USER MANUAL FOR MR TETRA.F: A MULTI-RESOLUTION FINITE ELEMENT-BOUNDARY INTEGRAL (FE-BI) CODE WITH HIERARCHICAL TETRAHEDRAL ELEMENTS

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User manual for MR_TETRA.f: A multi-resolution finite element - boundary integral (FE-BI) code with hierarchical tetrahedral elements

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Abstract

The following is a manual for a multi-resolution finite element / boundary integral code for analysis of printed antennas backed by material-filled (dielectric or ferrite) metallic cavities recessed in infinite metallic ground planes. The main characteristic of the code is several attractive options for expanding the electric field within the cavity using lowest order, higher order or any combination of lowest and higher order hierarchical mixed-order tangential vector finite elements for tetrahedra.

1 Introduction

The following is a manual for a multi-resolution finite element / boundary integral (FE/BI) code (referred to as MR_TETRA in the following) for analysis of printed metallic antennas backed by material-filled metallic cavities recessed in infinite metallic ground planes. It is based on a standard FE/BI formulation where the cavity volume is discretized into tetrahedral elements using a FE approach and the mesh is rigorously truncated at the aperture surface using a BI [1]. The electric field within each tetrahedral element is expanded using hierarchical mixed-order tangential vector finite elements (TVFEs) of order 0.5 or 1.5 (referred to as lowest and higher order TVFEs in the following) as presented by Andersen and Volakis [2]. The resulting linear equation system is solved using an iterative solver. The matrix-vector products within the iterative solver can be carried out either directly or via a two-dimensional discrete Fourier transform (DFT) in the spectral domain through application of the adaptive integral method (AIM) [3]. A quasi minimal residual (QMR) [4] or conjugate gradient squared (CGS) [4] solver can be used in case matrix-vector products are computed directly while a QMR or biconjugate gradient (BCG) [4] solver can be used in

case matrix-vector products are computed using a DFT. Upon solution of the resulting linear equation system, input impedance and, optionally, near and far fields (patterns, polarization characteristics) are computed.

The purpose of this manual is to familiarize the reader with the steps involved in running MR_TETRA. This is achieved via a general description as well as a set of examples. It is not the purpose of this manual to demonstrate the merits of the approach on which the code is based. For such a demonstration, see for instance [5].

This manual is organized as follows. Section 2 provides a general description of how to run MR_TETRA. All relevant steps needed by the user will be presented. Section 3 outlines the construction of the input file for MR_TETRA. Section 4 offers a few examples of how to run MR_TETRA. Surface meshes, input files as well as results are given in order to demonstrate various capabilities of the code. Section 5 summarizes the manual.

2 General description of how to run MR_TETRA

The root directory of MR_TETRA consists of two sub-directories. The sub-directory src contains all source files (.f), parameter files (.h) and a makefile MMR_TETRA. Upon compilation, it will also contain all object files (.o). The executable MR_TETRA will be in the sub-directory bin where the input file MR_TETRA in and the universal file describing the mesh (see below) must also be placed. Upon execution, this sub-directory will also contain all output files. The sub-directory bin further contains a sub-directory plot with MATLAB codes for plotting various output files. These MATLAB codes are not documented in this manual.

To run the code, the following steps must be followed:

- 1. A mesh in universal file format must be provided in the sub-directory bin.
- 2. In the sub-directory src, several dimensioning parameters in the file param1.h must be initialized. They must be estimated conservatively, otherwise the code will either crash or produce erroneous results. After a first run for a given mesh, they can manually be set to their smallest possible values for that particular mesh. The parameters are Nnomax (max number of nodes), Nsnomax (max number of metallic nodes), Nedmax (max number of edges), Nsedmax (max number of metallic triangular faces), Nmgmax (max number of material groups), Npredmax (max number of probe edges), Nhovolmax (max number of higher order sections), Nbifamax (max number of triangular BI faces), Nbiedmax (max number of BI edges), IAIMmax (max number of AIM grid points in the x-direction), JAIMmax (max number of AIM grid points in the y-direction), IJAIMmax (max of IAIMmax and JAIMmax) and Nnffamax (max number of near field triangular AIM faces). Note that the file param1.h contains several additional dimensioning parameters. These have been set conservatively so the code will run for fully higher order cavities without AIM. This, however, is a tremendous overkill if lowest order

TVFEs and AIM is applied. The user is highly advised to familiarize himself / herself with these parameters and set them optimally for each application. Specifically, BWmax1 and Nunkmax can be lowered from 100 and 2*Nedmax+2*Nfamax to 30 and Nedmax if only lowest order TVFEs are applied and Nmatmax can be lowered from BWmax1*Nunkmax+Nbiedmax**2 to BWmax1*Nunkmax if AIM is used. This will lead to significant memory savings without altering the results.

