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RADAR SYSTEM ANALYSES - I

Final Report

by

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1. INTRODUCTION AND CONCLUSION

In this report, the results of a study concerning the effect of atmosphere on the operation of the synthetic aperture radar (SAR) are summarized. It is well known that the success of SAR depends on the proper choice of the weighting function in the signal processing to account for the phase difference of the radar signals returned from different parts of the ground surface. [Cutrona, et al (1961), Brown (1967)]. So far, in most analysis concerning SAR, the weighting functions are constructed on the basis of free space propagation of signals. In reality, of course, the atmosphere is inhomogeneous spatially and temporally. The effect of atmospheric refraction and scattering causes the deviations and uncertainties on the amplitude, phase, and direction of the signal. In this report, based on the present state-of-the-art in the solutions of wave propagation in inhomogeneous (and/or random) media, several models of atmosphere are chosen, and the effect of these atmospheric models on the operation of SAR are discussed.

In Section 2 we describe a simple SAR system. The returned signal, and the principles of signal processing based on free space operation are outlined. Various quantities involved in this "free space" model that may be effected by the atmosphere are pointed out. In order to bring out primarily the effect of atmosphere on its operation, this simple system is idealized, and the system noise is ignored.

In Section 3 we review briefly the problem of wave propagation in inhomogeneous media. Several "popular models" of atmosphere are described, and our present knowledge on waves in such atmosphere models is reviewed. Results useful to the analysis of the operation of SAR system are outlined.

In Section 4, the results of Section 3 are introduced to the signal processing scheme of the simple SAR system given in Section 2. The effect of the atmosphere on the operation of this system are then deduced.

Due to the uncertainties and variations of the parameters of atmosphere involved, this report emphasizes the basic approach to the problem and the use of mutual cohence function and two frequency coherence function in the analysis of SAR resolution. For the case of weak turbulence, approximate sample calculations appears to indicate that the effect of turbulence on range resolution are negligible while the azimuth resolution is deteriorated slightly due to turbulence. This conclusion, of course, is based on the particular set of parameters used.

For the rainy and stormy weather, the theory of weak turbulence does not apply. In Section 4.4 we formulate the approach that could be used to analyze the SAR resolution. Due to excessive numerical work that is involved and lack of time, we did not proceed further.

2. A SIMPLE SAR SYSTEM

2.1 Geometry of a Simple SAR System

A simple model of a synthetic aperture side-looking radar system is used in this chapter to illustrate the basic operating principles of SAR system. The idealized geometry involved in the operation of this system is given in Figure 2-1. A moving transmitter A emits pulsed signals, illuminating a patch of ground. The transmitter is moving with velocity $v \ll c$, in the x-direction at a fixed height h. Thus, the vector position of the transmitter is given by $\{x_i = vt, 0, h\}$. The antenna of the transmitting beam is pointed toward the broadside with a depression angle γ, so that the direction of the center of the beam is

$$\hat{s}_o = \hat{a}_y \cos\gamma - \hat{a}_z \sin\gamma. \tag{2-1}$$

The antenna radiation pattern may be expressed in terms elevation angle α and azimuth angle β. As illustrated in Figure 2-1, the direction of any ray from the transmitter B is given by

$$\hat{s}_{\alpha\beta} = \hat{a}_x \sin\beta + \hat{a}_y \cos\beta \cos(\gamma - \alpha) - \hat{a}_z \cos\beta \sin(\gamma - \alpha) \tag{2-2}$$

For simplicity, the ground is taken as the plane $z = 0$, and the ground property is characterized by the ground reflectivity $\rho(x, y)$. For free space propagation, a ray in direction $\hat{s}_{\alpha\beta}$ would be reflected by a point on the ground with coordinates

$$\left.\begin{aligned} x &= x(\alpha, \beta) = h \tan\beta / \sin(\gamma - \alpha) + x_i \\ y &= y(\alpha, \beta) = h \cot(\gamma - \alpha) \end{aligned}\right\} \tag{2-3}$$

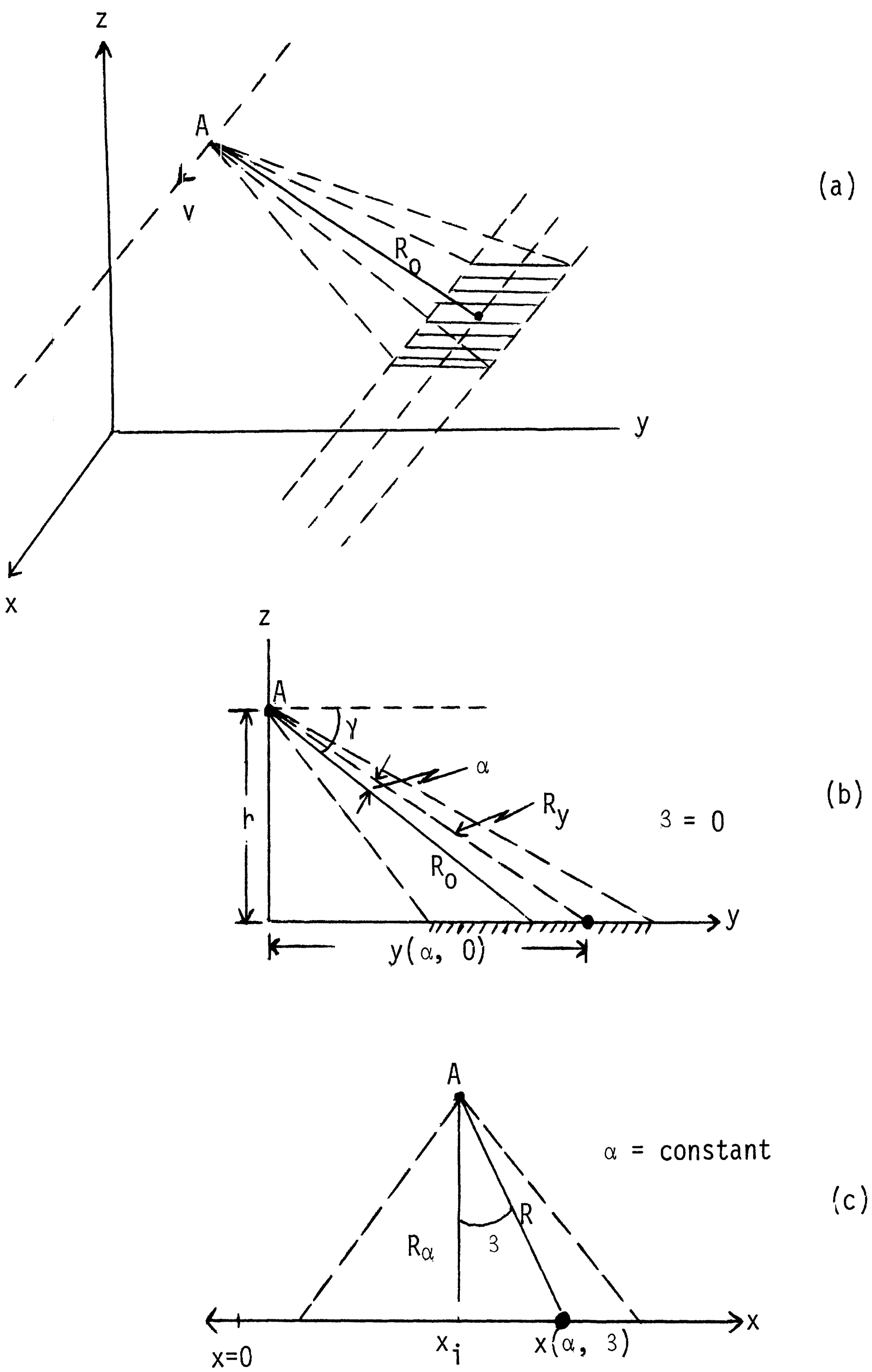

Figure 2-1. (a) Geometry for a side looking radar system.
(b) Beams in y-z plane through A.
(c) Beams in a slant plane.

