ANTENNA PATTERN MODIFICATION BY A NEARBY MOUNTAIN

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1. Introduction

This is the Final Report of a brief study carried out for the Megatek Corporation under Purchase Order SAN 18459 during the period 5 June to 15 August 1974.

The purpose of the study was to estimate the effect of a nearby mountain on shore station antenna patterns at HF. The geometry that was specified is shown in Fig. 1. For propagation in the plane of the diagram, the mountain is modelled by a perfectly conducting wedge of included angle 60^{o} with its vertex at a height of 2700 ft. above an infinite flat earth. The mountain is 4 miles from a quarter wave monopole antenna at the surface of the earth, and will, of course, affect the pattern of the antenna through shielding (shadowing) and reflection, as well as by diffraction at its apex. However, the aspects of concern are those close to grazing incidence on the apex, and for these, shadowing and diffraction are the relevant effects. In order to determine the modifications which the mountain produces, the far field antenna patterns must be computed with and without the mountain present.

Data were required at each of the three frequencies 4, 11 and 30 MHz for two different specifications of the earth: a) perfectly conducting, simulating "sea water", and b) lossy or "poor" ground whose relative permittivity ϵ_r is 15 and conductivity σ is 10^{-2} mhos/m.

In the following sections we summarize the derivation of analytical expressions for the perturbed patterns of the monopole, then describe the computations carried out, and conclude with the presentation and brief discussion of the results obtained. It is a pleasure to acknowledge the assistance of Professor C-T Tai and Dr. D. L. Sengupta in this investigation.

2. Analysis

The geometrical theory of diffraction (GTD) provides an effective and convenient method for determining the diffracted contribution of an edge such as

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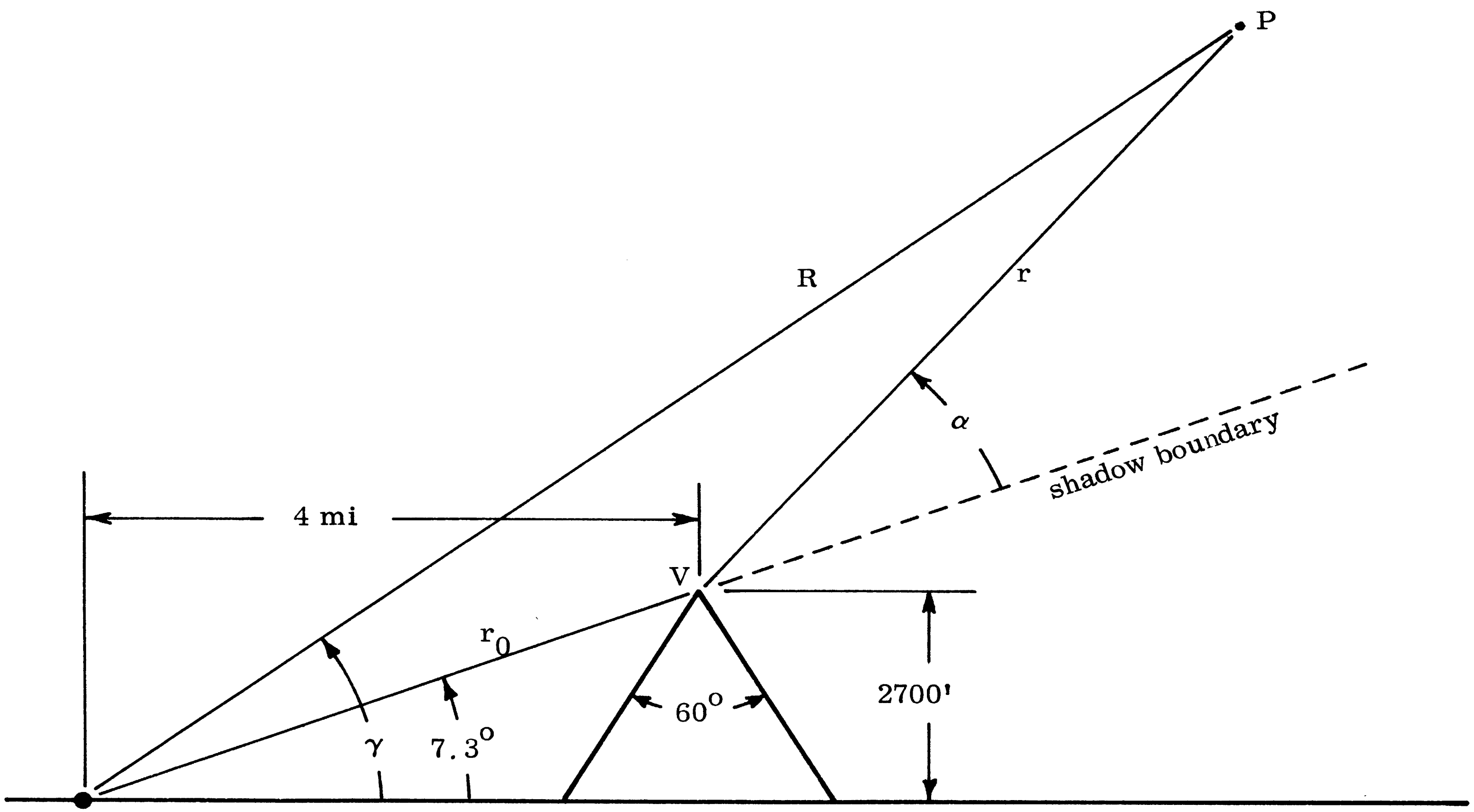

FIG. 1: Geometry of the problem.

the "vertex" of our mountain. In order to show how the method is employed, we consider first the problem of a point source T in the presence of a metallic wedge in isolation, i.e. of the mountain without the ground present.

