A NOTE ON THE EXACT AND APPROXIMATE SURFACE WAVE THEORY

by

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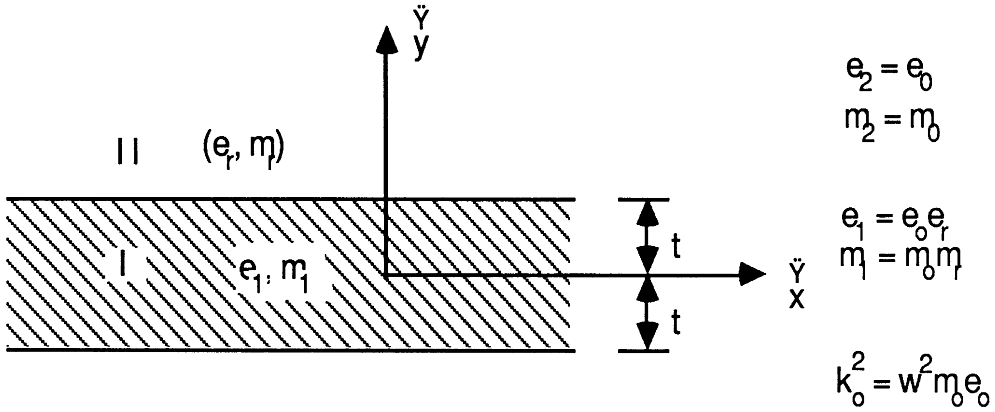
Abstract:

The computation of the surface wave field parameters on a grounded and an ungrounded permeable dielectric slab is discussed. A summary of the exact surface wave field theory is first presented. This is then followed by an approximate theory based on sheet and impedance boundary conditions. For the last case explicit expressions are derived for the propagation constant.

I. EXACT SURFACE WAVE THEORY

1. Dielectric Slab



e_r = relative permittivity

m_r = relative permeability

Figure 1. Geometry of the dielectric slab.

(a) <u>E_Z case (even mode)</u>

Assume the following surface wave fields exist in regions I and II satisfying the wave equation (an e^{jwt} time dependence is assumed and suppressed throughout):

$$E_Z^I = B\, e^{-fx} \cosh (g_1 y) \qquad (1)$$

$$E_Z^{II} = A\, e^{-fx}\, e^{-g_2 y} \qquad (2)$$

with

$$g_1^2 = - k_0^2 m_r e_r - f^2 , \qquad g_2^2 = - k_0^2 - f^2 \ . \tag{3}$$

From Maxwell's equations we now find that the corresponding tangential magnetic fields are:

$$H_x^I = - \frac{1}{jwm_0 m_r} \frac{\partial E_z^I}{\partial y} = - g_1 B e^{-fx} \sinh(g_1 y) \Big/ (jwm_0 m_r) \tag{4}$$

$$H_x^{II} = - \frac{1}{jwm_0} \frac{\partial E_z^{II}}{\partial y} = g_2 A e^{-fx} e^{-g_2 y} \Big/ (jwm_0) \ . \tag{5}$$

The propagation constants f, g_1 and g_2 and the ratio A/B can be found via the application of the boundary conditions requiring continuity of E_z and H_x at y = t. We find that

$$A e^{-g_2 t} = B \cosh(g_1 t) \tag{6}$$

and

$$A m_r g_2 e^{-g_2 t} = - g_1 B \sinh(g_1 t). \tag{7}$$

By dividing equations (6) and (7) we further find that

$$P = g_1 \tanh(g_1 t) + m_r g_2 = 0 \tag{8}$$

which is the characteristic equation for the E_z case even mode. We may now use equation (3) to eliminate g_1 and g_2 so that P is a function of f only. In so doing, we find that

$$P = \sqrt{-k_0^2 m_r e_r - f^2}\ \tanh\left(t\sqrt{-k_0^2 m_r e_r - f^2}\right) + m_r\sqrt{-k^2 - f^2} = 0 \qquad (9)$$

which can be solved numerically to find the propagation constant f. If we write f as

$$f = a + jb\ , \qquad (10)$$

where a and b are real, then a can be identified as the attenuation constant of the surface wave (1). In addition, if we let v denote the phase velocity of the surface wave then

$$b = k_0\left(\frac{c}{v}\right) \qquad (11)$$

where c is the speed of light. The computer program [1] in Figure 2 can be used to find the roots of P and thus determining the propagation constant f.