In particular, for the center of the beam,

$$\left.\begin{aligned} x_0 &= x(0, 0) = x_i \\ y_0 &= y(0, 0) = h \cot \gamma \end{aligned}\right\} \tag{2-4}$$

The slant range of any reflecting point on the direction $\hat{s}_{\alpha\beta}$ is

$$R = R(\alpha, \beta) = \frac{h}{\sin (\alpha - \gamma)} \sec \beta . \tag{2-5}$$

In particular, for the center of the beam,

$$R_0 = R(0, 0) = \frac{h}{\sin \gamma} \quad , \tag{2-6}$$

and in any slant plane with fixed α (or y), the shortest slant range is

$$R_y = R(\alpha, 0) = h/\sin (\gamma - \alpha) \tag{2-7}$$

In most analysis, we assure α, β to be small and employ the following approximate relation in signal processing

a. The azimuth range is $x - x_i \cong R_{\alpha\beta} \cong R_0$ (2-8a)

b. The broadside range $y - y_0 \cong \frac{R_\alpha}{\sin \gamma} \alpha \cong \frac{R_0}{\sin \gamma} \alpha$ (2-8b)

c. For fixed α, $R \cong R_y + \frac{(x - x_i)^2}{2R_y}$ (2-8c)

d. In the direction $\beta = 0$,

$$R(\alpha, 0) = R_0 + 2R_0 \cos \gamma \ (y - y_0) + \frac{(y - y_0)^2}{2R_0} \quad . \tag{2-8d}$$

These are commonly used in relating the transmitter radiation pattern (function of α, β), the ground reflection point, and estimating the time delay in the analysis of the returned signal. However, in an inhomogeneous medium, the rays may be curved, and the above relations must be modified according to the variation of the mean index of refraction of the atmosphere.

2.2 Transmitted and Returned Signal

The moving transmitter A in Figure 2-1 emits a sequence of pulses, and the returned signals for each pulse, after the carrier is removed, (or converted to IF) is stored for processing. Let us consider the m^{th} pulse, the transmitted signal may be represented by

$$S\ (t') = R_e\ A\ (t')\ e^{-i\omega_0 t'} \quad , \tag{2-9}$$

where ω_0 is the carrier frequency (Ra/sec), and A (t') is the complex pulse modulation. The time t' is a shift in t such that t' = 0, the transmitter is at $x_i = x_m$. Thus, in terms of t',

$$x_i = x_m + vt' \quad , \tag{2-10}$$

and the expression S (t') is same for each pulse. The returned pulse depends on the reflectivity of the ground and the medium between the transmitter and the ground. Let us assume that for a two way transmission from the transmitter A to a ground point (x', y') introduces phase of $\varphi(x_i, x', y')$ to the RF signal and modifies the complex amplitude (due to delay and possible dispersion) to $\tilde{A}\ (t', x_i, x', y')$. Then the returned signal from the illuminated patch of ground may be formally written as

$$S_m(t') = \int dx' \int dy'\ G(\alpha, \beta)\ \tilde{A}(t', x_i, x', y') e^{i\varphi\ (x_i,\ x',\ y')}\ \rho(x', y') \tag{2-11}$$

where $G(\alpha, \beta)$ is the power pattern of the transmitting antenna, and α, β are functions of x' and y'. For simplicity, we shall assume that $G\ (\alpha, \beta) \cong 1$.

The returned signal for all the pulses are stored in two dimensional format and may be considered as a function of two variables, x_m and t'.

$$S(x_m, t) \cong \int dx' \int dy' \tilde{A}(t', x_m, x', y') \, Exp[i\phi(x_m, x', y')]\rho(x', y') \qquad (2\text{-}12)$$

In (2-12), t' takes continuous values from $-\frac{\tau}{2}$ to $+\frac{\tau}{2}$, where τ is the pulse length while x_m takes discrete values mvT, where T is the interval between pulses and m are integers.

In signal processing, we want to obtain the best estimation of $\rho(x', y')$ from the information $S(x_m,t)$. This is usually accomplished optically or electronically using the principle of matched filters.

2.3 The Ideal Resolution

The two dimensional signal $S(x_m, t')$ may be considered as a two dimensional mapping of $\rho(x', y')$ with Kernel

$$K(x_m, t', x', y') = \tilde{A}(t', x_m, x', y') Exp[i\phi(t', x_m, x', y')] \qquad (2\text{-}13)$$

Based on the principles of matched filters, the resolution of the estimation of $\rho(x, y)$ may be obtained from the generalized ambiguity function

$$\chi(x, y, x', y') = \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} dt' \int_{-\frac{N}{2}vT}^{\frac{N}{2}vT} dx_m \, K(t', x_m, x', y') \, K^*(t', x_m, x, y) \qquad (2\text{-}14)$$

where N is the number of pulses summed to gether during processing (for convenience of analysis, the summation is approximately replaced by integration). The integrand in (2-14) may be factored into

$$KK^* = g_v \cdot g_h \qquad (2\text{-}15)$$

where

$$g_v \triangleq \tilde{A}(t', x_m, x', y')\, \tilde{A}^*(t', x_m, x, y), \quad (2\text{-}16a)$$

and

$$g_h = \exp[i\phi(x_m, x', y') - i\phi(x_m, x, y')] \,. \quad (2\text{-}16b)$$

It is seen that g_h depends on the RF phase of the signal and g_v depends on the modulation envelope distortion.

In free space, the

$$\tilde{A}(t, x_m, x', y') = A(t' - \frac{2R'}{c}) \quad (2\text{-}17)$$

where R' is the distance from the transmitter at $\{x_m, 0, h\}$ to the ground point $(x', y', 0)$.

From (2-8c) we may approximate

$$R' \cong R_{y'} + \frac{(x' - x_m)^2}{2R_{y'}} \,. \quad (2\text{-}18)$$

In the near forward direction, the second term can be neglected, so that g_v may be considered as a function of $R_{y'}$ only.

The phase delay, for free space propagation, is given by

$$= \frac{2\omega_0}{c} R_y = \frac{2\omega_0}{c} R_{y'} + \frac{2\omega_0}{c} \frac{(x' - x_m)^2}{2R_{y'}} \,. \quad (2\text{-}19)$$

From (2-18) and (2-19), it is seem that for simplicity, the analysis of the resolution near the forward direction may be decoupled into two one-dimensional problems.

a. In the forward direction, $x' = x = x_m$, the range resolution is obtained from the range ambiguity function

$$\chi_v(y, y') = \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} dt' \; g_v(t', y, y')dt' \; , \tag{2-20}$$

b. For fixed y (range R_y), the azimuth resolution may be determined from

$$\chi_h(x, x') = \int_{-\frac{N}{2}vT}^{\frac{N}{2}vT} dx_m \; g_h(x, x', x_m) \; . \tag{2-21}$$

It is expected that in real atmosphere, such decoupling is also valid, hence the azimuth resolution depends on the structure function of RF phase shift and the range resolution depends on the envelope distortion.

In free space, from (2-19) and (2-16b)

$$g_h = \mathrm{Exp}[-i \frac{2\omega_0}{cR_{y'}} (x' - x)x_m]\exp[i\frac{\omega_0}{cR_{y'}}(x'^2 - x^2)] \; . \tag{2-22}$$

Thus

$$\chi_h(x, x') = \chi_h(x - x') = \exp[i \frac{\omega_0}{cR_{y'}}(x'^2 - x^2)]$$

$$\int_{-\frac{N}{2}vT}^{\frac{N}{2}vT} \exp[-i \frac{2\omega_0}{cR_{y'}} (x' - x)x_m]dx_m \; . \tag{2-23}$$

This equation indicates that the synthetic aperture in the azimuth direction is equivalent to an aperture of effective length

$$\mathrm{Leff} = 2NvT \tag{2-24}$$

To estimate the resolution, we see that the pattern function of this array except and imaginary multiplicative constant, is given by

$$F_h(x') = \int_{-\frac{N}{2}vT}^{\frac{N}{2}vT} \exp[-i \frac{2\omega_0}{cR_{y'}} x_n x']dx_n .$$

If we <u>define</u> the resolution in terms of the equivalent rectangular pattern, then the azimuth resolution is given by

$$\beta_h = \frac{\int_{-\infty}^{\infty} |F(x')|^2 dx'}{|F(0)|^2} = \frac{\lambda R_{y'}}{2NvT} . \qquad (2\text{-}25)$$

The range resolution is usually achieved by chirping. If we let

$$A(t) = e^{i\alpha t^2} , \qquad (2\text{-}26)$$

then

$$g_v = \exp[i\alpha(t' - \frac{2R_{y'}}{c})^2 - i\alpha(t' - \frac{2R_y}{c})^2]$$

$$= \exp[-\frac{4i\alpha}{c} (R_{y'} - R_y)t']\exp[\frac{4i\alpha}{c^2} (R_{y'}^2 - R_y^2)] , \qquad (2\text{-}27)$$

so that

$$\chi_v(R_{y'}, R_y) = \exp[\frac{4i\alpha}{c^2} (R_{y'}^2 - R_y^2)]\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \exp[- \frac{4i\alpha}{c} (R_{y'} - R_y)t']dt' . \qquad (2\text{-}28)$$

Thus, the range resolution is,

$$\beta_v = \frac{\pi c}{2\alpha\tau} . \qquad (2\text{-}29)$$

The deterioration of these ideal resolutions due to atmospheric effect shall be discussed in Section 4.

