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STUDY OF TWO-DIMENSIONAL HIGH RESOLUTION

FREQUENCY ANALYSIS FOR APPLICATION TO

SYNTHETIC APERTURE RADAR

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STUDY OF TWO-DIMENSIONAL HIGH RESOLUTION FREQUENCY ANALYSIS FOR APPLICATION TO SYNTHETIC APERTURE RADAR

Philip L. Jackson

SUMMARY

A postdoctoral study at the University of Michigan was made on the potential application of high resolution autoregression techniques to synthetic aperture radar (SAR). The techniques include maximum entropy frequency analysis (MEM or MESA). The goal of such an application is to improve the resolution of SAR, or to maintain the currently achieved resolution with a reduced amount of data.

SAR imagery requires two-dimensional processing of the signal data. Therefore, a two-dimensional high resolution frequency analysis is required. During the course of the investigation a new method for two-dimensional frequency resolution was made available through Rome Air Development Command. The new method, termed a semicausal model by the originators, E. Jain and S. Ranganath, was tested on SAR-like simulated data. The method was found to produce two-dimensional spectra, giving better resolution than the FFT, even when a low autoregression order is used. It holds promise to improve the resolution of SAR.

Perspective plots of the tested two-dimensional functions, their autocorrelations, and FFT and semicausal spectra are illustrated. Additionally a two-dimensional de-chirping algorithm for SAR data was developed and programmed. De-chirping is necessary to transform SAR strip map data into a form which can be processed by two-dimensional frequency analysis.

1. INTRODUCTION

For short truncations of data with discrete frequencies, maximum entropy frequency analysis (MEM, or MESA) and similar autoregression techniques produce spectra with finer resolution than produced by conventional power spectrum analysis [1, 2, 3]. Current processing of synthetic aperture radar (SAR) is limited in resolution to that obtained by the bandwidth based on the Fourier transform [4, 5]. MEM appears to have the potential to obtain finer resolution than obtained by basic Fourier transform processing. It also appears particularly suitable for SAR processing because SAR data can be reduced to a form of short truncations of discrete frequencies. In fact, one type of conventional processing reduces SAR data to this form.

After such reduction a two-dimensional fast Fourier transform (FFT) is conventionally applied to SAR data. Applying MEM or similar processing in place of the FFT has the potential of improving the resolution or of achieving the FFT resolution with less data. Either is a highly desirable goal. Processing of radar data other than SAR would also be benefitted.

In addition to the improvement in resolution, negligible side lobes are produced with MEM processing. Successful application of MEM to SAR would, therefore, aid in detecting weak targets proximate to strong targets. Conventional processing produces considerable side lobes which often hide weak targets.

The new high resolution techniques of MEM and autoregression have heretofore been confined to a single dimension. MEM one-dimensional frequency analysis was applied to synthetic SAR rotating platform data, and significant improvement in resolution, as well as elimination of side lobes in one-dimension was shown [6]. The dimension perpendicular to the MEM-processed dimension was processed by the FFT. Much higher resolution was obtained in the MEM-processed direction than in the FFT-processed direction. Also, side lobes in the MEM-processed direction were undetectable.

However, to process SAR images with higher resolution requires a two-dimensional embodiment. At the outset of the study period the anticipated approach to achieve the resolution was to first obtain a MEM processing in the first direction and an FFT processing in the second (as previously obtained [6]). Then a duplicate image was to be processed, except with MEM processing in the second direction and FFT processing in the first direction. After both images are produced they would then be multiplied together so that only those points which share sufficiently large values in both images would be significant in the product of the images. This duplicating and multiplying method would, of course, produce unwanted amplitude distortions in the product image, but potentially would sharpen resolution in both directions.

Although the maximum entropy frequency concept was first presented in 1967 [1], no two-dimensional embodiment of this concept has been developed and illustrated, although investigations have been made [7,8 ,9]. In fact the direct two-dimensional embodiment of MEM may be an ill-posed problem, primarily because no phase information is retained in one direction for subsequent use in the second direction.

During this investigation a new two-dimensional high resolution frequency analysis algorithm became available. It is related to MEM in method, and makes use of a two-dimensional autoregression scheme. The algorithm was developed by E. Jain and S. Ranganath of the State University of New York, Buffalo. A portion of the algorithm was supplied by RADC, and its evaluation for SAR was made the prime goal of this study. The program supplied by RADC required extension so that the input was two-dimensional data and the output a two-dimensional spectrum. Most of the study time was expended on this extension.

The method developed by Jain and Ranganath is a remarkable achievement. It has not yet been properly and fully described in the literature, although a partial idea of the concepts can be obtained in references [10, 11]. It would be an extremely significant contribution to the frequency analysis literature if the authors were to completely and lucidly present this material.

The two-dimensional algorithm was tested on synthetic SAR data. A series of tests were run, and perspective plots were generated to illustrate the results. Even though very small two-dimensional autoregressive orders were employed, good results were obtained in separating proximate frequency components. Finer frequency resolution than found with the FFT was obtained by Jain and Ranganath's semicausal method. Questions of computing efficiency aside, the results are highly encouraging for eventual application to actual synthetic aperture radar data.

Much remains to be investigated before final conclusions about the method's special or applicational use for SAR. Such questions as noise degradation, amplitude accuracy, spectral splitting, and applicability to actual SAR signal data are some of the questions to be asked.

In this report a description of the frequency aspect of SAR data processing, a discussion of high resolution frequency analysis and its application to SAR processing, and illustrations of two-dimensional processing on synthesized data will be given.

2. TWO DIMENSIONAL FREQUENCY DEPENDENCE IN SAR IMAGE PROCESSING

Rayleigh resolution in a SAR image is inversely proportional to bandwidth, which is the spread between the lowest and highest frequencies in a signal. For example, the Rayleigh resolution, ρ, which can be obtained from a chirp pulse in radar is dependent upon the highest and lowest frequencies in the pulse:

$$\rho = C/2B$$

where C = free space EM velocity

$B = f_u - f_1$ (bandwidth)

f_u = highest frequency in pulse

f_1 = lowest frequency in pulse

To conceptualize the relationship between bandwidth and resolution, consider that an (unobtainable) spike pulse, or Dirac delta function, contains all frequencies, so it would produce infinitely fine resolution. It is obvious that radar resolution is increased in size as a transmitted rectangular pulse is increased in length; in turn the pulse contains lower and narrower frequency content with increasing length. Also, the bandwidth is determined by the size of the main lobe of the Fourier transform of a more complicated pulse such as a chirp. Hence, in conventional processing, the resolution of SAR is controlled by the frequency resolution of the Fourier transform. This control holds whether the processing is performed by matched filtering, cross-correlation (deconvolution), optical focussing SAR signal film, or dechirping and Fourier transforming (FFT).

The basis for the relationship between the Fourier transform, bandwidth, and resolution is lucidly stated in [12]. Many authors [4,5] make use of this relationship to define the resolution of radar. In fact the term "bandwidth" is often used in the radar discipline to imply resolution.

When a transmitted chirp pulse is used in SAR, one means of processing the received signal data is by a two-dimensional Fourier transform. Two kinds of SAR are processed by this means.

The first kind is the annular, or circular motion, SAR. An example is rotating platform data, in which an annular variable density signal film is generated, and a resulting Fraunhofer diffraction pattern is the image. Also, of course, the data may be digitized so that a two-dimensional complex array is the signal data in which the image is its two-dimensional FFT. This type of SAR is fully described in [13]. No data of this kind is available for this project.

The second kind, strip-map SAR, can be processed in the four ways described above. The last-mentioned method, dechirping and Fourier transforming, directly uses frequency analysis techniques for image formation. A description of this method is found in [14]. Basically it can be described as follows:

Consider first the transmitted chirp pulse, and the determination of range. The chirp is a pulse with a linearly changing frequency. The propagation time of the transmitted pulse to and from a reflecting object determines the range (distance) to the object. Now, a simple concept can be used to convert the propagation time (range) to frequency. A frequency-ramp reference function is used which has the same linear rate of change of frequency as the rate in the chirp pulse, and extends over a much longer time than a single chirp pulse. If the difference between a chirp pulse and the reference function is found, a continuous wave (discrete frequency) of the same duration as the chirp is formed. The frequency value will then represent the propagation time of the pulse. An early return will produce a continuous wave of a relatively low frequency, while a late return will produce a relatively high frequency; because the ramp reference function will have reached a higher frequency at the time of the late arrival, the frequency difference between the reference and the pulse will, therefore, be greater. The salient point is that each reflected chirp pulse is transformed into a pulse of a discrete frequency of the same length as the pulse.

As the discrete frequencies of the resulting continuous wave (cw) pulses represent propagation times, which in turn represent ranges to the reflecting objects, it follows that the frequency domain of these cw pulses represents the range image of the reflecting objects. An FFT of these cw pulses produces such an image.

The frequency resolution in the FFT limits the SAR image resolution by the width of its main lobe. The duration of the original chirp controls the width of the main lobe. In addition the FFT sidelobes are often large enough to obscure weak reflectance by strong reflectance even though they are separated by a distance greater than the Rayleigh resolution.

Given this known resolution limitation, it is apparent that a means of frequency processing which produces finer frequency resolution than the Fourier transform could potentially produce finer image resolution in SAR. Also, if sidelobes can be reduced or eliminated in such frequency processing, proximate weak targets could be resolved from strong targets. A glance at

Figure 1 should be sufficient to indicate that at least one frequency process exists which might have the potential to achieve higher SAR resolution and render sidelobes negligible. Also, the same conventional resolution achieved by the FFT might be achievable with less data (shorter pulses), in which case the effect of phase non-linearities could be reduced.

The above description was restricted to one dimension for conceptual purposes. However, a radar image is two-dimensional--range and azimuth. The signal data in the azimuth direction is formed in precisely the same manner as in the range direction discussed above. In a moving radar vehicle a Doppler signal from a target is generated which is related to the azimuth position of the vehicle. The Doppler signal is essentially a linear swept frequency if the angle from broadside is less than about 8°. Hence, in the azimuth direction a pulse of the same type as the range pulse is formed by the Doppler signal. A two-dimensional chirp is thereby formed, as shown in Figure 2. Dechirping results in a two-dimensional signal (shown in Figure 2c) in which the location of a reflector is found by two-dimensional frequency analysis. Currently FFT processing is employed for such analysis.

Two-dimensional frequency processing with finer resolution than the FFT would then potentially produce finer SAR resolution than is attainable with the FFT. In a continuing investigation, noise, accuracy, spectral splitting, spurious frequencies, efficiency, etc, would have to be thoroughly looked into before conclusive indications that the high resolution method would be applicable to SAR.

3. MODERN HIGH RESOLUTION FREQUENCY ANALYSIS

Beginning in 1967, with Burg's presentation [1] of MEM, new concepts of frequency analysis have arisen. Essentially these new concepts are similar, if not quite identical, to Burg's concept, and are termed autoregressive models, which were first presented by Parzen [3] in 1968. Full treatments of these new frequency analysis techniques can be found in many references [15, 16, 17] and will not be repeated here, except for a brief non-mathematical discussion of the differences between the new techniques and the Fourier transform.

Particularly for short truncation of discrete frequencies (precisely the form of SAR dechirped data), MEM and the other new techniques achieve very high resolution with essentially no side lobes. Again, Figure 1 illustrates this difference in resolution. This new capability has been employed in several disciplines, particularly in geophysics. For example, irregularities of the earth's motion were studied [15].

MEM produces a power spectrum. Conventionally power spectra are produced by the Wiener-Tukey method of Fourier transforming the autocorrelation of the data. The differences in approach using MEM is in the formation of an error prediction filter whose power spectrum is the inverse of that of the data. The Fourier transform is taken of the error prediction filter, and the inverse of each frequency value is, therefore, taken for the shape of the power spectrum of the data. The product of these values and a constant representative of the total power of the spectrum gives the power spectrum. It should be noted that the order (number of elements) in the error prediction filter affects not only the computing efficiency, but the shape of the resulting power spectra.

The unique manner in which the error prediction filter is computed is shown by Burg to make the minimum assumption about data outside the data sampling interval. This minimum assumption is shown to be the information theory maximum entropy representation of the extended data. By using certain constraints, such as the Toeplitz matrix of the extended autocorrelation function being semipositive definite, this minimum is made consistent with the known data.

Such a minimum assumption about the unknown data is not achieved with conventional frequency analysis: In the Fourier transform all data is assumed to be zero. In the Fourier series it is assumed to be infinitely periodic. Either assumption (usually unwarranted) causes edge effects which tend to be ameliorated by a maximum entropy extension. Such an "infinite" extension of the data appears as a virtually long data sample and, therefore, produces a narrow main lobe while severely attenuating the side lobes. It is revealing that a single rectangular pulse produces a similar spectrum with both MEM and the FFT. With the FFT a short truncation of a function is convolved with an enveloping rectangular pulse; with MEM it is not.

The extension of MEM to two-dimensional frequency analysis had been previously attempted and investigated [7, 8, 9], with results which were either inconclusive or not specified as a programmable algorithm.

Recently, however, Jain and Ranganath [10, 11] have developed a method to achieve high resolution two-dimensional analysis. Their method uses autoregression in two-dimensions, but the authors claim it is not a maximum entropy algorithm. They term it a "semicausal" model. However, it is an extension of the error-prediction or autoregression frequency analysis approach to two-dimensions.

Jain and Ranganath have developed a remarkable approach to the two-dimensional problem. Their primary contribution has been in treating the two-dimensions differently. They treat one direction as an initial value (causal) problem, and the other direction as a boundary value problem, hence the term "semicausal". As shown in the next section, the method was successfully applied to two-dimensional data which simulated two targets of SAR data.

The semicausal model requires the computation of an autocorrelation (autocovariance) function before the spectral estimation is made. Obtaining the autocorrelation function requires the computation of two two-dimensional FFT's. The data is transformed by the FFT, squared, and inverse FFT transformed. From an efficiency standpoint, these necessary preliminary FFT computations require more than twice the computation time of conventional FFT processing. Again, the order of the autoregression coefficients affects the shape of the power spectrum.

