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Diffraction by a Coated Wedge Using Second and Third Order Generalized Boundary Conditions

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Abstract

Second and Third Order Generalized Boundary Conditions are presented for simulating metal-backed dielectric coatings. A detailed quantitative assessment of their accuracy as a function of thickness and material parameters is first given. The higher order boundary conditions are subsequently used to derive corresponding diffraction coefficients for a coated wedge. The solution is obtained via a modification of Maliuzhinets method requiring the introduction of a particular solution which serves to produce the correct edge behavior and yield a reciprocal result.

Contents

I. Introduction

Of considerable interest in the computation of the radar cross section by complex targets is the simulation of metallic geometries coated with penetrable material. Such material are usually magnetic and serve to reduce the scattering of an otherwise perfectly conducting surface or junction. For thin coatings the standard impedance boundary condition[1] has been traditionally employed to provide a suitable mathematical simulation. However, the validity of this simulation is generally poor for oblique incidences, particularly in the case of low loss dielectrics. In a recent study[2] it was shown that higher order boundary conditions analogous to those proposed by Karp and Karal[3] can correctly predict the reflection by a dielectric layer at all angles of incidence, including grazing. These boundary conditions involve higher order derivatives of the surface field and as a first correction to the standard impedance boundary condition they account for the presence of currents normal to the dielectric layer. Notably, the maximum layer or coating thickness that can be accurately simulated depends on the highest derivative order kept in the boundary condition and this determines the order of the condition. When only the first derivative is kept they reduce to the standard impedance boundary condition and are, thus, accordingly referred to as generalized impedance boundary conditions(GIBC).

This report is concerned with the use of second and third order generalized impedance boundary conditions for computing the radar scattering by a fully coated wedge. The geometry, shown in Fig. 1, often occurs on aircraft structures and is thus of practical interest. An approximate solution to the diffracted field by the subject geometry using the standard impedance boundary condition is well known and has been given by Maliuzhinets[4]. The solution given here is more accurate by virtue of the employed GIBC and reduces to that in [4] for high loss coatings.

The second and third order boundary conditions to be employed in this analysis are presented in the next section along with data quantifying their accuracy as a function of the coating's thickness and refractive index. They were derived elsewhere[2, 5]and are analogous to those employed recently [6, 7] for the diffraction by a semi-infinite ferrite/dielectric layer. The diffracted field is obtained via a modification of the Maliuzhinets' method[4] requiring the addition of a particular solution as a direct implication of the employed higher order boundary condition. Such a particular solution is difficult to determine in a closed form for an arbitrary wedge angle. However, it has so far been derived for two specific wedge angles[5, 8] and, interestingly, its function has been to produce a reciprocal scattered field by cancelling out terms of the homogeneous solution that are of unacceptable order. Therefore, here we avoid the derivation of the particular solution by imposing reciprocity to obtain the "correct" diffracted field. As expected, it involves the usual Maliuzhinets' functions for which highly accurate approximate analytical expressions are available[9].

In the following, after presentation of the boundary conditions we proceed with the solution for the H-polarization. The E-polarization solution is then obtained directly from the H-polarization one via modification of the impedance parameters.

II. Boundary Conditions

For the problem at hand, a plane wave is assumed to be incident on the coated wedge configuration shown in Fig. 1. In the case of H-polarization, the only non-zero field components are H_z, E_x and E_y and for E-polarization the corresponding non-zero components are E_z, H_x and H_y. To proceed with a mathematical solution of the scattered field due to an incident plane wave excitation, it is necessary to replace the coating with an effective boundary condition. Recently [2, 5], higher order boundary conditions were proposed for this purpose. These can be generally written as (an $e^{-i\omega t}$ time convention is assumed)

$$\prod_{m=1}^{M}\left(\frac{\partial}{\partial n}+ik\Gamma_m\right)E_n=0 \qquad \prod_{m=1}^{M}\left(\frac{\partial}{\partial n}+ik\Gamma_m{}'\right)H_n=0 \tag{1}$$

were, E_n and H_n denote the respective normal components to the coating's surface and $\frac{\partial}{\partial n}$ implies differentiation with respect to the surface normal. They provide an improvement over the standard impedance boundary condition and are thus referred to as generalized impedance boundary conditions (GIBC).

For M = 1, these reduce to the standard impedance boundary conditions

$$\left(\frac{\partial}{\partial n}+ik\Gamma_1\right)E_n = 0 \qquad \left(\frac{\partial}{\partial n}+ik\Gamma_1'\right)H_n = 0 \tag{2}$$

with

$$\Gamma_1 = \frac{1}{\Gamma_1'} = -i\,\frac{N}{\varepsilon^{\pm}}\tan(Nk\tau) \tag{3}$$

in which $\varepsilon^{\pm}$ and $\mu^{\pm}$ are the relative permittivity and permeability of the coating on the upper (+) or lower (-) face of the wedge, $N = \sqrt{\mu^{\pm}\varepsilon^{\pm}}$, k is the wavenumber and τ denotes the coating's thickness. But an alternative and more common form of (2) is given in terms of tangential components as

$$\left(\frac{ik}{\rho}\gamma_1\frac{\partial}{\partial\phi}\pm k^2\right)\begin{Bmatrix}H_z\\ E_z\end{Bmatrix} = 0 \qquad \phi = \pm\Phi \tag{4}$$

where (ρ, ϕ) are the usual cylindrical coordinates and

$$\gamma_1 = \begin{cases}\Gamma_1 & \text{for H - polarization}\\ \dfrac{1}{\Gamma_1'} & \text{for E - polarization}\end{cases} \tag{5}$$

A solution for wedge diffraction based on the standard impedance boundary condition (1st order GIBC) has been given by Maliuzhinets [4]. However, as is well known, the standard impedance boundary condition is only applicable for high loss dielectric coatings, particularly for H-polarization, since it does not account for the presence of polarization currents normal to the coating.

The second order GIBC provides the next best simulation to the standard impedance boundary condition. It takes the form [5]

$$\left(\gamma_2 \frac{\partial^2}{\partial \rho^2} \mp \frac{ik}{\rho}\gamma_1 \frac{\partial}{\partial \phi} - k^2\right)\begin{Bmatrix} H_z \\ E_z \end{Bmatrix} = 0 \qquad \phi = \pm\Phi \tag{6}$$

where $\gamma_{1,2}$ are parameter functions of the material properties and specific expressions for these have been derived in [2,5].

A second order GIBC which has been found to yield reasonable accuracy has associated values of $\gamma_{1,2}$ given as

$$\gamma_1 = \frac{\Gamma_1 + \Gamma_2}{\Gamma_1\Gamma_2 + 1} = \frac{a_1}{a_0 + a_2}$$

$$\gamma_2 = -\frac{1}{1 + \Gamma_1\Gamma_2} = -\frac{a_2}{a_0 + a_2} \tag{7a}$$

for H-polarization and as

$$\gamma_1 = \frac{\Gamma_1' + \Gamma_2'}{\Gamma_1'\Gamma_2' + 1} = \frac{a_0 + a_2}{a_1}$$

$$\gamma_2 = -\frac{1}{1 + \Gamma_1'\Gamma_2'} = -\frac{a_2'}{a_0' + a_2'} \tag{7b}$$

for E-polarization. In these

$$
\begin{aligned}
a_0 &= \left(N - \frac{1}{2N}\right)\left[\tan(k\tau N) - \tan\left(\frac{k\tau}{2N}\right)\right] \\
a_1 &= i\epsilon^{\pm}\left[1 + \tan(k\tau N)\tan\left(\frac{k\tau}{2N}\right)\right] \\
a_2 &= \frac{1}{2N}\left\{\tan(k\tau N) - \tan\left(\frac{k\tau}{2N}\right) + k\tau\left(N - \frac{1}{2N}\right)\left[1 + \tan(k\tau N)\tan\left(\frac{k\tau}{2N}\right)\right]\right\} \\
a_0' &= (2N^2 - 1)\left[1 + \cot(k\tau N)\cot\left(\frac{k\tau}{2N}\right)\right] \\
a_1' &= i\,2N\mu^{\pm}\left[\cot(k\tau N) - \cot\left(\frac{k\tau}{2N}\right)\right] \\
a_2' &= 1 + \cot(k\tau N)\cot\left(\frac{k\tau}{2N}\right) + k\tau\left(N - \frac{1}{2N}\right)\left[\cot(k\tau N) - \cot\left(\frac{k\tau}{2N}\right)\right]
\end{aligned}
\tag{8}
$$

Figures 2 to7 show the maximum thickness for which the above second order GIBC is capable of predicting the coating's plane wave reflection coefficient within 10 degrees of its actual phase and/or 10% of its actual magnitude. As seen, in comparison with the standard impedance boundary condition, the second order GIBC provides substantially better accuracy for incidence angles away from normal. Notably, the simulation improves monotonically as one approaches grazing. In making this observation it should be also noted that the given curves are for lossless dielectrics and, therefore, represent a worse case. We may conclude from figures 2 to 4 that for H-polarization, the second order GIBC is capable of simulating coatings having thickness up to 1/5 of a wavelength for incidence angles greater than 35° from normal (55° from grazing). This is regardless of the dielectric's properties since the simulation improves substantially as N and/or the loss in the coating increases. In contrast, the 1st order GIBC provides a superior simulation, with respect to the second order GIBC, for the rest of the angular region (i.e. within 35° from normal). Turning to Figures 5 to 7, one again arrives at similar conclusions for E-polarization. However, it should be noted that for small N the deterioration of the simulation provided by the second order GIBC as normal incidence is approached, is now more rapid.

It is desirable for practical purposes to have a single boundary condition simulating the coating's presence for all incidence angles. From the above, the second order GIBC is obviously not adequate if our goal is to simulate coatings of at least 1/4 of a wavelength in thickness. Therefore, it is of interest to examine the suitability of a third order GIBC. The third order GIBC may be written as

$$\left(\pm \frac{1}{ik\rho}\gamma_3 \frac{\partial^3}{\partial\rho^2\partial\phi} - \gamma_2 \frac{\partial^2}{\partial\rho^2} \mp \frac{ik}{\rho}\gamma_1 \frac{\partial}{\partial\phi} - k^2 \right) \begin{Bmatrix} H_z \\ E_z \end{Bmatrix} = 0 \qquad \phi = \pm\Phi \tag{9}$$

and from [2]

$$\gamma_1 = \frac{a_1+a_3}{a_0+a_2} = \frac{\Gamma_1\Gamma_2+\Gamma_2\Gamma_3+\Gamma_1\Gamma_3+1}{\Gamma_1+\Gamma_2+\Gamma_3+\Gamma_1\Gamma_2\Gamma_3}$$

$$\gamma_2 = -\frac{a_2}{a_0+a_2} = -\frac{\Gamma_1+\Gamma_2+\Gamma_3}{\Gamma_1+\Gamma_2+\Gamma_3+\Gamma_1\Gamma_2\Gamma_3} \tag{10a}$$

$$\gamma_3 = -\frac{a_3}{a_0+a_2} = -\frac{1}{\Gamma_1+\Gamma_2+\Gamma_3+\Gamma_1\Gamma_2\Gamma_3}$$

for H-polarization and

$$\gamma_1 = \frac{a_1'+a_3'}{a_0'+a_2'} = \frac{\Gamma_1'\Gamma_2'+\Gamma_2'\Gamma_3'+\Gamma_1'\Gamma_3'+1}{\Gamma_1'+\Gamma_2'+\Gamma_3'+\Gamma_1'\Gamma_2'\Gamma_3'}$$

$$\gamma_2 = -\frac{a_2'}{a_0'+a_2'} = -\frac{\Gamma_1'+\Gamma_2'+\Gamma_3'}{\Gamma_1'+\Gamma_2'+\Gamma_3'+\Gamma_1'\Gamma_2'\Gamma_3'} \tag{10b}$$

$$\gamma_3 = -\frac{a_3'}{a_0'+a_2'} = -\frac{1}{\Gamma_1'+\Gamma_2'+\Gamma_3'+\Gamma_1'\Gamma_2'\Gamma_3'}$$

for E-polarization. In these, a_0, a_1, a_2, a_0', a_1' and a_2' are as defined in (8). The remaining constants a_3 and a_3' are given by

$$a_3 = -\frac{ik\tau\varepsilon^{\pm}}{2N}\left[\tan(k\tau N) - \tan\left(\frac{k\tau}{2N}\right)\right] \tag{11}$$

$$a_3' = -ik\tau\mu^{\pm}\left[1 + \cot(k\tau N)\cot\left(\frac{k\tau}{2N}\right)\right]$$

Clearly, (10) reduce to (7) when $a_3 = a_3' = 0$ or $\Gamma_3 = \Gamma_3' \to \infty$.

From Figure 8 to 10, it is now seen that the above third order GIBC provides an acceptable simulation for coating thicknesses of at least 0.4λ regardless of material properties, angle of incidence and polarization. This is, of course, a conservative statement since the simulation improves for lossy coatings having large refractive indices and for angles of incidence away from normal.

III. Solution

Consider a wedge whose faces are located at $\phi = \pm\Phi = \pm\frac{n\pi}{2}$. The wedge is illuminated by the plane wave

$$\begin{Bmatrix} H_z^i \\ E_z^i \end{Bmatrix} = u^i(\rho, \phi) = e^{-ik\rho\cos(\phi-\phi_0)} \tag{12}$$

and each face is subject to an Mth order boundary condition with impedances $\Gamma_m = \sin\theta_m^{\pm}$ for H-polarization and $\Gamma_m' = \sin\theta_m^{\pm}$ for E-polarization ($m = 1, 2, \ldots, M$) on the upper (positive) and lower (negative) face with $\mathrm{Re}\sin\theta_m^{\pm} \geq 0$. It is required that, for small kρ, the total field $u(\rho, \phi)$ satisfy the edge condition

$$\begin{Bmatrix} H_z(\rho,\phi) \\ E_z(\rho,\phi) \end{Bmatrix} = u(\rho,\phi) = 0\,\{(k\rho)^{\epsilon}\}$$

with $\epsilon > 0$.

Following Maliuzhinets [4] we write

$$u(\rho, \phi) = \frac{1}{2\pi i} \int_\gamma e^{-ik\rho\cos\alpha}\, s(\alpha+\phi)\, d\alpha \qquad (13)$$

where γ is the double loop Sommerfeld path. Application of the boundary conditions then gives

$$\int_\gamma e^{-ik\rho\cos\alpha} \left\{ \prod_{m=1}^{M} (\sin\alpha \pm \sin\theta_m^{\pm})\, s(\alpha \pm \Phi) \right\} d\alpha = 0 \qquad (14)$$

and the necessary and sufficient conditions for this to be satisfied is [10]

$$\prod_{m=1}^{M} (\sin\alpha \pm \sin\theta_m^{\pm})\, s(\alpha \pm \Phi) - \prod_{m=1}^{M} (-\sin\alpha \pm \sin\theta_m^{\pm})\, s(-\alpha \pm \Phi) = \sin\alpha \sum_{m=0}^{M} A_m^{\pm} \cos^m\alpha \qquad (15)$$

for appropriate constants $A_m^{\pm}$

As the solution of (15) we write

$$s(\alpha) = g(\alpha)\, \sigma_0(\alpha) + h(\alpha) \qquad (16)$$

where

$$g(\alpha) = \prod_{m=1}^{M} \frac{\Psi(\alpha, \theta_m^{+}, \theta_m^{-})}{\Psi(\phi_o, \theta_m^{+}, \theta_m^{-})} \qquad (17)$$

and Ψ is a product of four Maliuzhinets functions [4]. For large $|\mathrm{Im}\,.\,\alpha|$

$$g(\alpha) = O\left\{ \exp\left(\frac{M}{n} |\mathrm{Im}\,.\alpha| \right) \right\}$$

where $n = 2\Phi/\pi$, and for brevity we shall refer to this as "order M". To satisfy the edge condition it is necessary that $s(\alpha)$ be of order $-n\epsilon$. In order to reproduce the incident field (12) we choose

$$\sigma_0(\alpha) = \frac{\frac{1}{n} C_0}{S - S_0} \qquad (18a)$$

where

$$S = \sin\frac{\alpha}{n}\ , \qquad C = \cos\frac{\alpha}{n}\ ,$$

$$S_0 = \sin\frac{\phi_0}{n}\ , \qquad C_0 = \cos\frac{\phi_0}{n}\ , \tag{18b}$$

and thus the first term in (16) is of order M-1. The second term on the right hand side of (16) is a particular solution of (15) which is free of singularities in $|\text{Re.}\ \alpha| \leq \Phi$ and cancels all terms of excess order in $g(\alpha)\ \sigma_0\ (\alpha)$. In addition to these three requirements on $h(\alpha)$, there is a fourth one as we shall show later. Provided such an $h(\alpha)$ can be found, the resulting solution will satisfy the correct edge behavior and reciprocity.