(b) <u>H_z case (even mode)</u>

We now assume that the following H_z field exists in regions I and II:

$$H_z^{I} = B\,e^{-fx}\cosh(g_1 y) \qquad (12)$$

$$H_z^{II} = A\,e^{-fx}\,e^{-g_2 y} \qquad (13)$$

with g_1, g_2 and f again satisfying equation (3). Using Maxwell's equations we also find that the corresponding tangential E field is given by

$$E_x^I = \frac{1}{jwe_o e_r} \frac{\partial H_z^I}{\partial y} = g_1 B e^{-fx} \sinh(g_1 y) \Big/ (jwe_o e_r) \qquad (14)$$

$$E_x^{II} = \frac{1}{jwe_o} \frac{\partial H_z^{II}}{\partial y} = - g_2 A e^{-fx} e^{-g_2 y} \Big/ (jwe_o) \qquad (15)$$

The determination of the propagation constants can be again accomplished via the application of the boundary conditions requiring continuity of the tangential electric and magnetic fields. We find that

$$A e^{-g_2 t} = B \cosh(g_1 t) \qquad (16)$$

and

$$A e_r g_2 e^{-g_2 t} = - g_1 B \sinh(g_1 t), \qquad (18)$$

giving

$$P = g_1 \tanh(g_1 t) + e_r g_2 = 0 \qquad (19)$$

which is the dual of (8). By using (3), (19) can be written as a function of f only. We note that the computer program given in Figure 2 is still applicable for the solution of (19) by simply interchanging the values of m_r and e_r.

(c) E_z case (odd mode)

Assume the fields

$$E_z^I = B\,e^{-fx}\sinh(g_1 y) \qquad (20)$$

$$E_z^{II} = A\,e^{-fx}\,e^{-g_2 y} \qquad (21)$$

with g_1, g_2 and f again satisfying equation (3). The H_x field is found by

$$H_x^I = -\frac{1}{jw\mu_o\mu_r}\frac{\partial E_z^I}{\partial y} = g_1 B\,e^{-fx}\cosh(g_1 y)\Big/(jw\mu_o\mu_r) \qquad (22)$$

$$H_x^{II} = -\frac{1}{jw\mu_o}\frac{\partial E_z^{II}}{\partial y} = g_2 A\,e^{-fx}\,e^{-g_2 y}\Big/(jw\mu_o) \qquad (23)$$

By demanding continuity of the tangential fields at y = t we now find that

$$A\,e^{-g_2 t} = B\sinh(g_1 t) \qquad (24)$$

$$A\,\mu_r g_2\,e^{-g_2 t} = +B\,g_1\cosh(g_1 t) \qquad (25)$$

giving

$$P = g_1\coth(g_1 t) - \mu_r g_2 = 0 \qquad (26)$$

which can be solved numerically to find the propagation constant f in conjunction with (3).

(d) H_z case (odd mode)

Assume the fields

$$H_z^I = B\,e^{-fx}\sinh(g_1 y) \qquad (27)$$

$$H_z^{II} = A\,e^{-fx}\,e^{-g_2 y} \tag{28}$$

with g_1, g_2 and f satisfying (3). The tangential E fields are given by

$$E_x^{I} = -\,g_1 B\,e^{-fx}\cosh(g_1 y)\,\big/\,(jwe_o e_r) \tag{29}$$

$$E_x^{II} = -\,g_2 A\,e^{-fx}\,e^{-g_2 y}\big/(jwe_o) \quad . \tag{30}$$

Following the same procedure as before, we obtain the characteristic equation

$$P = g_1 \coth(g_1 t) - e_r g_2 = 0 \tag{31}$$

which can be used to compute f in conjunction with (3).

A graphical solution of equations (9), (19), (26) or (31) is illustrated in Figure 3 [2]. As seen a solution of equations (9) and (19) always exists for t Æ 0. However, a solution of (26) and (31) can only be possible for larger values of t. Thus, when we refer to surface waves one generally assumes the existance of even modes.