Despite the published work of Jain and Ranganath [10, 11] related to this subject, they have not published a thorough description and discussion of their method. It is hoped they will soon do so, as their algorithm is a remarkable and significant achievement in frequency analysis.

As promising as these new techniques appear to improve resolution in SAR, certain caveats should be made:

1. Variations in amplitude have been found.
2. Spectral splitting sometimes occurs.

3. Frequency shifts for short truncations occur similar to those found in the FFT.

4. Sensitivity to noise is substantially greater than in the FFT.

5. Theory has not yet been developed to account for all aspects found experimentally.

6. The choice of autoregressive order influences the spectral output.

7. Computing time, particularly for the two-dimensional case, becomes much larger than for the FFT.

4. TWO-DIMENSIONAL PROCESSING

Prior to adapting the algorithm of Jain and Ranganath, two-dimensional de-chirping was programmed and debugged. De-chirping is not a new development, as it is used in actual digital SAR systems. However, it was necessary to develop a dechirping program because none was available.

As described in Section 2, linear swept frequency pulses (chirps) form SAR signal data. Chirps are generated as a pulse in the range direction, and are produced by the Doppler effect in the azimuth direction. Frequency ramp functions are generated with the same rate of change of frequencies as in the transmitted chirp pulses and in the Doppler-formed pulse. By subtracting the reflected pulse from the ramp function the range position defines a frequency, as does the along-track position of the Doppler chirp. Subtraction is simply performed through multiplication of the chirps with the respective reference function and subsequent performance of low-pass filtering. This operation enables the location of reflectors, hence the forming of an image, by performing two-dimensional frequency analysis, as was explained more fully in Section 2.

To test the program a two-dimensional chirp was constructed, as shown in Figure 2a. This synthetic function differs from actual SAR data in two respects: The along-track data may be a different frequency with a different rate of change than the range pulse; also the Doppler rate of change is a function of range. These differences can be compensated for in the program developed for the symmetric case shown in Figure 2a.

Figure 2b shows the result of dechirping in one direction, while Figure 2c shows the results of dechirping in two directions. The two-dimensional frequency spectra of a function, such as shown in the two-dimensional dechirped result in Figure 2c, will produce the image of a reflector. The computer program was developed in anticipation of using actual strip-map signal data, but time did not allow this experiment.

Figures 3 through 7 are perspective plots illustrating two-dimensional functions and frequency analysis by the FFT and the semicausal model. In each figure the two-dimensional function, its autocovariance, the square of the FFT, and both the linear and decibel semicausal power spectrum are plotted. Note that the scale of the plots of the FFT and semicausal models are different in these plots. All plots were generated with a total of 64 x 64 samples. The FFT was taken with 128 x 128 samples by filling in zeros between 65 and 128 in the function. Note that the figures show decreasing frequency differences between Figure 3 and Figure 6, and an increase in additive white noise in Figure 7. Low autoregressive orders were used: 4 in one direction and 2 in the other direction.

In Figure 3a, where no white noise was added to the function, the interference effect between the two frequency components is clearly seen. In the FFT the two main lobes are clearly separated; the difference in height of the two main lobes is undoubtedly due to the sampling point of the peak. The semicausal spectrum clearly shows the two frequency component peaks, which are somewhat different in height and shape, but which are at the proper frequency locations.

In Figure 4a, where 7% white noise was added to the function, the difference in the frequency components was reduced to 55% of the difference in Figure 3a. In Figure 4a the interference effect between the two frequency components is still clearly seen by eye, but is less pronounced than in Figure 3a. The FFT barely resolves the two components in Figure 4c, while the semicausal spectrum in Figure 4d and 4e also resolves the components with peaks whose difference in amplitude is possibly due to the sampling interval. Note that the falloff of the two components away from the central peaks out to the edge of the figures is substantially greater in Figure 4d than in Figure 3d. This falloff distance is undoubtedly due to the addition of white noise in Figure 4d, as this type of frequency analysis is more stable with a moderate amount of added white noise.

The frequency difference was reduced to a value below the resolution capability of the Fourier transform in Figure 5a. Here the interference pattern is not apparent to the eye because the two components are so close together in frequency. The FFT spectrum shows only one central peak in Figure 5c, as would be expected, because $\Delta t \; \Delta f = 64 \; (1/8 - 1/8.2) \simeq (8-7.8) \simeq 0.2$. However, the semicausal spectrum in Figure 5d clearly shows two peaks. Although there is a small decrease in amplitude between them, they clearly indicate that Rayleigh resolution is maintained. The frequency peaks in Figure 4d are of unequal amplitude but the difference is probably less than 1.5 to 2 db.

In Figure 6a the frequency difference is reduced to approximately 50% of that in Figure 5a. Here the semicausal model clearly resolves the two frequency components (Figure 4d) while, of course, the FFT fails to do so (Figure 4c).

In Figure 7a the same frequency component difference is generated as that shown in Figure 6a, but the additive white noise is increased to 30%. The semicausal model shows two peaks in Figure 7d, but one peak is down 10-12 db below the other. The limit of usable resolution has been reached for the low autoregressive orders (4 and 2) used.

5. CONCLUSIONS AND RECOMMENDATIONS

This study has validated the fact that a two-dimensional high resolution frequency analysis method has the potential to be applied to SAR data. The achievement of Jain and Ranganath [10, 11] in developing this method is extremely significant in radar and other fields. In comparison with current SAR processing methods, the potential in SAR is to achieve higher resolution with the same amount of data, or to achieve the same resolution with fewer data. Fewer data generates less phase errors, allows the data to be gathered in a shorter time, and reduces communication bandwidth requirements.

Computer-generated perspective plots demonstrate the dechirping of SAR strip map signal data to prepare it for two-dimensional frequency analysis which produces the image. Perspective plots of two-dimensional functions, autocovariances, the FFT spectra and high resolution (semicausal) spectra demonstrate the relative resolving capabilities of the two methods with decreasing differences between the frequency components.

Although the actual operation of the two-dimensional semicausal frequency analysis method was demonstrated on synthetic data, the work reported here is preliminary: No application to actual SAR data was made; only two frequency components were used; the effect of autoregressive order was not investigated; the effect of noise was not completely shown; spectral splitting, amplitude variations and frequency shifts were not investigated. To thoroughly develop the potential for SAR requires these categories of investigations.

Computing time was not considered critical in this investigation. The applicability of the method is fundamental at this time. If high resolution frequency analysis methods can operationally be applied to SAR, development of efficiency and of hard-wired computation would be the next step. Also with the newness of the technique, new computational methods (not to mention the increasing capacity of computers) have a good probability of being developed.

The results of this preliminary study, then, show that a new two-dimensional high resolution frequency analysis method can be used on limited data sets which are similar to SAR data. Better resolution than the FFT can be achieved. The further above-mentioned investigative steps are required for the thorough investigation of the applicability of this frequency analysis method to SAR processing, as well as for development of the method for operational use in SAR.

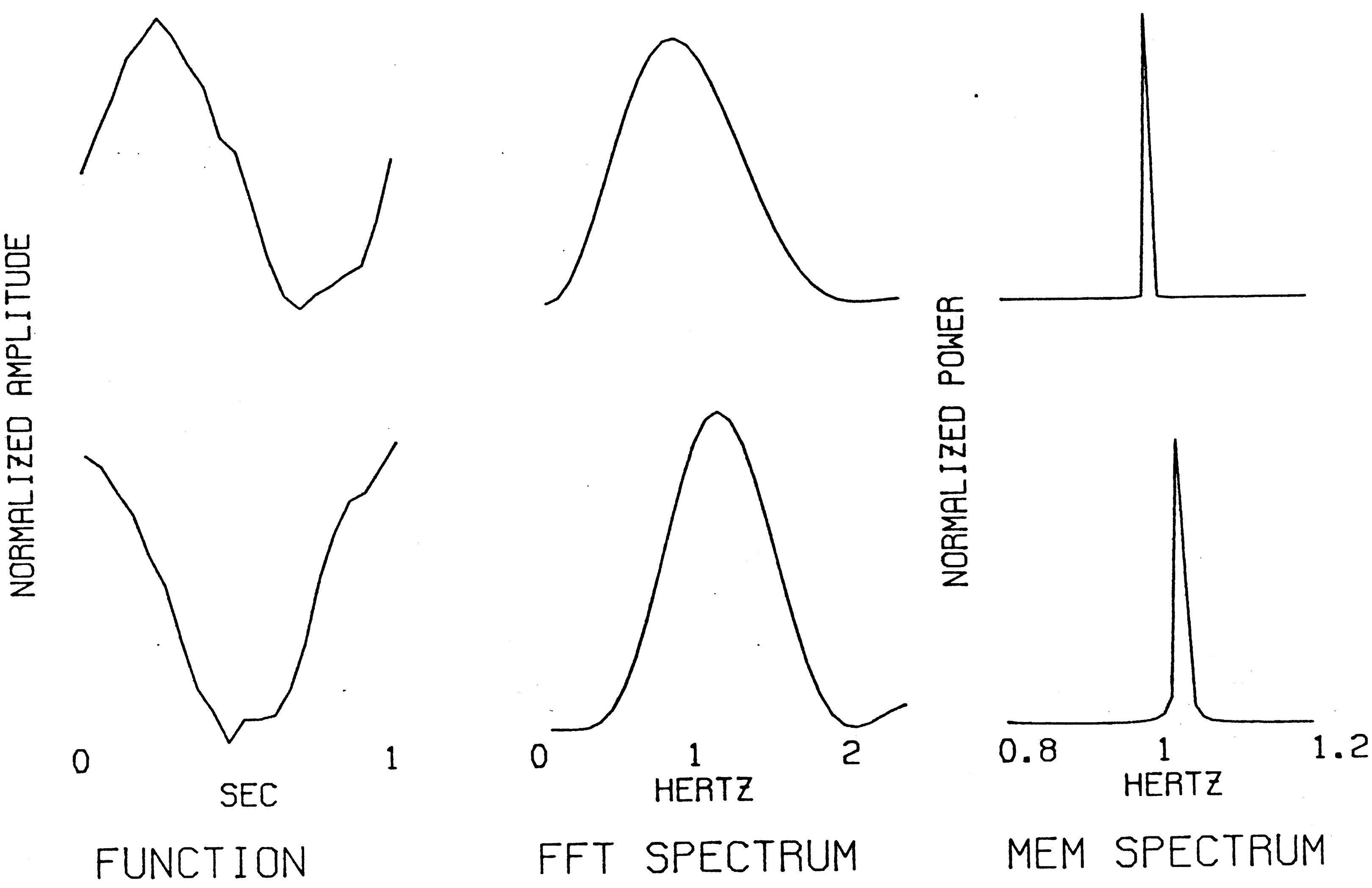

Figure 1. Comparison of FFT Spectrum and MEM Spectrum for Sine and Cosine Function Truncated to One Period.

CHIRP SIGNAL

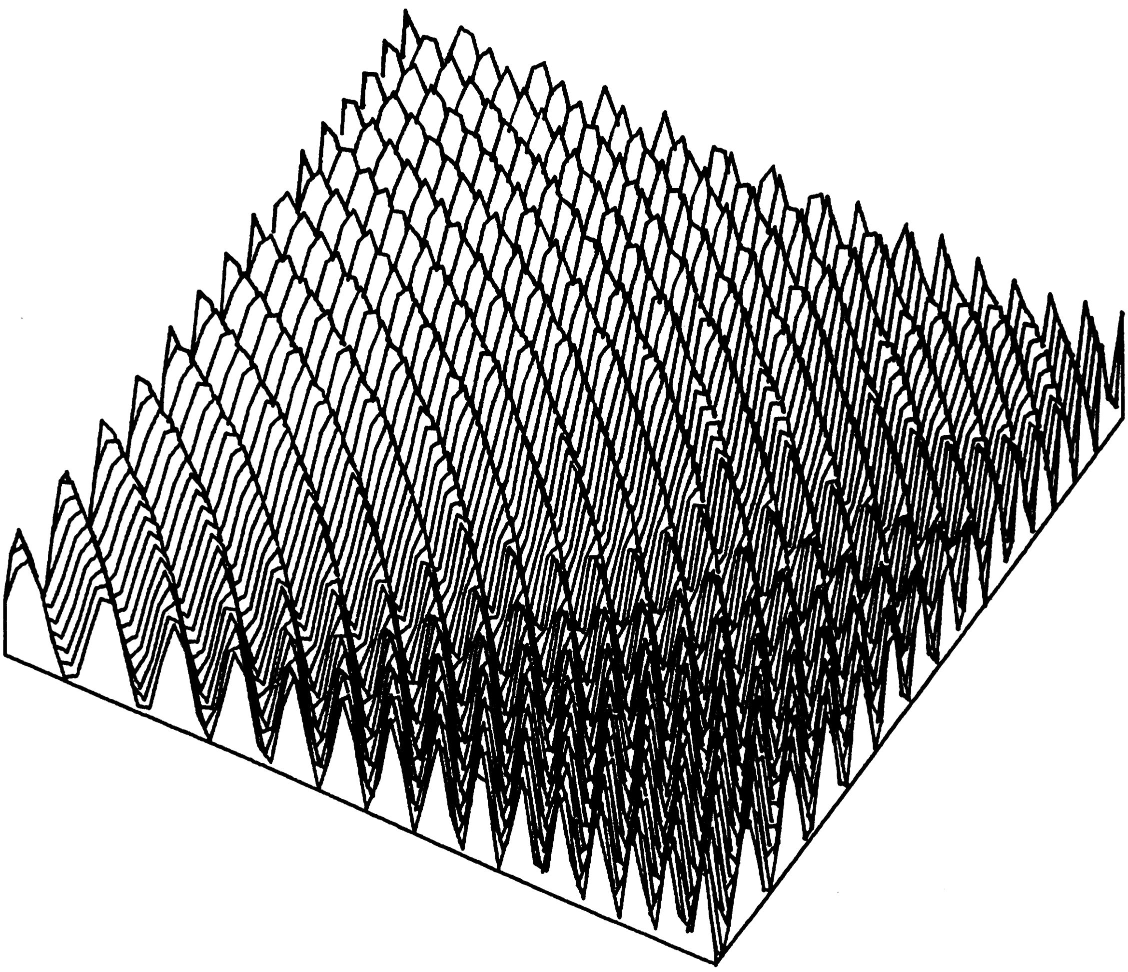

Figure 2a. Two-dimensional Chirp Signal.