When the expression for the diffracted field is reduced to an integral over a steepest descent path through the origin, the non-exponential part of the integrand is

$$s(\alpha+\pi) - s(\alpha-\pi) = f(\alpha) + h(\alpha-\pi) - h(\alpha-\pi) \tag{19}$$

where

$$f(\alpha) = g(\alpha+\pi)\,\sigma_0(\alpha+\pi) - g(\alpha-\pi)\,\sigma_0(\alpha-\pi) \tag{20}$$

and we have

$$g(\alpha+\pi) = X_M(\alpha, \phi_0)\prod_{m=1}^{M}(a_m - x)\,(b_m + x) \tag{21}$$

$$g(\alpha-\pi) = X_M\,(\alpha, \phi_0)\prod_{m=1}^{M}(a_m - y)\,(b_m + y)$$

where

$$a_m = \cos\frac{1}{n}(\theta_m^+ - \pi/2)\ , \qquad b_m = \cos\frac{1}{n}(\theta_m^- - \pi/2) \tag{22}$$

$$x = qS + pC\ , \qquad y = qS - pC \tag{23}$$

with

$$p = \sin\frac{\pi}{2n}\ , \qquad q = \cos\frac{\pi}{2n} \tag{24}$$

and

$$X_M(\alpha, \phi_0) = 4^{-N} \{\Psi_\Phi(\pi/2)\}^{8N} \prod_{m=1}^{M} \{ \Psi(\alpha, \theta_m^+, \theta_m^-) \Psi(\phi_0, \theta_m^+, \theta_m^-) \}^{-1} \quad (25)$$

Thus, $X_M(\alpha, \phi_0)$ is symmetric in α and ϕ_0 and of order -M. Further,

$$\sigma_0(\alpha \pm \pi) = \frac{C_0}{nD} \{(2q^2-1) S - S_0 \mp 2pq\, C\} \quad (26)$$

where

$$D = S^2 - 2(2q^2-1) SS_0 + S_0^2 - 4p^2 q^2 \quad (27)$$

and D is also symmetric in α and ϕ_0. It follows that

$$f(\alpha) = \frac{1}{n} X_M(\alpha, \phi_0) \frac{C_0}{D} \Big[\{ 2q^2-1)S - S_0\} \{ \prod_{m=1}^{M} (a_m - x)(b_m + x) - \prod_{m=1}^{M} (a_m - y)(b_m + y) \} - 2pqC \{ \prod_{m=1}^{M} (a_m - x)(b_m + x) + \prod_{m=1}^{M} (a_m - y)(b_m + y) \} \Big] \quad (28)$$

In general this is neither symmetric in α and ϕ_0 nor of acceptable order, and to see this we consider the cases M = 1, 2 and 3.

First Order Boundary Condition (M=1)

This is the simplest case. When M=1 it can be shown that the constants $A_m^{\pm}$ in (15) are all zero and hence $h(\alpha) = 0$. Since

$$x - y = 2pC, \qquad x + y = 2qS, \quad (29)$$

$$x^2 - y^2 = 4pqSC, \qquad x^2 + y^2 = 2(2q^2-1) S^2 + 2p^2,$$

we have

$$f(\alpha) = \frac{1}{n} X_1(\alpha, \phi_0) \frac{C_0}{D} \Big[\{ (2q^2-1)S-S_0\} \{a_1-x)(b_1+x) - (a_1-y)(b_1+y)\}$$

$$-2pqC \{(a_1-x)(b_1+x)+(a_1-y)(b_1+y) \Big]$$

$$= \frac{1}{n} X_1 (\alpha, \phi_0) \frac{1}{D} \Big[C_0 \{ (2q^2-1) S-S_0\} \{ -(x^2-y^2) + (a_1-b_1)(x-y) \}$$

$$- 2pqCC_0 \{ -(x^2+y^2) - (a_1-b_1)(x+y) + 2a_1 b_1 \} \Big]$$

which reduces to

$$f(\alpha) = \frac{1}{n} X_1 (\alpha, \phi_0)\, 2p \frac{CC_0}{D} \Big[2q(SS_0+p^2) - (a_1-b_1)(S+S_0) - 2qa_1 b_1 \Big] \tag{30}$$

This is symmetric in α and ϕ_0 and of order -1, implying $u(\rho,\phi_0) = O\{(k\rho)^{1/n}\}$ for small $k\rho$. Consequently, there is no need for any particular solution $h(\alpha)$.

Second Order Boundary Condition (M=2)

The constants $A_0^{\pm}$ and $A_1^{\pm}$ in (15) are no longer zero and hence $h(\alpha) \neq 0$. From (28)

$$f(\alpha) = \frac{1}{n} X_2 (\alpha, \phi_0) \frac{C_0}{D} \Big[\{ (2q^2-1) S-S_0\} \{ (a_1-x)(a_2-x)(b_1+x)(b_2+x) - (a_1-y)(a_2-y)(b_1+y)(b_2+y)\}$$

$$- 2pqC \{(a_1-x)(a_2-x)(b_1+x)(b_2+x) + (a_1-y)(a_2-y)(b_1+y)(b_2+y) \} \Big]$$

$$= \frac{1}{n} X_2 (\alpha, \phi_0) \frac{1}{D} \Big[C_0 \{ (2q^2-1) S-S_0\} \{ x^4-y^4 - (A_1-B_1(x^3-y^3) + (A_2+B_2-A_1B_1)(x^2-y^2)$$

$$+ (A_2B_1 - A_1B_2)(x-y) \} - 2pqCC_0 \{x^4+y^4 - (A_1-B_1)(x^3+y^3)$$

$$+ (A_2+B_2- A_1B_1)(x^2+y^2) + (A_2B_1 - A_1B_2)(x+y) + 2A_2B_2\} \Big] \tag{31}$$

where

$$A_1 = a_1 + a_2 , \qquad B_1 = b_1 + b_2 ,$$
$$A_2 = a_1 a_2 , \qquad B_2 = b_1 b_2 . \tag{32}$$

The first order term (involving $x \pm y$) in square brackets is

$$-2p\, CC_0\, (S+S_0) ,$$

and the second order term is

$$-4\, pq\, CC_0\, (SS_0 + p^2).$$

Both are symmetric in α and ϕ_0 and of allowed order in $|\mathrm{Im.}\ \alpha|$. Since

$$\begin{aligned}
x^3 - y^3 &= 2pC\, \{(4q^2-1)\, S^2 + p^2\} , \\
x^3 + y^3 &= 2qS\, \{4q^2 - 3)\, S^2 + 3p^2\} , \\
x^4 - y^4 &= 8pqSC\, \{\, (2q^2 - 1)\, S^2 + p^2\} , \\
x^4 + y^4 &= 2\, (8q^4 - 8q^2 + 1)\, S^4 + 4\, (4q^2-1)\, p^2 S^2 + 2p^4 ,
\end{aligned} \tag{33}$$

after much tedious trigonometric manipulations the third order term is found to be

$$2pCC_0\, \{S^3 - (4q^2-1)\, S^2 S_0 - p^2\, (4q^2+1) S - p^2 S_0\}$$

and this is not, obviously, symmetric. Moreover, the first term is not of allowed order, but the entire expression can be rewritten as

$$2p\, CC_0 SD - 2p\, CC_0 (S+S_0)(SS_0 + p^2) ,$$

and the second term of this <u>is</u> symmetric and of allowed order. Finally, after much simplification the fourth order term in square brackets is

$$\begin{aligned}
&4pq\, CC_0\, \{S^4 - 2(2q^2-1)\, S^3 S_0 - 4p^2 q^2 S^2 - 2p^2 SS_0 - p^4\} \\
&= 4pq\, CC_0 S^2 D - 4pq\, CC_0 (SS_0 + p^2)^2 ,
\end{aligned}$$

and the second term of this is both symmetric and of allowed order. Note that the "elimination" of the term S^4 has also eliminated the unacceptable one involving S^3. Thus

$$f(\alpha) = \frac{2p}{n} X_2(\alpha, \phi_0)\, CC_0 \{ 2qS^2 - S(A_1 - B_1) + K \}$$

$$- \frac{2p}{n} X_2(\alpha, \phi_0) \frac{CC_0}{D} \Big[2q\, (SS_0+p^2)^2 - (A_1-B_1)(S+S_0)(SS_0+p^2)$$

$$+ 2q\,(A_2+B_2 - A_1B_1)(SS_0+p^2) + (A_2B_1-A_1B_2)(S+S_0) \qquad (34)$$

$$+ 2q\, A_2B_2 + K \{ (S+S_0)^2 - 4q^2(SS_0+p^2) \Big]$$

where K is an arbitrary constant independent of ϕ_0. The second group of terms is symmetric in α and ϕ_0 and of order -1; however, the first group violates both the symmetry and order conditions, and it is therefore necessary that the particular solution $h(\alpha)$ be chosen such that

$$h(\alpha+\pi) - h(\alpha-\pi) = -\frac{2p}{n} X_2(\alpha, \phi_0)\, CC_0 \{2q\, S^2 - S\,(A_1-B_1) + K\} \; . \qquad (35)$$

This is the fourth and final condition to be satisfied by $h(\alpha)$.

To complete the solution it now remains to find the constant K and to show that $h(\alpha)$ exists as defined above . The first can be determined by examining the edge behavior of the solution for a given ϕ_0 and from this it can be conjectured that $K = 0$. However, the existence of a solution for $h(\alpha)$ has not yet been rigorously established for an arbitrary wedge angle $(2-n)\pi$, although at present we believe that a proof of its existance can be established for $\Phi = \frac{n\pi}{2} = \frac{L_1\pi}{2L_2+1}$ for integer values of $L_{1,2}$.

Third Order Boundary Condition (M=3)

This is similar to the previous case but just a little more complicated. From (28)

$$f(\alpha) = \frac{1}{n} X_3(\alpha, \phi_0) \frac{C_0}{D} \Big[\{(2q^2-1) S - S_0\} \{(a_1-x)(a_2-x)(a_3-x)(b_1+x)(b_2+x)(b_3+x)$$

$$- (a_1-y)(a_2-y)(a_3-y)(b_1+y)(b_2+y)(b_3+y)\} - 2pqC \{(a_1-x)(a_2-x)(a_3-x)(b_1+x)(b_2+x)(b_3+x)$$

$$+ (a_1-y)(a_2-y)(a_3-y)(b_1+y)(b_2+y)(b_3+y)\} \Big]$$

$$= \frac{1}{n} X_3(\alpha, \phi_0) \frac{1}{D} \Big[C_0 \{(2q^2-1)S-S_0\} \{-(x^6-y^6) + (A_1-B_1)(x^5-y^5) + (A_1B_2-A_2B_1)(x^4-y^4)$$

$$+ (A_3-B_3+A_1B_2-A_2B_1)(x^3-y^3) + (A_3B_1 + A_1B_3 - A_2B_2)(x^2-y^2)$$

$$+ (A_3B_2 - A_2B_3)(x-y)\} - 2pq\, CC_0 \{-(x^6+y^6) + (A_1-B_1)(x^5+y^5)$$

$$+ (A_1B_2 - A_2B_1)(x^4+y^4) + (A_3-B_3+A_1B_2 - A_2B_1)(x^3+y^3)$$

$$+ (A_3B_1 + A_1B_3 - A_2B_2)(x^2+y^2) + (A_3B_2 - A_2B_3)(x+y) + 2A_3B_3\} \Big] \qquad (36)$$

where now

$$A_1 = a_1+a_2+a_3 \;, \quad B_1 = b_1+b_2+b_3 \;,$$

$$A_2 = a_1a_2+a_2a_3+a_3a_1 \;, \quad B_2 = b_1b_2+ b_2b_3 + b_3b_1 \;,$$

$$A_3 = a_1a_2a_3 \;, \quad B_3 = b_1b_2b_3 \;. \qquad (37)$$

The first and second order terms in square brackets are, apart from the multiplying constants,

$$-2p\, CC_0 (S+S_0)$$

and

$$-4pq\, CC_0(SS_0+p^2)$$

respectively, as in the previous case. Similarly, the third and fourth order terms are

$$2p\, CC_0\, SD - 2p\, CC_0(S+S_0)(SS_0+p^2)$$

and

$$4pq\, CC_0\, S^2D - 4pq\, CC_0\, (SS_0+p^2)^2$$

respectively, but there is now a subtle point that should be noted. In the third order case even the first term is of allowed order, and its separation out is forced by symmetry considerations and not by order. Thus, for a third order impedance boundary condition, specification of the edge behavior is not sufficient to ensure reciprocity.

Since

$$\begin{aligned}
x^5 - y^5 &= 2pC\,\{(16q^4 - 12q^2+1)S^4 + 2(6q^2-1)p^2S^2 - p^4\} \\
x^5 + y^5 &= 2qS\,\{\,(16q^4 - 20q^2 + 5)S^4 + 10(2q^2-1)p^2S^2 + 3p^4\} \\
x^6 - y^6 &= 4pq\,CS\,\{(16q^4 - 16q^2+3)\,S^4 + 2(8q^2-3)p^2S^2 + 3p^4\} \\
x^6 + y^6 &= 2(32q^6-48q^4+18q^2-1)S^6 + 6(16q^4-12q^2+1)p^2S^4 \\
&\quad + 6(6q^2-1)p^4S^2 + 2p^6
\end{aligned} \tag{38}$$

we again find after much trigonometric manipulations that the fifth and sixth order terms in square brackets of the expression for $f(\alpha)$ are

$$2p\, CC_0\, S\,\{\,(4q^2-1)\,S^2 + SS_0 + 2p^2\}\,D\; - 2p\, CC_0\,(S+S_0)\,\{(SS_0+p^2)^2 - 2p^4\}$$

and

$$\begin{aligned}
&- 4pq\, CC_0\,\{2\,(2q^2-1)S^4 + S^3S_0 + 3p^2S^2\}\,D \\
&\qquad + 4pq\, CC_0\, SS_0\,(S^2S_0^2 + 3p^2\, SS_0 + 3p^4)
\end{aligned}$$

respectively. Clearly, in each case the second group of terms is symmetric and of allowed order.

The resulting expression for $f(\alpha)$ now is

$$f(\alpha) = \frac{2p}{n} X_3(\alpha, \phi_0)\, CC_0 \Big[-\ 2q\, \{2(2q^2-1)S^4 + S^3S_0 + 3p^2S^2\}$$

$$+ (A_1-B_1)\, S\, \{(4q^2-1)S^2 + SS_0 + 2p^2\} + 2q(A_1B_2 - A_2B_1)S^2$$

$$+ (A_3-B_3 + A_1B_2 - A_2B_1)\, S + K_1(S+S_0) + K_2 \Big]$$

$$- \frac{2p}{n} X_3(\alpha, \phi_0) \frac{CC_0}{D} \Big[-2q\, \{ (SS_0 + p^2)^3 - p^6\} + (A_1-B_1)(S+S_0)\, \{(SS_0+p^2)^2 - 2p^4\}$$

$$+ 2q\, (A_1B_2-A_2B_1)\, (SS_0+p^2)^2 + (A_3-B_3 + A_1B_2-A_2B_1)(S+S_0)(SS_0+p^2)$$

$$+ 2q\, (A_3B_1 + A_1B_3 - A_2B_2)(SS_0+p^2) + (A_3B_2-A_2B_3)(S+S_0)$$

$$+ 2q\, A_3B_3 + \{K_1(S+S_0) + K_2\}\, \{ (S+S_0)^2 - 4q^2\, (SS_0 + p^2) \} \Big] \qquad (40)$$

where K_1 and K_2 are arbitrary constants independent of ϕ_0. The second group of terms is symmetric in α and ϕ_0 and of order -1, but the first group violates both of these requirements. It is therefore necessary to choose the particular solution $h(\alpha)$ such that

$$h(\alpha+\pi) - h(\alpha-\pi) = -\frac{2p}{n}\, X_3(\alpha, \phi_0)\, CC_0 \Big[-2q\, \{2(2q^2-1)S^4 + S^3S_0 + 3p^2S^2\}$$
$$+ (A_1-B_1)\, S\, \{(4q^2-1)S^2 + SS_0 + 2p^2\} + 2q\, (A_1B_2-A_2B_1)S^2$$
$$+ (A_3-B_3+A_1B_2-A_2B_1)S + K_1(S+S_0) + K_2 \Big]. \qquad (41)$$

Comparison with (35) shows how rapidly the complexity of $h(\alpha)$ increases with the order of the imposed boundary condition . Similarly to the second order case we may again conjecture that $K_1 = K_2 = 0$ but the existence of $h(\alpha)$ for an arbitrary Φ remains to be shown.

IV. Determination of the Field

To determine the field $u(\rho, \phi)$ given by (13) we may close the contour γ by two steepest descent paths through $\alpha = \pm\pi$. In this process, we may capture the geometrical optics poles located at $\alpha = \alpha_1 = -\phi + \phi_0$ and $\alpha = \alpha_2 = -\phi - \phi_0 + 2\Phi$ as well as possible surface wave poles located at $\alpha = \alpha_3^{\pm} = -\phi + \pi + \Phi + \phi_1^{\pm}$ and $\alpha = \alpha_4^{\pm} = -\phi - (\pi + \Phi + \phi_2^{\pm})$.
located at $\alpha = a_3^{\pm} = -\phi + \pi + \Phi + \theta_1^{\pm}$ and $\alpha = a_4^{\pm} = -\phi - (\pi + \Phi + \theta_2^{\pm})$. If captured, the residue of these poles must be added to the total field $u(\rho, \phi)$. The remaining contribution (the non-residue contribution) is the diffracted field given by

$$u(\rho, \phi) = -\frac{1}{2\pi i}\int_{S(\phi)} e^{ik\rho\cos(\alpha-\phi)}\{s(\alpha+\pi) - s(\alpha-\pi)\}\, d\alpha \tag{42}$$

where $S(\phi)$ is a steepest descent path through $\alpha = \phi$. For the first order GIBC

$$\{s(\alpha+\pi) - s(\alpha-\pi)\} = f(\alpha) = f_1(\alpha, \phi_0) \tag{43}$$

with $f(\alpha)$ as given in (30). In case of the second order GIBC we have from (19), (34) and (35)

$$\begin{aligned}\{s(\alpha+\pi) - s(\alpha-\pi)\} &= f_2(\alpha, \phi_0)\\ &= -\frac{2p}{n} X_2(\alpha, \phi_0)\frac{CC_0}{D}\Big[\, 2q\,(SS_0+p^2)^2 - (A_1-B_1)(S+S_0)(SS_0+p^2)\\ &\quad + 2q\,(A_2+B_2 - A_1B_1)(SS_0+p^2) + (A_2B_1 - A_1B_2)(S+S_0)\\ &\quad + 2q\,A_2B_2 \Big]\end{aligned} \tag{44}$$

with $X_2(\alpha, \phi_0)$ as defined in (25), A_i and B_i as defined in (32), p and q as defined in (24), S, S_0, C and C_0 as defined in (18b) and D as given in (27). Finally, for the third order GIBC we obtain from (19), (40) and (41)

$$\{s(\alpha+\pi)-s(\alpha-\pi)\} = f_3(\alpha, \phi_0)$$

$$= -\frac{2p}{n} X_3(\alpha, \phi_o) \frac{CC_0}{D}\Big[-2q\{(SS_0+p^2)^3-p^6\}+(A_1-B_1)(S+S_0)\{(SS_0+p^2)^2-2p^4\}$$

$$+ 2q(A_1B_2-A_2B_1)(SS_0+p^2)^2+(A_3-B_3+A_1B_2-A_2B_1)(S+S_0)(SS_0+p^2)$$

$$+ 2q(A_3B_1+A_1B_3-A_2B_2)(SS_0+p^2)+(A_3B_2-A_2B_3)(S+S_0)$$

$$+ 2q\, A_3B_3 \tag{45}$$

with $X_3(\alpha, \phi_0)$ as defined in (25), A_i and B_i as defined in (37), p and q as defined in (24), S, S_0, C and C_0 as defined in (18b) and D as given in (27).