If the source is far from the vertex, so that the incident field is a locally plane wave with its electric vector in the plane of Fig. 1, the field at the point of observation P is made up of optics and diffracted parts. The former is

$$E^i(P) = \begin{cases} \dfrac{e^{ikR}}{kR} & \alpha > 0 \\ 0 & \alpha < 0 \end{cases} \tag{1}$$

where k is the free space propagation factor and a time factor $e^{-i\omega t}$ has been suppressed. For convenience we have normalized the source strength in the direction TP to unity. The diffracted contribution is

$$E^d(P) = E^i(V)\Gamma_{VP} D e^{ikr} \tag{2}$$

where $E^i(V)$ is the field incident at the vertex, Γ_{VP} is the divergence factor for the diffracted rays and D is the diffraction coefficient of the edge. If the source is omnidirectional,

$$E^i(V) = \frac{e^{ikr_0}}{kr_0} \tag{3}$$

and since the radius of curvature of the wavefront incident at V is r_0,

$$\Gamma = \sqrt{\frac{r_0}{(r_0+r)r}} \sim \frac{\sqrt{r_0}}{r} \qquad \text{if } r \gg r_0 . \tag{4}$$

The local diffraction coefficient D can be deduced from the solution for the canonical problem of the diffraction of a plane wave by a wedge, and for this polarization

$$D = \frac{e^{i\pi/4}}{\sqrt{2\pi k}} \; (X+Y) \tag{5}$$

with

$$X = \frac{\frac{1}{\nu}\sin\frac{\pi}{\nu}}{\cos\frac{\pi}{\nu} - \cos\frac{\pi-\alpha}{\nu}} \quad , \tag{6}$$

$$Y = \frac{\frac{1}{\nu}\sin\frac{\pi}{\nu}}{\cos\frac{\pi}{\nu} - \cos\frac{\pi-2\phi_0+\alpha}{\nu}} \quad , \tag{7}$$

provided α is not close to zero. The specific requirement is that $\sqrt{kr}\,|\sin\alpha| \gg 1$ and we observe that as $\sigma \to 0$, $X \to \infty$. In eqs. (6) and (7), ϕ_0 is the angle which TV makes with the interior bisector of the wedge, and

$$\nu = 2(1 - \frac{\Omega}{\pi})$$

where 2Ω is the included angle of the wedge. Thus, for the mountain specified,

$$\nu = 5/3 \quad .$$

On assembling the results of eqs. (1) through (7), the total field at P becomes

$$E(P) = \frac{e^{ikR}}{kR} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{\exp\left[ik(r+r_0) + \frac{i\pi}{4}\right]}{kr\sqrt{2\pi kr_0}} \; (X+Y)$$

and since $r \sim R$ for $r \gg r_0$,

$$E(P) = \frac{e^{ikR}}{kR} \left\{ \begin{matrix} 1 \\ 0 \end{matrix} + \frac{\exp\left(\frac{i\pi}{4} + 2ikr_0 \sin^2 \alpha/2\right)}{\sqrt{2\pi kr_0}} \; (X+Y) \right\} \quad . \tag{8}$$

The discontinuity in this formula at $\alpha = 0$ is produced by our use of the simple "wide angle" expression for the diffraction coefficient D, and can be removed by using in place of X a uniform representation X' valid even in the immediate vicinity of the shadow boundary:

$$X' = X + \frac{1}{2}\operatorname{cosec}\frac{\alpha}{2} + i\sqrt{2kr_0}\exp\left(-2ikr_0\sin^2\frac{\alpha}{2}\right)F\left[\sqrt{2kr_0}\sin\frac{\alpha}{2}\right] \tag{9}$$

(Oberhettinger, 1956), where F(x) is the Fresnel integral

$$F(x) = \int_x^\infty e^{i\mu^2}\,d\mu \quad . \tag{10}$$

Since

$$X' = \begin{cases} X & \text{if } \sqrt{2kr_0}\sin\frac{\alpha}{2} \gg 1 \\ -\sqrt{\frac{\pi kr_0}{2}}\, e^{-i\pi/4} & \text{if } \alpha = 0 \\ -\sqrt{2\pi kr_0}\, e^{-i\pi/4} & \text{if } \sqrt{2kr_0}\sin\frac{\alpha}{2} \ll -1 \end{cases}$$

we observe that the modified result

$$E(P) = \frac{e^{ikR}}{kR}\left\{1 + \frac{\exp\left(\frac{i\pi}{4} + 2ikr_0\sin^2\frac{\alpha}{2}\right)}{\sqrt{2\pi kr_0}}(X' + Y)\right\} \tag{11}$$

is continuous through $\alpha = 0$, and reduces to eq. (8) well away from the shadow boundary. It also predicts that

$$E(P) = \frac{1}{2}\frac{e^{ikR}}{kR}$$

when $\alpha = 0$, as required.

Equation (11) is in agreement with the "uniform" expression for the field of a point source in the presence of a wedge (see Bowman et al., 1969; p. 274) and is the basis for our subsequent analyses.