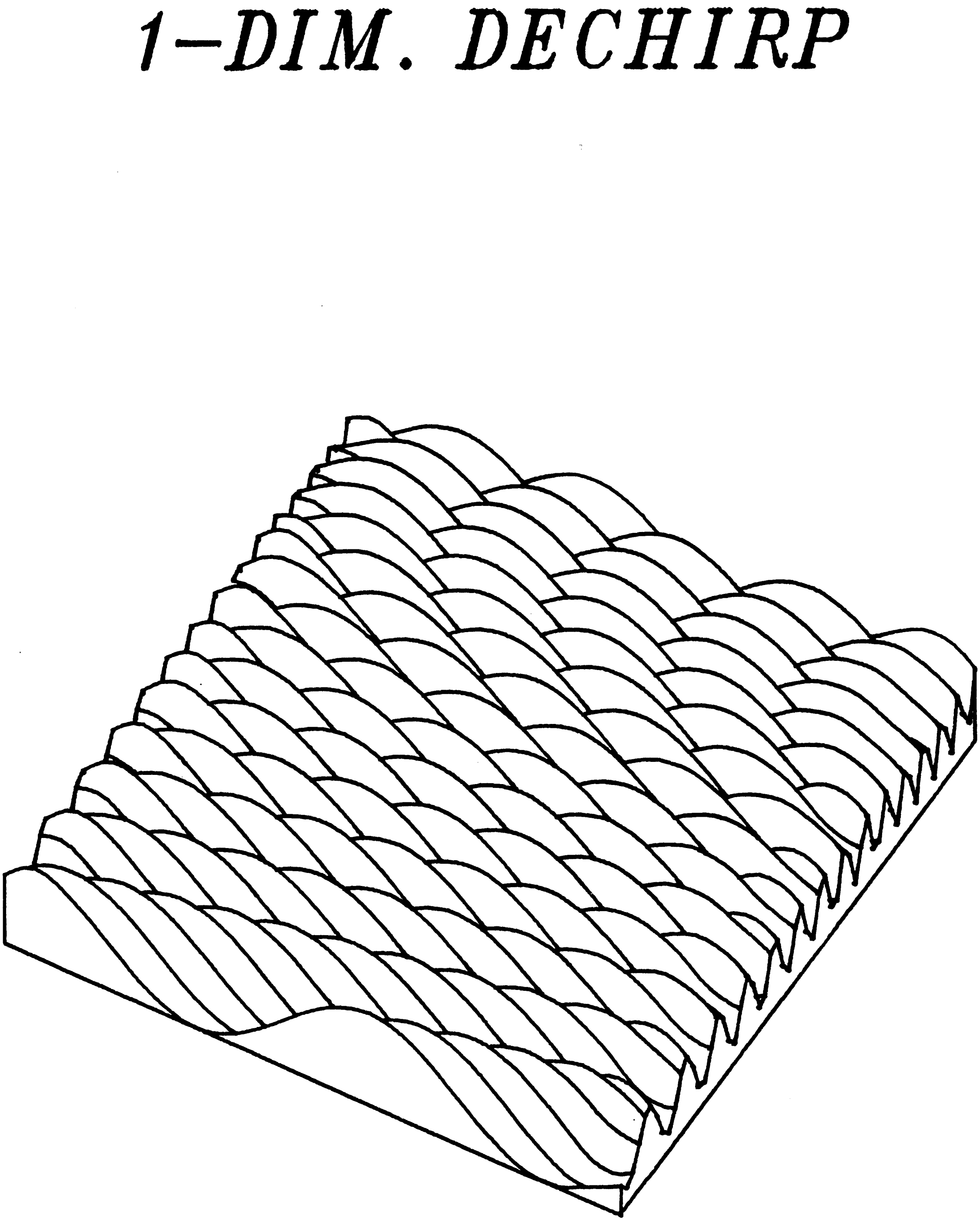

Figure 2b. Signal in 2a Dechirped in One Dimension.

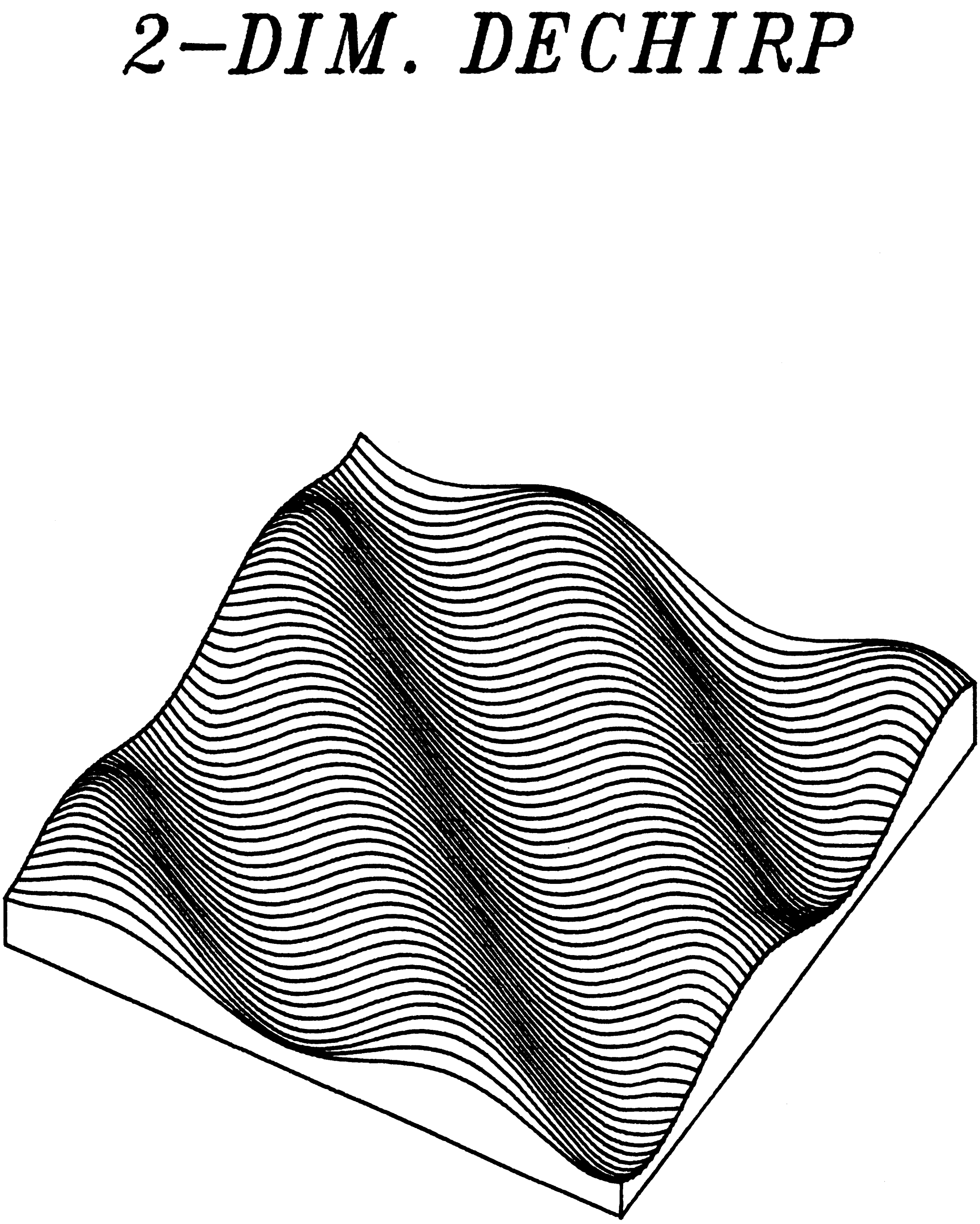

Figure 2c. Signal in 2a Dechirped in Both Directions.

2-D FUNCTION

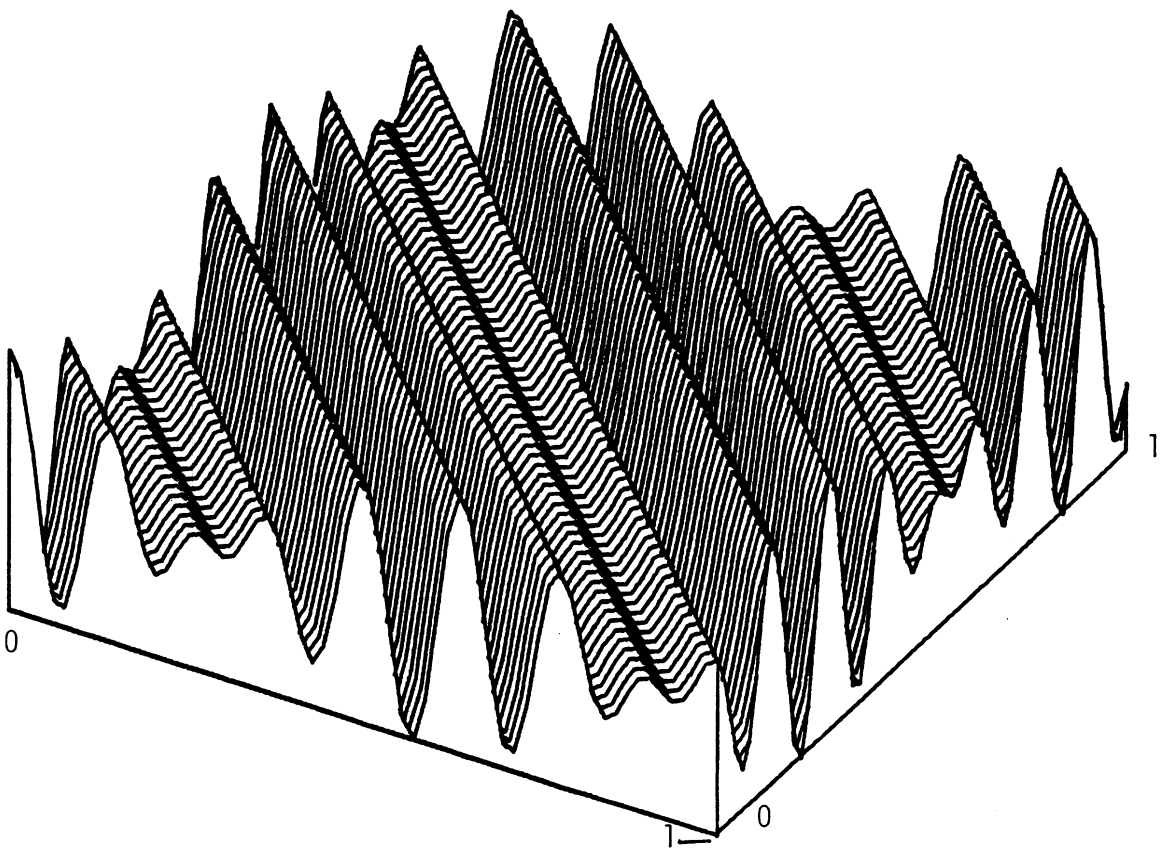

Figure 3a. Plot of $\sin \frac{2\pi}{8}(i+j) + \sin \frac{2\pi}{10}(i+j)$ $1 \leq i \leq 64; 1 \leq j \leq 64$

AUTOCOVARIANCE

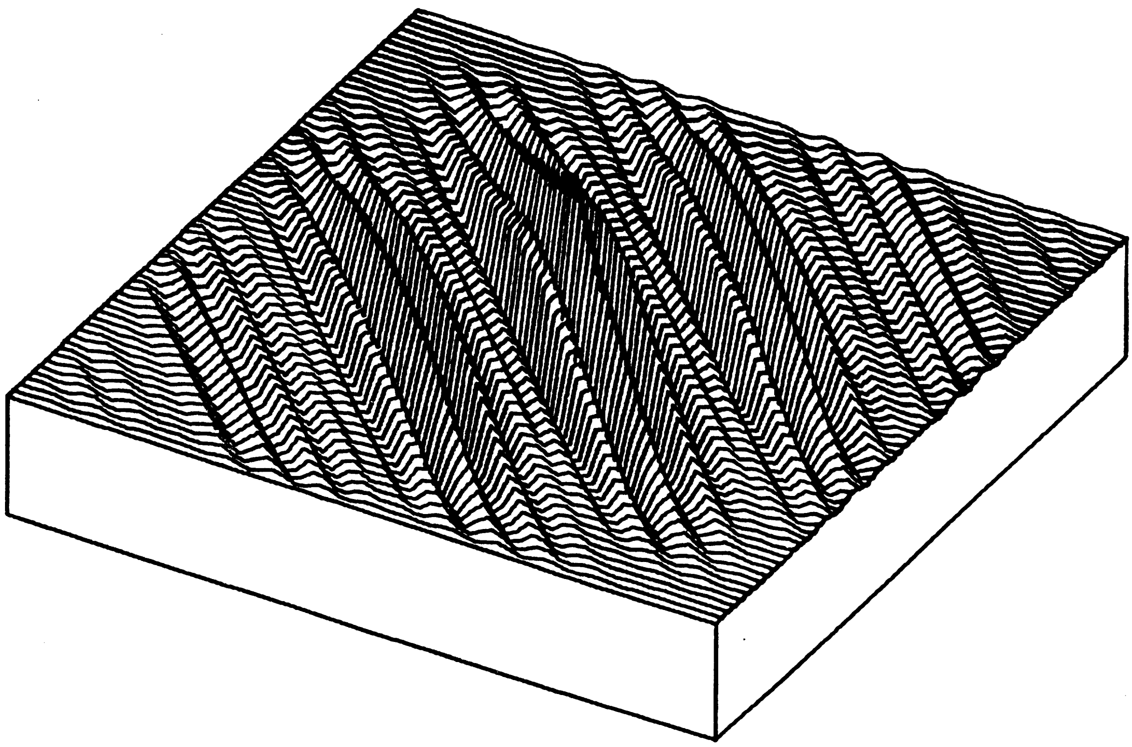

Figure 3b. Autocovariance of 3a.