A non-uniform evaluation of (42) now yields

$$u(\phi, \phi_0) \sim \frac{e^{ik\rho}}{\sqrt{\rho}} D(\phi' \phi_0) \quad , \tag{46}$$

where $D(\phi, \phi_0)$ is the associated diffraction coefficient given by

$$D(\phi,\phi_0) = -\frac{1}{\sqrt{2\pi k}} e^{i\pi/4} \{s(\phi+\pi)-s(\phi-\pi)\}$$

$$= -\frac{1}{\sqrt{2\pi k}} e^{i\pi/4} f_M(\phi, \phi_0) \tag{47}$$

for the Mth order GIBC. The corresponding echowidth is given by

$$\sigma = 2\pi\, |D(\phi, \phi_0)|^2 = \frac{|f_M(\phi,\phi_0)|^2}{k} \tag{48}$$

Figures 11 to 14 include several backscatter patterns for coated wedges with a variety of material coatings. As a simple verification of the validity of the derived second and 3rd order solutions, figures 11 and 12 present the echowidth patterns for a wedge with an electrically ($\varepsilon = 1+i\ 10^3$, $\mu=1$) or a magnetically ($\varepsilon=1$, $\mu=1+i\ 10^5$) conducting coating. It is seen that all solutions, regardless of order, are in agreement and predict the traditional known patterns. The patterns in figure 13 correspond to a wedge with an internal angle of 30 degrees (n=1.833) and coated with a uniform lossless dielectric layer having ε=4 and μ=1. It is now observed that although the first and second order solutions are reasonably close, the third order one deviates from both of these, particularly for the thicker coating. Interestingly, the three solutions do not display substantial variance near edge-on incidences, but in contrast, for incidences near grazing, the third order solution (which provides an accurate simulation of the reflected field) predicts much higher echowidth. This is surprising and is currently investigated on whether it is inherent to the employed simulation and not necessarily related to a physical phenomenon.

The patterns in figure 14 correspond again to a similar wedge which is now coated with an absorber layer having ε=7.4+i1.11 and μ=1.4+i0.672. As expected, the general level of the echowidth patterns is now lower, particularly for the thinner coating. However, we again observe a similar disagreement among the solutions for grazing incidences. This, of course, demonstrates the inadequacy of the first and second order solutions in simulating the coatings.

V. Summary

The problem considered was that of diffraction by a coated wedge of arbitrary angle. To obtain a solution for the diffracted field a simulation of the coating was first developed using higher order impedance boundary conditions, referred to here generalized impedance boundary conditions(GIBC). A qualitative assessment was then given on their accuracy. In particular, the second and third order GIBCs were examined in some detail. This examination amounted in comparing the coating's plane wave reflection coefficient predicted by the employed GIBC with the exact. It was found that if a phase error of 10 degrees (or 10% in magnitude, but the phase error criterion is usually more limiting) was acceptable, the second order GIBC was capable of simulating coatings up to 0.25 wavelengths in thickness regardless of the coating's material composition provided the incidence was away from normal. This last limitation prompted the need to consider simulations using a third order GIBC. It should be noted, however, that the second order GIBC is probably sufficient for most practical materials with some loss and under the same conditions the maximum allowed thickness for a 10 degree error could also be relaxed. The employed third order GIBC was found far superior to the second order one. In particular, the third order GIBC allowed simulation of coatings as thick as 0.4 wavelengths regardless of their composition and the angle of incidence for the same error criteria. Similarly, to the second order GIBC the maximum allowed thickness for a 10 degree error can again be relaxed for high contrast and/or lossy coatings.

The solution to the scattered and diffracted fields by the coated wedge were obtained via a generalization of Maliuzhinets ' method. This required the introduction of a particular solution $h(\alpha)$ to the difference equations which result upon application of the subject GIBC imposed on the wedge faces. Such a particular solution is zero for the standard impedance boundary condition, but is required for higher order GIBCs and causes substantial complexity in the analysis. It serves to correct the order of the spectral function under the Sommerfeld integral and allows for a reciprocal solution in the last phase of the analysis. Fortunately, it does not appear in the final result and, therefore, the only requirement (for a valid solution of the diffracted field) is a proof of its existence. Once established, the particular solution can be ignored and the solution for the diffracted field can then be obtained by retaining the terms of the homogeneous solution satisfying reciprocity as well as the edge condition.

The second part of this report presents solutions of the diffracted field based on the second and third order GIBCs. In both cases the necessary difference equations satisfied by $h(\alpha)$ are stated, but not solved. Essentially, the diffracted field is derived by ensuring reciprocity. As mentioned above, this amounts to the tedious task of partitioning the homogeneous solution into reciprocal and nonreciprocal terms. Since the last are to be cancelled by the particular solution $h(\alpha)$, the diffracted field is the contribution of the remaining reciprocal terms.

Finally, using the derived diffraction coefficients for the first, second, and third order GIBCs a number of backscatter echowidth patterns were presented. These included several coating configurations and demonstrated the strong variance of the third order solution from the first and second order ones for grazing incidences. The FORTRAN program used for generating the patterns is given in the Appendix.

References

1. T.B.A. Senior, "Approximate Boundary Conditions," *IEEE Trans. Antennas and Propagat.,* Vol. AP-29, No. 5, pp 826-829, 1981.

2. T.B.A. Senior and J.L. Volakis, "Derivation and Application of a Class of Generalized Boundary Conditions" submitted to *IEEE Trans. Antennas and Propagat.;* see also University of Michigan Radiation Laboratory 388967-7-T.

3. S.N. Karp and F.C. Karal, Jr., "Generalized Impedance Boundary Conditions with Applications to Surface Waves Structures," in Electromagnetic Wave Theory, Part 1, ed. J. Brown, pp. 479-483, Pergamon: New York, 1965.

4. G.D. Maliuzhinets, "Excitation, Reflection and Emission of Surface Waves from a Wedge with Given Face Impedances," *Sov. Phys. Dokl., Engl. Transl.,* Vol. 3, pp. 752-755, 1958.

5. J.L. Volakis and T.B.A. Senior, "Application of a Class of Generalized Boundary Conditions to Scattering by a Metal-Backed Dielectric Half-Plane," *Proceeding of t he IEEE,* May 1989 (in press); see also *University of Michigan Radiation Laboratory Report.* 388967-6-T.

6. J.L. Volakis and T.B.A. Senior, "Diffraction by a Thin Dielectric Half-Plane," *IEEE Trans. Antennas and Propagat.,* Vol. AP-35, pp. 1483-1487, Dec. 1987.

7. J.L. Volakis, "High Frequency Scattering by a Material Half-Plane and Strip," *Radio Sci.,* Vol. 23, pp. 450-463, May-June 1988.

8. T.B.A. Senior, "Diffraction by a Right-Angled Second Order Impedance Wedge," submitted to *Electromagnetics.*

9. M. I. Herman, J.L. Volakis and T.B.A. Senior, "Analytic Expressions for a Function Occurring in Diffraction Theory," *IEEE Trans. Antennas and Propagat.,* Vol. AP-35, Sept. 1987, pp. 1083-1086.

10. G.D. Maliuzhinets, "Inversion Formula for the Sommerfeld Integral," *Sov. Phys. Dokl., Eng. Transl.,* Vol. 3, pp. 52-56, 1958.

List of Figure Captions

12. Backscatter H_z echowidth for a plane wave incident on a perfectly conducting wedge having n=1.5 and coated on both faces with 0.1 wavelengths thick electrically ($\varepsilon=1+i10^3$) or magnetically ($\mu=1.+i10^5$) perfectly conducting layer. (a) Magnetically perfectly conducting layer. (b) Electrically perfectly conducting layer.

13. Backscatter H_z echowidth for a plane wave incident on a perfectly conducting wedge having n=1.833 and coated on both faces with a layer whose $\varepsilon=4$ and $\mu=1$. (a) Coating thickness $\tau=0.1$ wavelength. (b) Coating thickness $\tau=0.2$ wavelengths.

14. Backscatter H_z echowidth for a plane wave incident on a perfectly conducting wedge having n=1.833 and coated on both faces with an absorbing layer whose $\varepsilon=7.4+i1.1$ and $\mu=1.4+i0.672$. (a) Coating thickness $\tau=0.1$ wavelengths. (b) Coating thickness $\tau=0.2$ wavelengths.