2.1 Ideal earth

If the earth is perfectly conducting, the pattern factor for a vertical electric monopole at its surface is (Wolff, 1967)

$$A(\gamma) = \frac{\cos \frac{\pi}{2} \sin\gamma}{\cos\gamma} \tag{12}$$

where γ is the elevation angle shown in Fig. 1. When there is no mountain, the field observed at P is then

$$E(P) = A(\gamma) \frac{e^{ikR}}{kR} \quad , \tag{13}$$

whereas with the mountain present

$$E(P) = \frac{e^{ikR}}{kR} \left\{ A(\gamma) + A(\gamma_0) \frac{\exp\left(\frac{i\pi}{4} + 2ikr_0 \sin^2 \frac{\alpha}{2}\right)}{\sqrt{2\pi k r_0}} (X' + Y) \right\} \tag{14}$$

where $\gamma_0 = 7.3^0$ is the elevation of the mountain top. Apart from a normalization through the removal of the space factors, the fields in (13) and (14) are the ones whose moduli have been computed. We note that (14) does not take into account any reflection from the front face of the mountain, nor does it in any way simulate disturbances produced at the junction of the mountain with the earth. The latter would be a second order effect and strongly influenced by the precise details of the model used to simulate the foothills.

2.2 Lossy earth

The far field of a short vertical electric dipole at the surface of a lossy earth can be decomposed into a space wave and a surface wave as follows (Feinberg, 1967, chapter 5; Stratton, 1941, p. 587):

$$\underline{E}(P) = \underline{E}^{(1)}(P) + \underline{E}^{(2)}(P) \tag{15}$$

in which the space wave is

$$\underline{E}^{(1)}(P) = \frac{e^{ikR}}{kR} (1 + \Delta) \cos\gamma \hat{\gamma} \tag{16}$$

and the surface wave is

$$\underline{E}^{(2)}(P) = \frac{e^{ikR}}{kR} (1 - \Delta)(\cos\gamma - \beta \sin\gamma) f(w) \hat{\gamma} \quad , \tag{17}$$

where

$$\Delta = \frac{N \sin\gamma - \sqrt{N - \cos^2\gamma}}{N \sin\gamma + \sqrt{N - \cos^2\gamma}} \quad ,$$

$$N = \epsilon_r \left(1 + i \frac{\sigma}{\omega \epsilon_r \epsilon_0}\right) \quad ,$$

$$\beta = \frac{\cos\gamma}{N} \left(1 + \frac{1}{2} \sin^2\gamma\right) \sqrt{N - \cos^2\gamma}$$

and

$$f(\omega) = \frac{i}{4kR} \frac{N^2 (1-\Delta)^2}{N - \cos^2\gamma} \quad .$$

The strength of the dipole has been normalized to unity through absorption of a factor $\frac{i\omega k \mu_0 I\ell}{4\pi}$.

Computations of the space and surface waves for the specified ground show (see, for example, Fig. 2) that the relative strength of the latter is small at angles comparable to the elevation of the mountain top even at the lowest frequency of interest. It is therefore sufficient to confine attention to the space wave alone, thereby removing the conceptual difficulty posed by the radial component of the surface wave which has not been included in eq. (17).

In the absence of the mountain, the field is now given by (16) alone, but to ease comparison with the results for an ideal earth, we have renormalized the field through multiplication by a factor $1/2$ (since $\Delta \to 1$ as $\sigma \to \infty$ for $\gamma \neq 0$) and have replaced $\cos\gamma$ by the $A(\gamma)$ of eq. (12). The result is

$$E(P) = \frac{e^{ikR}}{kR} \frac{1}{2} (1+\Delta) A(\gamma) \qquad (18)$$

which differs from the eq. (13) for an ideal earth only in the presence of a modified pattern factor

$$A'(\gamma) = \frac{1}{2} (1+\Delta) A(\gamma) \qquad (19)$$

in place of γ. When the mountain is present, we can therefore use eq. (14) again provided A is replaced by the factor A'.