FFT SPECTRUM

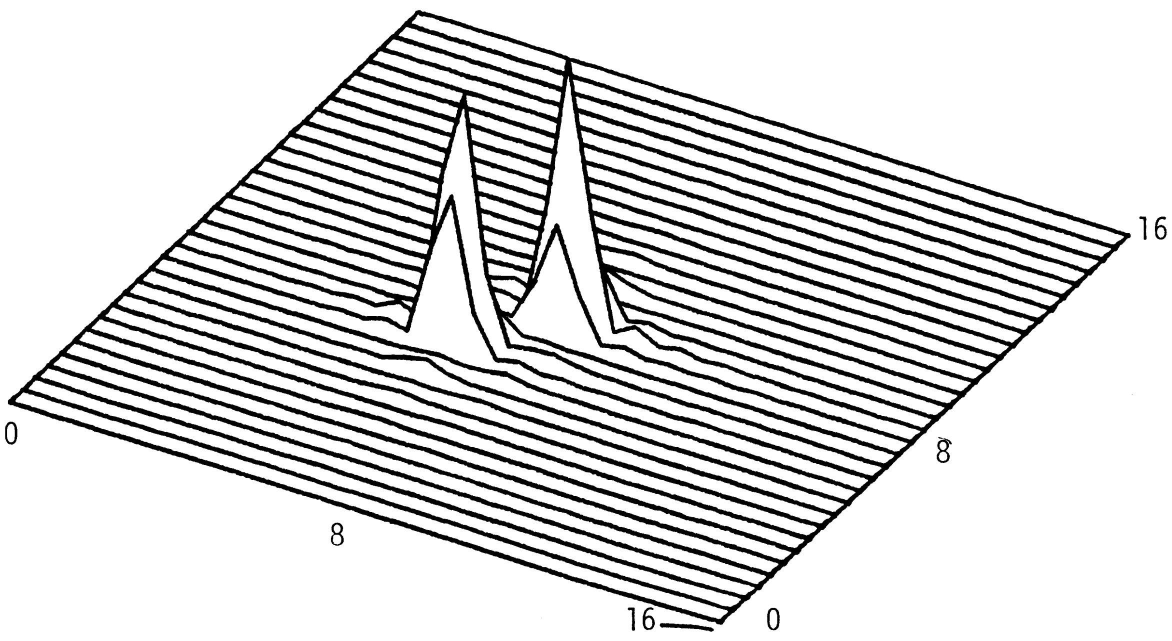

Figure 3c. FFT Spectrum of 3a.

S–C SPECTRUM

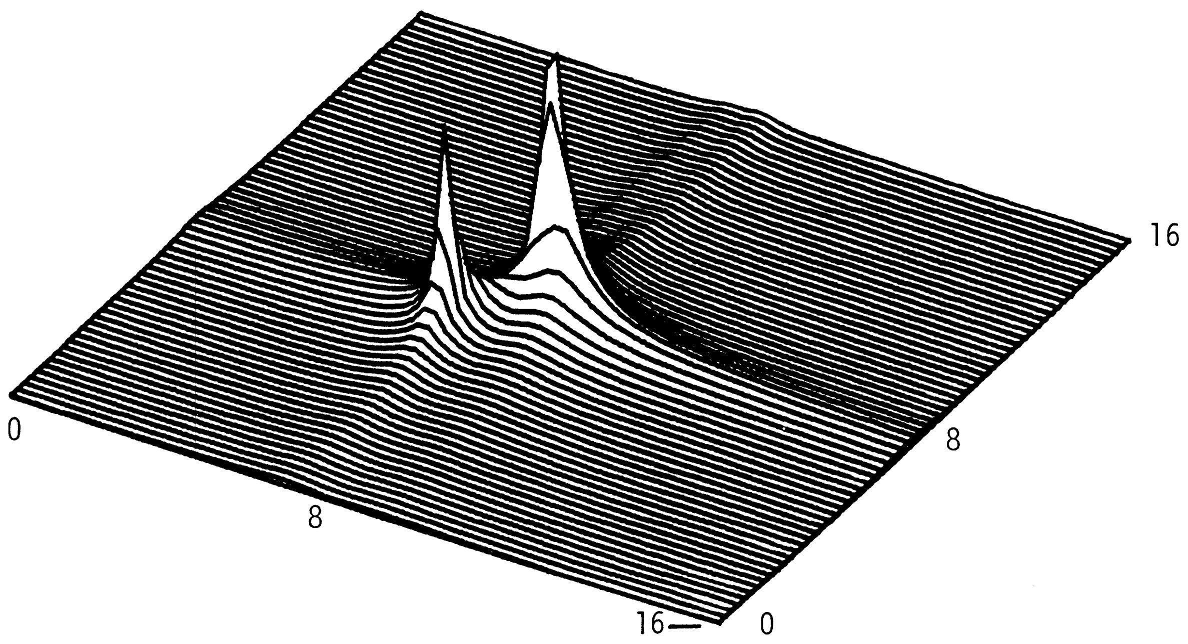

Figure 3d. Semicausal Spectrum of 3a (Amplitude).

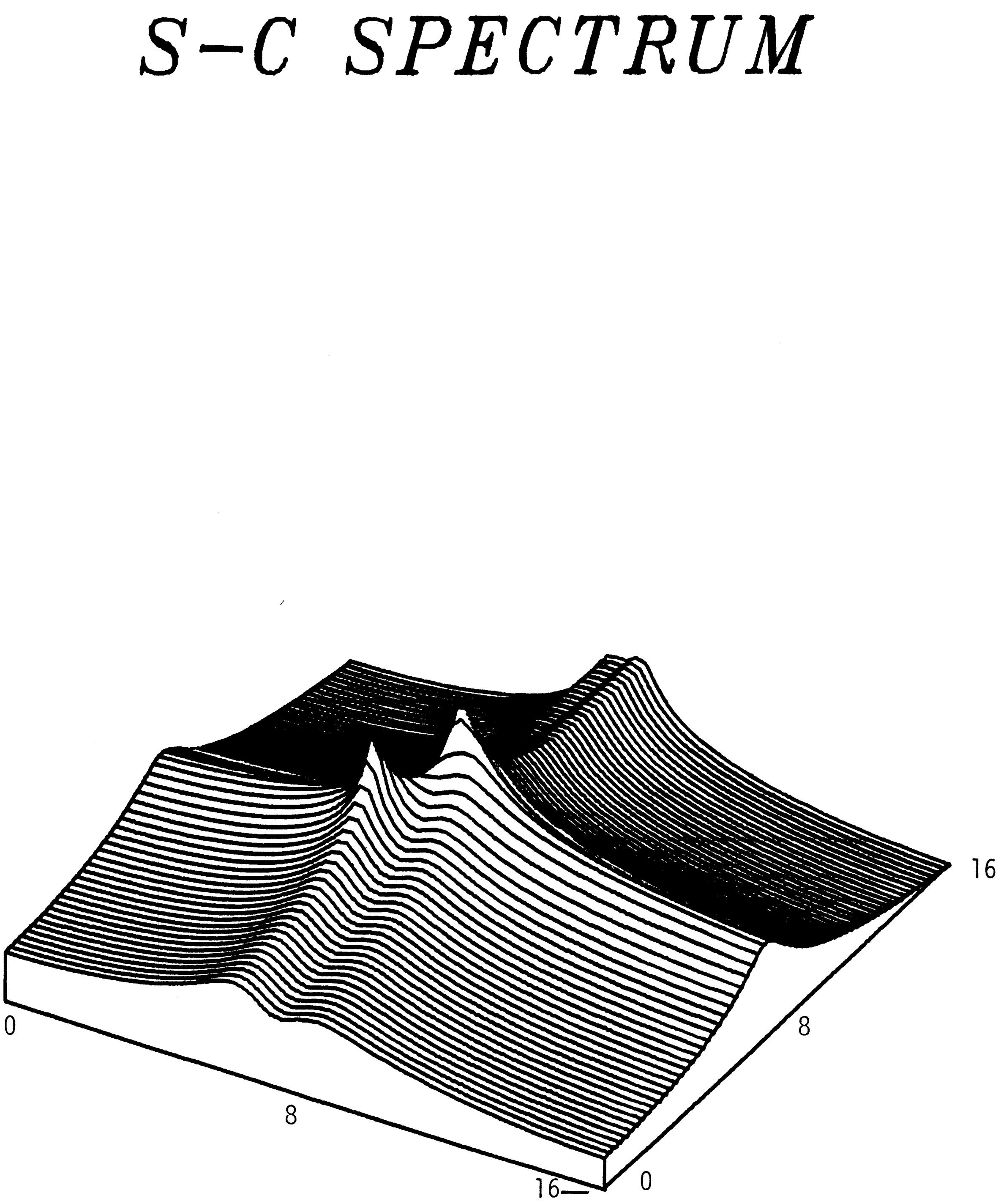

Figure 3e. Semicausal Spectrum of 3a (db).

2-D FUNCTION

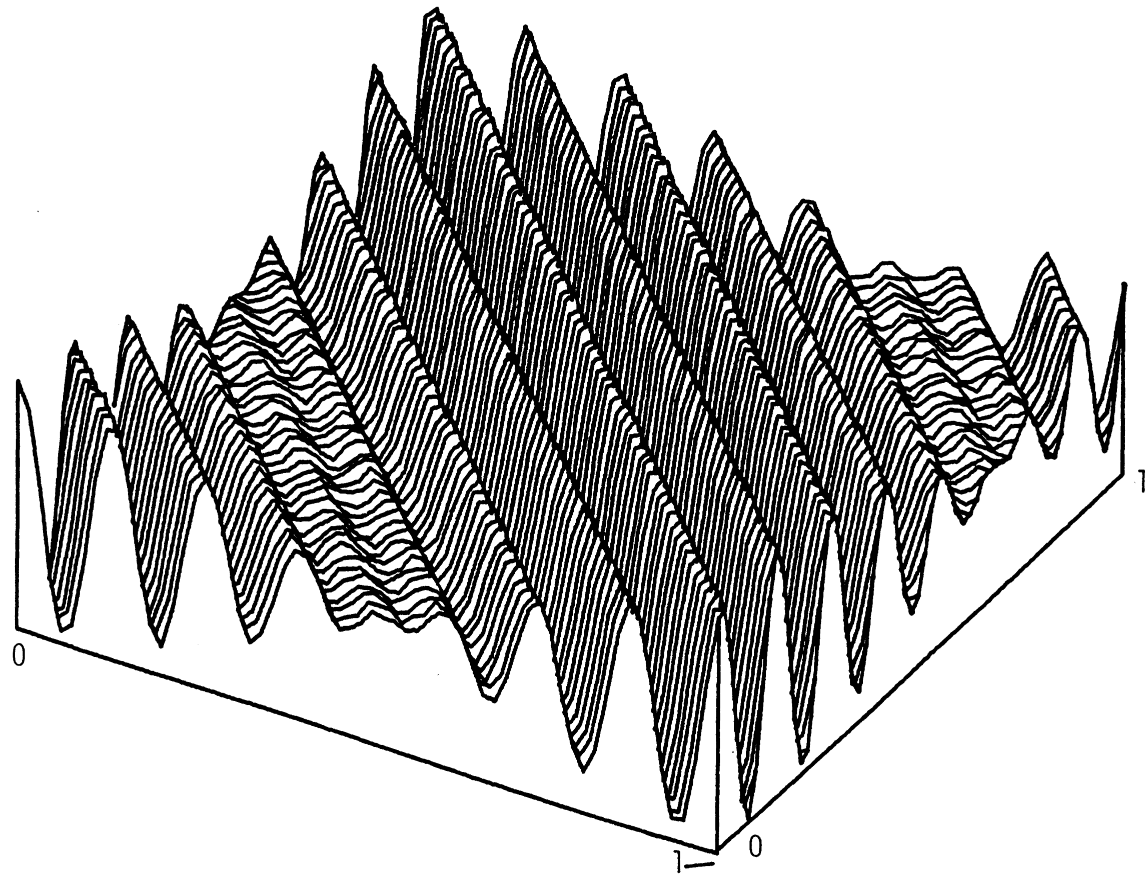

Figure 4a. Plot of $\sin \frac{2\pi}{8}(i+j) + \sin \frac{2\pi}{9}(i+j)$ + 7% Additive White Noise.

AUTOCOVARIANCE

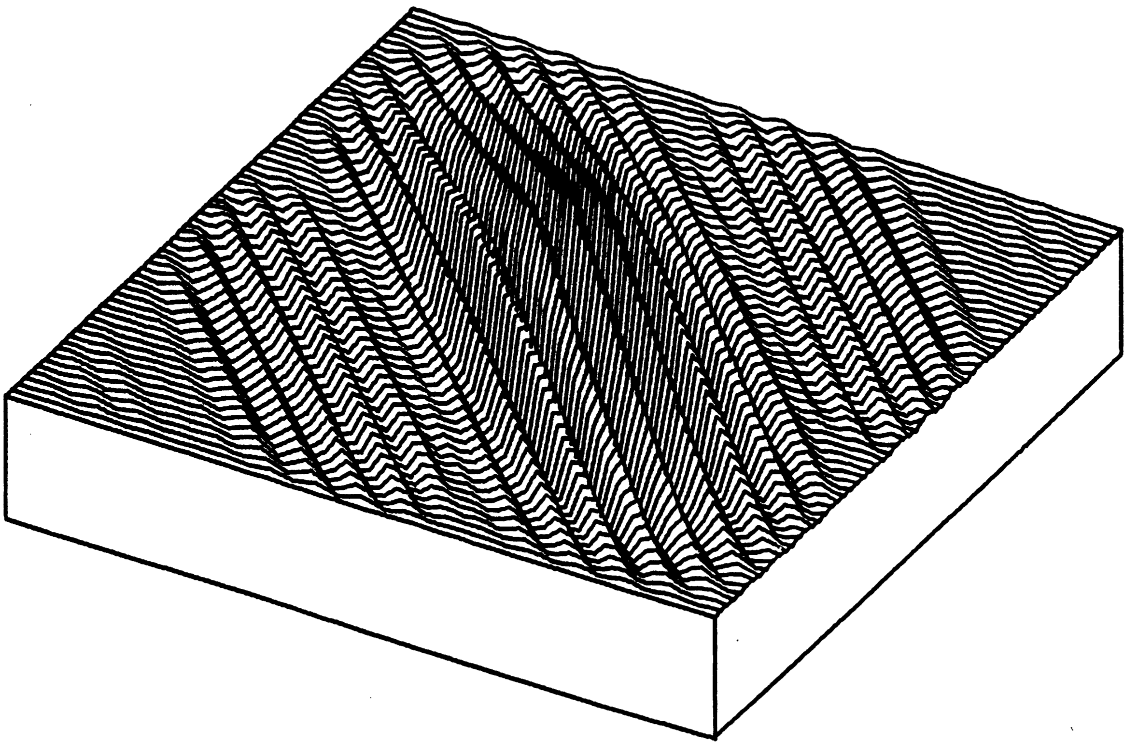

Figure 4b. Autocovariance of 4a.