APPENDIX

FORTRAN Listing of Program GIBCWEDGE

```
C                PROGRAM   GIBCWEDGE                                  C
C     THIS PROGRAM COMPUTES THE DIFFRACTED/SCATTERED FIELD FROM   C
C     A DIELECTRICALLY COATED WEDGE                                   C
C     IF FAR ZONE IS CHOSEN ONLY DIFFRACTED FIELD IS GIVEN            C
C     IF NEAR ZONE IS CHOSEN THE TOTAL UNIFORM FIELD IS COMPUTED  C
C     PROGRAM WRITTEN BY J.L. VOLAKIS, AUGUST 1988                    C
C                                                                     C
      COMPLEX A0,A1,A2,A3,A4,G1,G2,G3,C0,C1,C2,C3,RI
      COMPLEX B1,B2,B3,A11,B11,CC,DD,EE,ETA4,DDC
      COMPLEX*8 AU,BU,AL,BL,QQ,PP,RR,XM,THX,CPOWER,GEE,PSIPH0,PSIPH
      COMPLEX PSIPI2,PSI,PSIPHI,AC2(3),BC2(3),AC3(3),BC3(3),CONST
      COMPLEX THP1(3),THP2(3),THP3(3),THM1(3),THM2(3),THM3(3)
      COMPLEX REFL1,REFL2,REFL3,REFL4,HZ1,HZ2,HZ3
      COMPLEX TH1,TH2,TH3,ER,UR,CF1,CF2,CF3,AA,BB,AB,APB,CFF,DFC
      COMPLEX CI,CI4,ETA,ETA1,ETA2,ETA3,HEE,RINDX,DEN,TEMP,ETAN
      COMPLEX CSQRC,LGEE,PHC,CFFG,ARG,FFCL,HA,GINCB,GINCT,REFLN
      COMPLEX GUP,GBOT,FKP1,FKP2,FKP3,SUM,SGEE,TH1L,TH2L,REFLH
      COMPLEX SWRES1,SWRES2,HAP,FKP4,FKP5,COSW1,COSW2,SWD1,SWD2
      COMPLEX GAM1,GAM2,TEMPC,ROOT,R1,R2,EXREFL,CTAN,REFLL,PHASE
      DIMENSION HZDB1(361),HZDB2(361),HZDB3(361),ANG(361)
      COMMON /CONS/AA,BB,AB,CF3
      COMMON /PS/PSIPI2
      DATA CI,PI,CI4/(0.,-1.),3.1415926,(.707107,-.707107)/
      PI2=PI/2.
      TPI=2.*PI
      PRINT *,'ENTER WEDGE ANGLE IN DEGREES:'
      READ(5,*) WA
      DTR=PI/180.
      WA=WA*DTR
      WN=2-(WA/PI)
      PHIW=WN*PI/2.
      PRINT *,'WEDGE CAP PHI AND N:',PHIW/DTR,WN
      PSIPI2=PSIPHI(CMPLX(PI2,0.),PHIW)
C      PRINT *,'PSIPI2',PSIPI2
      PRINT *,'ENTER PH,PH0,DPH,DPH0(IN DEG):'
      READ(5,*) PH,PH0,DPH,DPH0
C      PRINT *,'NUMBER OF PATTERNS:'
C      READ(5,*) NPLOTS
      DTR=PI/180.
      NPTS=WN*PI/DPH/DTR
      PH=PH*DTR
      PH0=PH0*DTR
      PHI=PH
      PH0I=PH0
      DPH=DPH*DTR
      DPH0=DPH0*DTR
C BEGIN PLOT LOOP
C      DO 2000 J=1,NPLOTS
      PH=PHI
      PH0=PH0I
      PRINT *,'ENTER PERMIT.,PERMEAB.(EXP(+JWT)) AND THICK:'
      READ(5,*) ER,UR,THK
C EXACT REFLECTION COEFFICIENT
      SPH0=SIN(PH0)
      CPH0=COS(PH0)
      CPH0S=CPH0*CPH0
      SPH0S=SPH0*SPH0
C ENSURE THE CORRECT BRANCH
```

```
      ER=ER+CI*1.E-6
      UR=UR+CI*1.E-6
C COMPUTE EXACT REFLECTION COEFFICIENT
      RINDX=CSQRC(ER*UR)
      ROOT=CSQRC(RINDX*RINDX-CPH0S)
      ARG=2.*PI*THK*ROOT
      CTAN=CSIN(ARG)/CCOS(ARG)
      R1=ROOT*CTAN
      R2=CI*ER*SPH0
      EXREFL=-(R1-R2)/(R1+R2)
      CTAN=CSIN(2.*PI*THK*RINDX)/CCOS(2.*PI*THK*RINDX)
      ETA=-CI*RINDX*CTAN/ER
C      PHASE=CEXP(-2.*CI*2*PI*THK*SPH0)
C      EXREFL=EXREFL*PHASE
C "A" COEFFICIENTS OF GIBC
      AR1=2.*PI*THK
      RI=RINDX
      C1=CSIN(AR1*RI)/CCOS(AR1*RI)
      C2=CSIN(.5*AR1/RI)/CCOS(.5*AR1/RI)
      C3=C1*C2+1.
      C1=C1-C2
      A0=(RI-(.5/RI))*C1
      A1=CI*ER*C3
      A2=(C1+AR1*C3*(RI-(.5/RI)))/(2.*RI)
      A3=-CI*AR1*ER*C1/(2.*RI)
      A4=AR1*C3/(4.*RI*RI)
      REFL4=-(A4*(SPH0**4)-A3*(SPH0**3)+A2*(SPH0**2)-A1*SPH0+A0)
      TEMPC=(A4*(SPH0**4)+A3*(SPH0**3)+A2*(SPH0**2)+A1*SPH0+A0)
      REFL4=REFL4/TEMPC
      REFL2=-(A2*(SPH0**2)-A1*SPH0+A0)/(A2*(SPH0**2)+A1*SPH0+A0)
      REFL3=-(-A3*(SPH0**3)+A2*(SPH0**2)-A1*SPH0+A0)
      TEMPC=(A3*(SPH0**3)+A2*(SPH0**2)+A1*SPH0+A0)
      REFL3=REFL3/TEMPC
C FIRST ORDER BOUNDARY CONDITION
      THP1(1)=PI2-HEE(ETA,1,1.)
      THM1(1)=THP1(1)
      PRINT *,'1ST ORDER TEST on THETA:',ETA,CSIN(THP1(1))
      PRINT *,'1st ORDER THETA:',THP1(1)
      REFL1=-(ETA-SPH0)/(ETA+SPH0)
C SECOND ORDER BOUNDARY CONDITION
      G1=A1/(A0+A2)
      G2=-A2/(A0+A2)
      TEMPC=CSQRT(G1*G1+4.*G2*(1.+G2))
      ETA1=0.5*(-G1+TEMPC)/G2
      ETA2=0.5*(-G1-TEMPC)/G2
      PRINT *,'2ND ORDER ETAS:',ETA1,ETA2
C REFLECTION COEF. USING ETAS
      REFL2=-(ETA1-SPH0)*(ETA2-SPH0)/((ETA1+SPH0)*(ETA2+SPH0))
      THP2(1)=PI2-HEE(ETA1,1,1.)
      THP2(2)=PI2-HEE(ETA2,1,1.)
      IF(REAL(THP2(1)).LT.0.)THP2(1)=PI-THP2(1)
      IF(REAL(THP2(2)).LT.0.)THP2(2)=PI-THP2(2)
C     THP2(2)=THP2(2)-REAL(THP2(2))
      THP2(2)=THP2(2)
      THM2(1)=THP2(1)
      THM2(2)=THP2(2)
      PRINT *,'TH1,TH2:',THP2(1),THP2(2)
      PRINT *,'CHECK:',CSIN(THP2(1)),ETA1,CSIN(THP2(2)),ETA2
```

```
C 3RD ORDER GIBC
      G1=(A1+A3)/(A0+A2)
      G2=-A2/(A0+A2)
      G3=-A3/(A0+A2)
C     find cubic roots
      PP=-G2/G3
      QQ=-(G1+G3)/G3
      RR=(1.+G2)/G3
      AL=(3.*QQ-PP*PP)/3.
      BL=(2.*PP*PP*PP-9.*PP*QQ+27.*RR)/27.
      XM=2.*(0.,1.)*CSQRC(AL/3.)
      ARG=3.*BL/(AL*XM)+CI*1.E-6
      THX=HEE(ARG,1,1.)/3.
      ETA1=XM*CCOS(THX)-PP/3
      ETA2=XM*CCOS((PI/1.5)+THX)-PP/3
      ETA3=XM*CCOS((4.*PI/3.)+THX)-PP/3
      PRINT *,'3RD ORDER ETAS:',ETA1,ETA2,ETA3
C     reflection coefficient using etas
      REFL3=-(ETA1-SPH0)*(ETA2-SPH0)*(ETA3-SPH0)
      REFL3=REFL3/((ETA1+SPH0)*(ETA2+SPH0)*(ETA3+SPH0))
      THP3(1)=PI2-HEE(ETA1,1,1.)
      THP3(2)=PI2-HEE(ETA2,1,1.)
      THP3(3)=PI2-HEE(ETA3,1,1.)
      DEF1=CABS(THP3(1)-THP1(1))
      DEF2=CABS(THP3(2)-THP1(1))
      DEF3=CABS(THP3(3)-THP1(1))
      IF((DEF1.LT.DEF2).AND.(DEF1.LT.DEF3)) THEN
       TH1=THP3(1)
       TH2=THP3(2)
       TH3=THP3(3)
      ENDIF
      IF((DEF2.LT.DEF1).AND.(DEF2.LT.DEF3)) THEN
       TH1=THP3(2)
       TH2=THP3(1)
       TH3=THP3(3)
      ENDIF
      IF((DEF3.LT.DEF2).AND.(DEF3.LT.DEF1))THEN
       TH1=THP3(3)
       TH2=THP3(2)
       TH3=THP3(1)
      ENDIF
      THP3(1)=TH1
      THP3(2)=TH2
      THP3(3)=TH3
      THP3(2)=PI-THP3(2)
      THP3(3)=PI-THP3(3)
      THM3(1)=THP3(1)
      THM3(2)=THP3(2)
      THM3(3)=THP3(3)
      PRINT *,'3RD ORDER THETAS:',THP3(1),THP3(2),THP3(3)
      PRINT *,'CHECK:',CSIN(THP3(1)),ETA1,CSIN(THP3(2)),ETA2
C PRINT REFLECTION COEFFICIENTS:
      PRINT *,'          REFLECTION COEFFICIENTS:'
      EXMAG=CABS(EXREFL)
      EXPHAS=BTAN2(AIMAG(EXREFL),REAL(EXREFL))
      ELMAG=CABS(REFL2)
      ELPHAS=BTAN2(AIMAG(REFL2),REAL(REFL2))
      EHMAG=CABS(REFL3)
```

```
      EHPHAS=BTAN2(AIMAG(REFL3),REAL(REFL3))
      ENMAG=CABS(REFL4)
      ENPHAS=BTAN2(AIMAG(REFL4),REAL(REFL4))
      E1MAG=CABS(REFL1)
      E1PHAS=BTAN2(AIMAG(REFL1),REAL(REFL1))
      PRINT *,'EXACT,4TH ,3RD, 2ND & 1ST ORDER:'
      PRINT *,EXREFL,EXMAG,EXPHAS*180/PI
      PRINT *,REFL4,ENMAG,ENPHAS*180./PI
      PRINT *,REFL3,EHMAG,EHPHAS*180./PI
      PRINT *,REFL2,ELMAG,ELPHAS*180/PI
      PRINT *,REFL1,E1MAG,E1PHAS*180/PI
C      PRINT *,'CHOOSE 1ST, 2ND OR 3RD ORDER B.C:'
C      READ(5,*) M
C GENERATE A & B   CONSTANTS
      A11=CCOS((THP1(1)-PI2)/WN)
      B11=CCOS((THM1(1)-PI2)/WN)
      PRINT *,'THETA:',THP1(1)
      DO 10 MS=1,2
      AC2(MS)=CCOS((THP2(MS)-PI2)/WN)
10    BC2(MS)=CCOS((THM2(MS)-PI2)/WN)
      DO 11 MS=1,3
      AC3(MS)=CCOS((THP3(MS)-PI2)/WN)
11    BC3(MS)=CCOS((THM3(MS)-PI2)/WN)
      P1=SIN(PI2/WN)
      Q1=COS(PI2/WN)
C BEGIN PATTERN LOOP
      DO 1000 I=1,NPTS
      A1=A11
      B1=B11
      SPN=SIN(PH/WN)
      S0N=SIN(PH0/WN)
      CPN=COS(PH/WN)
      C0N=COS(PH0/WN)
      CONST=CI4/(2.*PI)
      CONST=CONST/WN
      P2=P1*P1
      Q2=Q1*Q1
      DENOM=SPN*SPN-2.*(2.*Q2-1.)*SPN*S0N+S0N*S0N-4.*P2*Q2
      DNOM1=SIN((PH-PI)/WN)-S0N
      DNOM2=SIN((PH+PI)/WN)-S0N
      DNOM3=DNOM1*DNOM2
      D1=DNOM1
      HZ1=CI4/(2.*PI)
      HZ1=(HZ1*C0N/WN)
      CONST=HZ1/DENOM
C FISRT ORDER STADARD MALIUZHINETS DIFFR. COEFF.
C      PSIPH0=PSI(CMPLX(PH0,0.),PHIW,THP1(1),THM1(1))
C      PSIPH=PSI(CMPLX(PH,0.),PHIW,THP1(1),THM1(1))
C      HZ1=HZ1/PSIPH0
C      TEMPC=PSI(CMPLX(PH-PI,0.),PHIW,THP1(1),THM1(1))/DNOM1
C      TEMP=PSI(CMPLX(PH+PI,0.),PHIW,THP1(1),THM1(1))
      HZ1=HZ1*(TEMPC-TEMP/DNOM2)
C 1ST ORDER IMPEDANCE WEDGE DIFFRACTION COEFFICIENT
      TEMPC=(2*Q1*(SPN*S0N+P2)-(A1-B1)*(SPN+S0N)-2.*Q1*A1*B1)
      TEMPC=TEMPC*CPN*2.*P1
      XM=GEE(CMPLX(PH,0.),PH0,PHIW,THP1,THM1,1)
      HZ1=-XM*TEMPC*CONST
C 2ND ORDER IMP. WEDGE DIFFRACTION COEFFICINET
```

```
      A1=AC2(1)+AC2(2)
      B1=BC2(1)+BC2(2)
      A2=AC2(1)*AC2(2)
      B2=BC2(1)*BC2(2)
      ST=SPN*S0N+P2
      SP0N=SPN+S0N
      TEMPC=2.*Q1*(ST**2)-(A1-B1)*SP0N*ST+2.*Q1*(A2+B2-A1*B1)*ST
      TEMPC=TEMPC+(A2*B1-A1*B2)*SP0N+2.*Q1*A2*B2
      TEMPC=TEMPC*2.*P1*CPN
      XM=GEE(CMPLX(PH,0.),PH0,PHIW,THP2,THM2,2)
      HZ2=CONST*XM*TEMPC

C 3RD ORDER IMP. DIFFR. COEFFICIENT
      A1=AC3(1)+AC3(2)+AC3(3)
      B1=BC3(1)+BC3(2)+BC3(3)
      A2=AC3(1)*AC3(2)+AC3(2)*AC3(3)+AC3(3)*AC3(1)
      B2=BC3(1)*BC3(2)+BC3(2)*BC3(3)+BC3(3)*BC3(1)
      A3=AC3(1)*AC3(2)*AC3(3)
      B3=BC3(1)*BC3(2)*BC3(3)
      ST=SPN*S0N+P2
      SP0N=SPN+S0N
      TEMPC=-2.*Q1*(ST**3-P2**3)+(A1-B1)*SP0N*(ST*ST-2.*P2*P2)
      TEMPC=TEMPC+2.*Q1*(A1*B2-A2*B1)*(ST*ST)
      TEMPC=TEMPC+(A3-B3+A1*B2-A2*B1)*SP0N*ST
      TEMPC=TEMPC+2.*Q1*(A3*B1+A1*B3-A2*B2)*ST+(A3*B2-A2*B3)*ST
      TEMPC=TEMPC+2.*Q1*A3*B3
      TEMPC=TEMPC*2.*P1*CPN
      XM=GEE(CMPLX(PH,0.),PH0,PHIW,THP3,THM3,3)
      HZ3=CONST*XM*TEMPC
      HZA=CABS(HZ1)
      HZDB1(I)=10.*ALOG10(2.*PI*HZA*HZA)
      HZA=CABS(HZ2)
      HZA2=10*ALOG10(2.*PI*HZA*HZA)
      HZDB2(I)=HZA2
      HZA=CABS(HZ3)
      HZA3=10.*ALOG10(2.*PI*HZA*HZA)
      HZDB3(I)=HZA3
      ANG(I)=PH/DTR
C     PRINT *,'PH,HZ:',ANG(I),HZ1,HZ2,HZ3
      PH=PH+DPH
      PH0=PH0+DPH0
1000  CONTINUE
C      IEND=0
C      IF(J.EQ.NPLOTS)IEND=1
      CALL GENPLO(ANG,HZDB3,NPTS,0,0)
      CALL GENPLO(ANG,HZDB2,NPTS,1,0)
      CALL GENPLO(ANG,HZDB1,NPTS,2,1)
2000  CONTINUE
      CALL EXIT
      END
C
      COMPLEX FUNCTION GEE(ARG,PH0,PHIW,THP,THM,M)
      COMPLEX PSI,ARG,THP(3),THM(3),PSIPI2
      COMMON /PS/PSIPI2
      DATA PI,PI2/3.1415927,1.5707963/
      M8=8.*M
      GEE=PSIPI2**M8/(4.**M)
      DO 10 MM=1,M
```

```
      GEE=GEE/PSI(ARG,PHIW,THP(MM),THM(MM))
      GEE=GEE/PSI(CMPLX(PH0,0.),PHIW,THP(MM),THM(MM))
c     if(M.eq.3) print *,'gee:',gee
10    CONTINUE
      RETURN
      END
C
      COMPLEX FUNCTION PSI(ALPHA,PHI,THETAO,THETA1)
      COMPLEX PSIPHI
      COMPLEX THETAO,THETA1,UJ,CJ,ALPHA
      COMPLEX CN1,CN2,CN3,CN4
      DATA PI2/1.5707963/
      DATA CJ,UJ/(0.,1.),(1.,0.)/
      CN1=ALPHA+(PHI+PI2)*UJ-THETAO
      CN2=ALPHA-(PHI+PI2)*UJ+THETA1
      CN3=ALPHA+(PHI-PI2)*UJ+THETAO
      CN4=ALPHA-(PHI-PI2)*UJ-THETA1
c     print *,'cn1,2,3,4###:',cn1,cn2,cn3,cn4
      CN1=PSIPHI(CN1,PHI)
      CN2=PSIPHI(CN2,PHI)
      CN3=PSIPHI(CN3,PHI)
      CN4=PSIPHI(CN4,PHI)
      PSI=CN1*CN2*CN3*CN4
c     print *,'cn1,cn2,cn3:',cn1,cn2,cn3,cn4
      RETURN
      END
C
      COMPLEX FUNCTION PSIPHI(CANG,PHI)
C     WRITTEN BY MARTIN HERMAN, UNIVERSITY OF MICHIGAN
C     BASED ON THE SUBMITTED ARTICLE BY HERMAN, VOLAKIS
C     AND SENIOR.    12/1/86
C
C     CANG : Complex Argument of the function
C
C     THIS CALCULATES THE MALUIZHINETS FUNCTON
C     FOR ANY ARBITRARY WEDGE ANGLE PHI
C
C     IF THE IMAGINARY PART IS LESS THAN 10 THEN A
C     REIMANN SUM IS PERFORMED, OTHERWISE THE LARGE ARG
C     FORM IS USED
C
C     PSIPI2 IS THE COMPLEX MAL. FUNCTION OF THE IDENTITY
C     GIVEN IN HERMAN, VOLAKIS, AND SENIOR
C
      COMPLEX CANG,CANG1,COEF
      COMPLEX CN1,CN2,CN3,CN4,CN5
      COMPLEX PSISQ,U,UJ
      DATA PI,PI2/3.141592654,1.5707963/
      DATA UJ/(1.,0.)/
C
C  CALCULATE PSISQ USING REIMAN SUM
C
C|||
C|||  MIDPOINT METHOD 5 POINTS (INTERVAL 0,1.5)
C|||
      UR=PI/2.
      UI=0.
      SUM=0.
```

```
      SUM1=0.
      FH=1.5/5.
      FH2=FH/2.
      DO 10 I=1,5
      S=FLOAT(I-1)*FH+FH2
      FS=(COSH(UR*S)*COS(UI*S)-1.)
      DENOM=S*COSH(PI*S/2.)*SINH(PHI*2.*S)
      FS=FS*FH/DENOM
      FS1=(SINH(UR*S)*SIN(UI*S))
      FS1=FS1*FH/DENOM
      SUM=SUM+FS
      SUM1=SUM1+FS1
   10 CONTINUE
      CN1=-.5*CMPLX(SUM,SUM1)
      CN1=CEXP(CN1)
      PSISQ=CN1*CN1
C
      AR=REAL(CANG)
      AI=AIMAG(CANG)
      ITTT=0
      IF(AR.GT.0)GO TO 30
       AR=-AR
       ITTT=1
30    ITT=0
      IF(AI.GT.0)GO TO 40
       ITT=1
       AI=-AI
40    IT=0
      CANG1=CMPLX(AR,AI)
      IF(AR.LT.PI2)GO TO 90
       AR=AR-PI
C
       CN5=UJ*(PI*PI/(8.*PHI))
       CN4=UJ*PI
       R5=PI/(4.*PHI)
C
       COEF=PSISQ*CCOS(CANG1*R5-CN5)
       IT=1
      IF(AR.LT.PI2)GO TO 90
       AR=AR-PI
       COEF=(CCOS(CANG1*R5-CN5))/CCOS((CANG1-CN4)*R5-CN5)
       IT=2
      IF(AR.LT.PI2)GO TO 90
       AR=AR-PI
       COEF=PSISQ*CCOS(CANG1*R5-CN5)*CCOS((CANG1-2.*CN4)*R5-CN5)
       COEF=COEF/CCOS((CANG1-CN4)*R5-CN5)
       IT=3
      IF(AR.LT.PI2)GO TO 90
      AR=AR-PI
      COEF=CCOS((CANG1-2.*CN4)*R5-CN5)*CCOS(CANG1*R5-CN5)
      COEF=COEF/(CCOS((CANG1-3.*CN4)*R5-CN5)*CCOS((CANG1-CN4)*R5-CN5))
      IT=2
      IF(AR.LT.PI2)GO TO 90
      AR=AR-PI
      COEF=CCOS((CANG1-2.*CN4)*R5-CN5)*CCOS(CANG1*R5-CN5)
      COEF=COEF/(CCOS((CANG1-3.*CN4)*R5-CN5)*CCOS((CANG-CN4)*R5-CN5))
      COEF=PSISQ*COEF*CCOS((CANG1-4.*CN4)*R5-CN5)
      IT=3
```

```
  90   CONTINUE
       IF(ABS(AR).GT.PI2)PRINT *,'AR > PI2 ',AR
C
       U=CMPLX(AR,AI)
       UR=AR
       UI=AI
       IF(UI.LE.10.)THEN
C|||
C|||   SMALL ARG APPROACH USING
C|||   MIDPOINT METHOD 5 POINTS (INTERVAL 0,1.5)
C|||
       SUM=0.
       SUM1=0.
       FH=1.5/5.
       FH2=FH/2.
       DO 100 I=1,5
       S=FLOAT(I-1)*FH+FH2
       FS=(COSH(UR*S)*COS(UI*S)-1.)
       DENOM=S*COSH(PI*S/2.)*SINH(PHI*2.*S)
       FS=FS*FH/DENOM
       FS1=(SINH(UR*S)*SIN(UI*S))
       FS1=FS1*FH/DENOM
       SUM=SUM+FS
       SUM1=SUM1+FS1
  100  CONTINUE
       CN1=-.5*CMPLX(SUM,SUM1)
       PSIPHI=CEXP(CN1)
       ELSE
C|||
C|||    large APPROX
C|||
       CN1=U*PI/(4.*PHI)
       CN2=CCOS(CN1)
       AMP=CABS(CN2)
       AMP=SQRT(AMP)
       R1=REAL(CN2)
       R2=AIMAG(CN2)
       PH=ATAN2(R2,R1)
       IF(PH.LT.0.)PH=2.*PI+PH
       PH=PH/2.
       R1=AMP*COS(PH)
       R2=AMP*SIN(PH)
       CN1=CMPLX(R1,R2)
        B=2.556343
        C=-3.259678
        D=1.659306
        E=-.3883548
        F=.03473964
       PSIPHI=CN1*EXP(-(B*PHI+C*PHI**2+D*PHI**3+E*PHI**4+
      1F*PHI**5)/PI)
       IF(REAL(PSIPHI).LT.0.)PSIPHI=-PSIPHI
       END IF
C
       IF(IT.EQ.1)PSIPHI=COEF/PSIPHI
       IF(IT.EQ.2)PSIPHI=COEF*PSIPHI
       IF(IT.EQ.3)PSIPHI=COEF/PSIPHI
       IF(ITT.EQ.1)PSIPHI=CONJG(PSIPHI)
       IF(ITTT.EQ.1)PSIPHI=CONJG(PSIPHI)
```

```
      RETURN
      END

      COMPLEX FUNCTION HEE(ETA,IUD,SB0)
C||| NEW VOLAKIS VERSION
CIII COMPUTES THE INVERSE COSINE OF COMPLEX NUMBER
      COMPLEX ETA,ETA1,CJ
      DOUBLE PRECISION RE,AE,REP,REM,AA,BB,SGN,RAA
      DATA SRT2,FPI,CJ/1.414213562,12.56637061,(0.,1.)/
      DATA PSIPI2,PI/.9656228,3.14159265/
      ETA1=1./(ETA*SB0)
      IF(IUD.EQ.1)ETA1=ETA/SB0
      RE=REAL(ETA1)
      AE=AIMAG(ETA1)
      REP=RE+1.
      REM=RE-1.
      AA=.5*(DSQRT(REP*REP+AE*AE)+DSQRT(REM*REM+AE*AE))
      BB=.5*(DSQRT(REP*REP+AE*AE)-DSQRT(REM*REM+AE*AE))
      IF(AE.NE.0.D0)THEN
       SGN=AE/DABS(AE)
      ELSE
       SGN=1.D0
      ENDIF
      RAA=AA*AA-1.
      IF(RAA.LT.1.E-6)RAA=0.
      HEE=DARSIN(BB)+CJ*DLOG(AA+DSQRT(RAA))*SGN
      HEE=.5*PI-HEE
300   RETURN
      END

          COMPLEX FUNCTION CSQRC(Z)
          COMPLEX Z
          ZR=REAL(Z)
          ZI=AIMAG(Z)
          PHAS=BTAN2(ZI,ZR)
          CSQRC=SQRT(CABS(Z))*CEXP(.5*(0.,1.)*PHAS)
          RETURN
          END
      REAL FUNCTION BTAN2(Y,X)
      DATA PI/3.1415926/
      IF(ABS(X).GT.1.E-6) GO TO 20
      IF(ABS(Y).GT.1.E-6) GO TO 10
      BTAN2=0.
      RETURN
10    BTAN2=.5*PI
      IF(Y.LT.0.)BTAN2=-BTAN2
20    BTAN2=ATAN2(Y,X)
      RETURN
      END
```