3. WAVE PROPAGATION IN INHOMOGENEOUS MEDIA

3.1 Introduction

The problem of wave propagation in an inhomogeneous medium (such as atmosphere) has been investigated extensively in the past few decades. Substantial useful results have been obtained from the early investigation of the propagation problems in stratified media, in weak turbulent media and in the more recent works involving media with strong turbulence and discrete scattering particles. The results of recent works are summarized in a paper by A. Ishimaru (1977). In this section, some of the results that are directly applicable to SAR operations are given. From the analysis of Section 2, the resolution of SAR system is effected most by the phase shift of the carrier and distortion of the pulse modulation, the results we quote in this section therefore concern these two types of problems.

It is to be noted that due to complicated spatial and temporal index of refraction of the atmosphere, most results are approximate, and the validity of each approximate approach depends on the frequency and distance of propagation. In this work, we are arbitrarily limiting our application to a propagating distance of 10 km and in the frequency range of 8 GHz to 26.5 GHz. In this range the parameters that may decide the particular choice of model are tabulated below.

Frequency	8×10^9 Hz	26.5×10^9 Hz
wavelength λ	0.0375 m	0.011 m
k	167.55 1/m	550.0 (1/m)
$\sqrt{\lambda \cdot 10^4}$	19.36 m	10.63 m

Within this range of parameters, it appears that the ray and weak turbulence approach may be appropriate.

3.2 Stratified Media

It is known since early days of radar operation that the index of refraction (r) of atmosphere, on the average is nearly unity but exhibits variations with height due to pressure, temperature and moisture variations. The variation of r with height (h) is usually very small, and may be functionally represented in the form

$$r = 1 + \delta f(z) \ , \tag{3-1}$$

where δ is a small quantity, and f(z) is the profile of the variation of r. In the exponential atmosphere model, we have

$$\delta = 313 \times 10^{-6}$$

and

$$f(z) = \exp[-(z)] \ , \tag{3-2}$$

with $\quad c \cong 0.1439/\text{km}.$

In general, due to excess moisture near sea and clouds, and large temperature gradient near deserts, f(z) takes different forms, and may be approximated by sections of straight lines.

Exact solutions for wave propagation in a stratified medium is difficult and ray theory (geometric optics) is commonly used. For a transmitter located at (x_i, y_i, h), the solution of the ray equation indicates that a ray starts from the transmitter in the direction

$$\hat{s} = s_x\hat{a}_x + s_y\hat{a}_y - s_z\hat{a}_z \ , \tag{3-3}$$

would be reflected from a point on the ground plane (z=0) with coordinates:

$$x = x_i + \int_0^h \frac{s_x r(h)dz}{[r^2(z) - r^2(h)(s_x^2 + s_y^2)]^{1/2}} ,$$

$$y = y_i + \int_0^h \frac{s_y r(h)\, dz}{[r^2(z) - r^2(h)(s_x^2 + s_y^2)]^{1/2}} . \qquad (3\text{-}4)$$

The phase delay of the ray from the transmitter to the ground along this ray is

$$\phi = \frac{\omega_o}{c} \int_0^h \frac{r^2(z)dz}{[r^2(z) - r^2(h)(s_x^2 + s_y^2)]^{1/2}} . \qquad (3\text{-}5)$$

For r decreasing with height, the ray bends toward the transmitter as illustrated in Figure 3 1. This refraction phenomena has an important effect on astronomical observations, and computer programs have been developed [Garifinkel (1957)] for the integration of the ray path and angle of arrival.

For approximate analysis of the ray path (3-3) through (3-5) may be approximately evaluated to the first order or δ, the results can be easily shown

$$x - x_i = \frac{s_x}{s_y} h + \delta \frac{s_x h}{s_z^3} [f(h) - \frac{1}{h}\int_0^h f(z)dz] , \qquad (3\text{-}6)$$

$$y - y_i = \frac{s_y}{s_z} h + \delta \frac{s_y h}{s_z^3} [f(h) - \frac{1}{h}\int_0^h f(z)dz] , \qquad (3\text{-}7)$$

and

$$\phi = \frac{\omega_o}{c}\left\{[\frac{h}{s_z} + \delta \frac{h}{s_z^3} [1 - s_z^2]f(h) - (1 - 2s_z^2)\frac{1}{h}\int_0^h f(z)dz]\right\} . \qquad (3\text{-}8)$$

These equations shall be utilized in the next section in the investigation of the distortion of the ground image.

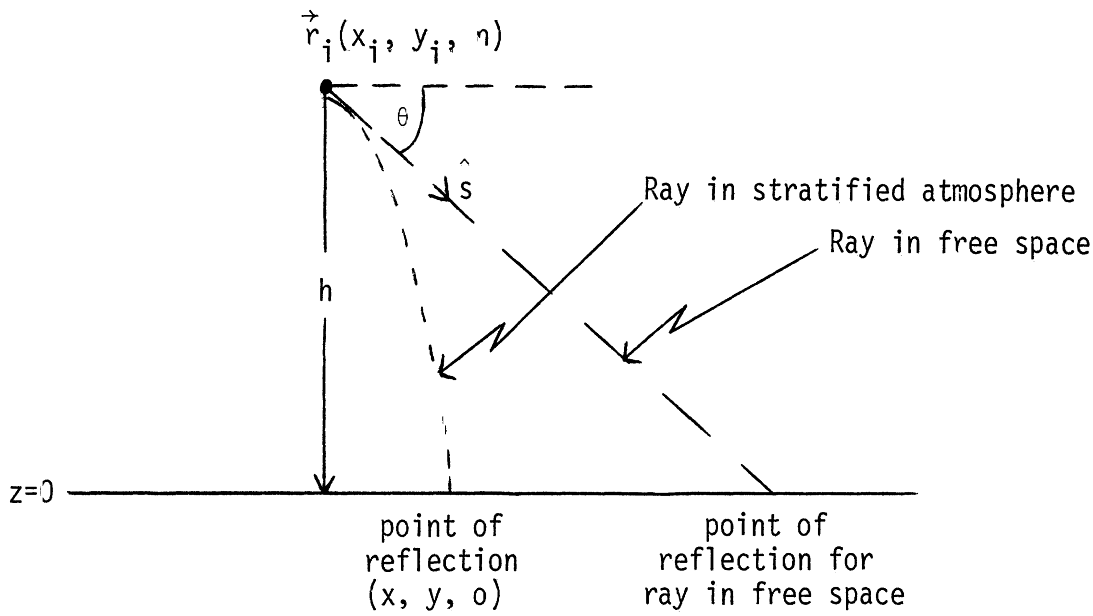

Figure 3-1. Bending of a ray due to refraction.