- 3. Build the executable MR_TETRA (type make -f MMR_TETRA in the sub-directory src 1).
- 4. Construct the input file MR_TETRA.in in the sub-directory bin (see below).
- 5. Run the code (type MR_TETRA in the sub-directory bin).

3 Construction of input file

The input file for MR_TETRA can be broken into 12 different sections. The format of each of these 12 sections will be described below. "I" denotes an integer, "R" denotes a double precision real number and "C" denotes a double precision complex number. "Text" denotes a text line that allows the user to describe the following input parameter(s). Information in all such lines is irrelevant to the code but it makes the input file easier to read and hereby easier to modify. We note that some parts seem unnecessary or redundant. This is partly due to the fact that the code has more options than those described in this manual and partly a result of convenient choices made while the code was under development. We also note that the following description is very general and that specific examples follow later in the report.

Section 1

This section describes the universal file that represents the mesh. Users unfamiliar with universal file formats are referred to SDRC I-DEAS manuals. A universal file section with the descriptor 2411 must give the node coordinates in cm (the ground plane and the metallic antenna must be in the plane z=0 and the cavity must be in the half space z<0). A universal file section with the descriptor 2412 must give the element connectivity and a material group for each element. Optionally, a universal file section with descriptor 2417 (used to describe groups of nodes) and name starting with "F" can be added. This section is used for describing the metallic triangular faces forming the printed antenna. Each node triplet describes a metallic face and hence the group will contain three times as many nodes as there are metallic faces. Note that this is the only way to uniquely describe the metallic antenna. For simple geometries, it can be done with nodes alone or with node doublets (edges) but in the general case node triplets (triangular faces) must be used. MR_TETRA allows easy specification of rectangular and circular patches via simple geometrical parameters (see Section 2). In these cases, a universal file section with descriptor 2417 is unnecessary. Note that the universal file can be generated using SDRC I-DEAS, the automatic mesher written at

¹We note that the compilation of source files was tested on a SUN ULTRA30 work station. Certain parts of the makefile MMR_TETRA as well as the timing commands in the code may have to be modified on other platforms.

the University of Michigan or any other meshing package capable of producing a tetrahedral mesh described in universal file format. The format of this section is the following:

Text

"filename" : Universal file

Section 2

This section describes the metallic cavity and, optionally, the metallic patch. A cavity code is 1/2 for a rectangular / circular cavity. In the former case, $(x,y,z)_{start}$ and $(x,y,z)_{stop}$ for the rectangular cavity as well as $(x,y)_{start}$ and $(x,y)_{stop}$ for a rectangular patch in the plane z=0 is given. In the latter case, the radius r_{cavity} and height h_{cavity} of a circular cavity centered at $(0,0,-h_{cavity}/2)$ as well as the radius r_{patch} of a circular patch centered at (0,0,0) is given. For cavities where the metallic faces are described in the universal file, all patch parameters should be set to zero. The format of this section is either of the following:

Text

1 : Cavity code (rectangular cavity)

Text

R: x_cavity_start
R: y_cavity_start
R: z_cavity_start
R: x_cavity_stop
R: y_cavity_stop
R: z_cavity_stop
R: x_patch_start
R: y_patch_start
R: x_patch_start
R: x_patch_stop
R: y_patch_stop
R: y_patch_stop

Text

2 : Cavity code (circular cavity)

Text

R: r_cavity
R: h_cavity
R: r_patch

Section 3

This section describes the probe excitation. Each probe has unit magnitude and zero phase and must start and end at a node in the mesh. A probe code (must be 1) and the number of probes is given along with $(x, y, z)_{start}$ and $(x, y, z)_{stop}$ for each probe. The format of this section is the following:

```
Text
1 : Probe code
Text
I : Number of probes
Text
R : x_probe_start
R : y_probe_start
R : z_probe_start
R : x_probe_stop
R : y_probe_stop
R : z_probe_stop
R : z_probe_stop
```