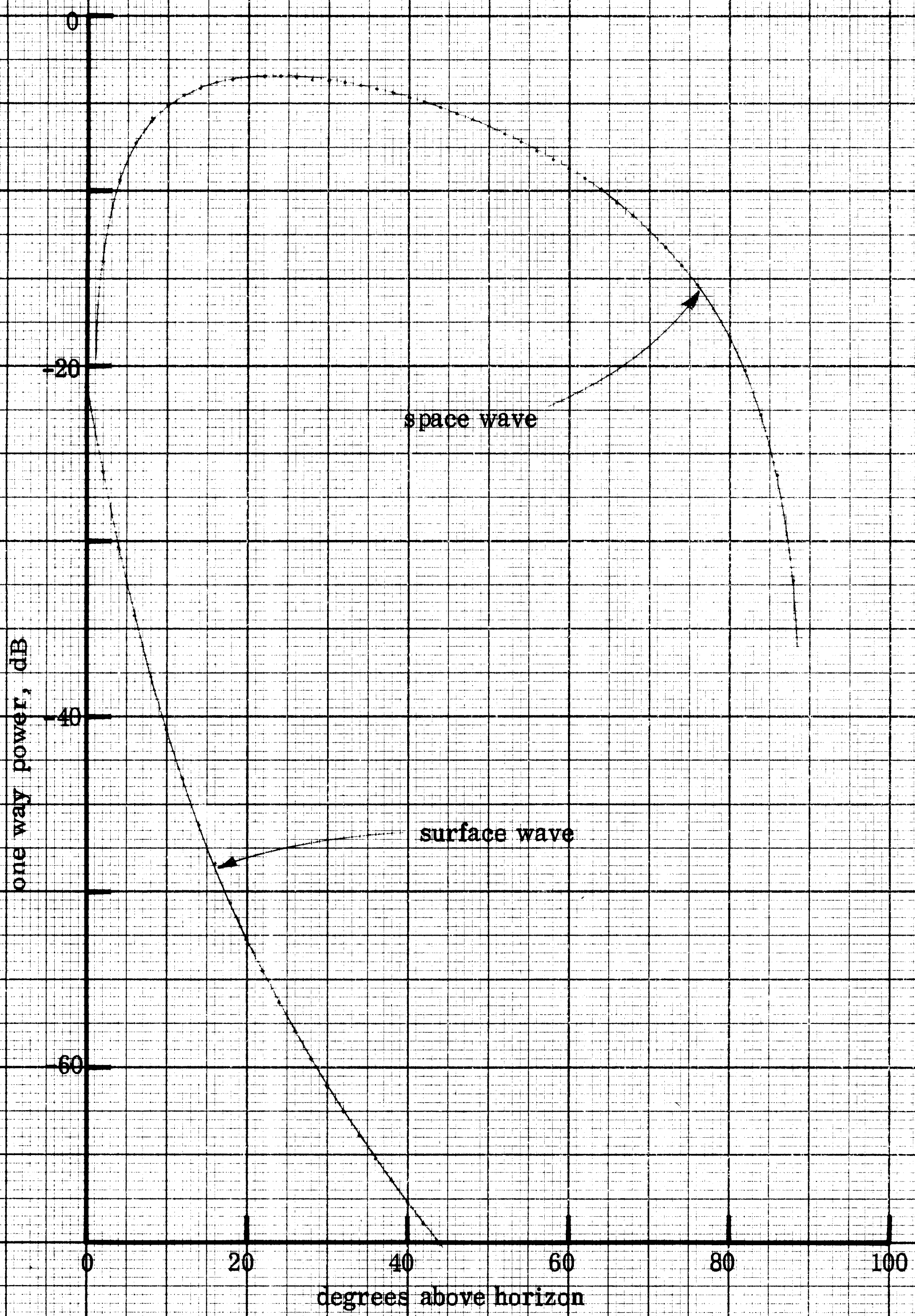

FIG. 2: Comparison of space wave and surface wave levels for imperfectly conducting earth at 4 MHz.

3. Computer Programs

Because of the highly specific nature of the problem, the computer program developed for the calculations was also specific, and therefore limited in scope. Since the geometry of the source and wedge is fixed at the dimensions given in Fig. 1, appropriate distances and angles are embedded in the program as constants. The total far field pattern (consisting of the sum of direct and diffracted rays) was initially calculated from the horizon to the zenith at intervals of 1 degree, but after it became apparent that the perturbation to the direct field was 1 dB or less for elevation angles above 30 degrees, the program was modified to cover only the first 30 degrees at intervals of 0.25 degree. The modification involves only two statements in the program and was a trivial one; although an equally trivial modification could be contrived to accommodate any angular range and sampling rate, the highly specific nature of the program hardly warranted a more sophisticated treatment.

A FORTRAN listing of the program is given on pages 16 and 17. It first reads the frequency of the source and the dielectric property N of the ground from the input stream, then computes the intensity of the excitation of the wedge apex, stored in the variable B. The program then indexes through 121 elevation angles, computing the direct and diffracted far fields, which are called A and ED. The total field E is the sum of the two and, after converting all three to decibels, the program prints them on the output record along with the elevation angle. When the computations have been completed for one frequency, the program returns to the input stream for a new data card. Specific instructions for an end-of-file condition are not included because The University of Michigan computing system provides for automatic program shut-down in this event.

Because the formulation of eq. (11) was chosen as a uniform representation of the diffracted pattern through the transition region and into the shadow, the intensity B at the apex must be added to the diffracted field instead of the pattern factor A when the fields in the shadow are calculated. The program tests for this condition and, depending whether the elevation angle γ is above or below the shadow boundary, A or B is added to the diffracted field, respectively.

The one subroutine called by the program is that supplied by IBM in its Scientific Subroutine Package. The subroutine CS returns the real and imaginary parts C and S, respectively, of the complementary Fresnel integral, and is required for the computation of the diffraction coefficient for each far field direction. Two scratch variables (SDUM and ARG) are used to hold intermediate results which, after being used in subsequent instructions, are no longer needed.

Although the program is restrictive because of its specificity, and therefore of limited application, a copy of the FORTRAN source deck is available upon request.

4. Computed Results

The far field radiation patterns are displayed in Fig. 3 for the perfectly and imperfectly conducting cases in the absence of the wedge. The pattern for perfectly conducting earth is the familiar cosinusoidal one characteristic of a half-wave dipole and is the same for all three frequencies (assuming a quarter-wave vertical monopole in each case). The maximum radiation is, of course, toward the horizon.