2. Grounded Dielectric Slab

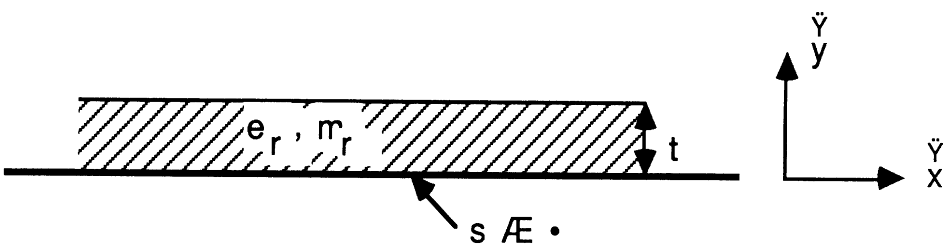


Figure 4. Geometry of the grounded dielectric slab.

(a) <u>E_z case</u>

Only the odd mode can be supported by this geometry since they are the only modes satisfying the boundary condition $E_z = 0$ at $y = 0$. Thus, the solution given in section 1(c) is applicable to this case since $d = 2t$, where d is the thickness of the ungrounded slab.

(b) <u>H_z case</u>

Only the even mode can be supported in this case since it produces a vanishing tangential E field (E_x) at $y = 0$. Thus, the solution given in section 1(b) is applicable here.

II. APPROXIMATE SURFACE WAVE THEORY

1. Dielectric Slab

If we assume the dielectric slab shown in Figure 1 has a very small thickness d = 2t we can then model it by coincident resistive and conductive sheets associated with a resisitivity

$$R = \frac{-jZ_o}{kd(e_r - 1)} \tag{32}$$

and a conductivity

$$R^* = \frac{-jY_o}{kd(m_r - 1)}, \tag{33}$$

respectively. In the above $Z_0 = 1/Y_0$ is the free space intrinsic impedance.

It is know that these sheets can support a surface wave field [3] of the form (C is a constant)

$$\begin{Bmatrix} E_z^{sw} \\ H_z^{sw} \end{Bmatrix} = C \exp(-jk_o x \cos q_{\substack{e \\ m}}) \tag{34}$$

depending on whether an E_z or H_z excitation is assumed. The parameters $q_{e,m}$ are found from diffraction theory to be given by

$$q_e = \sin^{-1}\left(\frac{1}{h_e}\right); \quad h_e = 2R / Z_0 = 2R\ Y_0 \tag{35}$$

and

$$q_m = \sin^{-1}\left(h_m\right); \quad h_m = 2R^* \,/\, Y_0 = 2R^* \, Z_0 \tag{36}$$

Clearly, the resistive sheet supports a surface wave only with E_z excitation, whereas the conductive sheet suports a surface wave only with H_z excitation.

From (34), one easily identifies the propagation constant $g = jk_0\cos q_{e,m}$ of the surface waves associated with the resistive and conductive sheets. As before, if we write $g = a + jb$, then a is the attenuation constant of the surface wave and is easily found from a knowledge of $q_{e,m}$. A computer program [10] for evaluating q_e given the parameters h_e and h_m is shown in Figure 5.

2. Grounded Dielectric Slab

Assuming that the ground plane is coated with a very thin dielectric layer of material, it can then be modeled as an impedance surface. Employing transmission line theory, we find that the surface impedance of this plane is given by

$$Z_S = j\, Z_o \sqrt{\frac{m_r}{e_r}}\, \tan\left(k_o \sqrt{m_r e_r}\; t\right) = Z_o h \ , \tag{37}$$

where t is the thickness of the coating and h is the normalized impedance relative to the free space intrinsic impedance Z_o.