FFT SPECTRUM

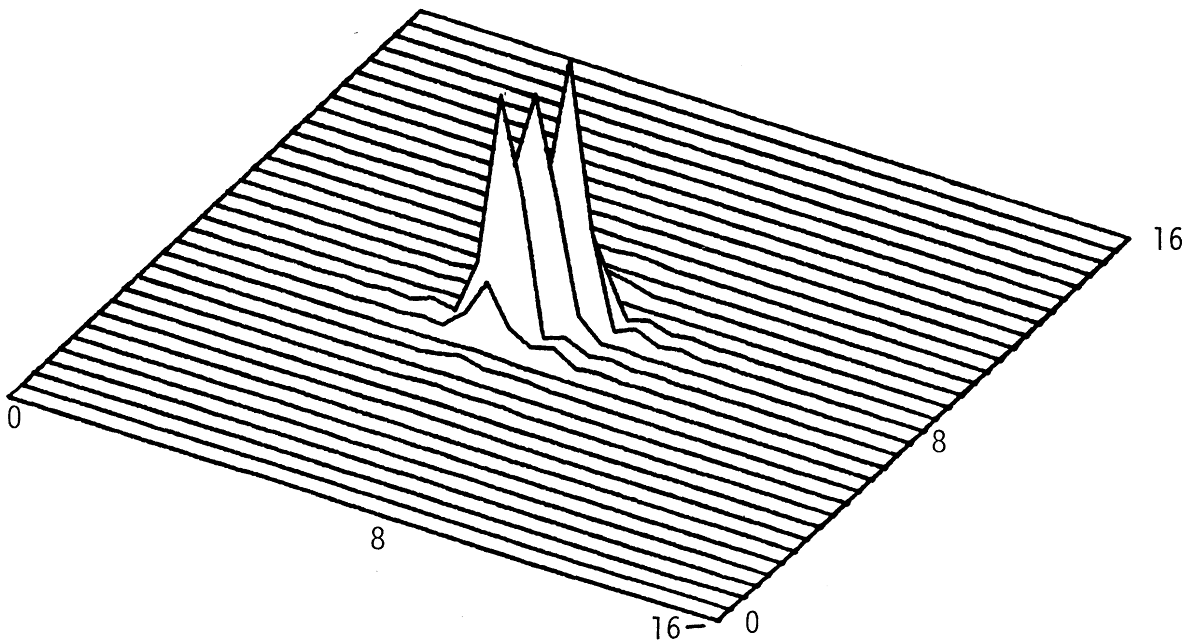

Figure 4c. FFT Spectrum of 4a.

S-C SPECTRUM

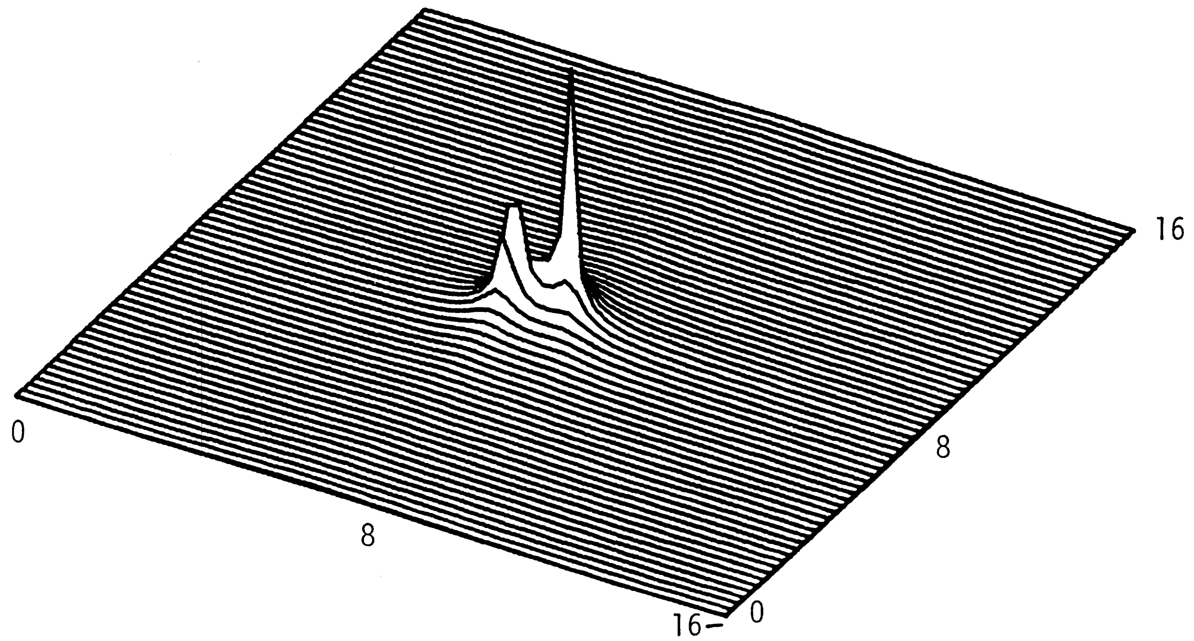

Figure 4d. Semicausal Spectrum of 4a (Amplitude).

S−C SPECTRUM

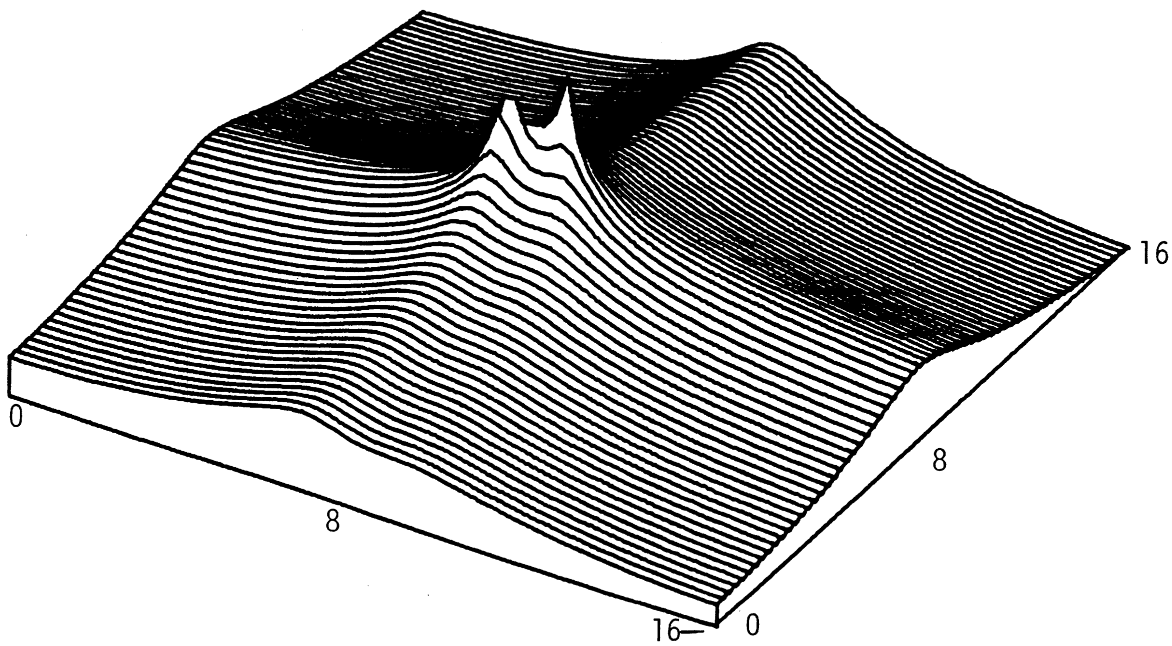

Figure 4e. Semicausal Spectrum of 4a (db).

2-D FUNCTION

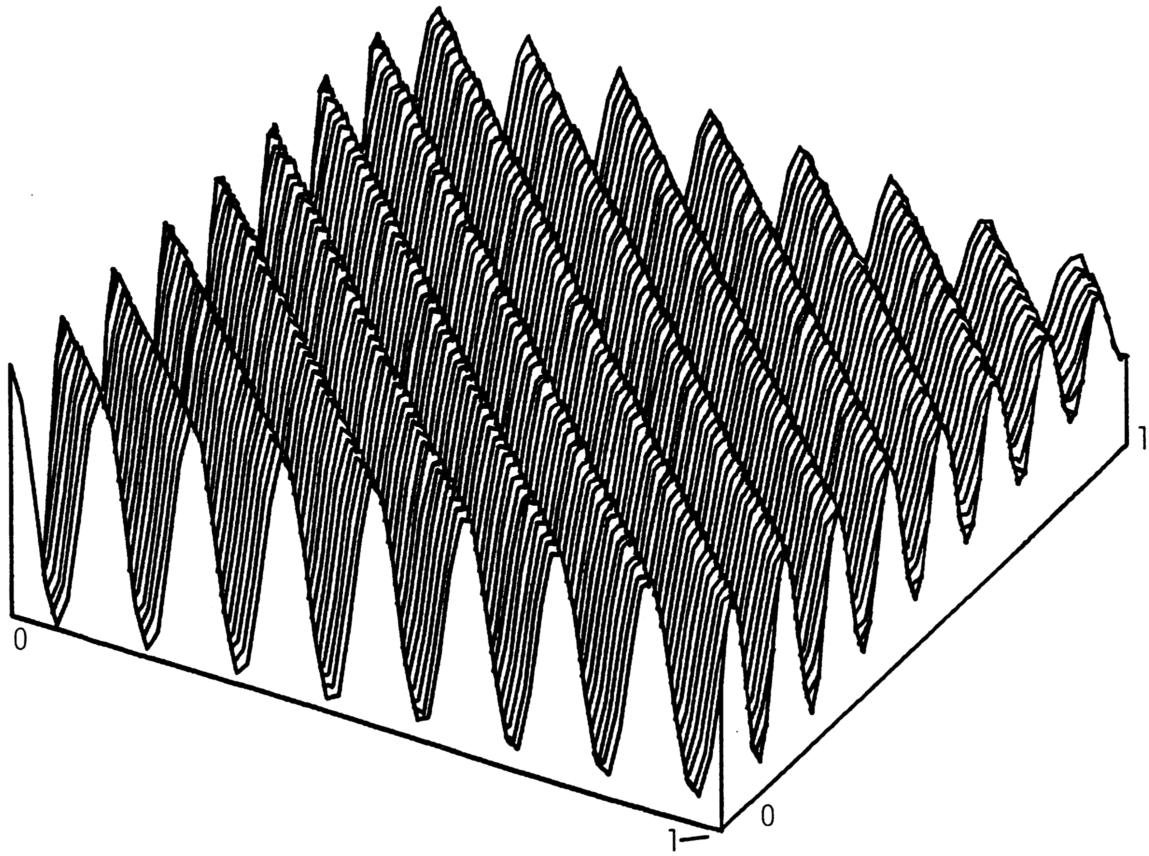

Figure 5a. Plot of $\sin \frac{2\pi}{8}(i+j) + \sin \frac{2\pi}{8.2}(i+j)$ + 7% Additive White Noise.

AUTOCOVARIANCE

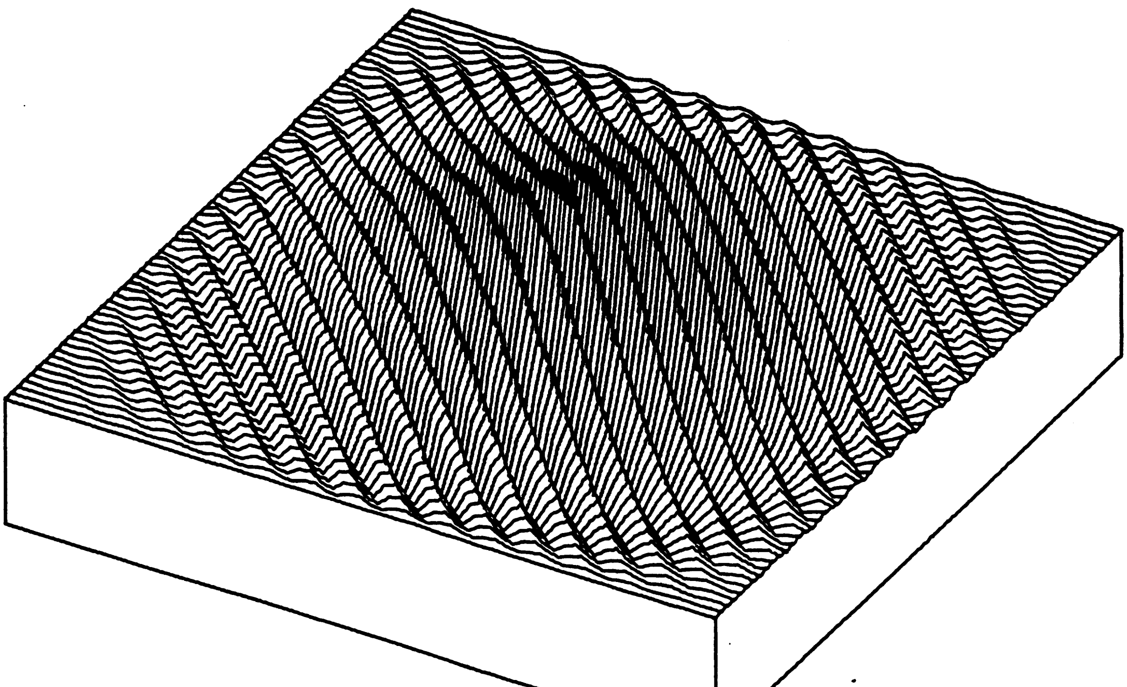

Figure 5b. Autocovariance of 5a.