This section describes the higher order sections in the mesh. A higher order code of 0 / 1 / 2 indicates that no higher order TVFEs are used / that higher order TVFEs are contained within rectangular brick sections of the mesh (for rectangular cavities) / that higher order TVFEs are contained within cylindrical shell sections of the mesh (for circular cavities). In case higher order TVFEs are used, the number of higher order sections is given along with a geometrical description of each higher order section. For rectangular brick sections, $(x,y,z)_{start}$ and $(x,y,z)_{stop}$ are given. The higher order section is then the brick having these points as opposite corners. For cylindrical shell sections, the inner and outer radii r_{start} and r_{stop} and the azimuthal angles ϕ_{start} and ϕ_{stop} are given. ϕ_{start} and ϕ_{stop} must be in degrees and be in the interval $[0^{\circ}, 360^{\circ})$. The higher order section is then the shell from r_{start} to r_{stop} radially, from ϕ_{start} to ϕ_{stop} (counter-clockwise) azimuthally and from $-h_{cavity}$ to 0 in the z-direction. The format of this section is either of the following:

```
Text
0 : Higher order code (no higher order TVFEs)

Text
1 : Higher order code (rectangular cavity)
Text
I : Number of higher order sections
Text
R : x_ho_start
R : y_ho_start
R : z_ho_start
R : x_ho_stop
R : y_ho_stop
R : z_ho_stop
R : z_ho_stop
```

```
Text
2 : Higher order code (circular cavity)
Text
I : Number of higher order sections
Text
R : r_ho_start
R : r_ho_stop
R : phi_ho_start
R : phi_ho_start
R : phi_ho_stop
R : phi_ho_stop
```

This section describes the TVFEs used for field expansion. A TVFE code 1 / 3 indicates that lowest order TVFEs / a combination of lowest and higher order TVFEs are used. Note that for a TVFE code of 1, the information about higher order TVFEs in Section 4 is not used. This allows the user to toggle between lowest order TVFEs and a combination of lowest and higher order TVFEs by changing only the TVFE code. The format of this section is the following:

Text

I : TVFE code

Section 6

This section describes the different material groups. The number of material groups is given. For each material group, the relative permittivity and permeability tensor is described. The former is described by the nine elements $\varepsilon_{xx}, \dots, \varepsilon_{zz}$ and a conductivity σ in S/cm that gives rise to a frequency dependent loss. The latter is either the unit tensor or a Polder tensor for a gyrotropic material. It is described by a bias code (0 means a unit tensor while 1/2/3 means a Polder tensor with x-/y-/z-bias) and the value of the precession and saturation frequencies f_0 and f_m in Hz [6]. The format of this section is the following:

```
Text
I : Number of material groups
Text
C : epsilon_xx
C : epsilon_xy
C : epsilon_xz
C : epsilon_yx
C : epsilon_yy
C : epsilon_yz
                      Repeated for each material group
C : epsilon_zx
C : epsilon_zy
C : epsilon_zz
R : sigma
I : Bias axis
R:f_0
R:f_m
```

This section describes the orders of the Gauss-Legendre integrations for computing the FEM matrix elements, BI near zone elements, BI far zone elements and far fields. The first type of integration is a volume integration over a tetrahedron where an integer 1 / 2 / 3 / 4 denotes 1- / 4- / 5- / 11-point Gauss-Legendre integration. The last three types of integrations are surface integrations over triangles where an integer 1 / 2 / 3 / 4 denotes 1- / 3- / 4- / 7-point Gauss-Legendre integration. The integration order for FEM should be at least 2 for lowest order TVFEs and 4 for higher order TVFEs if exact results are desired. Since accurate evaluation of the near zone elements in the BI matrix is crucial for accurate analysis, the integration order for the BI near zone elements should be set to 4. The integration order for the far zone elements in the BI matrix is usually set to 1 but can be increased to give slightly more accurate results at the price of a significantly slower code. The integration order for far fields is usually set to 4. The format of this section is the following:

Text

I : Integration order for FEM
I : Integration order for BI_near
I : Integration order for BI_far
I : Integration order for far field

Section 8

This section describes the frequencies at which the geometry is analyzed. The start and stop frequency in Hz as well as the number of frequency points is given. The format of this section is the following:

Text

R : f_start
R : f_stop

Text

I : Number of frequency points

Section 9

This section describes the iterative solver used for solving the resulting matrix equation system as well as the tolerance imposed on the relative residual as a stopping criterion. A solver code of 1 indicates a QMR solver. A solver code of 2 indicates a CGS solver for the case where matrix-vector products are computed directly and a BCG solver for the case where AIM is applied and matrix-vector products are computed using a DFT. The format of this section is the following:

Text

I : Solver code

Text

R : Solver tolerance

This section describes whether AIM is used as well as all AIM parameters. An AIM code 0 / 1 indicates that AIM is not / is used. Regardless of the AIM code, $(x, y)_{start}$ and $(x, y)_{stop}$ for the AIM grid are given along with the number of AIM grid points in the x- and y-direction and the number of near zone AIM grid points in both directions. This information is not used when AIM is not used. Reading the information anyway allows the user to toggle between direct and DFT computation of matrix-vector products by changing only the AIM code. The format of this section is the following:

Text

I : AIM code

Text

R: x_AIM_start
R: y_AIM_start
R: x_AIM_stop
R: y_AIM_stop
I: N_AIM_x
I: N_AIM_y
I: N_AIM_p

Section 11

This section describes whether near and far field distributions are computed. For near fields, a near field code of 0 / 1 indicates that they are not / are computed. For far fields, a far field code of 0 / 1 / 2 / 3 indicates that nothing / patterns only / polarization characteristics only / patterns and polarization characteristics are computed. For near fields, the cavity is sampled in 33×33 points in 4 equidistant planes from the bottom of the cavity to the top, as illustrated in Fig. 1. The file xpyp.dat has $33 \cdot 33 = 1089$ lines containing the values of (x,y) for the sample points in the plane z=0. The file FxFy.dat has $33\cdot 33=1089$ lines containing the corresponding values of the real part of the x- and y-directed electric field as well as the imaginary part of the x- and y-directed electric field. This surface electric field is the one that is being radiated (as an equivalent magnetic current) to give the far field. The file xpypzp.dat has $33 \cdot 33 \cdot 4 = 4356$ lines containing the values of (x, y, z) for the sample points in the 4 planes. The file Fmgn.dat has $33 \cdot 33 \cdot 4 = 4356$ lines containing the corresponding magnitudes $\sqrt{|E_x|^2 + |E_y|^2 + |E_z|^2}$ of the electric field. Note that an expert user can easily alter the routine nearfielddist.f to compute whatever near field quantity might be of interest or to change the sampling rate. For far fields, the half space above the antenna is sampled in $36 \times 16 = 576$ far field points (36 θ -values in $[0^{\circ}, 90^{\circ}]$ and 16 ϕ -values in $[0^{\circ}, 360^{\circ}]$ with θ and ϕ being the traditional polar and azimuthal angles of a spherical coordinate system related to the (x, y, z)-coordinate system). The file ff.dat has $36 \cdot 16$ lines containing the values of θ and ϕ along with a quantity proportional to $\sqrt{|E_{\theta}|^2 + |E_{\phi}|^2}$ in the far field. Note that an expert user can easily alter the routine farfielddist.f to compute whatever far field quantity might be of interest or to change the sampling rate. The file pol.dat contains information about the polarization of the far field normal to the plane of the antenna. It gives the axial ratio (in dB) of the polarization ellipse, the phase

difference (in degrees) between the two components and the tilt angle (in degrees) of the polarization ellipse measured from the x-axis. The format of this section is the following:

Text

I : Near field code

Text

I : Near field code

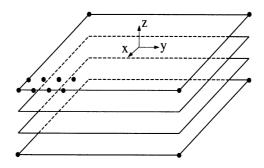


Figure 1: Illustration of cavity sampling.