The impedance plane can support a surface wave of the form

$$\begin{Bmatrix} E_z^{sw} \\ H_z^{sw} \end{Bmatrix} = C \exp(-j\, k_o x \cos q_{\substack{e \\ m}}) \tag{38}$$

depending on whether an E_z or H_z excitation is assumed. The parameters $q_{e,m}$ are found from diffraction theory [3] to be given by

$$q_e = \sin^{-1}(1/h) \tag{39}$$

and

$$q_m = \sin^{-1}(h) \ . \tag{40}$$

The similarity of (35) - (36) with (39) - (40) should be noted. It should be further noted that a surface wave cannot be supported on an impedance surface for all values of h. The condition that a surface wave is supported by the impedance surface is [5]

$$-\operatorname{Re}(q) + \operatorname{gd}\left(|\operatorname{Im}(q)|\right) \operatorname{sgn}\left(\operatorname{Im}(q)\right) > 0 \tag{41}$$

where $gd(x) = \cos^{-1}\{1/\cosh(x)\}$ is the Gudermann function and q can denotes q_e or q_m as given in (39) - (40). Thus, in the case of E_Z excitation it is necesary (but not sufficient) that h be capacitive whereas in the case of H_Z excitation it is necessary that h be inductive. We remark that h_e in (35) is always capacitive and h_m in (36) is always inductive.

The propagation constant of the surface wave is again given by

$$g = a + jb = j\, k_o \cos q_{\substack{e\\m}} \tag{42}$$

and from (39)

$$g = \begin{cases} j\, k_o \sqrt{1 - (1/h)^2} & \text{for } E_z \text{ case} \\ j\, k_o \sqrt{1 - h^2} & \text{for } H_z \text{ case} \end{cases} \tag{43}$$

If we write

$$R_s + j\, X_s = \begin{cases} 1/(Z_o h) & E_z \text{ case} \\ Z_o h & H_z \text{ case} \end{cases} \tag{44}$$

we find that

$$a = \mathrm{Re}(g) = \mathrm{Im}\left(k_o \sqrt{1 + X_s^2 - R_s^2 - j\, 2X_s R_s} \right) \tag{45}$$

and

$$b = \mathrm{Im}(g) = \mathrm{Re}\left(k_o \sqrt{1 + X_s^2 - R_s^2 - j\, 2X_s R_s} \right) \tag{46}$$

The attenuation of the surface wave power per unit length can now be written as

$$L = 20 \log\left(e^{ax}\right) = 8.69a \ \text{dB/meter} \tag{47}$$

and Figure 6 shows the constant L (loss) contours as a function of R_s and X_s [6].

The definition (44) can also be employed for the parameters h_e and h_m appearing in (35) and (36). In that case (47) will also be applicable for the computation of the surface wave power loss in an ungrounded dielectric slab.

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C  SURFACE WAVES ON LOSSY DIELECTRIC SLAB.
      COMPLEX CGT,EGT,EPR,EP2,F,FE,FF,FGG,FS
      COMPLEX G1,G2,G12,GM1,GM2,GS1,GS2,GT1,GT2
      COMPLEX MUR,MU2,P,PP,SGT
      DATA E0,U0/8.85418533677E-12,1.25663706144E-6/
      DATA PI,TP/3.14159265359,6.28318530718/
C  DM = SLAB THICKNESS, METERS.
C  UR,ER = RELATIVE PERMEABILITY AND PERMITTIVITY OF SLAB.
C  TDE,TDM = ELECTRIC AND MAGNETIC LOSS TANGENTS.
C  FGC,FMC = FREQUENCY IN GIGAHERTZ, MEGAHERTZ.
C  DD = SLAB THICKNESS / SKIN DEPTH.
C  NE = NUMBER OF NEWTON-RAPHSON ITERATIONS.
C  DBE,VCE = ATTEN CONST AND PHASE VEL FOR SURF WAVE WITH PERP POL.
    2 FORMAT(1X,1I5,8F12.5)
    5 FORMAT(1H0)
      DM=.025
      TM=DM/2.
      ER=4.
      UR=1.
      TDE=.1
      TDM=.0
      NAX=20
      FMC=300.
      FGC=FMC/1000.
      WAV0=300./FMC
      DL=DM/WAV0
      BET0=TP/WAV0
      OMEG=TP*FMC*1.E6
      EPR=ER*CMPLX(1.,-TDE)
      EP2=E0*EPR
      MUR=UR*CMPLX(1.,-TDM)
      MU2=MUR*U0
      GS1=-OMEG*OMEG*U0*E0
      GS2=-OMEG*OMEG*MU2*EP2
      GM1=CMPLX(.0,BET0)
      GM2=CSQRT(GS2)
      ALP2=REAL(GM2)
      DEL=.0
      DD=.0
      IF(ALP2.LE..0)GO TO 12
      DEL=1./ALP2
      DD=DM/DEL
   12 CONTINUE
      BET2=AIMAG(GM2)
      TK=TP*TM/WAV0
      G1=(MUR*EPR-1.)*TK*TK/(MUR*TM)
      FF=GM1*(1.-.5*G1*G1/GS1)
      F=FF
C
      DO 60 N=1,NAX
```

Figure 2. Computer program for surface wave parameters.