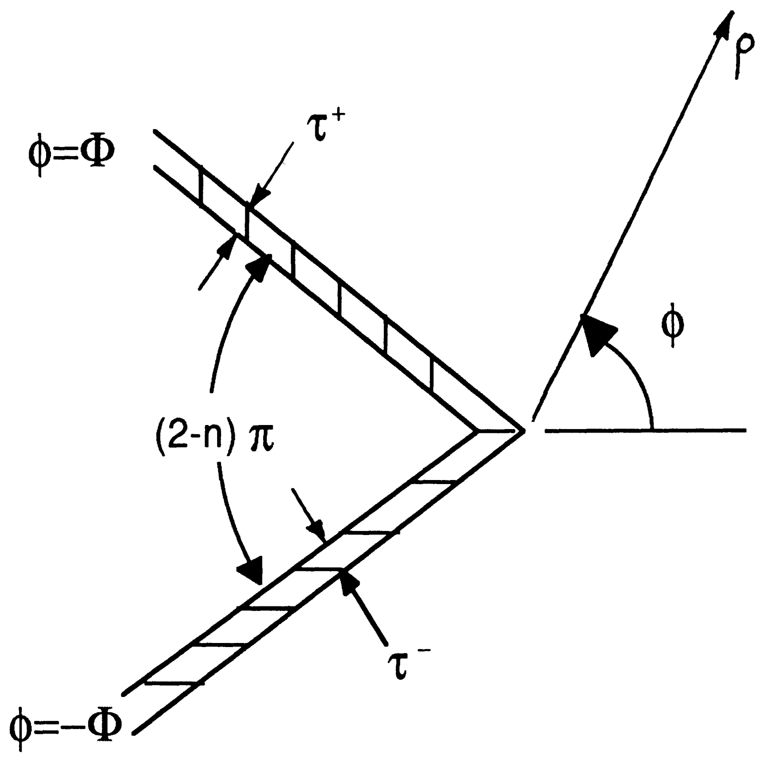
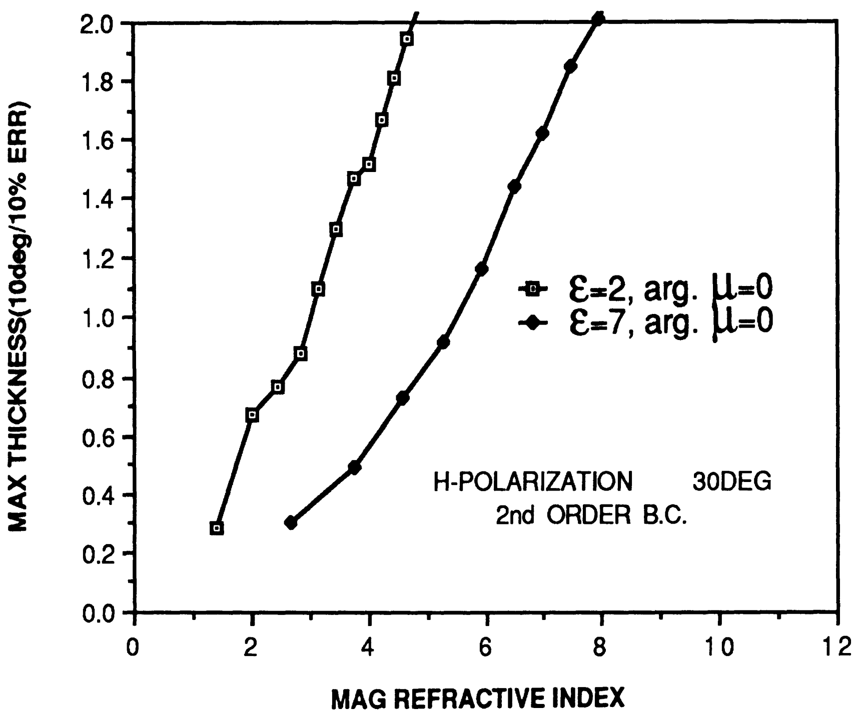
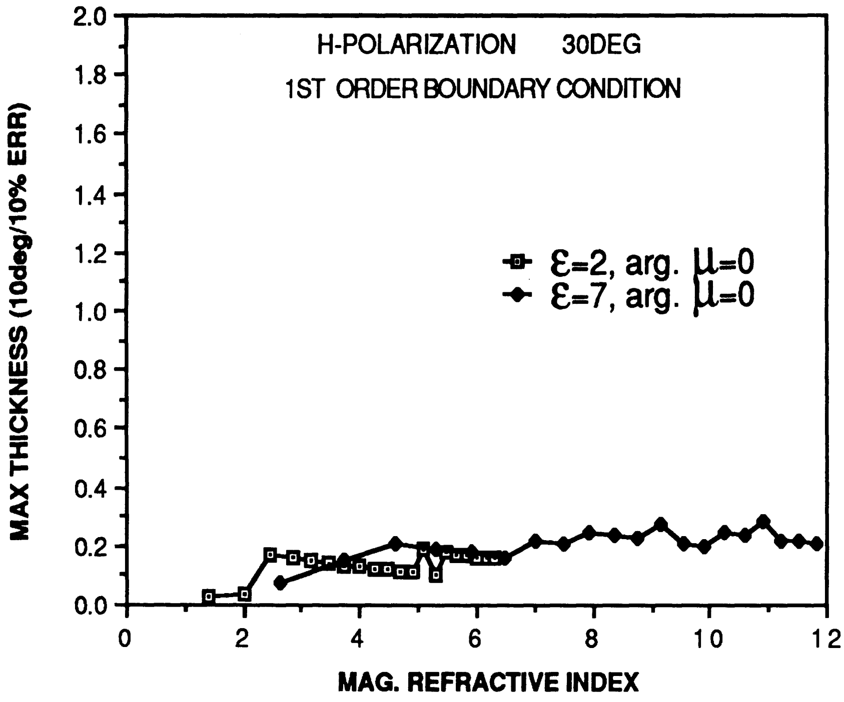


Fig. 1. Geometry of the coated wedge.



MAX THICKNESS(10deg/10% ERR)
MAG REFRACTIVE INDEX
ε=2, arg. μ=0
ε=7, arg. μ=0
H-POLARIZATION 30DEG
2nd ORDER B.C.

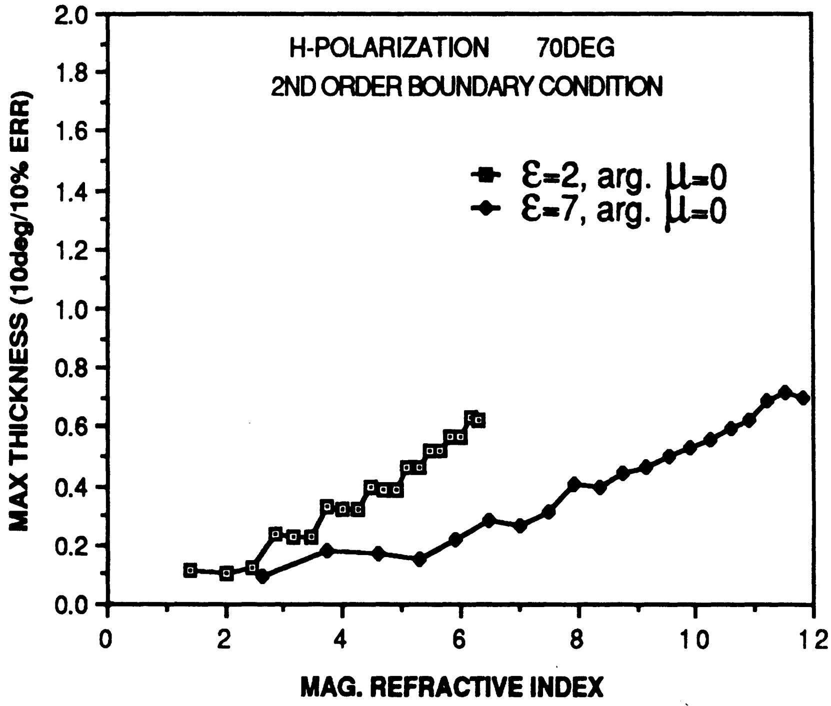
(a)



MAX THICKNESS (10deg/10% ERR)
MAG. REFRACTIVE INDEX
H-POLARIZATION 30DEG
1ST ORDER BOUNDARY CONDITION
ε=2, arg. μ=0
ε=7, arg. μ=0

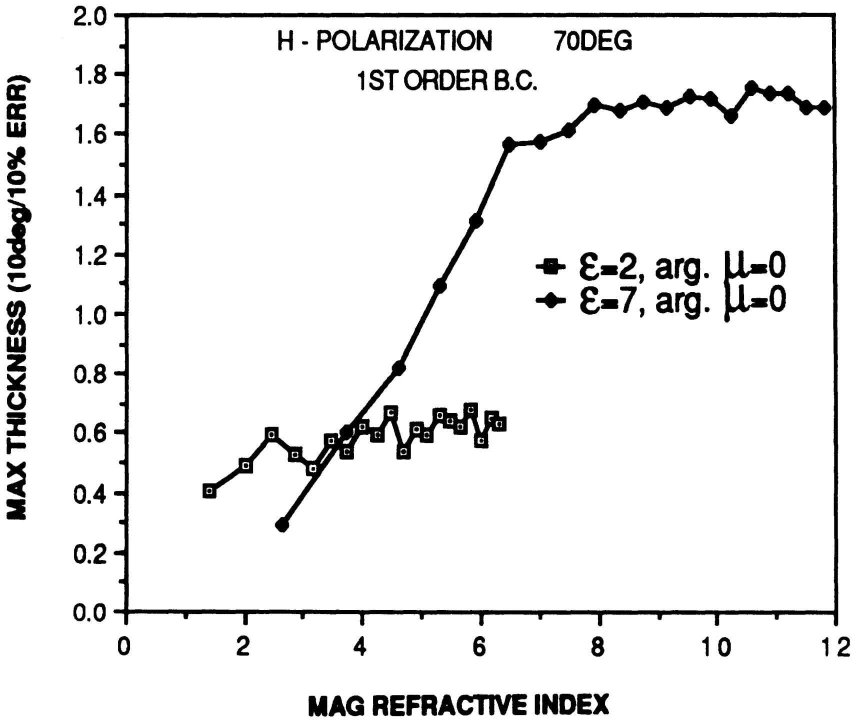
(b)

Figure 2



H-POLARIZATION 70DEG
2ND ORDER BOUNDARY CONDITION
ε=2, arg. μ=0
ε=7, arg. μ=0
MAX THICKNESS (10deg/10% ERR)
MAG. REFRACTIVE INDEX

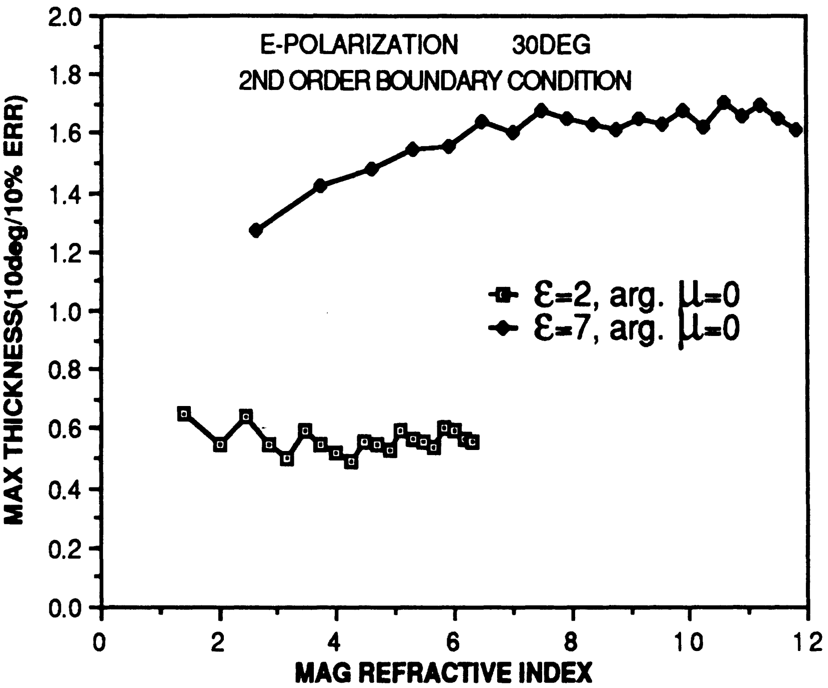
(a)



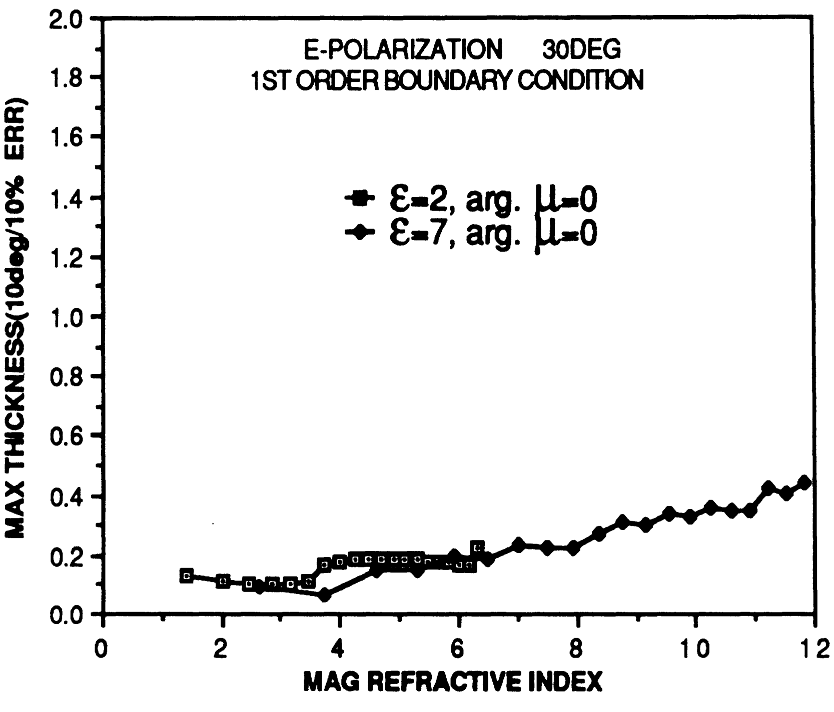
H - POLARIZATION 70DEG
1ST ORDER B.C.
ε=2, arg. μ=0
ε=7, arg. μ=0
MAX THICKNESS (10deg/10% ERR)
MAG REFRACTIVE INDEX

(b)

Figure 4

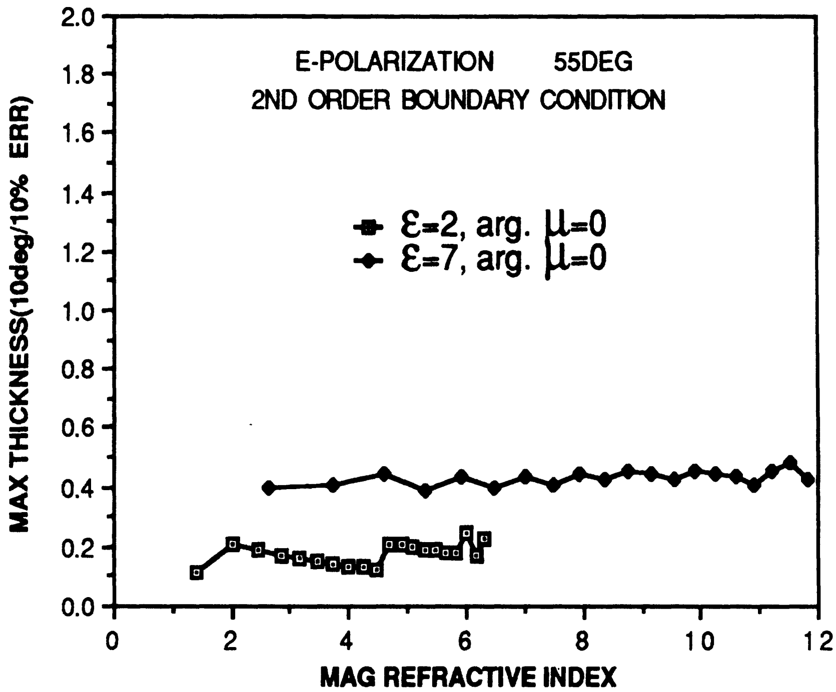

E-POLARIZATION 30DEG
2ND ORDER BOUNDARY CONDITION
ε=2, arg. μ=0
ε=7, arg. μ=0
MAX THICKNESS(10deg/10% ERR)
MAG REFRACTIVE INDEX
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
0
2
4
6
8
10
12

(a)

