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ELECTROMAGNETIC SCATTERING

This interim scientific report summarizes the work carried out under the Air Force Office of Scientific Research Grant No. 77-3188(C) during the year ended 31 December 1979.

Our most productive studies have been those concerned with resistive sheet materials whose properties are of interest for cross section reduction purposes. Strips made of such a material have lower backscattering cross sections than the corresponding perfectly conducting ones, and this is true in particular when the illumination is edge-on with the electric vector parallel to the edge. For an E-polarized plane wave incident on a uniform resistive strip of width w it is a relatively simple matter to obtain an integral equation for the total induced electric current and, hence, solve the equation by the moment method. From an examination of the data for the backscattered far field for edge-on incidence, the contributions of the front and rear edge of the strip can then be extracted. The front edge contribution is the same as for a half plane having the same resistance, but the surprising result (Senior, 1978, 1979a) is that for strips of width greater than about $\lambda/10$ the rear edge contribution is proportional to the square of the current at that point on a half plane corresponding to the rear edge of the strip. For at least a range of real resistivities, an empirical expression for the constant of proportionality was obtained, and the desire to verify this analytically was the motivation for the first of the studies performed. In the event, considerably more was achieved (Senior, 1979b,c).

For an E- or H-polarized plane wave incident on a resistive strip of electrical width kw, a high frequency asymptotic technique was used to derive expressions for the far field amplitude through second order terms. The results are valid for all angles of incidence and scattering other than grazing, and were cast in terms of functions analogous to those appearing in the uniform expressions derived by Fialkovskiy (1966) and

Khaskind and Vainshteyn (1964) for E- and H-polarizations respectively in the special case of perfect conductivity. It was shown that each function is directly related to the diffracted portion of the relevant current on a half plane, and using representations for the currents valid for all angles of incidence, uniform expressions for the far field amplitude of the strip were obtained which hold even at grazing angles. In the particular case of edge-on backscattering, the contribution from the rear edge of the strip is proportional to the square of the current at the corresponding point on a half plane. The constant of proportionality involves the 'split' function produced by the Wiener-Hopf method of solution of the half plane problem and, for E-polarization and real resistivities, differs by no more than 6 percent from the constant previously derived (Senior, 1979a) empirically. Moreover, the result is valid for complex "resistivities" as well as real.

To create a viable structure the strip must be rigidized, for example, by encasing it in a thin plastic layer. The effect is to make the apparent resistivity complex, and the reactive part of the resulting impedance can be either inductive or capacitative. The presence of the imaginary part not only produces a strip whose electrical properties are frequency dependent, but also diminishes the cross section reduction that can be achieved. This has been illustrated (Senior, 1979d) using data obtained by numerical solution of the integral equation for a variety of complex resistivities and strip widths.

To exploit the previously developed expressions for the edge contributions of a strip in terms of the half plane currents as a means of predicting the cross section reduction that a material would produce, it is necessary to have available an efficient method for computing the half plane currents. We have developed (Senior, 1979d) an expression for the current that is valid for any resistivity, real or complex, and is amenable to computation, and a program for carrying out the computation is now almost complete.

Another problem that has been investigated is the scattering of a plane wave by a wedge composed of two resistive sheets. Two different mathematical approaches have been considered, both of which are effective when applied to the similar but simpler problem of a solid impedance wedge. The first is that of Maliuzhinets (1959) and is based on the representation of the total field as a Sommerfeld integral of the form

$$E(\rho,\phi) = \frac{1}{2\pi i} \int_{\gamma} e^{ik\rho\cos\alpha} s(\alpha - \phi)\, d\alpha$$

where γ is a contour in the complex α plane. Application of the boundary conditions then leads, in principle at least, to a difference equation for $s(\alpha)$. Unfortunately, difficulties arise when the wedge is resistive, and though we succeeded in solving the problem of a resistive half plane (wedge of zero angle) using this technique, the extension to the wedge has not yet been carried out. The second method which has been explored is based on the integral transform of Kontorovich and Lebedev (1939) and this has been used to obtain uncoupled difference equations for the functions characterizing the exterior and interior fields of a resistive wedge. Because the equations are of second order, practical solution would be difficult; nevertheless, an iterative solution appears eminently feasible and, in addition, the process by which the equations were decoupled may be relevant for the coupled equations that are generated by the Maliuzhinets technique.

A resistive wedge is an appropriate model for a leading edge treatment of a low cross section wing or fin, but to make the model more realistic, it is necessary to choose a structure of finite dimensions. This naturally leads to a consideration of a (finite) resistive plate. A straightforward formulation of the problem using, for example, Hertz vectors, leads to two coupled integral equations for the orthogonal components of the total electric current supported by the plate, and by application of the moment method, a numerical solution could be obtained. Unfortunately, for small resistivities and/or for incidence close to grazing the matrix is ill-conditioned, and in addition, the sheer size of the matrix effectively limits the applicability of this technique to plates whose area is no more than about a square wavelength. This is too small to be of interest for practical applications. To overcome the ill-conditioning it had been our hope to discover a formulation analogous to that developed by Rahmat-Samii and Mittra (1974) for a perfectly conducting plate that also has the advantage of reducing the orders of the singularities of the integral equations by eliminating derivatives. In recent publications several of these boundary limits have been interpreted as Hadamard finite parts, but we have now shown (Cho and Senior, 1979) that this interpretation is incorrect. Though we have so far failed to find an appropriate extension of the Rahmat-Samii/Mittra approach, we have discovered some alternative boundary conditions for a resistive sheet that are valid if the sheet is flat, and these provide hope that an effective formulation of the resistive plate problem can be developed. The work is continuing.

Grant-Supported Publications

Senior, T.B.A. (1978), "Backscattering from resistive strips," talk presented at National Radio Science Meeting, Boulder, CO, 6-9 November.

Senior, T.B.A. (1979a), "Backscattering from resistive strips," IEEE Trans. Antennas Propagat. 27 (6), pp. 808-813.

Senior, T.B.A. (1979b), "Scattering by resistive strips," talk presented at National Radio Science Meeting, Seattle, WA, 18-22 June.

Senior, T.B.A. (1979c), "Scattering by resistive strips," Radio Sci. 14 (5), pp. 911-924, September-October 1979.

Senior, T.B.A. (1979d), "Scattering by impedance strips," talk presented at National Radio Science Meeting, Boulder, CO, 5-8 November.

Cho, S. K. and T.B.A. Senior (1979), "The boundary limits of certain integrals in scattering theory," submitted to IEEE Trans. Antennas Propagat.

References

Fialkovskiy, A. T. (1966), "Diffraction of planar electromagnetic waves by a slot and a strip," Radio Eng. Electr. 12, 150-157.

Khaskind, M. D. and L. A. Vainshteyn (1964), "Diffraction of plane waves by a slit and a tape," Radio Eng. Electron. 10, 1492-1502.

Kontorovich, M. J. and N. N. Lebedev (1939), "On a method of solution of some problems of the diffraction theory," J. Phys. USSR 1, 229-241.

Maliuzhinets, G. D. (1959), "Excitation, reflection and emission of surface waves from a wedge with given face impedance," Sov. Phys.-Dokl. 3 (4), 752-755.

Rahmat-Samii, Y. and R. Mittra (1976), "Integral equation solution and RCS computation of a thin rectangular plate," IEEE Trans. Antennas Propagat. 22 (4), 608-610.

Personnel

The Grant has provided partial salary support of the project director and three graduate students two of whose Ph.D. theses are on topics covered by the Grant.