```
   20 FS=F*F
      G1=CSQRT(GS1-FS)
      G2=CSQRT(GS2-FS)
      G12=G1*G2
      A12=CABS(G12)
      IF(A12.LE..0)GO TO 100
      GT2=G2*TM
      EGT=CEXP(GT2)
      CGT=(EGT+1./EGT)/2.
      SGT=(EGT-1./EGT)/2.
      GT1=G1*TM
      P=G2*SGT/CGT+MUR*G1
   40 FGG=F/(G12*CGT)
      PP=-FGG*(MUR*G2*CGT+G1*SGT+G1*GT2/CGT)
      ALP=REAL(F)
      BET=AIMAG(F)
      VC=BET0/BET
      AP=CABS(P)
      WRITE(6,2)N,ALP,VC,AP
      F=F-P/PP
      FE=F
      NE=N
      IF(N.LT.3)GO TO 60
      APP=CABS(P/(F*PP))
      IF(APP.LT..0001)GO TO 62
   60 CONTINUE
   62 CONTINUE
      WRITE(6,5)
      ALPE=REAL(FE)
      VCE=BET0/AIMAG(FE)
      DBE=8.686*ALPE
      WRITE(6,2)NE,FMC,DL,DD,DBE,VCE
  100 CONTINUE
      CALL EXIT
      END
```

Figure 2. (continued)