When the ground becomes imperfectly conducting, the strong lobe along the horizon becomes a null and the direction of maximum radiation shifts to about 25 degrees above the horizon for the dielectric properties specified (10^{-2} mhos/m, $\epsilon/\epsilon_0 = 15$). The complex relative dielectric constant N is assumed to have the form

$$N = \epsilon_r + i\frac{\sigma Z_0}{k} \quad ,$$

where ϵ_r is the relative permittivity and σ the conductivity of the earth, Z_0 is the impedance of free space and k is the free space wave number. Table I lists the values taken by N for the three frequencies used in the calculations; these were the values specified on input to the computer program.

Table 1
Electrical Properties of Earth Used in the Computations

frequency, mHz	N
4.0	15.0+i44.96888
11.0	15.0+i16.35232
30.0	15.0+i 5.99585

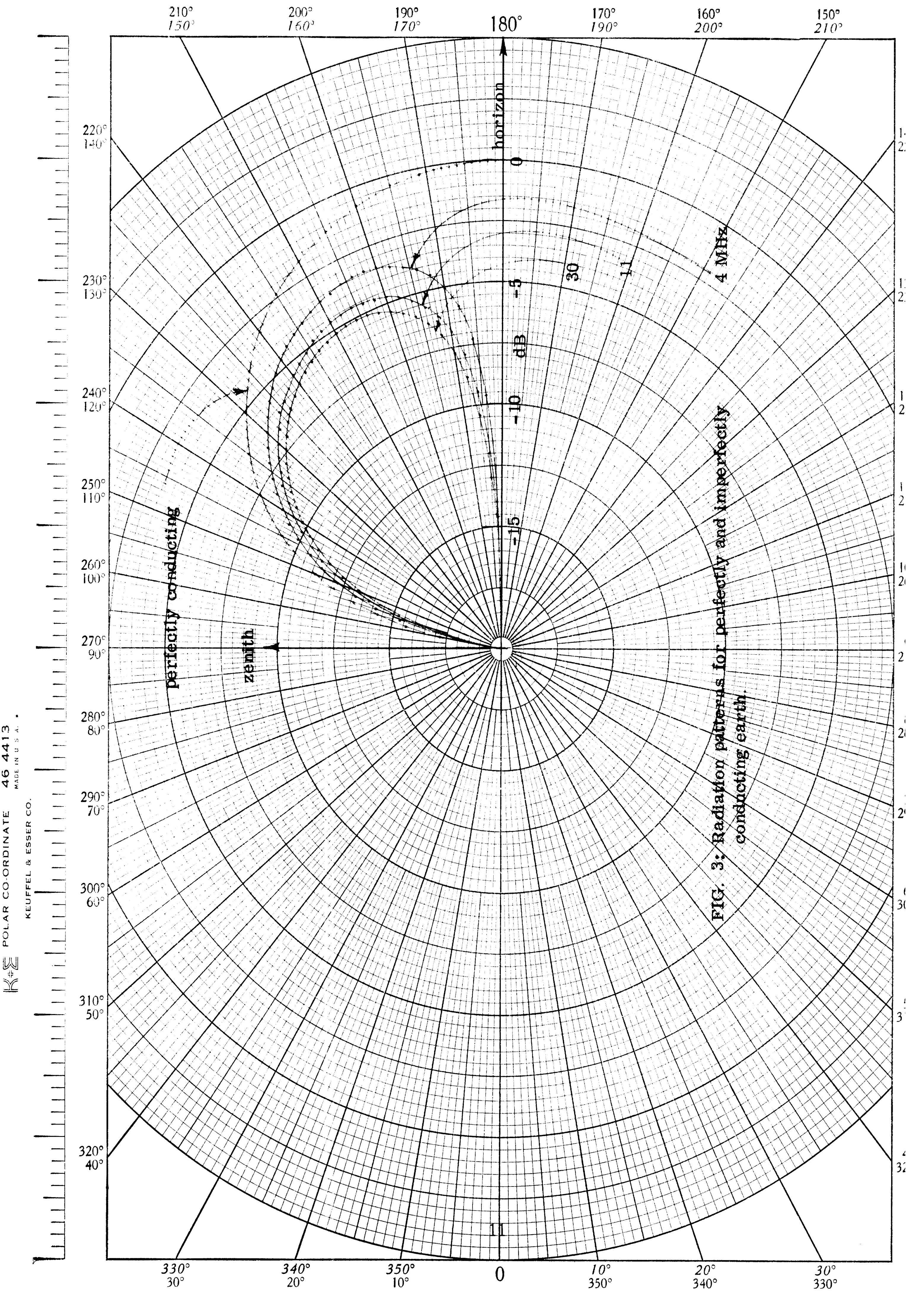

FIG. 3: Radiation patterns for perfectly and imperfectly conducting earth.

The total far field patterns are plotted in Figs. 4 through 6 for the three frequencies as solid lines, along with the reference patterns of Fig. 3 as dashed lines. In each case there is a reduction of the direct pattern from the perfectly to imperfectly conducting case and at an elevation angle of 25 degrees amounts to 2.2, 3.4 and 3.9dB, respectively, for 4.0, 11.0 and 30.0 MHz. The oscillatory nature of the total pattern is due to path length differences (between direct and diffracted rays) that change rapidly with increasing angle.

The amount of the perturbation is slightly smaller for the imperfectly conducting case than for the conducting case. This is because the excitation of the wedge apex is reduced by 6 to 9 dB over the perfectly conducting case while the radiation pattern (above the shadow boundary) is reduced by only the 2.2, 3.4 and 3.9dB mentioned above. Thus, in a relative sense, the source of perturbation (i.e., the diffracted ray) becomes weaker in going from perfectly to imperfectly conducting earth.