Section 12

This section describes file names of output files containing geometrical parameters, input impedance information (the real and imaginary part of the input impedance for each frequency), convergence information (the number of iterations and the final relative residual for each frequency) and timing information (the time for the FEM part, BI part and solver for each frequency as well as time for pre-processing and "the rest of the code"). The format of this section is the following:

Text

"filename" : Geometry file

Text

"filename" : Impedance file

Text

"filename" : Convergence file

Text

"filename" : Timing file

4 Examples

Consider a square metallic patch antenna backed by a rectangular cavity recessed in an infinite metallic ground plane, as illustrated in Fig. 2 (side view) and Fig. 3 (top view). The cavity-backed patch antenna is situated in free space characterized by the permittivity ε_0 and the permeability μ_0 . The cavity is of dimensions 1.85 cm \times 1.85 cm \times 0.15 cm and filled with a dielectric material of permittivity 10 ε_0 and conductivity 0.0003 S/cm. The patch is of side length 0.925 cm and centered in the cavity aperture. It is fed by a vertical coaxial

line whose outer conductor is attached to the ground plane and whose inner conductor is attached to the patch at the mid point of an edge, as illustrated in Fig. 2 and Fig. 3. The coaxial feed will be modeled as a vertical probe of constant current.

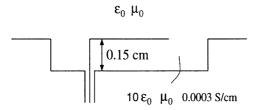


Figure 2: Side view of square metallic patch antenna backed by a dielectric-filled rectangular cavity recessed in an infinite metallic ground plane.

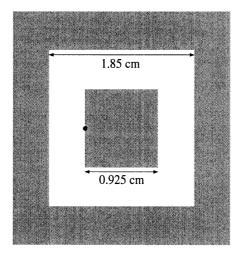


Figure 3: Top view of square metallic patch antenna backed by a dielectric-filled rectangular cavity recessed in an infinite metallic ground plane.

A coarse surface mesh is given in Fig. 4. For mixed-order TVFEs of order 0.5 and 1.5, the particular mesh is too coarse to yield the exact resonant frequency of 4.43GHz as obtained by Schuster and Luebbers [7] and confirmed by Andersen and Volakis for finer meshes [5]. Nevertheless, the mesh is very useful for illustrating the capabilities of MR_TETRA.

Let us analyze the above antenna at 21 frequency points in the interval [3.5GHz,4.5GHz]. We will solve resulting linear equation system using a QMR solver with tolerance 10⁻³. AIM will not be used. The input file is given below.

```
Name of universal file
    "mesh.unv"
PEC code
    1
(x,y,z)_start, (x,y,z)_stop, (x_pa,y_pa)_start, (x_pa,y_pa)_stop
    -0.4625d0
    -0.4625d0
```

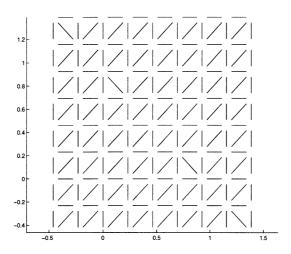


Figure 4: Top view of coarse surface mesh for square metallic patch antenna backed by a dielectric-filled rectangular cavity recessed in an infinite metallic ground plane.

```
-0.15d0
  1.3875d0
  1.3875d0
  0d0
  0d0
  0d0
  0.925d0
  0.925d0
Probe code
  1
Number of probes
(x_pr,y_pr,z_pr)_start, (x_pr,y_pr,z_pr)_stop for first probe
  0d0
  0.4625d0
  -0.15d0
  0d0
  0.4625d0
  0d0
HO code
  0
TVFE code
Number of material groups
Parameters for the first group
  (10d0, -0d0)
  (0d0, -0d0)
  (0d0, -0d0)
  (0d0, -0d0)
```

```
(10d0, -0d0)
  (0d0, -0d0)
  (0d0, -0d0)
  (0d0, -0d0)
  (10d0, -0d0)
  0.0003d0
  0
  0d0
  0d0
Integration order : FEM, BI_near, BI_far and farfield
  4
  1
Start and stop frequencies
  3.50d9
  4.50d9
Number of frequency points
  21
Solvercode
Tolerance
  1d-3
AIM code
AIM parameters (x_start, y_start, x_stop, y_stop, I, J, K)
  0d0
  0d0
  0d0
  0
  0
Near field distribution code
Far field distribution code
Name of file with geometrical data
  "3D-FEM-BI.out1"
Name of file with input impedances
  "3D-FEM-BI.imp1"
Name of file with convergence data
  "3D-FEM-BI.con1"
Name of file with timing data
  "3D-FEM-BI.tim1"
```