FFT SPECTRUM

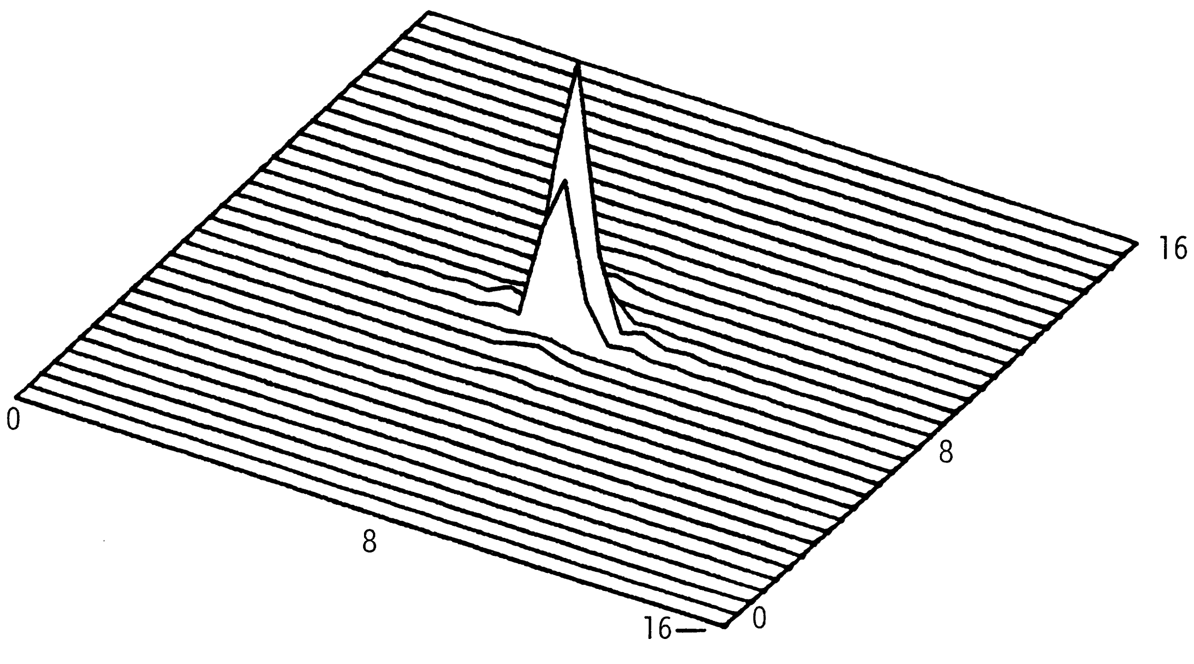

Figure 5c. FFT Spectrum of 5a.

S–C SPECTRUM

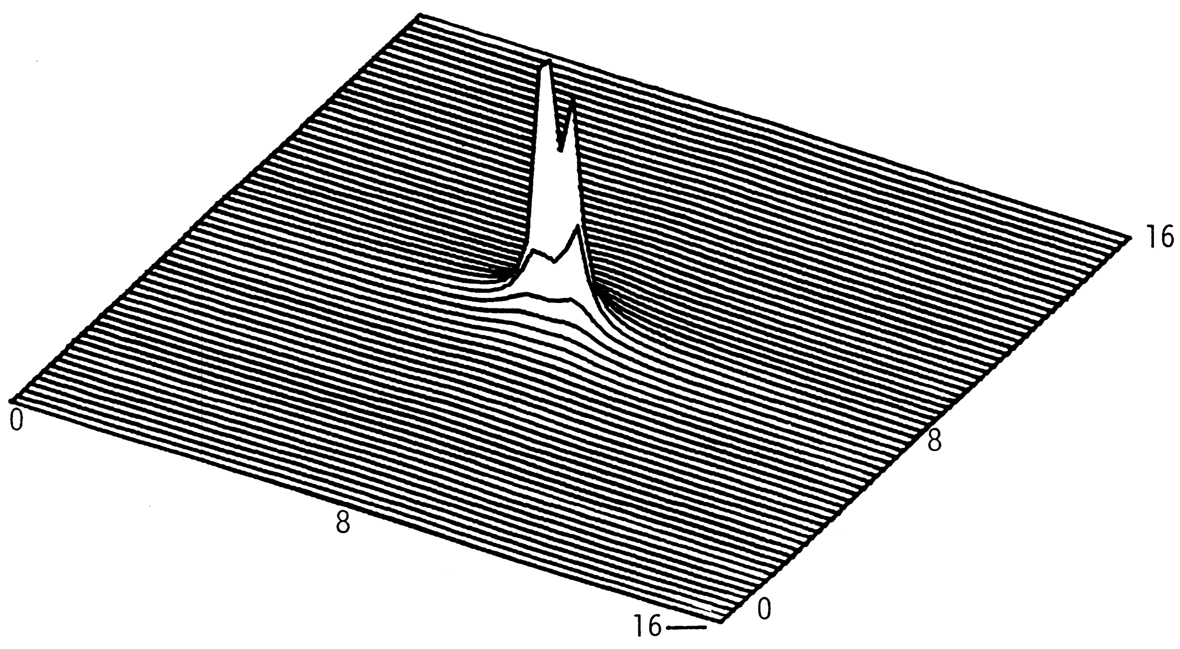

Figure 5d. Semicausal Spectrum of 5a (Amplitude).

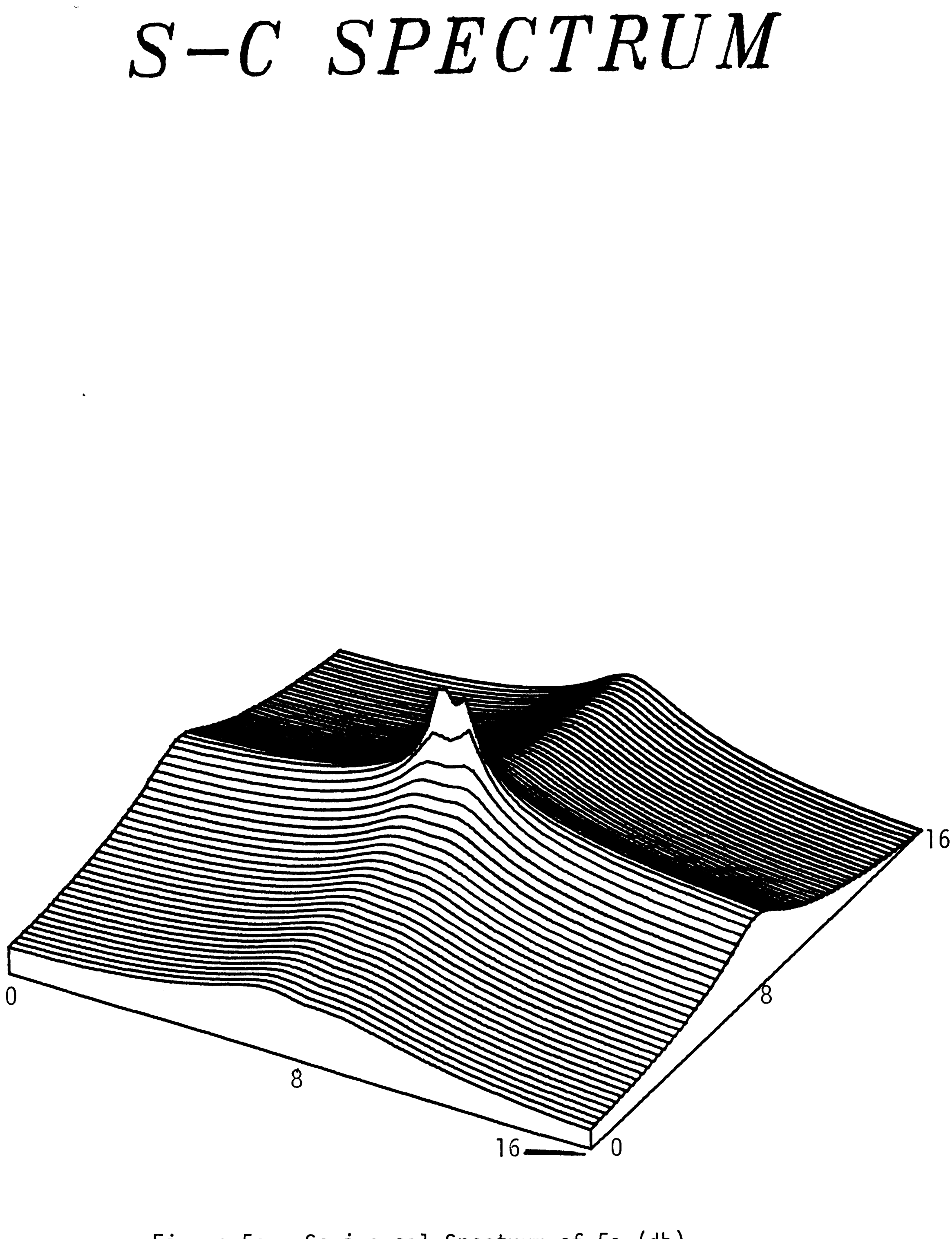

Figure 5e. Semicausal Spectrum of 5a (db).

2-D FUNCTION

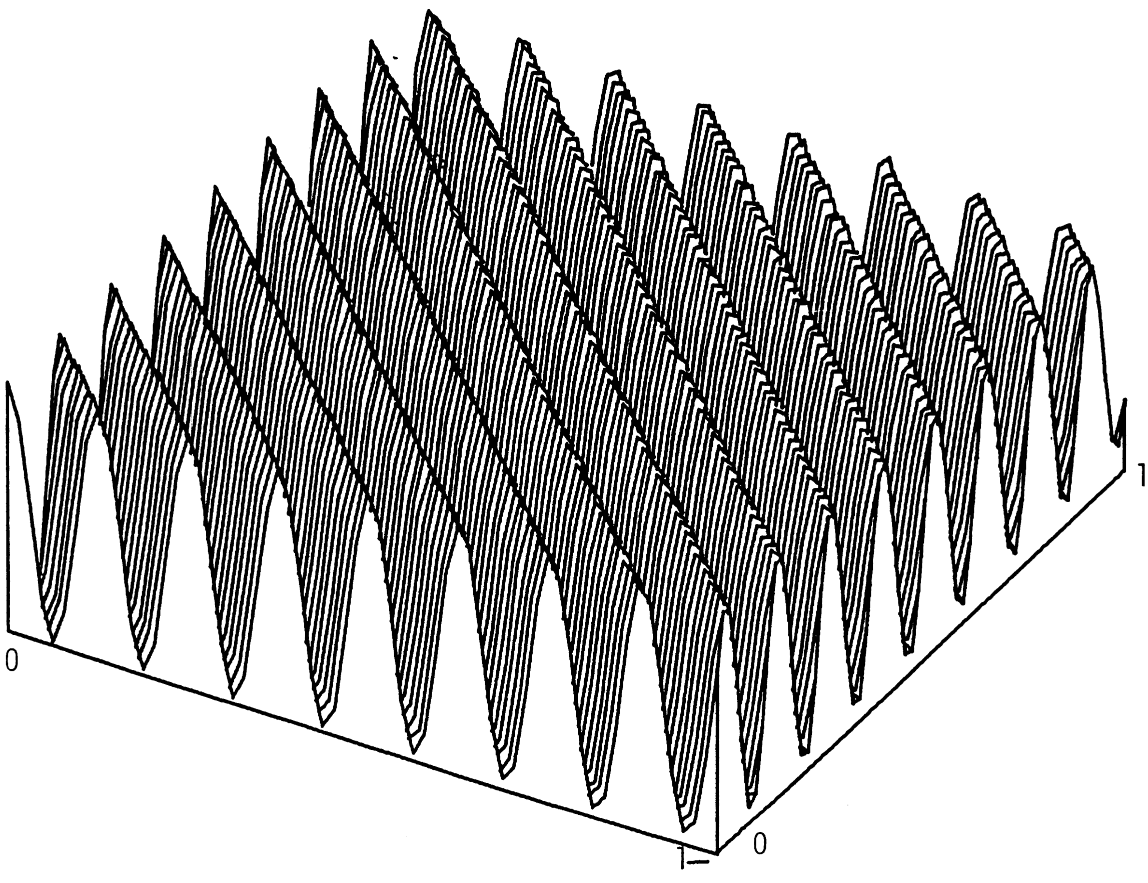

Figure 6a. Plot of $\sin \frac{2\pi}{8}(i+j) + \sin \frac{2\pi}{8.05}(i+j)$ + 7% Additive White Noise.