3.3 Models of Turbulent Atmosphere

The index of refraction of a turbulent atmosphere is represented by

$$n(\vec{r}, t) = n + n_1(\vec{r}, t)$$

when n is the mean value (used in (3-2) and $n_1(\vec{r}, t)$ is a random function of zero mean. In wave propagation problems, for simplicity, $n_1(\vec{r}, t)$ is assumed to be isotropic, stationary and may statistically be described by correlation function, structure function or spectral density function. To avoid confusion we briefly state the notations used:

a) Given any isotropic random function, $f(\vec{r}, t)$, we represent

$$f(\vec{r}, t) = f(\vec{r}) + f_1(\vec{r}, t), \tag{3-9}$$

where $f(\vec{r}) = \langle f(\vec{r}, t)\rangle$ (3-10)

is the ensemble average and $f_1(\vec{r}, t)$ is a random function of zero mean.

b) The correlation function is denoted by the statistical average

$$B_f(\vec{r}_1 - \vec{r}_2) = \langle f(\vec{r}_1, t)\, f(\vec{r}_2, t)\rangle . \tag{3-11}$$

For stationary and spatially homogeneous random field,

$$B_f(\vec{r}_1 - \vec{r}_2) = B_f(\vec{r}_1 - \vec{r}_2) = B_f(r) , \tag{3-12}$$

where $r = \vec{r}_1 - \vec{r}_2$

The mean square variation of $f_1(r, t)$ is denoted by

$$\langle f_1^{\,2}\rangle = B_f(0) . \tag{3-13}$$

c) If a random function is not homogeneous, it is sometimes meaningful [Tarkarsi (1961)] to introduce a structure function defined by

$$D_f(\vec{r}_1, \vec{r}_2) = \langle |f_1(\vec{r}_1, t) - f_1(\vec{r}_2, t)|^2 \rangle . \tag{3-14}$$

For stationary, locally homogeneous isotropic random field,

$$D_f(\vec{r}_1, \vec{r}_2) = D_f(\vec{r}_1 - \vec{r}_2) = D_f(r) . \tag{3-15}$$

d) For a homogeneous random field we have

$$D_f(r) = 2B\ (0) - 2B_f(r) , \tag{3-16}$$

$$B_f(r) = \frac{1}{2} D_f(r) - \frac{1}{2} D_f(\infty) . \tag{3-17}$$

e) The three dimensional Fourier transform of $B_f(r)$ is the spectral density

$$\Phi(K) = \frac{1}{2\pi k} \int_0^\infty r B_f(r) \sin(Kr) dr . \tag{3-18}$$

The inverse is

$$B_f(r) = \frac{4\pi}{r} \int_0^\infty K\, \Phi(K) \sin Kr\, dK . \tag{3-19}$$

In terms of spectral density,

$$D_f(r) = 4\pi \int_0^\infty \left(1 - \frac{\sin Kr}{Kr}\right) \Phi(K)\, K^2\, dK . \tag{3-20}$$

Various models of correlation functions are postulated for atmosphere in the study of tropospheric scattering [see for example Staras and Wheelon (1959)]. Solution of the propropation problems however has carried out in detail only for the Gaussian model [Chernov (1960)]. For this model,

$$B_n(r) = B_n(0)\, e^{-r^2/\ell_0^2} . \tag{3-21}$$

Chernov assumed the value of

$$B_n(0) = \langle n_1^2 \rangle = 5 \times 10^9 ,$$

and $\ell_0 \cong 0.6$ m.

The model of locally homogeneous turbulence is a more realistic description of the atmosphere. Correlating with the theory of homogeneous turbulence and meteorological measurements, the model postulates the structure function of the index of refraction to be [Tartarski (1961)]

$$\begin{aligned} D_n(r) &= C_n^2\, r^{2/3} & \ell_0 \ll r \lll L_0 \\ D_n(r) &= C_n^2\, \ell_0^{2/3} \left(\frac{r}{\ell_0}\right)^2 & r \ll \ell_0 \end{aligned} \tag{3-22}$$

L_0 is the outer scale of the turbulence, ℓ_0 is smallest size of eddies, and Cn is known as the structure constant. These parameters are usually inferred from meteorological measurements and varies over a wide range. L_0 is generally on the order of 100 m and ℓ_0 is on the order of 10^{-3} to 10^{-4} m [Strohbehn (1968)]. The measured values of structure "constant" Cn varies with altitude and depends on the frequency (microwave or optical). Some data of C_n^2 was given by Hafnagel (1966). In sample calculation for this work, we shall arbitrarily take the value $C_n^2 \cong 2 \times 10^{-13}/m^{2/3}$.

3.4 Wave Propagation in Weak Turbulence

Meaningful approximate solutions of wave propagation in atmosphere with weak turbulence were obtained by the method of smooth perturbations. [Tartaski (1961)]. Using this approximation, the structure functions of the phase and amplitude fluctuations of a plane wave (or spherical waves) at points transverse to the direction of propagation were expressed formally in terms of the spectral density of the index of refraction. A summary of available results are given by Lawrence and Strohbehn (1970). Because these results appear to be directly applicable (approximately) in investigations of azimuth resolution, a brief description of the method of smooth perturbation is outlined in this section. We also derived the two frequency choerence function which is to be used in the analysis of range resolution.

The essential steps in studying the fluctuation of EM wave in turbulent media are given below.

a) Neglecting polarization effects, and consider the solution of the scalar Helmholtz equation

$$\nabla^2 u + k^2(1 + n_1)^2 u = 0 . \tag{3-23}$$

b) For plane wave propagation in the x-direction, assuming

$$u = \exp[-i\omega t + ikx] \exp \psi \tag{3-24}$$

If we write

$$\psi = \chi + is , \tag{3-25}$$

then χ is the log-amplitude fluctuation and s is the phase fluctuation.

c) Assuming $n << 1$, $|\nabla\psi| << k$, and neglecting the term $\frac{\partial^2 \psi}{\partial x^2}$ (parabolic approximation), it is shown that ψ satisfies

$$\frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + 2ik \frac{\partial \psi}{x} + 2ik^2 n_1 \psi = 0 . \tag{3-26}$$

Thus, using Born's approximation, $\psi(\vec{r})$ can be expressed by

$$\psi(x,y,z) \cong \frac{k^2}{2\pi}\iiint \frac{n_1(x',y',z')}{|x-x'|}\exp\left[ik\frac{(y-y')^2+(z-z')^2}{2|x-x'|}\right] dx'dy'dz' \qquad (3\text{-}27)$$

d) From (3-27) we may express the spectral density of the random functions χ and s in terms of the spectral density of n_1. If we define the two dimensional partial Fourier transform of any random function $f(\vec{r})$ by*

$$\Phi_f(x, \vec{K}_\perp) = \frac{1}{2\pi}\int_0^\infty J_0(K_\perp\rho)B_f(x, \vec{\rho})\,\rho\,d\rho \qquad (3\text{-}28)$$

then, it can be shown that for two points $\{L,\vec{\rho}\}$ and $\{L + \Delta L, \rho = 0\}$,

$$\Phi_\chi(\Delta L, \vec{K}_\perp) = k^2\int_0^L dx' \int_0^L dx''\ \Phi_n(x'-x'', K_\perp)$$

$$\sin\left[\frac{K_\perp^2(L+\Delta L-x')}{2k}\right]\sin\left[\frac{K_\perp^2(L-x'')}{2k}\right] \qquad (3\text{-}29)$$

For phase fluctuations, the expression for $\Phi_s(\Delta L, \vec{K}_\perp)$ is obtained by replacing the sine function in (3-29) with cosine functions. Equation (3-29) is a slight generalization of the result given by Tartarski, who considered the special case of $\Delta L = 0$.

e) If $\sqrt{\lambda L} \ll L_0$, (3-29) can be simplified to

$$\Phi_{\chi \atop s}(\Delta L, \vec{K}_\perp) = \pi k^2 L\left[\cos\frac{K_\perp^2\Delta L}{2k} \mp \frac{2k}{K_\perp^2 L}\cos\frac{K_\perp^2(L+\Delta L)}{2k}\sin\frac{K_\perp^2 L}{2k}\right]\Phi_n(0, \vec{K}_\perp)$$

(3-30)

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* we denote $\vec{r} = \{x, \vec{\rho}\}$, and $\vec{K} = \{K_1, \vec{K}_\perp\}$ to separate the effects in transverse and longitudinal direction.

f) From (3-30) we may obtain the correlation and structure functions for phase and amplitude by the integrals given below

$$B_{\chi/S}(\Delta L, \vec{\rho}) = 2\pi \int_0^\infty J_0(k_\perp \rho)\, \Phi_{\chi/S}(\Delta L, \vec{K}_\perp)\, K_\perp dK_\perp \qquad (3\text{-}31)$$

and

$$D_{\chi/S}(\Delta L, \vec{\rho}) = 4\pi \int_0^\infty [\Phi_{\chi/S}(0, \vec{K}_\perp) - J_0(K_\perp \rho)\Phi_{\chi/S}(\Delta L, \vec{K}_\perp)]\, K_\perp dk_\perp \qquad (3\text{-}32)$$