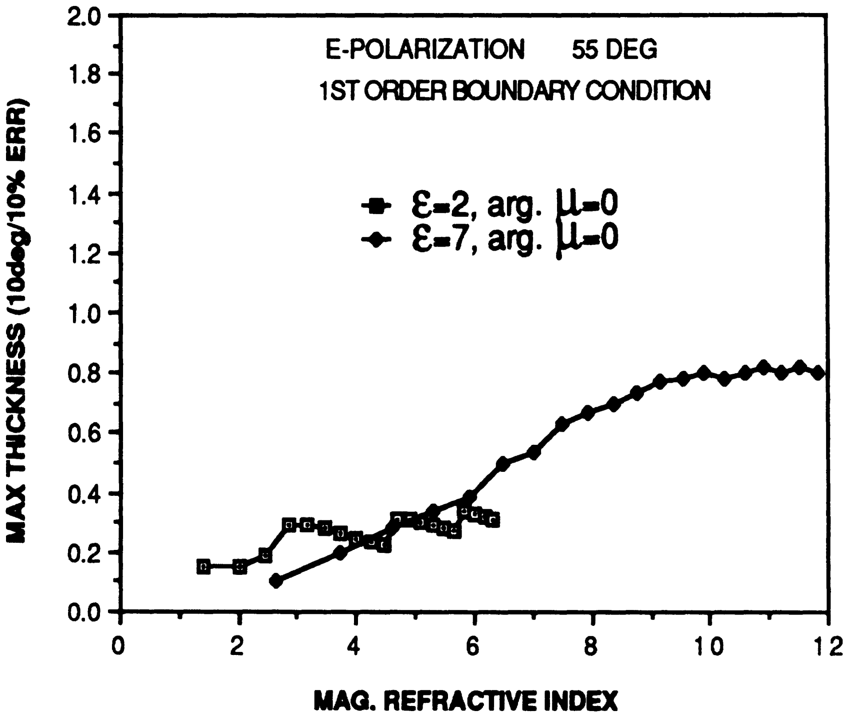

E-POLARIZATION 30DEG
1ST ORDER BOUNDARY CONDITION
ε=2, arg. μ=0
ε=7, arg. μ=0
MAX THICKNESS(10deg/10% ERR)
MAG REFRACTIVE INDEX
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
0
2
4
6
8
10
12

(b)

Figure 5

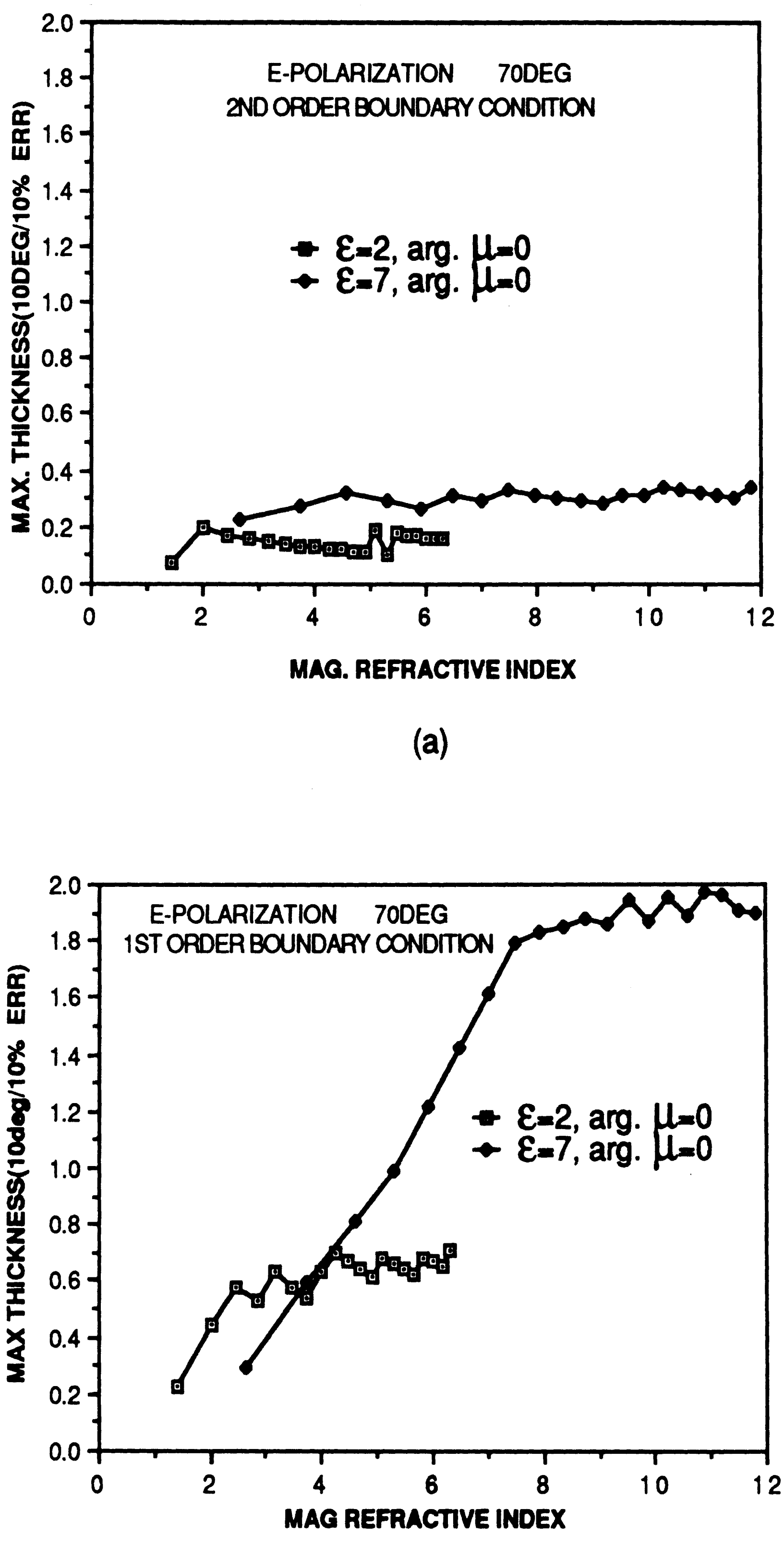

E-POLARIZATION 55DEG
2ND ORDER BOUNDARY CONDITION
ε=2, arg. μ=0
ε=7, arg. μ=0
MAX THICKNESS(10deg/10% ERR)
MAG REFRACTIVE INDEX

(a)


E-POLARIZATION 55 DEG
1ST ORDER BOUNDARY CONDITION
ε=2, arg. μ=0
ε=7, arg. μ=0
MAX THICKNESS (10deg/10% ERR)
MAG. REFRACTIVE INDEX

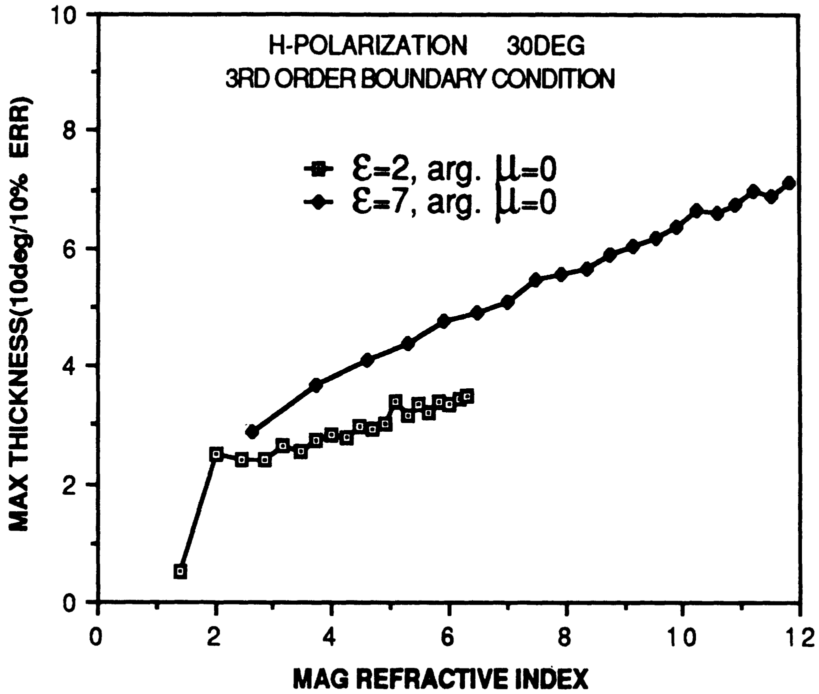
(b)

Figure 6

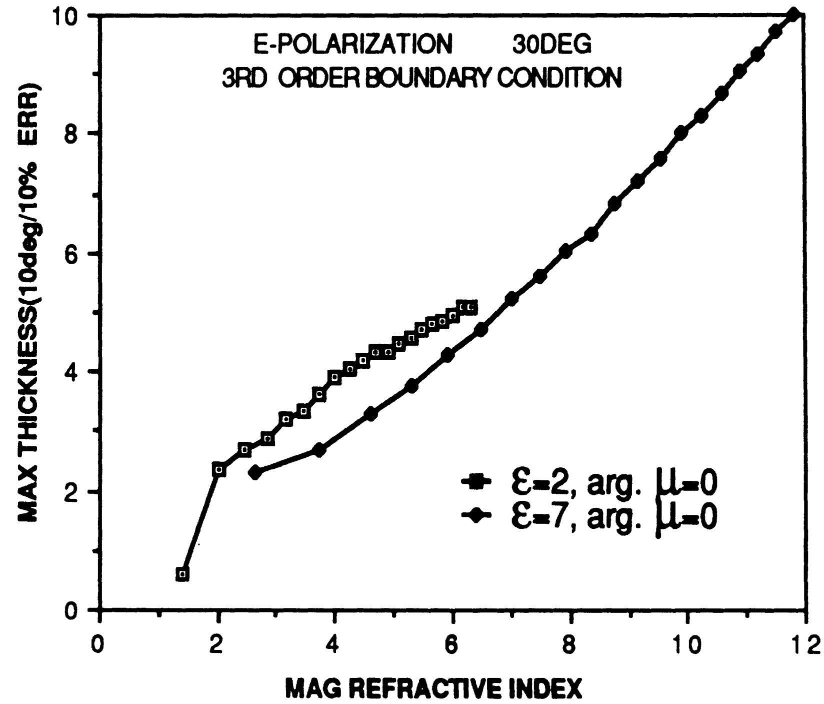


E-POLARIZATION 70DEG
2ND ORDER BOUNDARY CONDITION
ε=2, arg. μ=0
ε=7, arg. μ=0
MAX. THICKNESS(10DEG/10% ERR)
MAG. REFRACTIVE INDEX
(a)
E-POLARIZATION 70DEG
1ST ORDER BOUNDARY CONDITION
ε=2, arg. μ=0
ε=7, arg. μ=0
MAX THICKNESS(10deg/10% ERR)
MAG REFRACTIVE INDEX
(b)

Figure 7

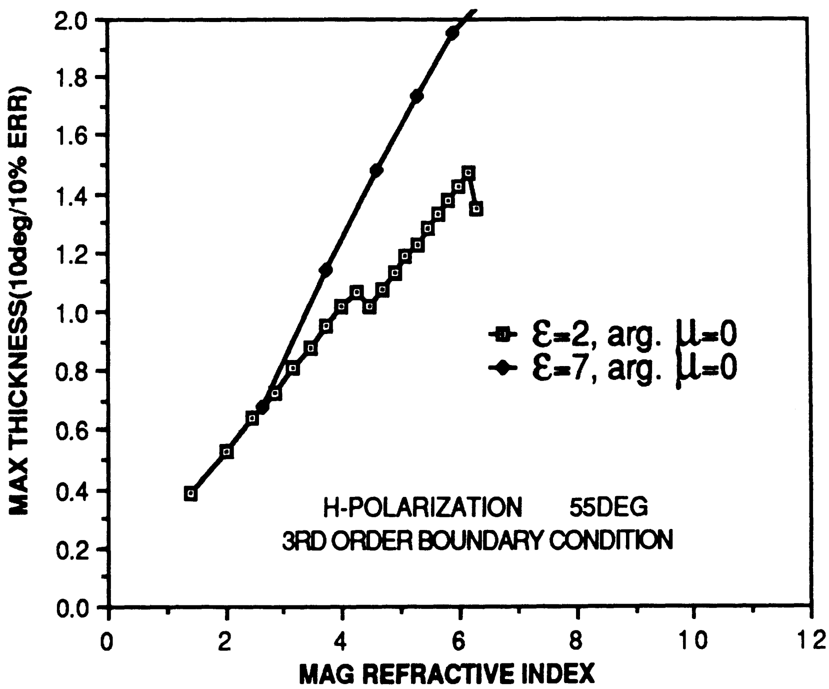

H-POLARIZATION 30DEG
3RD ORDER BOUNDARY CONDITION
ε=2, arg. μ=0
ε=7, arg. μ=0
MAX THICKNESS(10deg/10% ERR)
MAG REFRACTIVE INDEX
0
2
4
6
8
10
12

(a)


E-POLARIZATION 30DEG
3RD ORDER BOUNDARY CONDITION
ε=2, arg. μ=0
ε=7, arg. μ=0
MAX THICKNESS(10deg/10% ERR)
MAG REFRACTIVE INDEX
0
2
4
6
8
10
12

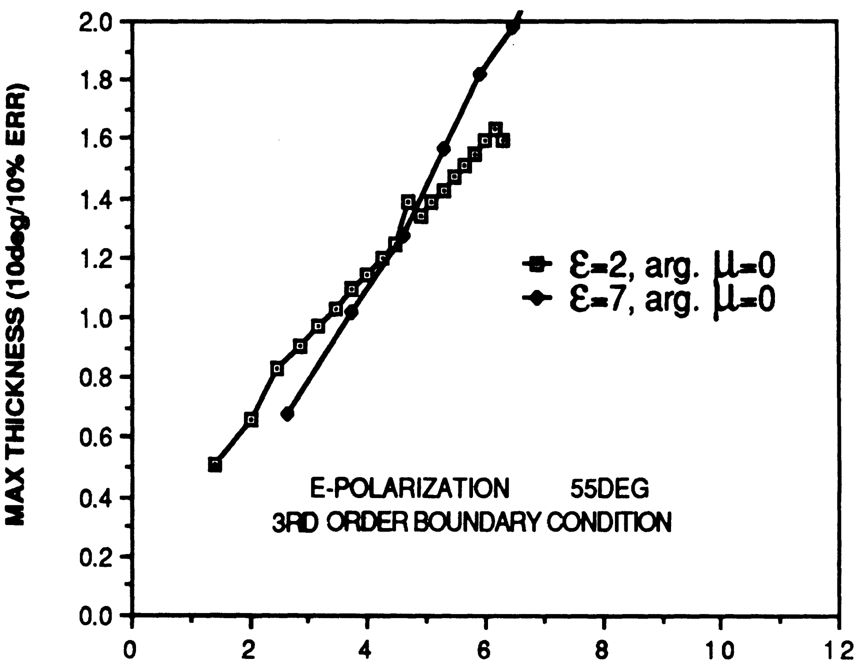
(b)

Figure 8



MAX THICKNESS(10deg/10% ERR)
MAG REFRACTIVE INDEX
ε=2, arg. μ=0
ε=7, arg. μ=0
H-POLARIZATION 55DEG
3RD ORDER BOUNDARY CONDITION

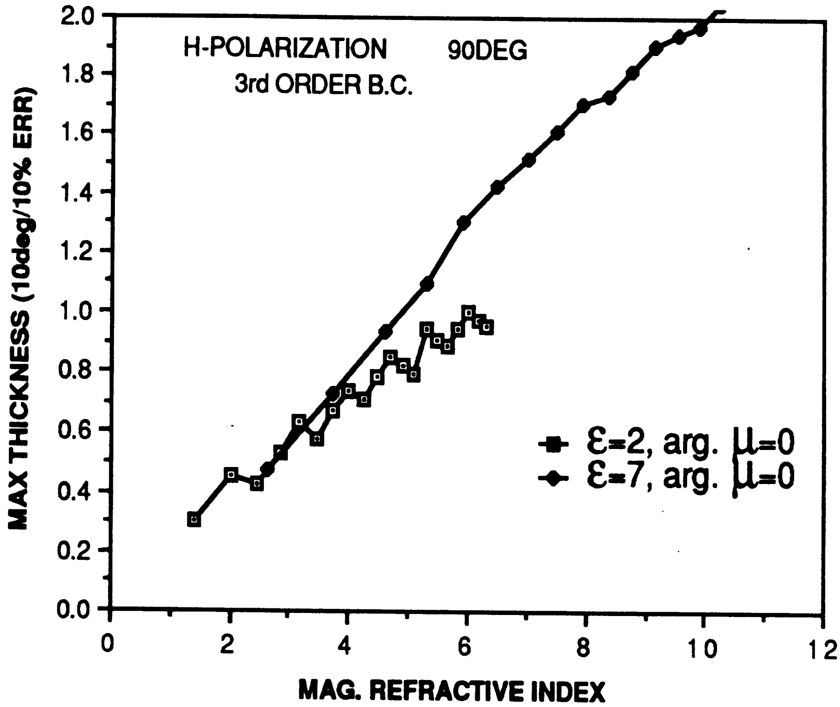
(a)