AUTOCOVARIANCE

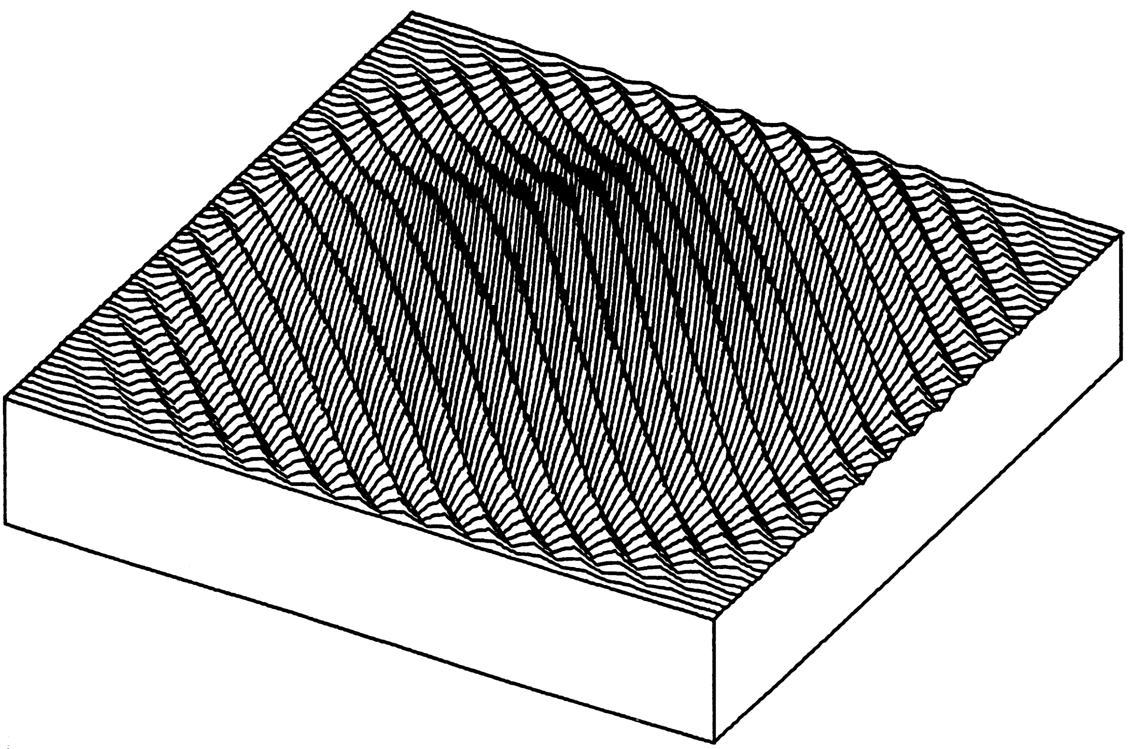

Figure 6b. Autocovariance of 6a.

FFT SPECTRUM

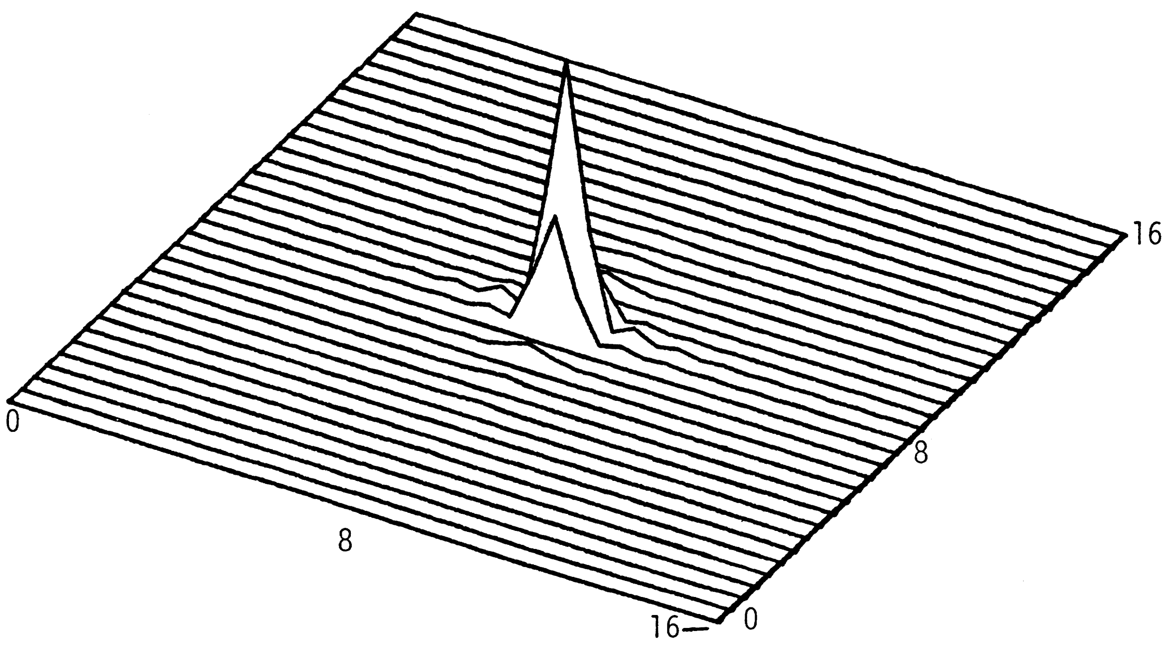

Figure 6c. FFT Spectrum of 6a.

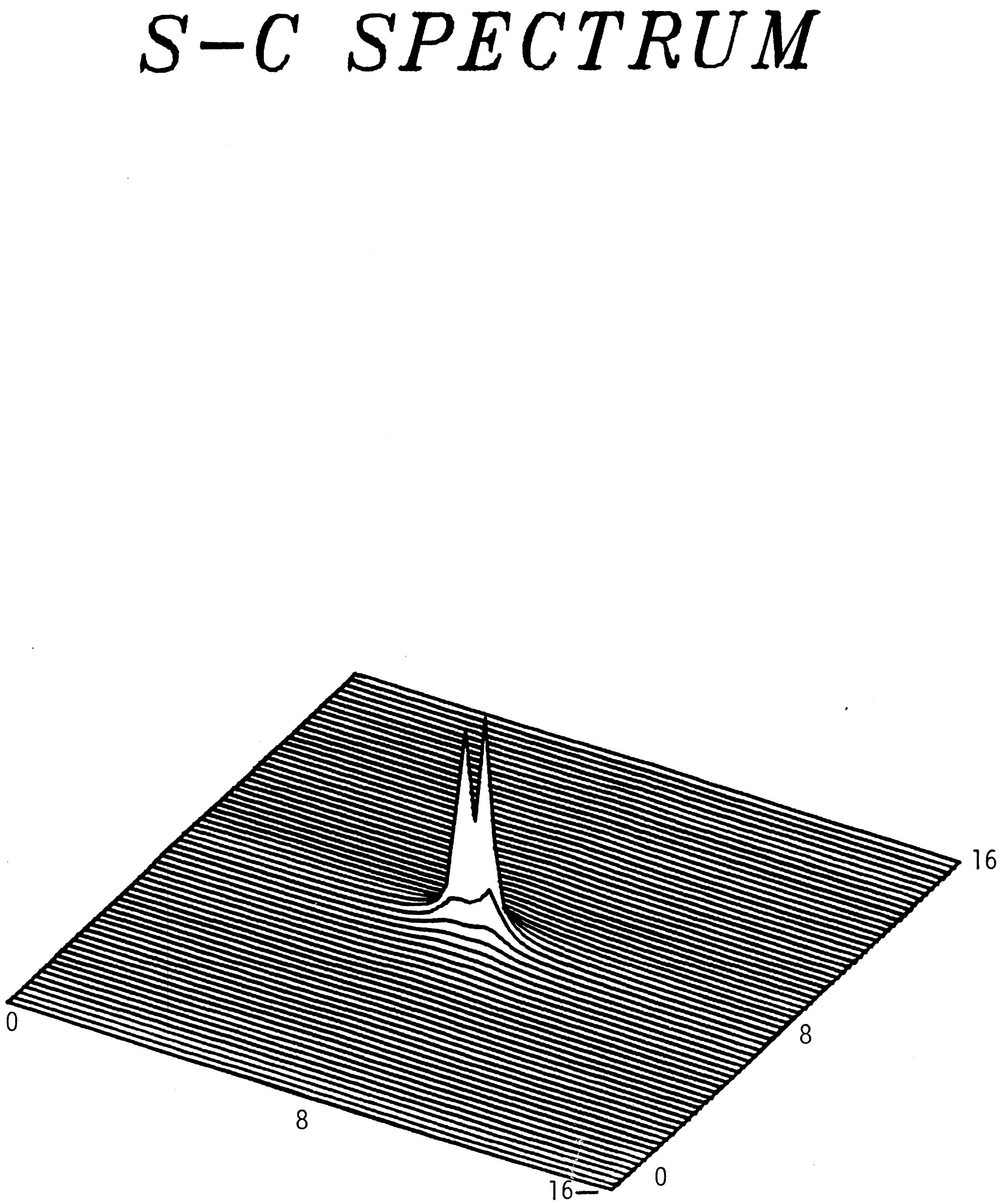

Figure 6d. Semicausal Spectrum of 6a (Amplitude).

S–C SPECTRUM

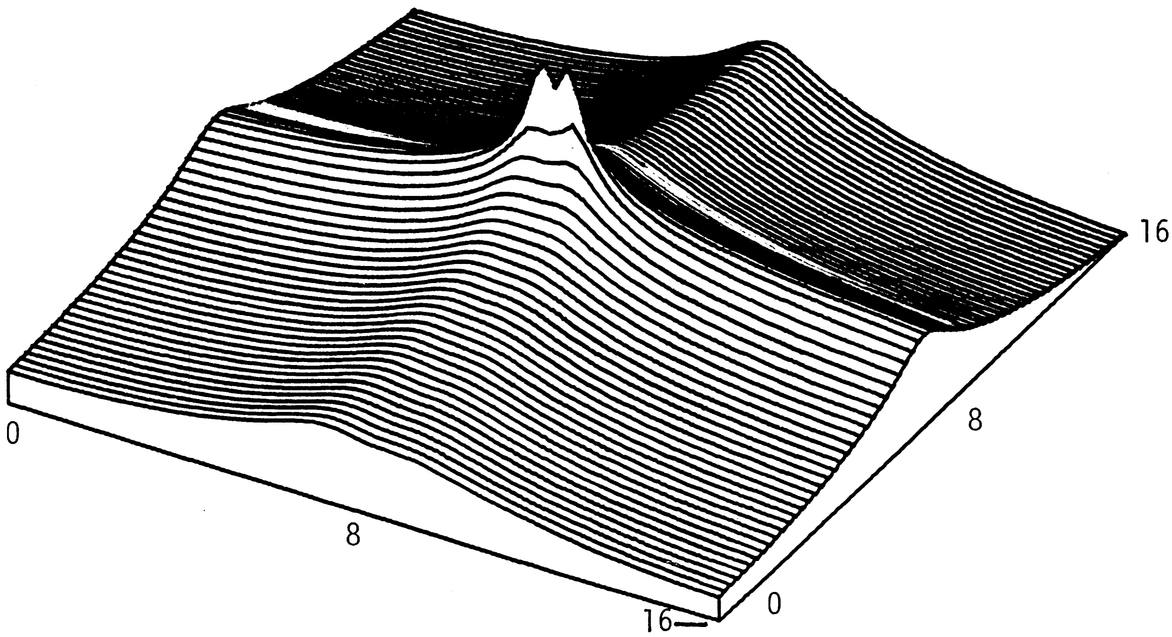

Figure 6e. Semicausal Spectrum of 6a (db).

2-D FUNCTION

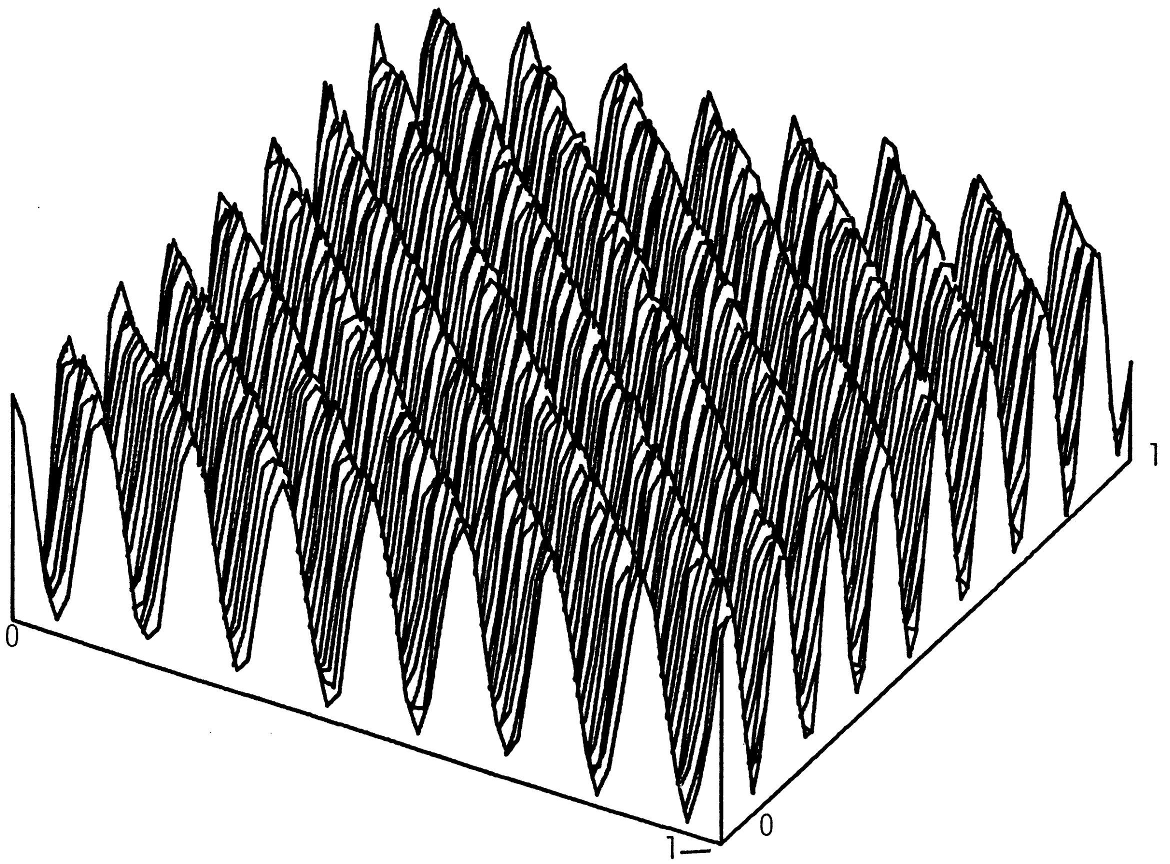

Figure 7a. Plot of $\sin \frac{2\pi}{8}(i+j) + \sin \frac{2\pi}{8.05}(i+j)$ + 30% Additive White Noise.