References

Bowman, J.J., T.B.A. Senior and P.L.E. Uslenghi (1969), Electromagnetic and Acoustic Scattering by Simple Shapes, North Holland Publishing Co., Amsterdam.

Feinberg, Ye.L. (1967), "The propagation of radio waves along the surface of the earth", translation from the Russian prepared by the Foreign Technology Division, Wright-Patterson Air Force Base, Ohio. AD 660971.

Oberhettinger, F. (1956), "On asymptotic series for functions occurring in the theory of diffraction of waves by wedges", J. Math. Phys., 34, 245-255.

Stratton, J.A. (1941), Electromagnetic Theory, McGraw-Hill Book Co., Inc., New York.

Wolff, E.A. (1967), Antenna Analysis, John Wiley and Sons, Inc., New York.

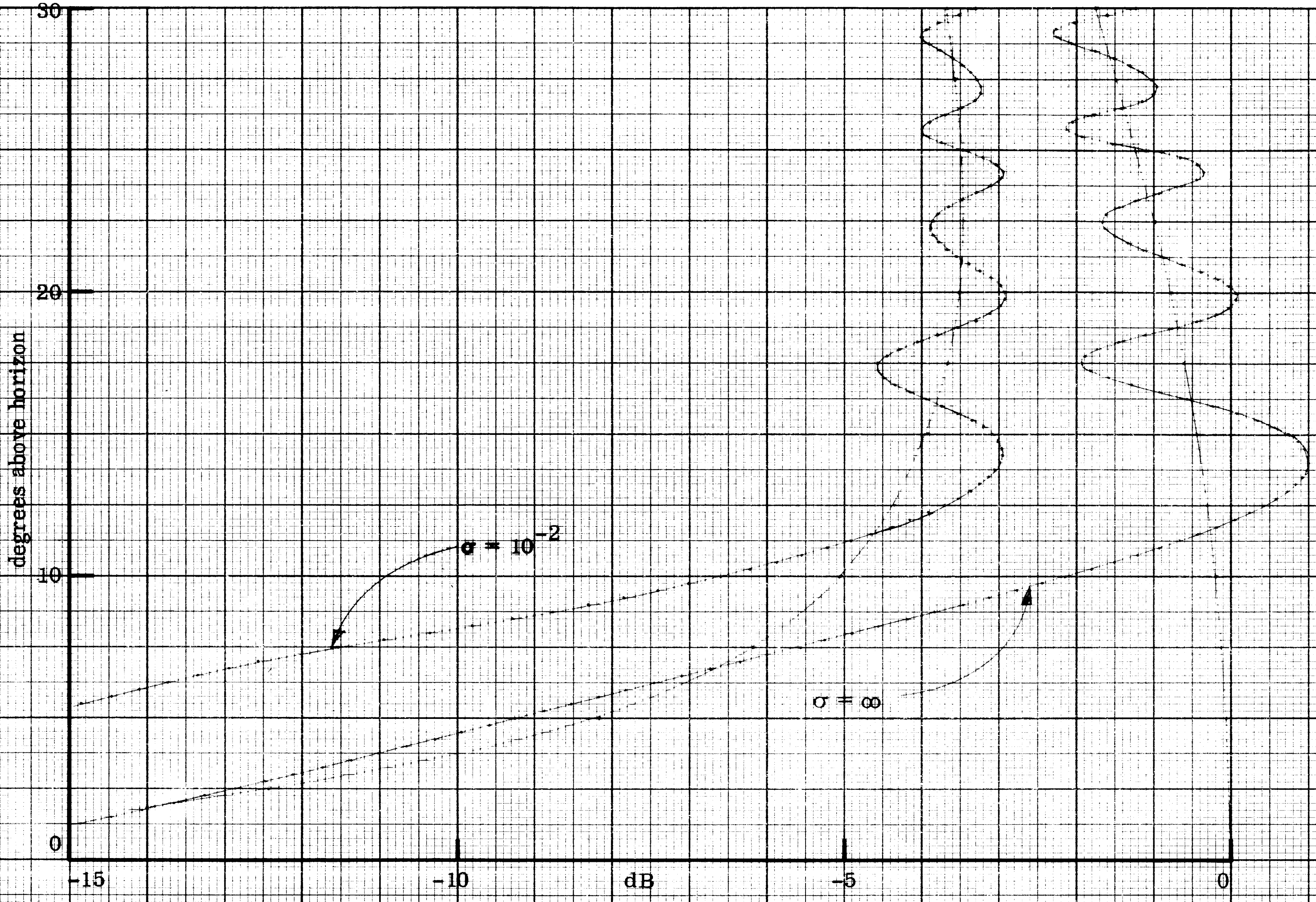

FIG. 4: Far field power patterns at 4 MHz.