For the case $L = 0$, (i.e., for points lie in a plane transverse to the direction of propagation), various expressions have been derived for the correlation and structure functions for log-amplitude and phase fluctuations. The results given by Tartarski, using the structure function given in (3-22), corresponding to the structure function

$$\Phi_r(K) = \begin{cases} 0.033 C_n^2\, K^{-11/3} & K < K_n = 5.48/\ell_0 \\ 0 & K > K_n \end{cases} \qquad (3\text{-}33)$$

appears to correlate fairly well with some experiments. We shall therefore use them in the present study. The results that shall be used in this report are

$$\langle\chi^2\rangle = B_\chi(0, 0) = 0.31\, C_n^2\, k^{7/6} L^{11/6} \qquad \sqrt{\lambda L} > \ell_0 \qquad (3\text{-}34)$$

$$D_s(0, \vec{\rho}) = \begin{cases} 3.44\, k^2 C_n^2 \ell_0^{1/2} L \rho^2 & \rho < \ell_0 \\ 2.91\, k^2 C_n^2 L\, \rho^{5/3} & \rho > \sqrt{\lambda L} \\ 1.46\, k^2 C_n^2 L\, \rho^{5/3} & \ell_0 < \rho < \sqrt{\lambda L} \end{cases} \qquad (3\text{-}35)$$

For the range of frequency and distance of interest in this work, $\lambda = 0.0375 - 0.011$ m, and for a distance of $L \cong 10^4$ m,

$$\sqrt{\lambda L} = 19.36 \text{ to } 10.63 \text{ m},$$

so that the last expression seems to be valid. For this range of parameters, it is also to be noted that the highest value of $<\chi^2>$ is about 2.24×10^{-3}, hence the theory of weak turbulence is valid, and the attenuation may be, to the first order, neglected.

In investigating pulse propagation in random media, it is usually convenient to use the two frequency mutual coherence function [see for example Ishimaru and Hong (1975)]. Although recent investigations on the mutual coherence function are mostly for the case of strong turbulence and discrete scatters, for weak turbulence, the two frequency coherence functions that we shall use in Section 4 can be derived by a method of smooth perturbations. For a description of coherence function we need, let us consider a plane wave propagating into a random medium. At any distance L, the phase fluctuation at two different frequencies, $s(\omega_1, L)$ and $s(\omega_2, L)$ are different, and we need for pulse propagation, the two frequency coherence structure function

$$D_s(\omega_1, \omega_2, L) = < s(\omega_1, L) - s(\omega_2, L)^2>.$$

From the spectral density function of $s(w_1, L)$ obtained by the method of smooth perturbations, it is easily seen that

$$\phi_s(\omega_1, \omega_2, L, \vec{\rho}= 0) = 2\pi\int_0^\infty K_\perp dK_\perp \phi_s(\omega_1, \omega_2, L, K_\perp) , \tag{3-36}$$

and $\phi_s(\omega_1, \omega_2, L, K_\perp) \cong$

$$\pi\, \phi_n(0, \vec{K}_\perp)\{k_1^2L + \frac{k_1^3}{K_\perp^2} \sin(\frac{K_\perp^2 L}{k_1}) + k_2^2L + \frac{k_2^3}{K_\perp^2} \sin(\frac{K_\perp^2 L}{k_2})$$
$$- \frac{4\, k_1^2k_2^2}{(k_1+k_2)\, K_\perp^2} \sin[\frac{K_\perp^2 L(k_1+k_2)}{2k_1k_2}] - \frac{4\, k_1^2k_2^2}{(k_1-k_2)K_\perp^2} \sin[\frac{K_\perp^2(k_1-k_2)}{2k_1k_2}]\} \tag{3-37}$$

To simplify the integration we choose the Gaussian model for $B_n(r)$ [(3-21)], the spectral density for which is

$$\Phi_n(K) = \frac{\langle n_1^2\rangle \ell_o^3}{8\pi\sqrt{\pi}} \exp\left[-\frac{K^2\ell_o^2}{4}\right] \tag{3-38}$$

Using (3-38) and carry out the integration, we have

$$\Gamma_s(\omega_1,\omega_2,L,\vec{\rho}=0) = \frac{\sqrt{\pi}\,\langle n_1^2\rangle \ell_o^3}{8}\Big\{\frac{L}{\ell_o^2}\, k_1^2 L + \frac{4}{\ell_o^2}\, k_2 \ell + k_1^3 \tan^{-1}\frac{4L}{\ell_o^2 k_1}$$

$$+ k_2^3 \tan^{-1}\frac{4L}{\ell_o^2 k_2} - \frac{4k_1^2 k_2^2}{k_1+k_2}\tan^{-1}\left[\frac{4L}{\ell_o^2}\left(\frac{k_1+k_2}{2k_1k_2}\right)\right] - \frac{4k_1^2k_2^2}{k_1-k_2}\tan^{-1}\left(\frac{k_1-k_2}{2k_1k_2}\right)\Big] . \tag{3-39}$$

For small frequency differences we introduce

$$k_1 = k_o + \Delta k \, ,$$
$$k_2 = k_o - \Delta k_1 , \tag{3-40}$$

and expand Γ as series of Δk. Approximately, then Γ is represented by the first term of the series which is given by

$$\Gamma_s(\omega_1,\omega_2,L,\vec{\rho}=0) \cong \sqrt{\pi}\,\frac{\langle n_1^2\rangle \ell_o^3}{8}\,\Delta k^2\Big\{4p + 16k_o \tan^{-1}\frac{p}{k_o} - 10\,\frac{k_o^2 p}{(p^2+k_o^2)}$$

$$+ 4\,\frac{k_o^4 p}{(p^2+k_o^2)^2} + \frac{2}{3}\,\frac{p^3}{k_o^2}\Big\} \tag{3-41}$$

where

$$p = \frac{4L}{\ell_o^2} \tag{3-42}$$

is the distance parameter, while the other factors in (3-41) depend on frequency only. Equation (3-41) reveals that within this approximate formulation Γ varies as L^3 for large L, and varies linearly with L for small L. Moreover, for the case that

$$p = \frac{4L}{\ell_o^2} \ll k_o \, , \tag{3-43}$$

Γ is almost independent of frequency. It is interesting to note that the condition given in (3-43) is precisely the condition often quoted for the validity of geometric optics. For $p \gg k_o$, (3-41) may be simplified to

$$\Gamma_s(\omega_1,\omega_2,L,\rho=0) \cong \frac{\sqrt{\pi}}{2} \langle n_1^2 \rangle \, \ell_o L \left[1 + \frac{8L^2}{3k_o^2 \ell_o} - 1 \right] \Delta k^2. \tag{3-44}$$

On the other hand for $p \ll k_o$, (3-41) may be approximated by

$$\Gamma_s(\omega_1, \omega_2, L, \rho = 0) \cong 7\sqrt{\pi} \, \langle n_1^2 \rangle \, \ell_o \, L \Delta k^2 \tag{3-45}$$

We shall use this equation in connection with the discussion of range resolution in Section 4.

It is to be noted that (3-41) is obtained by using the Gaussian model because the Tartarski model has a singularity at K = 0 to cause the integral in (3-36) to diverge. If we assume a value of the outer scale L_o and use a modified spectrum suggested by Strohbehn (1968), the integral of (3-36) converges, but no simple analytical form is possible. In our numerical calculation, we shall choose n_1^2 and L_o for the Gaussian model such that the Gaussian spectrum is approximately equivalent to the modified spectrum for small values of K. The condition to be satisfied for such a choice can be shown to be

$$C_n^2 = 26.13 \, \langle n_1^2 \rangle \ell_o^{-2/3} \tag{3-46}$$

For the sample computation carried out in Section 4 we choose

$$\ell_o \cong 80 \text{ m}$$

and

$$\langle n_1^2 \rangle \cong 1.4 \times 10^{-13}$$

For this choice of parameters, for $L = 10^4$ m, $p \ll k$, for the frequency range of our interest, hence (3-45) is used in Section 4.

4. ATMOSPHERIC EFFECTS ON SAR OPERATION

4.1 Introduction

In this section, the results of wave propagation given in Section 3 are applied to the analysis of the operation of the simple SAR system postulated in Section 2.

The effect of bending of rays due to straitification on the error in imaging is discussed in section 4.2. The effect of turbulence on the return signal in general is formulated, and the deterioration of resolution in azimuth and range are analyzed in section 4.3.

Considerations were given for the effects of rain and storm on the SAR operation. General formulation of the problem are outlined in section 4.4. But due to complicated numerical procedures involved, no attempt has been made to carry out a numerical solution. This would be an appropriate area for further research.