The output file containing the input impedance for each frequency is given below and also plotted in Fig 5. The antenna is seen to experience the expected resonant behavior. Also, the resonant frequency is seen to be around 3.97GHz which is much smaller than the true resonant frequency 4.43GHz. This is due to the coarse mesh and the fact that lowest order TVFEs have been used.

```
3500000000.0000
                   2.5588456293470
                                       41.628451744725
                   3.2429206914211
                                       45.778072533465
3550000000.0000
                                       50.904979316993
360000000.0000
                   4.2286657863228
                   5.7163554782377
                                       57.440189012613
3650000000.0000
                   8.1003200176110
                                       66.098467821245
370000000.0000
3750000000.0000
                   12.236883071922
                                       78.132689480470
                   20.294341985730
                                       95.967863359927
380000000.0000
3850000000.0000
                   38.929881217035
                                       124.44216366128
390000000.0000
                   94.338002704657
                                       169.57056063719
3950000000.0000
                   278.78777970206
                                       152.06654344528
400000000.0000
                   248.05473459466
                                      -138.59626984899
4050000000.0000
                   87.641136274513
                                      -133.27080997795
410000000.0000
                   39.664558360104
                                      -92.517092077889
4150000000.0000
                   22.234424951724
                                      -66.678035812629
420000000.0000
                   14.241706452950
                                      -50.108946578041
4250000000.0000
                   9.9540096442860
                                      -38.749056086777
430000000.0000
                   7.3944826938964
                                      -30.493855847262
4350000000.0000
                   5.7400309760047
                                      -24.197511427906
440000000.0000
                   4.6063185864701
                                      -19.211509191283
4450000000.0000
                   3.8071120800757
                                      -15.235725559360
4500000000.0000
                   3.2156270709780
                                     -11.848352913871
```

At the resonant frequency 3.97GHz, let us now look at the polarization of the far field normal to the antenna as well as the far field patterns. Hence, we change Section 8 and 11 of the input file to read

```
Start and stop frequencies
3.97d9
3.97d9
Number of frequency points
1
Near field distribution code
0
Far field distribution code
3
```

The polarization of the far field normal to the antenna is described in the file pol.dat:

```
397000000.0000
26.122202028302 -85.365890875233 0.23007593744868
```

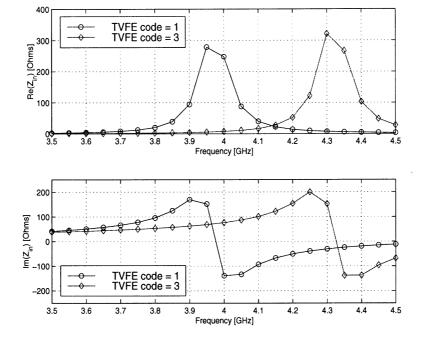


Figure 5: Real and imaginary part of the input impedance of the antenna in Fig. 2 and Fig. 3.

This output implies that the polarization ellipse is tilted 0.23° from the x-axis and has an axial ratio of 26.12dB. That is, we essentially have linear polarization along the x-axis which is what we expect for this patch. The patterns in the planes y = 0 and x = 0 at 3.97GHz are given in Fig. 6 (polar angles in $[0^{\circ}, 90^{\circ}]$ on the plot corresponds to ϕ -values in $[0^{\circ}, 180^{\circ}]$ while polar angles in $[90^{\circ}, 180^{\circ}]$ on the plot corresponds to ϕ -values in $[180^{\circ}, 360^{\circ}]$; also, the polar angle 90° on the plot corresponds to $\theta = 0^{\circ}$ and the polar angles 0° and 180° correspond to $\theta = 90^{\circ}$). These are consistent with the polarization of the far field normal to the antenna and exactly what we expect for this patch.

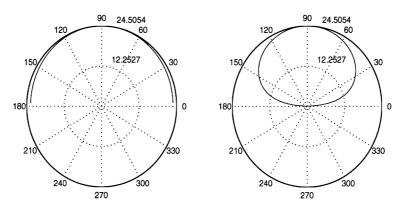


Figure 6: Patterns in the planes y = 0 and x = 0 at 3.97GHz for the antenna in Fig. 2 and Fig. 3.