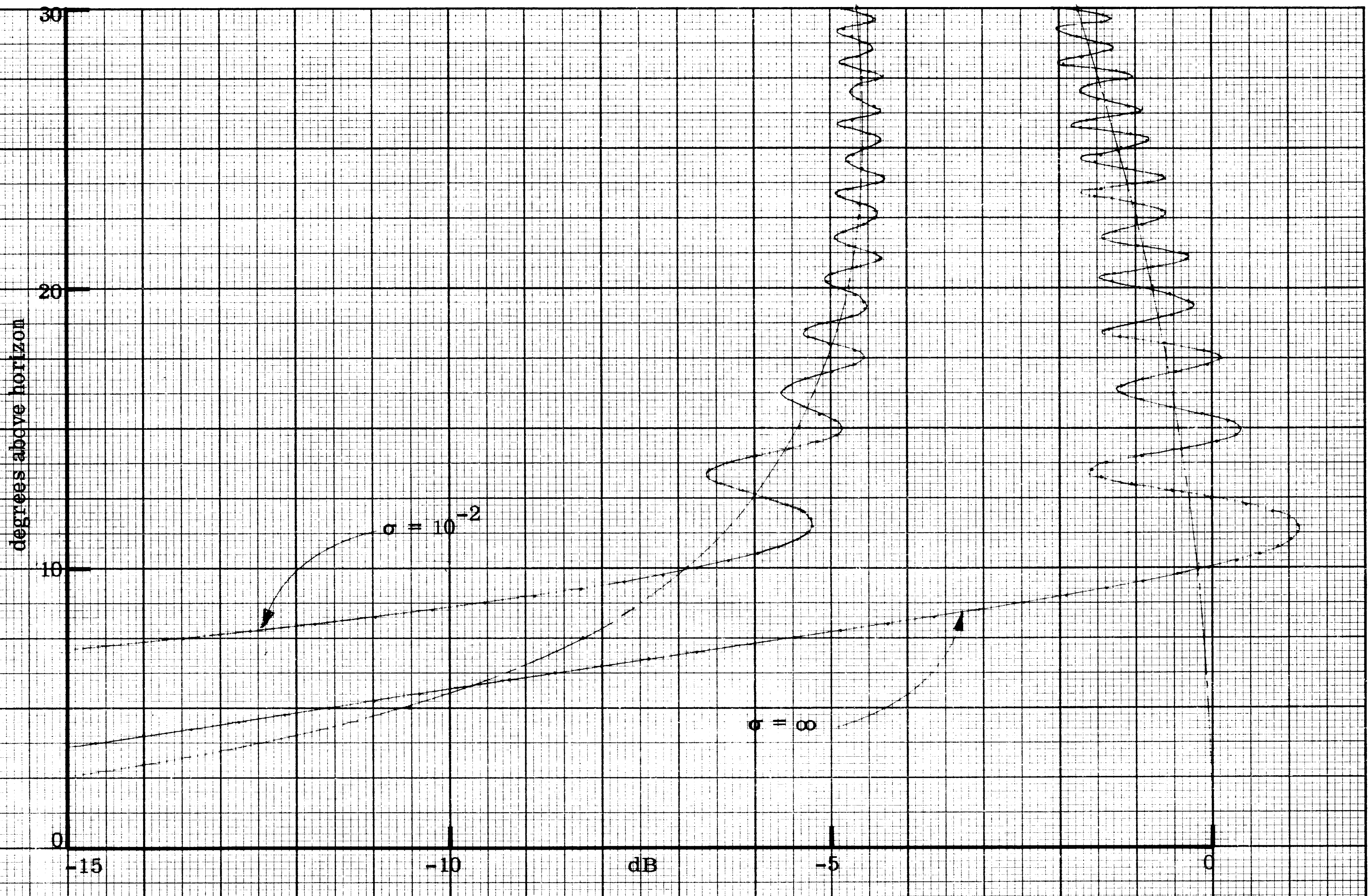

FIG. 5: Far field power patterns at 11 MHz

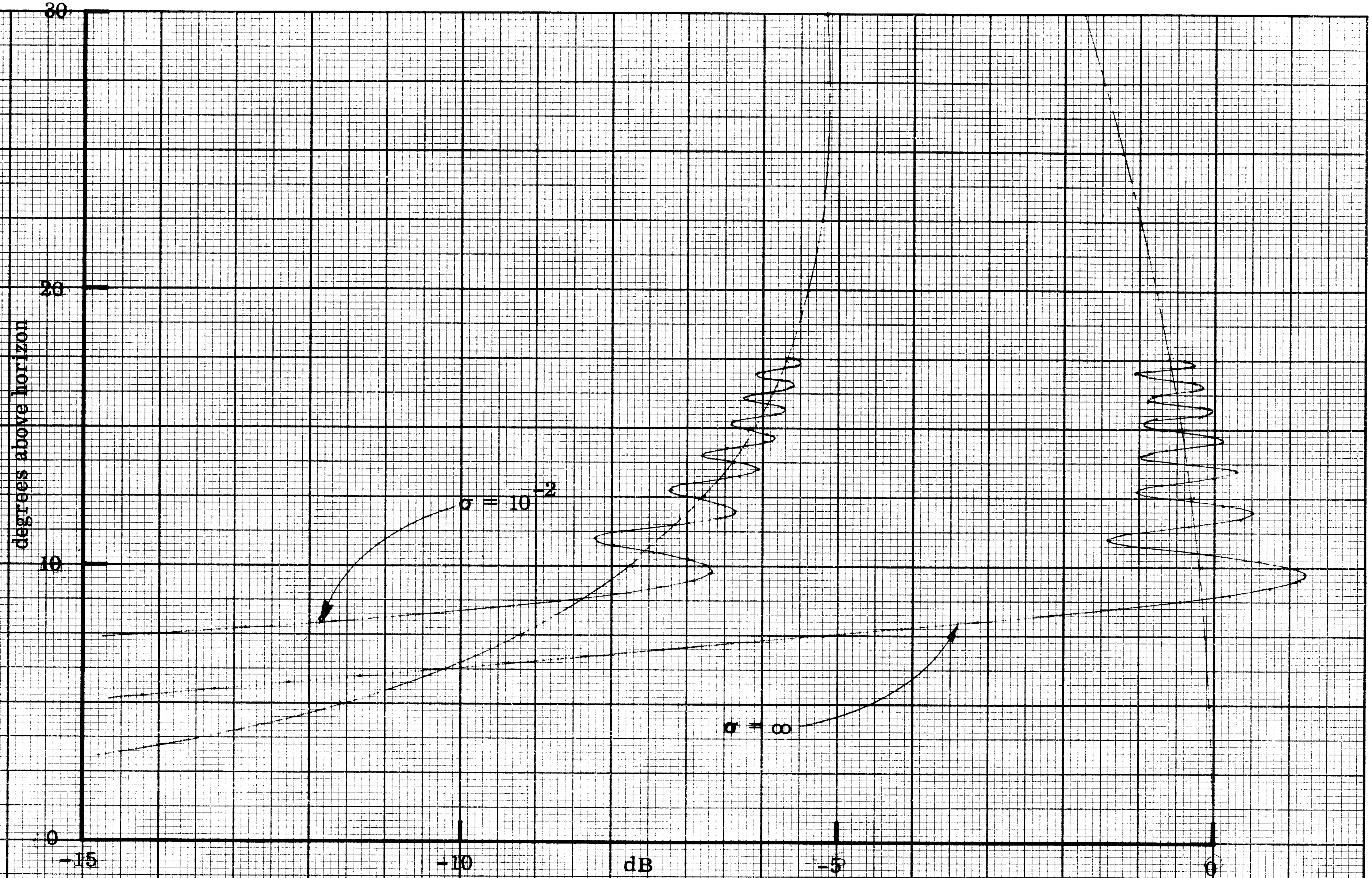

FIG. 6: Far field power patterns at 30 MHz.

```
      IMPLICIT REAL(K)
      DATA RED,PI/0.01745329,3.141593/
      DATA SP,CP,Q/0.5706339,-0.309017,983.5708/
      DATA R,F,K/21201.89,1.443387,0.006388137/
      DATA T1,T2,T3/1.961401,3.065147,0.127409/
      COMPLEX SDUM,A,B,XP,ED,E,DEN,N,L
      DIMENSION P(3)
   10 READ (5,100) FREQ,N
      WRITE (6,400) FREQ
      KR=K*R*FREQ
      ROW=R*FREQ/Q
      FAP=PI*SQRT(2.0*ROW)
      FFF=ATAN2(2700.0,21120.0)
      CG=COS(FFF)
      SG=SIN(FFF)
      SDUM=CSQRT(N-CG*CG)
      L=(N*SG-SDUM)/(N*SG+SDUM)
      C=COS(1.570796*SG)/CG
      B=0.5*(1.0+L)*C
      DEN=B/SQRT(2.0*PI*KR)
      DO 50 I=1,121
      GAM=0.25*(I-1)
      G=RED*GAM
      IF (I.EQ.41.OR.I.EQ.81) WRITE (6,400) FREQ
      CG=COS(G)
      SG=SIN(G)
      SDUM=CSQRT(N-CG*CG)
      L=(N*SG-SDUM)/(N*SG+SDUM)
      A=0.5*(1.0+L)*COS(1.570796*SG)/CG
      IF (CABS(A).LT.0.00001) A=CMPLX(0.00001,0.0)