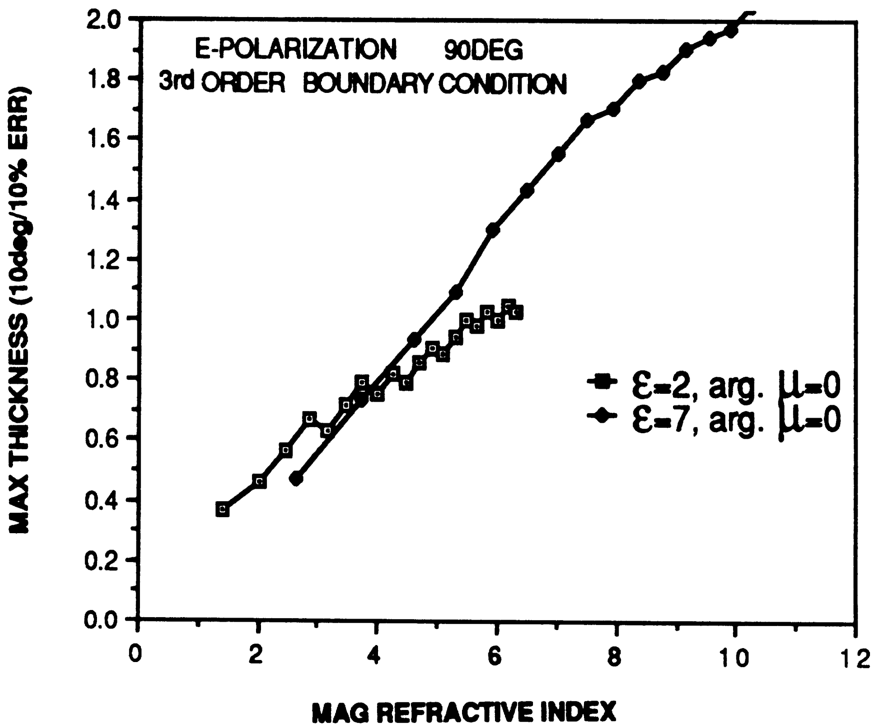
MAX THICKNESS (10deg/10% ERR)
MAG REFRACTIVE INDEX
ε=2, arg. μ=0
ε=7, arg. μ=0
E-POLARIZATION 55DEG
3RD ORDER BOUNDARY CONDITION

(b)

Figure 9

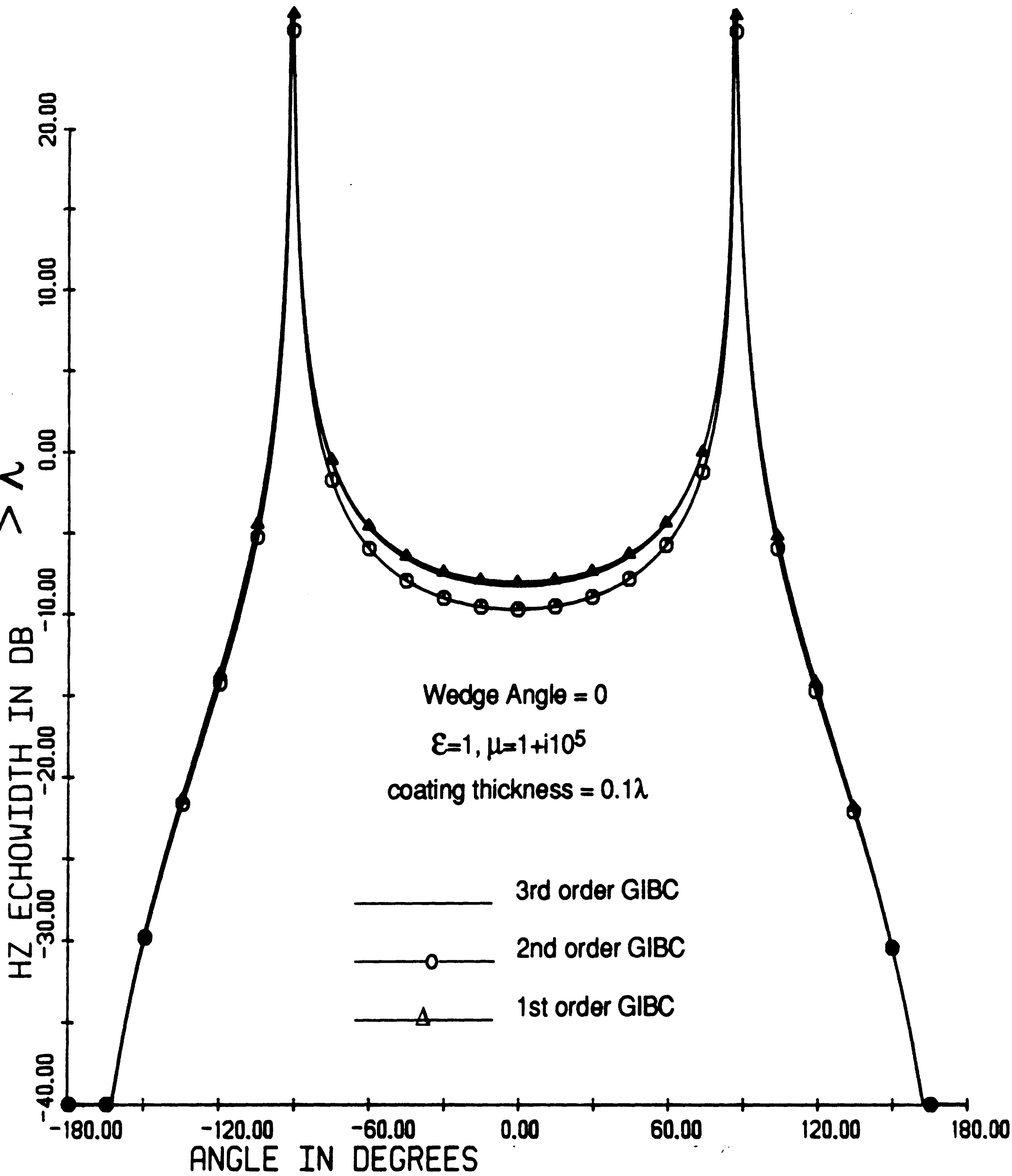

H-POLARIZATION 90DEG
3rd ORDER B.C.
MAX THICKNESS (10deg/10% ERR)
MAG. REFRACTIVE INDEX
ε=2, arg. μ=0
ε=7, arg. μ=0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
0
2
4
6
8
10
12

(a)


E-POLARIZATION 90DEG
3rd ORDER BOUNDARY CONDITION
MAX THICKNESS (10deg/10% ERR)
MAG REFRACTIVE INDEX
ε=2, arg. μ=0
ε=7, arg. μ=0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
0
2
4
6
8
10
12

(b)

Figure 10



20.00
10.00
0.00
-10.00
-20.00
-30.00
-40.00
HZ ECHOWIDTH IN DB -> λ
-180.00
-120.00
-60.00
0.00
60.00
120.00
180.00
ANGLE IN DEGREES
Wedge Angle = 0
ε=1, μ=1+i10^5
coating thickness = 0.1λ
3rd order GIBC
2nd order GIBC
1st order GIBC

Figure 11a

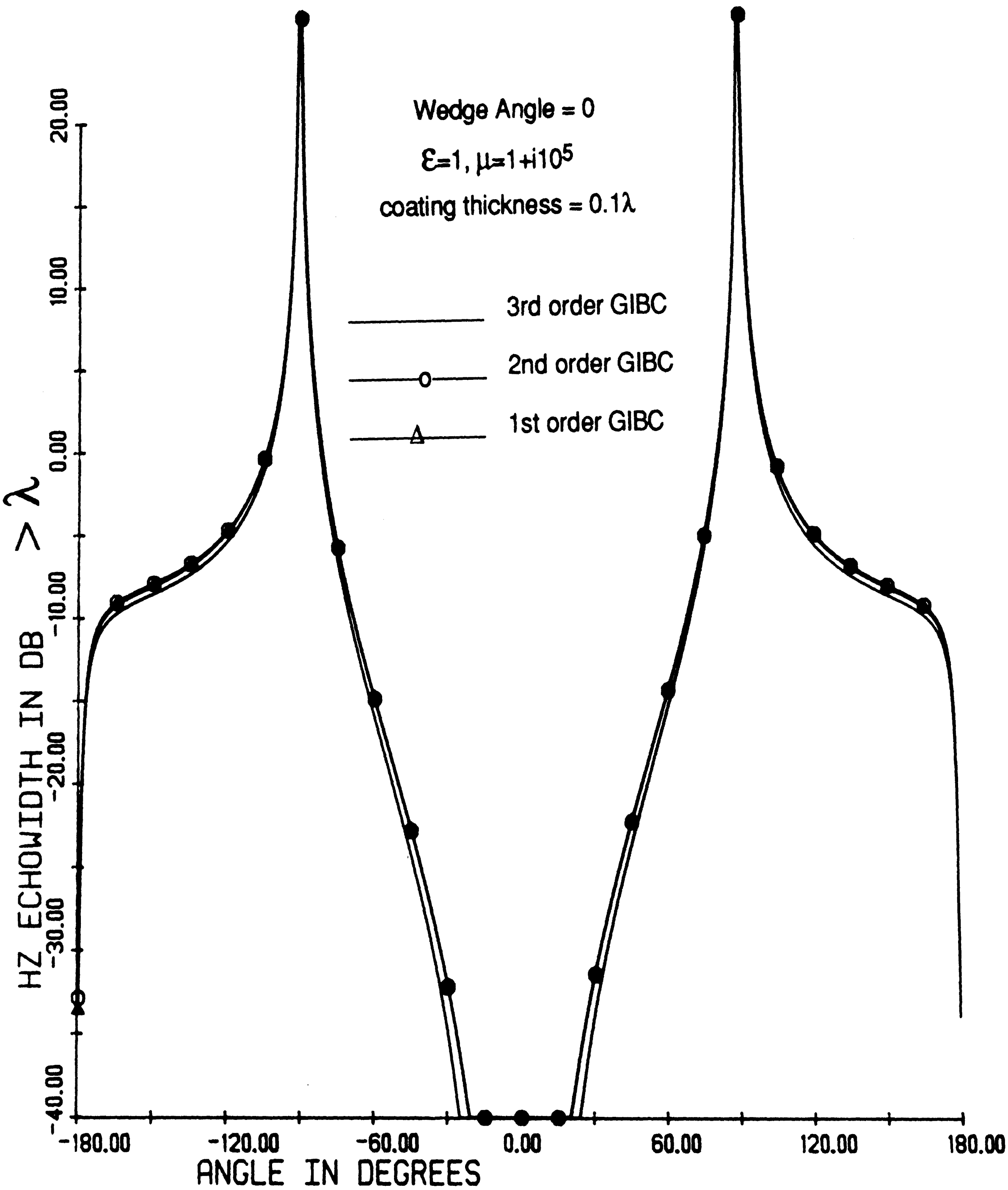

Wedge Angle = 0
ε=1, μ=1+i10^5
coating thickness = 0.1λ
3rd order GIBC
2nd order GIBC
1st order GIBC
HZ ECHOWIDTH IN DB > λ
20.00
10.00
0.00
-10.00
-20.00
-30.00
-40.00
-180.00
-120.00
-60.00
0.00
60.00
120.00
180.00
ANGLE IN DEGREES

Figure 11b

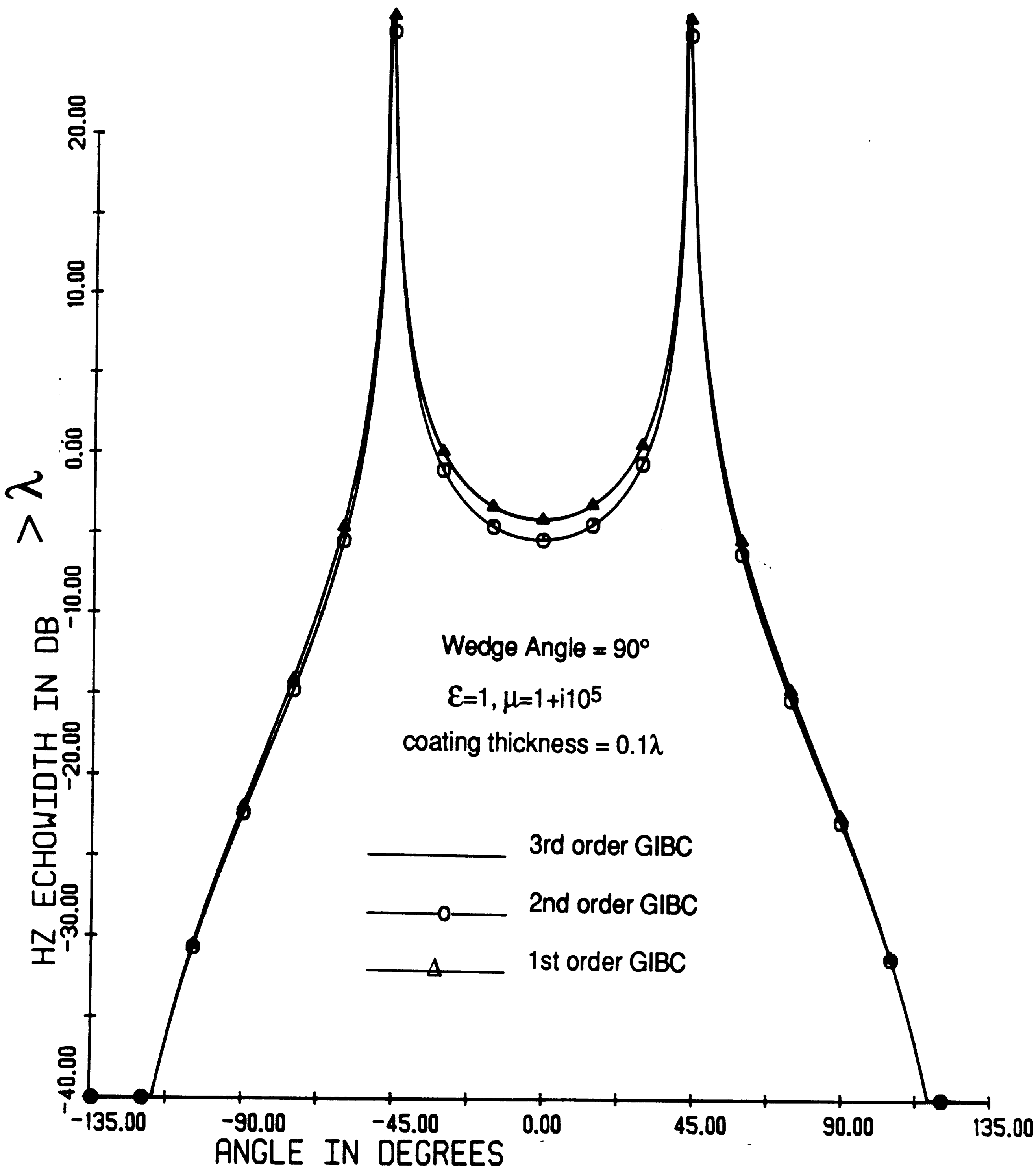

HZ ECHOWIDTH IN DB > λ
20.00
10.00
0.00
-10.00
-20.00
-30.00
-40.00
-135.00
-90.00
-45.00
0.00
45.00
90.00
135.00
ANGLE IN DEGREES
Wedge Angle = 90°
ε=1, μ=1+i10^5
coating thickness = 0.1λ
3rd order GIBC
2nd order GIBC
1st order GIBC

Figure 12a

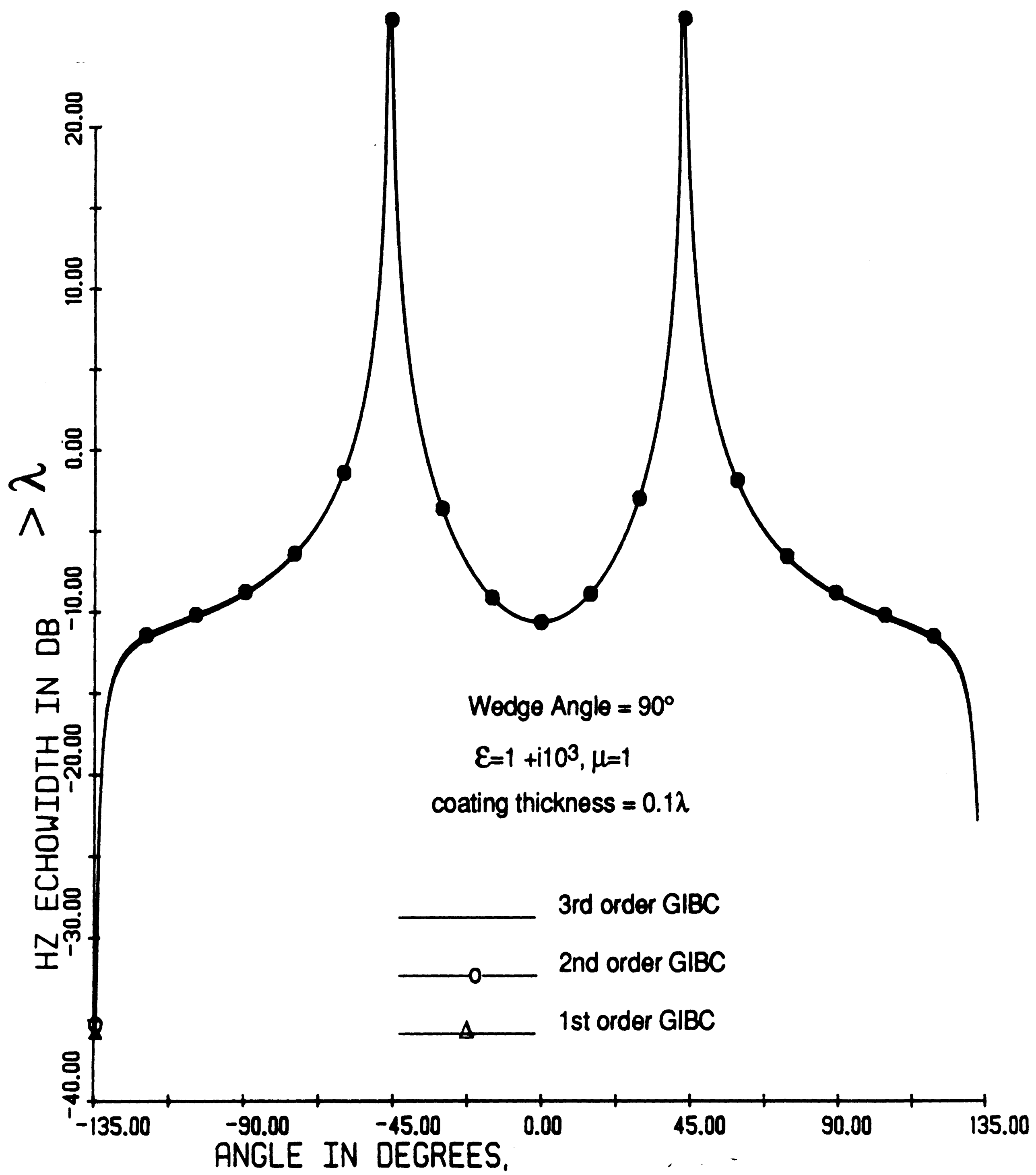

20.00
10.00
0.00
-10.00
-20.00
-30.00
-40.00
HZ ECHOWIDTH IN DB > λ
-135.00
-90.00
-45.00
0.00
45.00
90.00
135.00
ANGLE IN DEGREES,
Wedge Angle = 90°
ε=1 +i10³, μ=1
coating thickness = 0.1λ
3rd order GIBC
2nd order GIBC
1st order GIBC