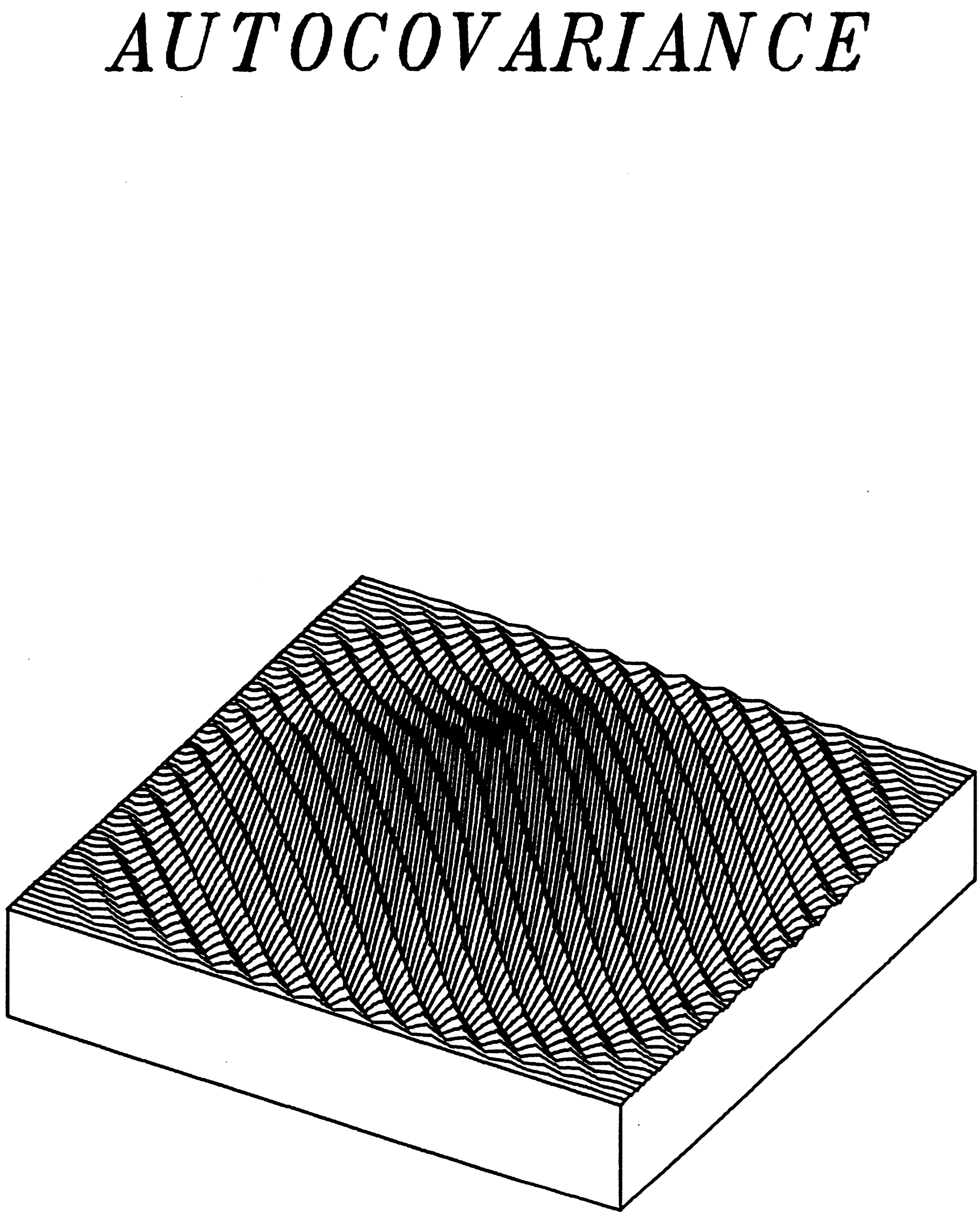

Figure 7b. Autocovariance of 7a.

FFT SPECTRUM

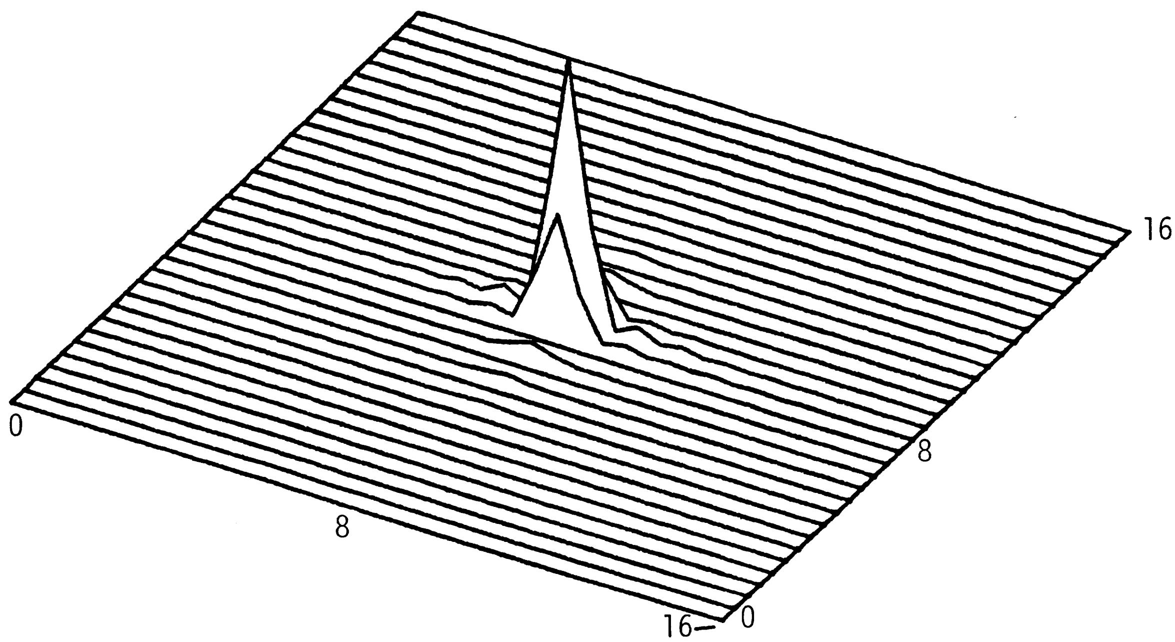

Figure 7c. FFT Spectrum of 7a.

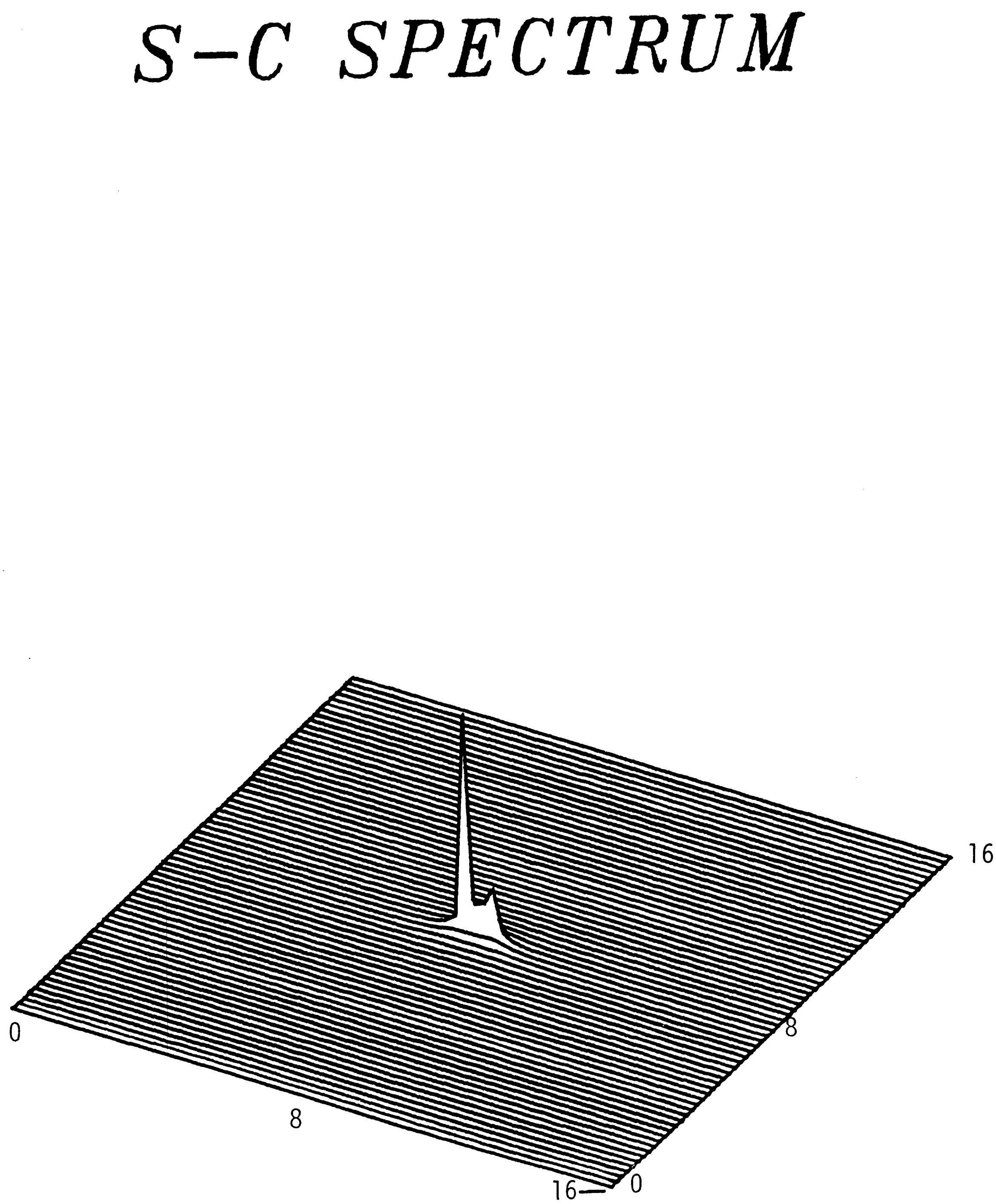

Figure 7d. Semicausal Spectrum of 7a (Amplitude).

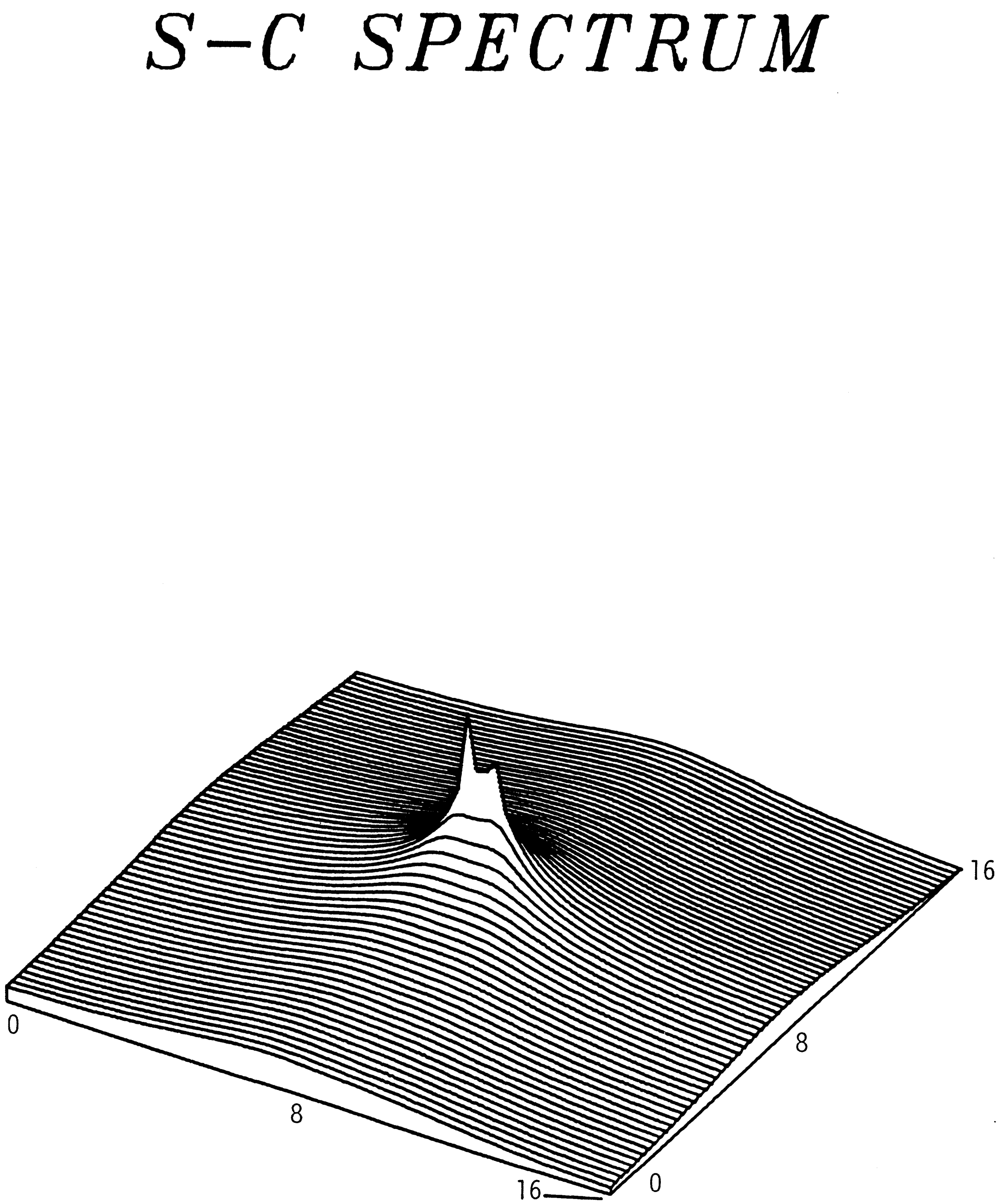

Figure 7e. Semicausal Spectrum of 7a (db).

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