4.2 Atmospheric Stratification

The effect of atmospheric straitification on the wave propagation has been studied extensively in problems of the angle of arrival in astronomical observations [see for example Weil (1973)]. If scattering effect is neglected, then from the ray theory, the bending of the rays causes deviations of the position of the ground point to be mapped [Eq. (3-4)] and derivation of time delay and phase shift [Eq. (3-5). If $n(z)$ is known exactly, then for a given α, β

$$\left.\begin{aligned} s_x &= \sin\beta \\ s_y &= \cos\beta\cos(\gamma - \alpha) \\ s_z &= \cos\beta\sin(\gamma - \alpha) \end{aligned}\right\} \qquad (4\text{-}1)$$

one may integrate (3-4) to obtain the relation between the image points x, y and the apparent image points [Eq. (2-3)]

$$x = \frac{h\tan\beta}{\sin(\gamma - \alpha)} + x_i$$

and (4-2)

$$y = h\cot(\gamma - \alpha) \quad .$$

Similarly given any ground point x, y and direction of main beam γ, we may determine numerically the corresponding values of α and β from (2-4). These values of α, β are then used in (3-4) to determine the correct time delay and RF phase shift for proper signal processing. This probably could be done, in principle, by digital processing.

If n(z) is not known exactly, but we know that the deviation of n(z) from unity is small, then the approximate equations [Eq. (3-6), (3-7) and (3-8)] can be used to estimate the errors. Let us consider a specific example for which the transmitter is at a height h = 5 km and the direction of the main beam is at γ = 20°. For this case, assuming α, β small,

$$\left.\begin{aligned} s_x &\cong \beta \\ s_y &\cong \cos\gamma \\ s_z &\cong \sin\gamma \end{aligned}\right\} \qquad (4\text{-}3)$$

the reflection point is approximately given by

$$\left.\begin{aligned} x - x_i &\cong \frac{\beta}{\sin\gamma} h + \delta \frac{\beta h}{\sin^3\gamma}\left[f(h) - \frac{1}{h}\int_0^h f(z)dz\right] \\ y &\cong \frac{h}{\tan\gamma} + \delta \frac{h\cos\gamma}{\sin^3\gamma}\left[f(h) - \frac{1}{h}\int_0^h f(z)dz\right] \end{aligned}\right\} \qquad (4\text{-}4)$$

and

$$R = \frac{h}{\sin\gamma} + \delta\frac{h}{\sin^3\gamma}\left[\cos^2\gamma t(h) - (1 - 2\sin^2\gamma)\frac{1}{h}\int_0^h f(z)dz\right. . \qquad (4\text{-}5)$$

In the above equations, the second term (involving δ) is due to atmospheric effects. For standard exponential variation

$$f(z) - \exp[-0.1439 \times 10^{-3}\ z]$$

and $\delta = 313 \times 10^{-6}$, we find that

$\Delta x = -8.8423$ m,

$\Delta y = -8.309$ m, and

$\Delta R = 4.545$ m.

It should be noted that (4-4) and (4-5) can also be used to estimate the azimuth and range errors due to cloud layers between the transmitter and the ground. From the water content of the cloud, if the increase in the index of refraction due to each layer is represented by $\delta_i f_i(z)$, the increase of the errors in Δx, Δy and ΔR are

$$\Delta x_{inc} = -\frac{3}{\sin^3\gamma}\sum_i \delta_i \int_{T_i} f_i(z)dz$$

$$\Delta y_{inc} = -\frac{\cos\gamma}{\sin^3\gamma}\sum_i \delta_i \int_{T_i} f_i(z)dz$$

$$\Delta R_{inc} = \frac{\delta}{\sin^3\gamma}(1 - 2\sin^2\gamma)\sum_i \delta_i \int_{T_i} f_i(z)dz ,$$

where T_i is the thickness of each layer.

4.3 Effect Due to Weak Turbulence

In Section 3.2 we postulated that for a transmitted signal $A(t)\mathrm{Exp}(-i\omega_0 t)$, the part of the returned signal reflected by the ground at point $\{x',y'\}$ takes the form

$$\tilde{A}(t,R')\mathrm{Exp}[-i\omega_0 t + i\phi(x_r,R')]$$

where R' is the distance from the transmitter to the reflecting point when the nth pulse is radiated. For azimuth processing, y' is constant, so that approximately

$$R' \cong R_{y'} + \frac{(x' - x_n)^2}{2R'} \tag{4-6}$$

while for range processing, $x' = x_r$,

$$R' \cong R_0 + (y' - y)\cos\gamma \ . \tag{4-7}$$

The functional forms of $\tilde{A}$ and $\tilde{\phi}$ are now examined based on the results of Section 3-4.

Let us represent the transmitted signal by its Fourier transform.

$$A(t)\, e^{-i\omega_0 t} = \int d\omega\, a(\omega)\, \exp\,[-i\,(\omega + \omega_0)t \tag{4-8}$$

The returned signal can then be expressed in terms of a log-amplitude fluctuation χ and a phase fluctuation s. The result is

$$A(t)\, \exp\,[-i\omega_0 t + \phi(x_r,\, R')]$$

$$= \int d\omega\, a(\omega)\, \exp\,[-i(\omega + \omega_0)t + \frac{2R'}{c}\,(\omega + \omega_0)]$$

$$\times\ \exp\,[\chi(x_m,\, \omega + \omega_0,\, R') + i\ s(x_r,\, \omega + \omega_0,\, R')]\ . \tag{4-9}$$

For azimuth processing R_y' is constant, and we may write (4-9) in the form

$$\tilde{A}(t)\exp[-i\omega_o t + i\phi(x_m, R')] \cong \exp[-i\omega_o t + \frac{2R'\omega_o}{c} + \chi(x_m, \omega_o, R')$$

$$+ is(x_m, \omega_o, R')] \int d\omega\, a(\omega)\exp[-i\omega t + i\frac{2R'}{c}\omega]$$

(4-10)

Approximately, therefore, for azimuth processing, if the variation of χ and s with the frequency is ignored, the distortion of the pulse shape is negligible. Since it has been noted that for fixed y, the term involved in the integration is not sensitive to the variation of x', the azimuth resolution is determined by the function [see Equation (2-22)]

$$g_h = \mathrm{Exp}[-i\frac{2\omega_o(x' - x')x_m}{cR_y'} + \chi(x_m, \omega_o, R') + \chi(x_m, \omega_o, R)$$

$$+ i\, s(x_m, \omega_o, R') - i\, s(x_m, \omega_o, R)] \qquad (4-11)$$

[Here we have neglected the quadratic terms in x and x' since they can be eliminated by focussing.] Therefore, the study of the azimuth resolution is equivalent to the study of the pattern of a linear aperture with random error. The pattern of the linear array is

$$F_h(x') = \int_{-\frac{NvT}{2}}^{\frac{NvT}{2}} \exp[-i\frac{2\omega_o}{cR_y'}x'x_m + \chi(x_m, \omega_o, R') + i\, s(x_m, \omega_o, R')]dx_m \qquad (4-12)$$

For range resolution, $x' = x_m$, we may write (4-12) in the form

$$\tilde{A}(t)\exp[-i\omega_o t + i\phi(x_m, R') = \exp[-i\omega_o t + \frac{2R'}{c}\omega_o]$$

$$\{\int d\omega\, a(\omega)\exp[i\omega t + i\frac{2R'}{c}]\exp[\chi(x_m, \omega+\omega_o, R') + i\, s(x_m, \omega+\omega_o, R')]\}$$

(4-13)

Consider the term in the curled bracket of (4-12) as A(t, y'), the arbiguity function χ_v becomes

$$\chi_v(R' - R_o, R - R_o) = \int d\omega\ a(\omega)a^*(\omega)\ \exp\left[\frac{2i\omega}{c}(R' - R)\right]$$

$$\exp\left[\chi(x_r, \omega + \omega_o, R') + \chi(x_m, \omega + \omega_o, R)\right]$$

$$\exp\left[i\ s(x_m, \omega + \omega_o, R') - is(x_m, \omega + \omega_o, R)\right]. \quad (4\text{-}14)$$

For chirpped signal,

$$A(t) = e[i\alpha t^2] \ , \quad (4\text{-}15)$$

so that, except for a constant (assume large time-bandwidth product), $a(\omega)$ may be approximated by Rect $(\frac{\omega}{2\alpha\tau})$. Thus, the resolution in range may be estimated from the pattern function

$$\tilde{F}_v(R' - R_o) = \int_{-\alpha\tau}^{\alpha\tau} \mathrm{Exp}\Big[i\ \frac{2\omega}{c}(R' - R_o) + \chi(x_m, \omega + \omega_o, R')$$

$$+ i\ s(x_m, \omega + \omega_o, R')\Big]d\omega\ . \quad (4\text{-}16)$$

For no perturbation, (4-16) yields the same resolution as given by (2-27).