Figure 12b

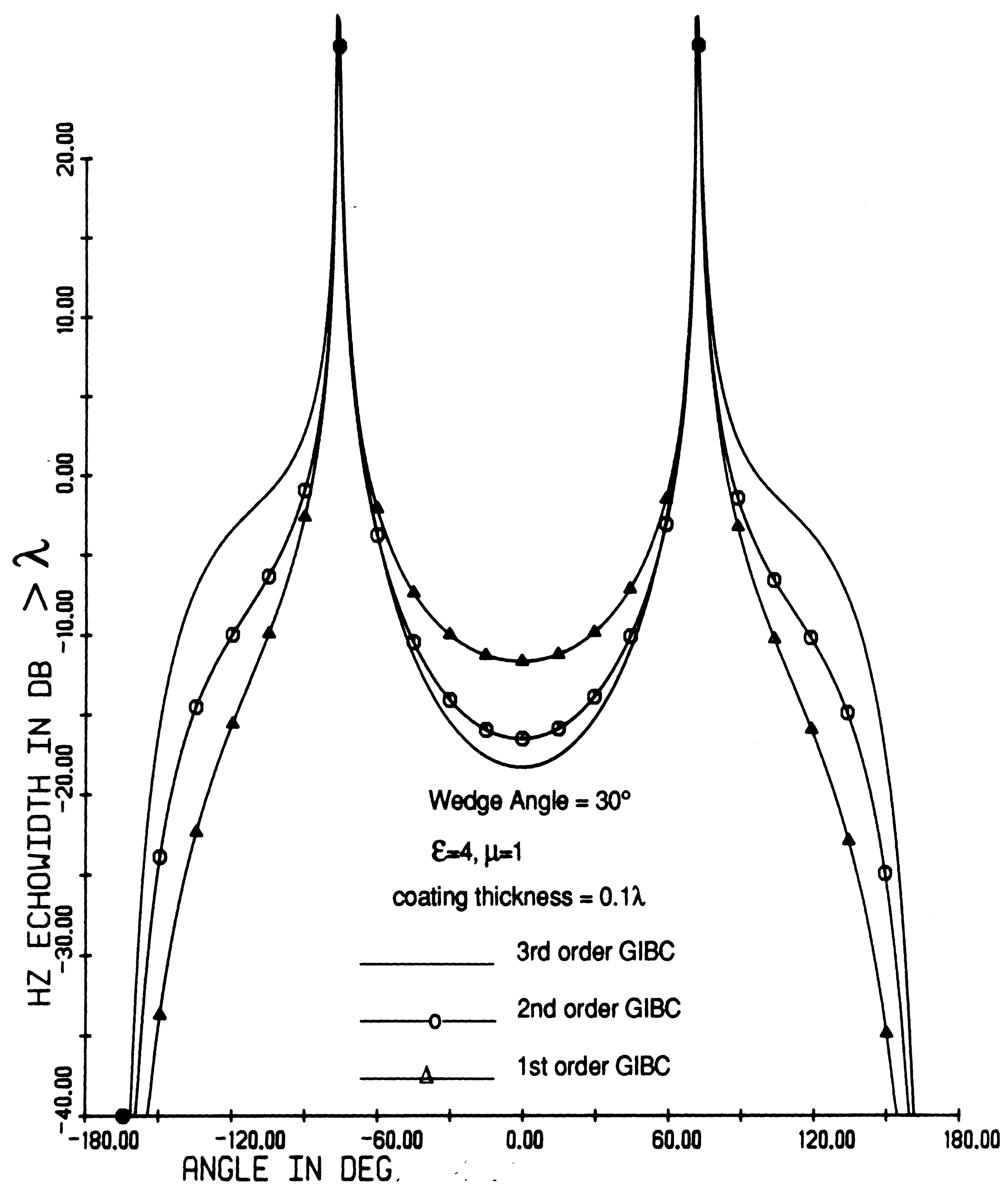
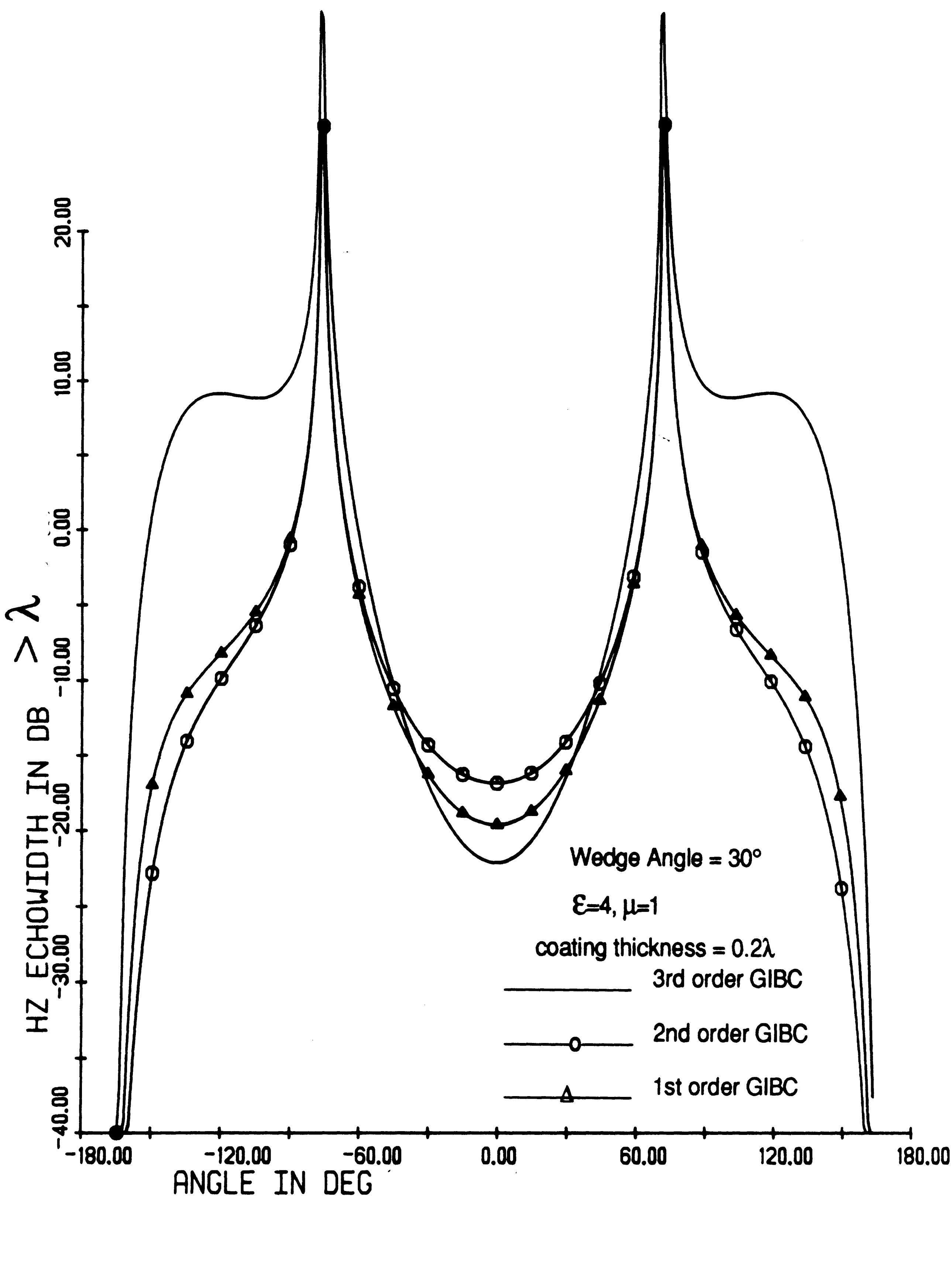

20.00
10.00
0.00
-10.00
-20.00
-30.00
-40.00
HZ ECHOWIDTH IN DB > λ
-180.00
-120.00
-60.00
0.00
60.00
120.00
180.00
ANGLE IN DEG.
Wedge Angle = 30°
ε=4, μ=1
coating thickness = 0.1λ
3rd order GIBC
2nd order GIBC
1st order GIBC

Figure 13a



HZ ECHOWIDTH IN DB >λ
ANGLE IN DEG
20.00
10.00
0.00
-10.00
-20.00
-30.00
-40.00
-180.00
-120.00
-60.00
0.00
60.00
120.00
180.00
Wedge Angle = 30°
ε=4, μ=1
coating thickness = 0.2λ
3rd order GIBC
2nd order GIBC
1st order GIBC

Figure 13b

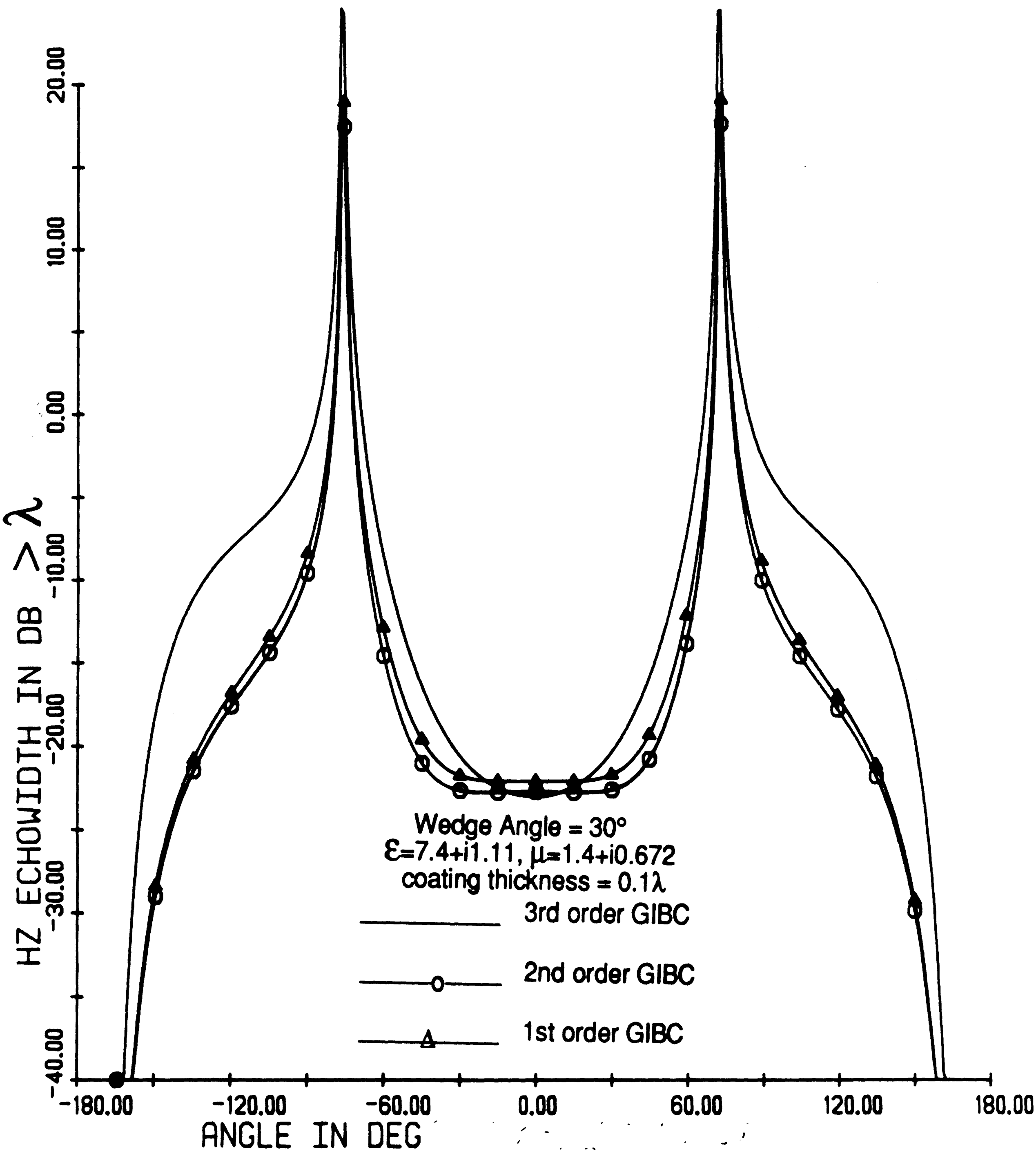
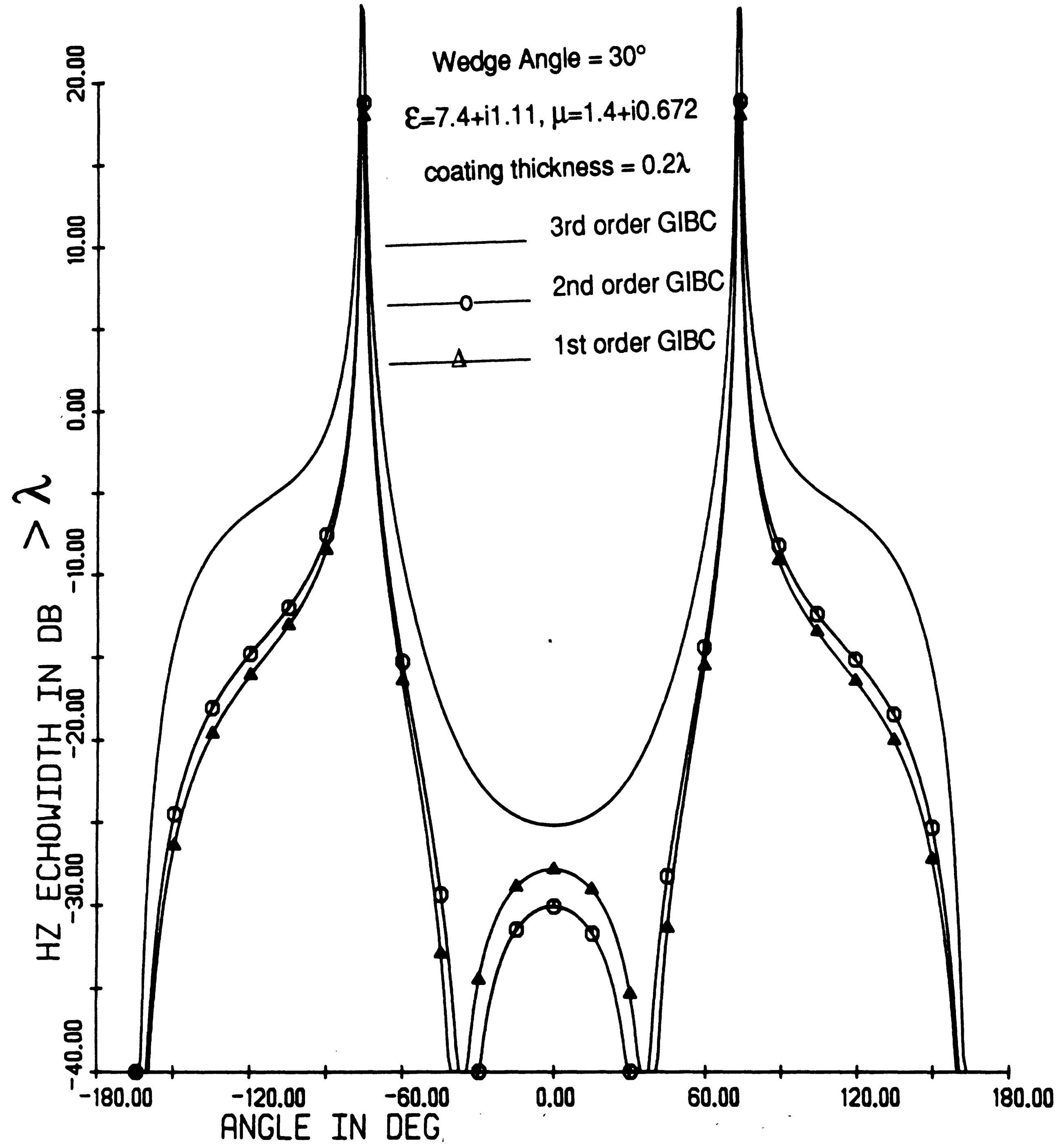

HZ ECHOWIDTH IN DB >λ
20.00
10.00
0.00
-10.00
-20.00
-30.00
-40.00
-180.00
-120.00
-60.00
0.00
60.00
120.00
180.00
ANGLE IN DEG
Wedge Angle = 30°
ε=7.4+i1.11, μ=1.4+i0.672
coating thickness = 0.1λ
3rd order GIBC
2nd order GIBC
1st order GIBC

Figure 14a



Wedge Angle = 30°
ε=7.4+i1.11, μ=1.4+i0.672
coating thickness = 0.2λ
3rd order GIBC
2nd order GIBC
1st order GIBC
20.00
10.00
0.00
-10.00
-20.00
-30.00
-40.00
HZ ECHOWIDTH IN DB > λ
-180.00
-120.00
-60.00
0.00
60.00
120.00
180.00
ANGLE IN DEG

Figure 14b