . \tag{3}$$

These bounds are valid for all homogeneous dielectric bodies having $\tau > 0$ and are optimum in the sense that the equality is obtained in (2) by the elements X_{33} for a vanishingly thin oblate spheroid and X_{11} for a disc and in (3) by the element X_{33} for a disc. We also remark that the present results have indicated that more stringent bounds may be obtained in such special cases as, for example, rotational symmetry, but this has not yet been fully investigated.

Applications

The formulation has been applied to a rotationally symmetric body of homogeneous isotropic material, and integral equations have been developed for this case in terms of both the potentials and their normal derivatives at the surface of the body. Computer programs have been written for the numerical solution of the integral equations, and hence the computation of X_{11} and X_{33}, and their validity has been checked by comparison with the results obtained from the known analytical expressions for a sphere. The programs are applicable to any rotationally symmetric body whose profile can be constructed from straight line and circular arc segments. Data have been obtained for bodies of various shapes, e.g. spheroids, ogives and right circular cylinders, and although the relation (1) has been proved analytically only for spheroids, these data have indicated that (1) may

be valid for rotationally symmetric bodies in general. The dipole moments per unit volume for an ogive arc closely approximated by those of the corresponding spheroid of the same length-to-width ratio and material parameter τ, the difference being about one percent or less, and for all practical purposes an ogive may therefore be represented by a spheroid. In addition, we remark that the formulation in terms of the potentials is far superior to that in terms of their normal derivatives from the standpoint of numerical convergence.

We have also examined the case of a homogeneous isotropic rectangular parallelepiped. The basic integral equations of attachment A have been specialized for this geometry with the result (c.f. eq. (16) attachment B)

$$
\frac{1+\tau}{1-\tau}\,\Psi_i(\underline{r}) = -2x_i + \frac{1}{2\pi}\sum_{m=1}^{3}\int_{S_m}\Psi_i(\underline{r}_m)K_{im}(\underline{r},\underline{r}_m)\,dS_m \tag{4}
$$

and we remark that although the integral equations are in general weakly singular, the kernels K_{im} in (4) are bounded provided the observation point (whose position vector is $\underline{r}$) does not lie on an "edge" of the body. The numerical solution of (4) by the moment method can be achieved by dividing the surfaces of integration into sampling cells over which the entire integrand in (4) is assumed constant. Computer programs have been written to compute X_{11} ($= X_{22}$) and X_{33} by this method for a rectangular parallelpiped of square cross section, and data obtained for the special case of a cube. The results are presented in attachment B and differ substantially from those previously reported in the literature. For rectangular parallelepipeds of large length-to-width ratio the numerical convergence properties of this method are rather poor, and the large number of sampling cells required to obtain accurate results greatly increases the expense of the computations. Since the kernels in (4) may be integrated analytically, we actually need only assume the potential Ψ to be constant over the surface of a cell, and a second program has been written to compute the tensor elements by this method. A comparison of the two programs has been made using the cubic geometry and shows the latter to be far superior numerically, yielding greater accuracy with 48 cells than was

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previously obtained using 75 cells. In addition to reducing the required number of sampling cells, we have also found that for bodies of large length-to-width ration, where the use of square cells is no longer feasible, the task of selecting the configuration of sampling cells is simplified in the second method since we are there concerned only with the potential, which is in general a slowly varying function over the surface of the body.

A third geometry which we have examined is a cylinder of hexagonal cross section. The mathematical formulation has been derived, and since it does not differ substantially from that for a rectangular parallelepiped, the development of numerical techniques for computing the tensor elements has been postponed pending further investigation of the methods described above.

A proposal seeking support for the continuation of this work was submitted to the National Science Foundation on 25 March 1976.

Publications Supported by the Grant

Senior, T.B.A. (1976), "Low frequency scattering by a dielectric body," to appear in Radio Sci. 11(5), 477-482 (attachment A).

Herrick, D. F. and T.B.A. Senior (1976), "The dipole moments of a dielectric cube," submitted to IEEE Trans. Antenna and Propagation (attachment B).

Internal Memoranda

Senior, T.B.A., "Low frequency scattering by a dielectric ellipsoid," Memo 013714-501 (20 July 1975).

Senior, T.B.A., "Low frequency scattering by a homogeneous dielectric body," Memo 013714-502 (29 July 1975).

Senior, T.B.A., "Computed low frequency scattering by a dielectric body," Memo 013714-503 (6 August 1975).

Senior, T.B.A., "Correction of an error," Memo 013714-504 (22 August 1975).

Senior, T.B.A., "Low frequency scattering data for dielectric bodies," Memo 013714-505 (3 September 1975).

Senior, T.B.A., "Upper and lower bounds," Memo 013714-506 (5 September 1975).

Senior, T.B.A., "The polarizability tensor $\overline{\overline{X}}$," Memo 013714-507 (10 September 1975).

Herrick, D. F., "Rectangular parallelepiped," Memo 013714-508 (18 November 1975).

Senior, T.B.A., "Some fundamental inequalities," Memo 013714-509 (28 February 1976).

Herrick, D. F. "Solutions of the integral equations," Memo 013714-510 (26 March 1976).

Administrative

Apart from the project director, the only personnel receiving support from the Grant were Mr. D. F. Herrick (a research student), programmers and a secretary.