      X=SP/(CP-COS(T1-0.6*G))
      Y=SP/(CP-COS(T2-0.6*G))
      ARG=1.0-SIN(F+G)
      FAC=0.707107/SQRT(ARG)
      ARG=KR*ARG
      SDUM=CMPLX(COS(ARG),-SIN(ARG))
      ARF=0.6366198*ARG
      CALL CS(C,S,ARF)
      IF (G.LE.FFF) GO TO 20
      XP=X+FAC+FAP*CMPLX(-0.5+S,0.5-C)*SDUM
      GO TO 30
   20 XP=X-FAC+FAP*CMPLX(-0.5-S,0.5+C)*SDUM
   30 ARG=ARG+0.25*PI
      SDUM=CMPLX(COS(ARG),SIN(ARG))*DEN
      ED=SDUM*(XP+Y)
      IF (G.LE.FFF) GO TO 35
      E=A+ED
      GO TO 40
   35 E=B+ED
   40 P(1)=20.0*ALOG10(CABS(A))
      P(2)=20.0*ALOG10(CABS(ED))
      P(3)=20.0*ALOG10(CABS(E))
   50 WRITE (6,300) GAM,P
      GO TO 10
  100 FORMAT (3F10.5)
```

```
  300  FORMAT (F13.2,F20.2,2F18.2)
  400  FORMAT ('1POWER PATTERN FOR ',F5.2,' MHZ, IN DB:'/
      &'0ANGLE ABOVE HORIZON    DIRECT PATTERN    DIFFRACTED PATTERN',
      &'    TOTAL PATTERN'/)
       END
IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
IN EFFECT*  NAME = MAIN     , LINECNT =         57
      SOURCE STATEMENTS =        58,PROGRAM SIZE =      3040
    NO DIAGNOSTICS GENERATED
```