Since both $\tilde{F}_v$ and $\tilde{F}_h$ are now random functions, we may only estimate the epxected values of the resolution by defining [see Equation (2-20)]

$$\langle\tilde{\beta}_h\rangle = \left\langle \frac{\int_{-\infty}^{\infty} |\tilde{F}_h(x')|^2 dx'}{\tilde{F}_h(o)^2} \right\rangle. \quad (4\text{-}17)$$

Similarly

$$\langle\tilde{\beta}_v\rangle = \left\langle \frac{\int_{-\infty}^{\infty} \tilde{F}_v(R')^2 dR'}{\tilde{F}_v(o)^2} \right\rangle. \quad (4\text{-}18)$$

In general, in order to find the expected values of $\tilde{\mathfrak{Z}}$, we need more statistical descriptions of the random variables χ and s. If we assume that they are Gaussian, then in principle we may find their correlation coefficient, from the joint probability density function, and compute the expected values of $\tilde{\mathfrak{Z}}_v$ and $\tilde{\mathfrak{Z}}_h$. However, since $<\chi^2>$ is very small within the range of parameter we are interested (see Section 3.4), they may be neglected. The resolution problem is then reduced to the problem of pattern deterioration due to random phase variation across a linear aperture. This problem is discussed thoroughly by Brown and Riordan (1970), their approach can be readily adopted in the present investigation.

By neglecting χ, we have from (4-12),

$$\tilde{F}_h(x') = \int_{-\frac{NvT}{2}}^{\frac{NvT}{2}} \exp\left[-\frac{i\omega_0}{cR_y'} x' x_\eta + i\, s(x_\eta, \omega_0, R')\right] dx_\eta \tag{4-19}$$

From Parseval's relation,

$$\int |\tilde{F}_h(x')|^2 \, dx' = \int_{-\frac{NvT}{2}}^{\frac{NvT}{2}} = NvT \tag{4-20}$$

which is independent of the random phase shift. Thus it is seen from (2-5) that the expected resolution due to random phase is

$$<\tilde{\mathfrak{Z}}_h> = \frac{\mathfrak{Z}_h |F_h(o)|^2}{<|\tilde{F}_h(o)|^2>} = \frac{\mathfrak{Z}_h (NvT)}{<|\tilde{F}_h(o)|^2>} . \tag{4-21}$$

Now,

$$|\tilde{F}_h(o)|^2 = \int_{-\frac{NvT}{2}}^{\frac{NvT}{2}} dx_\eta \int_{-\frac{NvT}{2}}^{\frac{NvT}{2}} dx_\eta' \exp[i\, s(x_\eta, R_y') - i s(x_\eta', R_y')] \tag{4-22}$$

hence, the expected value of $|\tilde{F}_h(o)|$ depends on the phase structure function between two points x_m and x'_m. If we approximate the antenna beam in the forward direction as a part of the plane wave, then these two points are transverse to the direction of propagation, with

$$L = R_y'$$

and a separation distance of

$$\rho = |x_m - x_m'| . \tag{4-24}$$

The expected value of $|F_h(o)|^2$ can then be expressed in terms of the phase structure function $D_s(\Delta L, \rho)$ [see Equation (3-35)]. The result is

$$<|\tilde{F}_h(o)|^2> = \int_{-\frac{NvT}{2}}^{\frac{NvT}{2}} dx_m \int_{-\frac{NvT}{2}}^{\frac{NvT}{2}} dx_m' \exp[- \frac{1}{2} D_s(o, |x_m' - x_m|)] . \tag{4-25}$$

Similarly, neglecting x in (4-16)

$$|\tilde{F}_v(o)|^2 = \int_{-\alpha\tau}^{\alpha\tau} d\omega_1 \int_{-\alpha\tau}^{\alpha\tau} d\omega_2 \exp[i\, s(x_m, \omega_1 + \omega_0, y_0) - i\, s(x_m, \omega_2 + \omega_0, yo)]$$

(4-26)

Hence, the expected value of $<|\tilde{F}_v(o)|^2>$ may be expressed as

$$<\tilde{F}_v(o)> = \int_{-\alpha\tau}^{\alpha\tau} d\omega_1 \int_{-\alpha\tau}^{\alpha\tau} d\omega_2 \exp[- \frac{1}{2} \Gamma_s(\omega_1, \omega_2, y_0)] \tag{4-27}$$

where $\Gamma_s(\omega_1, \omega_2, y_0)$ is the two frequency coherence structure function derived in (3-45).

Knowing the functional forms of D_s and Γ_s, (4-22) and (4-25) can be integrated at least numerically. From (3-35), we may represent

$$D_s(o, |x_m' - x_m|) \cong b\,|x_m' - x_m|^n$$

where

$$n = 5/3 \tag{4-28}$$

and for two-way propagation

$$b \cong 4 \times 1.46\, k^2 C_n^2 L\ . \tag{4-29}$$

For the parameters of interest for this work, $b \sim 10^{-3}$, hence approximate integration may be carried out by series expansion. Approximately therefore, the deterioration of azimuth resolution is given by

$$\frac{<\tilde{\beta}_h>}{\beta_h} = [1 = \frac{b(\sqrt{v}T)^n}{(n+1)(n+2)}]^{-1} \tag{4-30}$$

Similarly, for the azimuth resolution (3-44) may be represented by

$$\Gamma_s(\omega_1, \omega_2, -) = b'\,|\omega_1 - \omega_2|^{n'}\ , \tag{4-31}$$

where

$$n' = 2.$$

and

$$b' \cong \frac{28\sqrt{\pi}}{c^2} <n_1^2> \ell_o L\ \Delta\omega^2\ . \tag{4-32}$$

Again, b' is a very small quantity for the ranges of parameter of the present investigation. Therefore the deterioration of the range resolution is given by

$$\frac{<\tilde{\beta}_v>}{\beta_v} \cong [1 - \frac{b'(2\alpha\tau)^{n'}}{(n'+1)(n'+2)}]^{-1}\ . \tag{4-33}$$

Based on (4-30) and (4-33), one may also investigate the problem of optimal choice of the system parameters (ΔvT) and $(\alpha\tau)$ to achieve the best expected resolution. Since

$$\delta_h = \frac{\lambda R_{y'}}{2\Delta vT} \ , \tag{4-34}$$

and

$$\delta_v = \frac{\pi c}{2\alpha\tau} \ , \tag{4-35}$$

the mathematical problem of optimization is the same. The result is

$$(\Delta vT)_{opt} = \left(\frac{n'+2}{b'}\right)^{\frac{1}{n'}} \tag{4-36}$$

Similarly,

$$(\alpha\tau)_{opt} = \frac{1}{2}\left(\frac{n'+2}{b'}\right)^{\frac{1}{n'}} \tag{4-37}$$

The optimum resolutions are given by

$$<\tilde{\delta}_h>_{opt} = \frac{\lambda R_{y'}}{2} b^{\frac{1}{n}} C(n) \tag{4-38}$$

and

$$<\tilde{\delta}_v>_{opt} = \pi c \ b'^{\frac{1}{n'}} \ C(n') \tag{4-39}$$

where

$$C(n) = \frac{(n+1)}{n(n+2)^{\frac{1}{n}}} \ . \tag{4-40}$$

For $n = 2$, $C(n) = 0.75$ while for $n = 5/3$, $C(n) = 0.0733$. For a sample calculation using the above results, let us consider an idealized system with the following parameters: $f = 8 \times 10^9$ Hz and $L = 10^4$ m, and the ideal resolution ignoring the turbulence effect are

$$\mathfrak{z}_v = \mathfrak{z}_h \cong 3 \text{ m}$$

corresponding to values of

$$NvT \cong 62 \text{ m}$$

and

$$\alpha\tau \cong 0.524\, c \ \ (\sec^{-1}) \ .$$

Under the influence of weak turbulence specified by the following parameters

$$C_n^{\,2} = 2 \times 10^{-13} \ (m^{-2/3})$$

$$\langle n_1^{\,2} \rangle = 1.4 \times 10^{-13}$$

and

$$\ell_0 = \delta_0 r \ .$$

The following numerical results may be easily computed

a) $b = 3.28 \times 10^{-4}$ [Eq. (4-29)]