To demonstrate the use of higher order TVFEs for improved field modeling, we apply higher order TVFEs around the radiating edges of the patch where the field is known to be the strongest and experience the most variation. Hence, we change Section 4 and 5 of the input file to read

```
HO code
  1
Number of higher order volumes
(x,y,z)_start, (x,y,z)_stop for first higher order volume
  -0.23125d0
  -0.23125d0
  -0.15d0
  0.23125d0
  1.15625d0
(x,y,z)_start, (x,y,z)_stop for second higher order volume
  0.69375d0
  -0.23125d0
  -0.15d0
  1.15625d0
  1.15625d0
  0d0
TVFE code
  3
```

Let us consider the application of AIM and use a DFT to compute the matrix-vector products in the iterative solver. This is terribly inefficient for small problems like the one considered here and only serves the purpose of demonstrating how AIM is used. To invoke AIM, we change Section 10 of the input file to read

```
AIM code

1
AIM parameters (x_start, y_start, x_stop, y_stop, I, J, K)
-0.668055555555500
-0.668055555555500
1.593055555555500
1.593055555555500
23
23
8
```

The output file containing the input impedance for each frequency is given below and also plotted in Fig 5. The antenna is again seen to experience the expected resonant behavior. The resonant frequency is seen to be 4.32GHz which is much closer to the true resonant frequency 4.43GHz than the 3.97GHz found when lowest order TVFEs were used throughout the cavity.

```
3500000000.00001.032524557119937.8248862785033550000000.00001.178760291522639.7443519438873600000000.00001.363005971966541.855292671137
```

3650000000.0000	1.5908198292434	44.211407702366
3700000000.0000	1.8827809279585	46.855809984373
3750000000.0000	2.2511366594008	49.890077408460
3800000000.0000	2.7452949855043	53.404271011424
3850000000.0000	3.4128718362669	57.554844531757
390000000.0000	4.3376897449660	62.542584004342
3950000000.0000	5.6748167344092	68.707629601590
4000000000.0000	7.6934532336221	76.555049192618
4050000000.0000	10.930495769666	86.945891370685
4100000000.0000	16.607146734682	101.292500931489
4150000000.0000	27.615471055343	122.03417410905
4200000000.0000	52.159382628536	154.29204802524
4250000000.0000	122.49555861383	200.49906174576
430000000.0000	320.78659000805	152.29837557693
4350000000.0000	267.24188214807	-137.93155070714
4400000000.0000	103.160187785883	-137.62354804365
4450000000.0000	48.490909355324	-96.045352685244
4500000000.0000	27.621340718090	-67.895949125991

5 Summary

This manual describes how to run MR_TETRA, a multi-resolution finite element / boundary integral code for analysis of printed antennas backed by material-filled (dielectric or ferrite) metallic cavities recessed in infinite metallic ground planes. Both a general description as well as examples are given.

References

- [1] J.L. Volakis, A. Chatterjee, and L.C. Kempel. Finite element method for electromagnetics. IEEE Press, USA, 1998.
- [2] L.S. Andersen and J.L. Volakis. 'Hierarchical tangential vector finite elements for tetrahedra'. *IEEE Microwave and Guided Wave Letters*, vol. 8, pp. 127-129, March 1998.
- [3] E. Bleszynski, M. Bleszynski, and T. Jaroszewicz. 'AIM: Adaptive integral method for solving large-scale electromagnetic scattering and radiation problems'. *Radio Sci.*, vol. 31, pp. 1225-1251, 1996.
- [4] Y. Saad. Iterative methods for sparse linear systems. PWS Publishing Company, 1996.
- [5] L.S. Andersen and J.L. Volakis. 'Accurate and efficient simulation of antennas using hierarchical mixed-order tangential vector finite elements for tetrahedra'. Accepted for publication in *IEEE Trans. Antennas Propagat*.
- [6] D.M. Pozar. Microwave engineering. Addison-Wesley Publishing Company, Inc., 1990.
- [7] J.W. Schuster and R. Luebbers. Private communication.