$b' = \dfrac{5.56}{c^2} \times 10^{-6}$ [Eq. (4-32)]

b) Deterioration of the expected azimuth resolution [Eq. (4-30)]

$$\frac{\langle \tilde{\mathfrak{z}}_h \rangle}{\mathfrak{z}_h} \cong 1.03$$

c) Deterioration of the expected range resolution [Eq. (4-33)]

$$\frac{\langle \tilde{\mathfrak{z}}_v \rangle}{\mathfrak{z}_v} \cong 1$$

d) Optimum choice of NvT for best expected resolution [Eq. (4-35)]

$$(NvT)_{opt} = 268.5 \text{ m}$$

e) Optimum range resolution [Eq. (4-38)]

$$<\tilde{\xi}_h>_{opt} \cong 2.2 \text{ m}$$

f) Optimum choice of $\alpha\tau$ for best expected resolution [Eq. (4-37)]

$$(\alpha\tau)_{opt} = 424 \text{ c}$$

g) Optimum range resolution [Eq. (4-39)]

$$<\tilde{\xi}_v>_{opt} \cong 7.4 \times 10^{-3} \text{ m}$$

Although the numerical values concerning the range resolution appears to be unrealistic due to the set of turbulence parameters we assumed, it appears that in general, weak turbulence has negligible effect in range resolution and slight effect on the azimuth resolution.

4.4 Effects of Strong Turbulence and Discrete Scatters

Based on the analysis of Section 4.3, it is seen that the two functions relevant to the analysis of SAR resolution are:

a) The spatial correlation between the fields $u(\vec{r})$ at two points transverse to the direction of propagation. This is a spatial mutual coherence function commonly denoted by

$$<u(L, \vec{\rho})\, u^*(L, \vec{\rho}*)> = \Gamma(\omega, L, \vec{\rho} - \vec{\rho}\,'). \qquad (4\text{-}41)$$

b) The frequency correlation of the field $u(\vec{r})$ at two different frequencies, ω_1 and ω_2. This is commonly known as two frequency mutual coherence function [Hong, Screenivasiah and Ishimaru (1977)], denoted by

$$\Gamma(\omega_1, \omega_2, \vec{r}_1, \vec{r}_2, t_1, t_2) = <u(\omega_1, \vec{r}_1, t_1)u^*(\omega_2, \vec{r}_2, t_2)>. \qquad (4\text{-}42)$$

(In our application, we need only the function for $z_1 = z_2 = L$, $\vec{\rho}_1 = \vec{\rho}_2$ and $t_1 = t_2$.)

In our analysis, due to the assumption of weak turbulence, we find approximate solution for $u(\vec{r}, t)$ first by Born approximation, and then obtain the Γ's by assuming Gaussian distribution of the fluctuation. For strong turbulence, and for the analysis in a medium with discrete scatters (rain, snow), this analysis is inadequate. Improving the approximate solution for u by using iteration would be too complicated to get meaningful results. Recent research work in the field, therefore is concerned in obtaining approximate differential equations for Γ. The resulting equations are more complicated than the Helmholtz equation and numerical techniques are required to obtain the solutions. In this section, the equations for Γ of our interest are briefly reviewed.

Let us start with the scalar Helmholtz equation

$$\nabla^2\psi + k^2(1 + \varepsilon_1)\psi = 0 \ . \qquad (4\text{-}43)$$

For plane wave propagation, let

$$\psi = u\, e^{ikz} \ , \qquad (4\text{-}44)$$

and obtain approximately

$$ik\,\frac{\partial u}{\partial z} + \nabla_\rho^2\, u + k^2\varepsilon_1 u = 0 \ . \qquad (4\text{-}45)$$

The average field $\langle u \rangle = \bar{u}$ cannot be obtained directly without further approximations. The usual assumption is that u is a Markov process in z, or equivalently ε_1 is delta correlated in x, i.e.,

$$B_\varepsilon(z - z', \vec{\rho}) \cong \delta(z - z')\, A(\vec{\rho}) \ , \qquad (4\text{-}46)$$

$A(\vec{\rho})$ is related to the spectral density of ε by

$$A(\vec{\rho}) = 2\pi \iint \Phi_\varepsilon\,(K_-)\, e^{i\,\vec{K}_-\cdot\vec{\rho}}\, d\vec{K}_- \ . \qquad (4\text{-}47)$$

Using this assumption we can deduce that

$$2ik \frac{\partial \langle u \rangle}{\partial z} + \nabla_\rho^2 \langle u \rangle + \frac{ik^3}{4} A(o) \langle u \rangle = 0 , \tag{4-48}$$

$$2ik \frac{\partial \Gamma(z, \vec{\rho}_d)}{\partial z} + \nabla_{\rho_d}^2 \Gamma(z, \vec{\rho}_d) + \frac{ik^3}{2} [A(o) - A(\vec{\rho}_d)] \Gamma(z, \vec{\rho}_d) = 0 \tag{4-49}$$

where

$$\vec{\rho}_d = \vec{\rho}_1 - \vec{\rho}_2 .$$

Equation (4-49) introduces an attenuation to the average field which is consistent to the physical picture of that coherent field is attenuated due to scattering.

Equation (4-49) may be used for the problem involving discrete scatterers by proper interpretation of $A(o)$ and $A(\vec{\rho})$ for such medium. From (4-48), we see that $\langle u \rangle$ is attenuated according to $\exp[-A(o)\frac{k^2}{8}z]$, and from transport theory, we know that $\langle u \rangle$ is attenuated according to

$$e^{-\frac{n\sigma_t}{2} z} ,$$

where σ_t is extinction cross section of a particle, and $n\sigma_t$ is the average extinction cross section per unit volume. Given a distribution of scatterers, we can therefore compute $n\sigma_t$, and identify

$$A(o) = \frac{4n\sigma_t}{k^2} , \tag{4-50}$$

the function $A(\vec{\rho})$ is not directly identified for discrete scatterers without further assumptions. From cross section theory of scattering, we know

$$\sigma_t = \frac{4\pi}{k} \text{Im } f(\hat{i}, \hat{i}) \tag{4-51}$$

where $f(\hat{s}, \hat{i})$ is the complex amplitude of single scattering of waves into direction $\hat{s}$ due to a wave of unit amplitude incident from direction $\hat{i}$. Also, from transport theory, we know the field satisfies approximately

$$\nabla^2 \langle\psi\rangle + K^2 \langle\psi\rangle = 0 \;, \tag{4-52}$$

where

$$K = k + 2\,n\,f(\hat{i}, \hat{i})/k \tag{4-53}$$

$$\triangleq K_r + i\,K_i \;,$$

which is related to the scattered amplitude in the forward direction. By considering $f(\hat{s}, \hat{i})$ as the angular spectrum of scattered field, and $A(\vec{\rho})$ as the spatial transform of the angular distribution, one can then argue that approximately

$$A(\vec{\rho}_d) = n \int f(\hat{s}, \hat{i})\, f^*(\hat{s}, \hat{i})\, e^{iK\hat{s}\cdot\vec{\rho}_d}\, d\Omega \;. \tag{4-54}$$

Thus, knowing the size and density distribution of scatterers, one may compute $A(\vec{\rho})$ and solve Γ from (4-49). It is to be noted, however, due to the assumption that u is a Markov process in z, we are physically limiting our problem to forward scattering, so that (4-49) is to be used for large (compared to wave length) particles.

Differential equations for the two frequency mutual coherence functions are deduced from (4-49). The resulting equation is

$$\left[\frac{\partial}{\partial z} - \frac{i}{2}\left(\frac{1}{k_1} - \frac{1}{k_2}\right)\nabla_{\rho_d}^2 - iK_1 - (iK_2)^* - A(\vec{\rho}_d)\right]\Gamma(\omega_1, \omega_2, z, \vec{\rho}_d) = 0. \tag{4-55}$$

In literature, available solutions of (4-49) and (4-55) are all by numerical methods, and not enough data is available for immediate application to our problem. Some consideration has been given to choosing a model for the size and particle distributions of rain, and then carrying out a numerical solution of Equations (4-49) and (4-55) for the analysis of SAR resolution, but the task is too elaborate to accomplish in this period of research.

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