```
      COMPLEX FUNCTION HEE(ETA,IUD,SB0)
C|||  NEW VOLAKIS VERSION
      COMPLEX ETA,ETA1,CJ
      DATA SRT2,FPI,CJ/1.414213562,12.56637061,(0.,1.)/
      DATA PSIPI2,PI/.9656228,3.14159265/
      ETA1=SB0/ETA
      IF(IUD.EQ.1)ETA1=SB0*ETA
      RE=REAL(ETA1)
      AE=AIMAG(ETA1)
      REP=RE+1.
      REM=RE-1.
      AA=.5*(SQRT(REP*REP+AE*AE)+SQRT(REM*REM+AE*AE))
      BB=.5*(SQRT(REP*REP+AE*AE)-SQRT(REM*REM+AE*AE))
      SGN=AE/ABS(AE)
      RAA=AA*AA-1.
      IF(RAA.LT.1.E-6)RAA=0.
      HEE=ARSIN(BB)+CJ*ALOG(AA+SQRT(RAA))*SGN
C     HEE=.5*PI-HEE
      GO TO 300
      ETAM=CABS(ETA1)
      ETAA=ATAN(AIMAG(ETA1)/REAL(ETA1))
      ETAM2=ETAM*ETAM
      SA=SIN(ETAA)
      CA=COS(ETAA)
      F1=ETAM2-1.+SQRT((ETAM2-1.)**2+4.*ETAM2*SA*SA)
      F1=F1/(2.*ETAM2)
      IF(F1.LT.0.)F1=0.
      HEER=ASIN(SQRT(F1))
      SHEER=SIN(HEER)
      CHEER=COS(HEER)
      IF(CABS(ETA1).GT.1.)GO TO 100
      HEEI=CA/(ETAM*CHEER)
      HEEI=ALOG(HEEI+SQRT(ABS(HEEI*HEEI-1.)))
      IF(ETAA.LT.0.)HEEI=-HEEI
      GO TO 200
  100 HEEI=SA/(ETAM*SHEER)
      HEEI=ALOG(HEEI+SQRT(HEEI*HEEI+1.))
  200 HEE=CMPLX(HEER,HEEI)
  300 RETURN
      END
```

Figure 5. Computer program for the computation of $\theta = \sin^{-1}(1/\eta)$ or $\theta = \sin^{-1}(\eta)$.

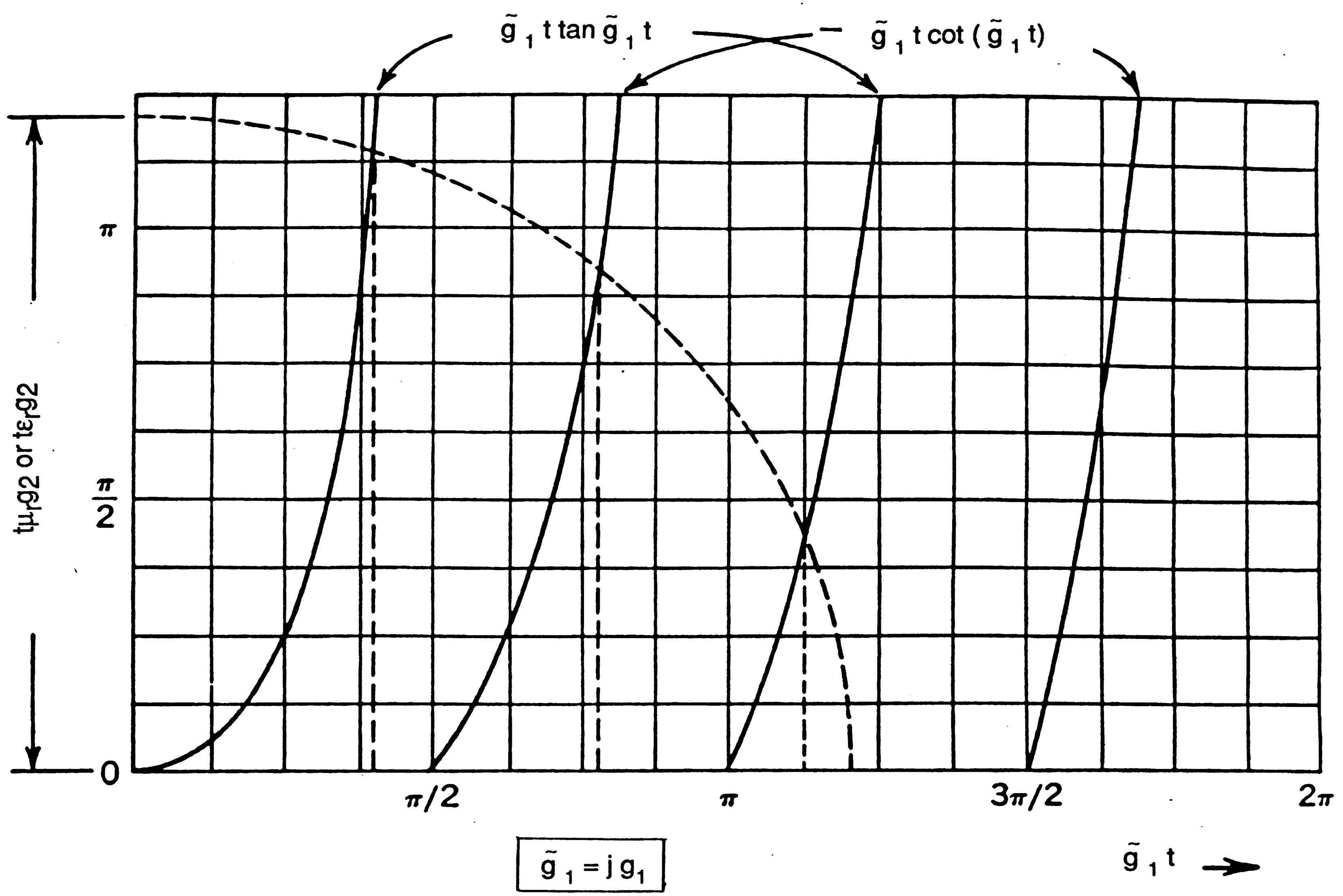

Figure 3. Graphical solution of the characteristic equation for the slab waveguide.

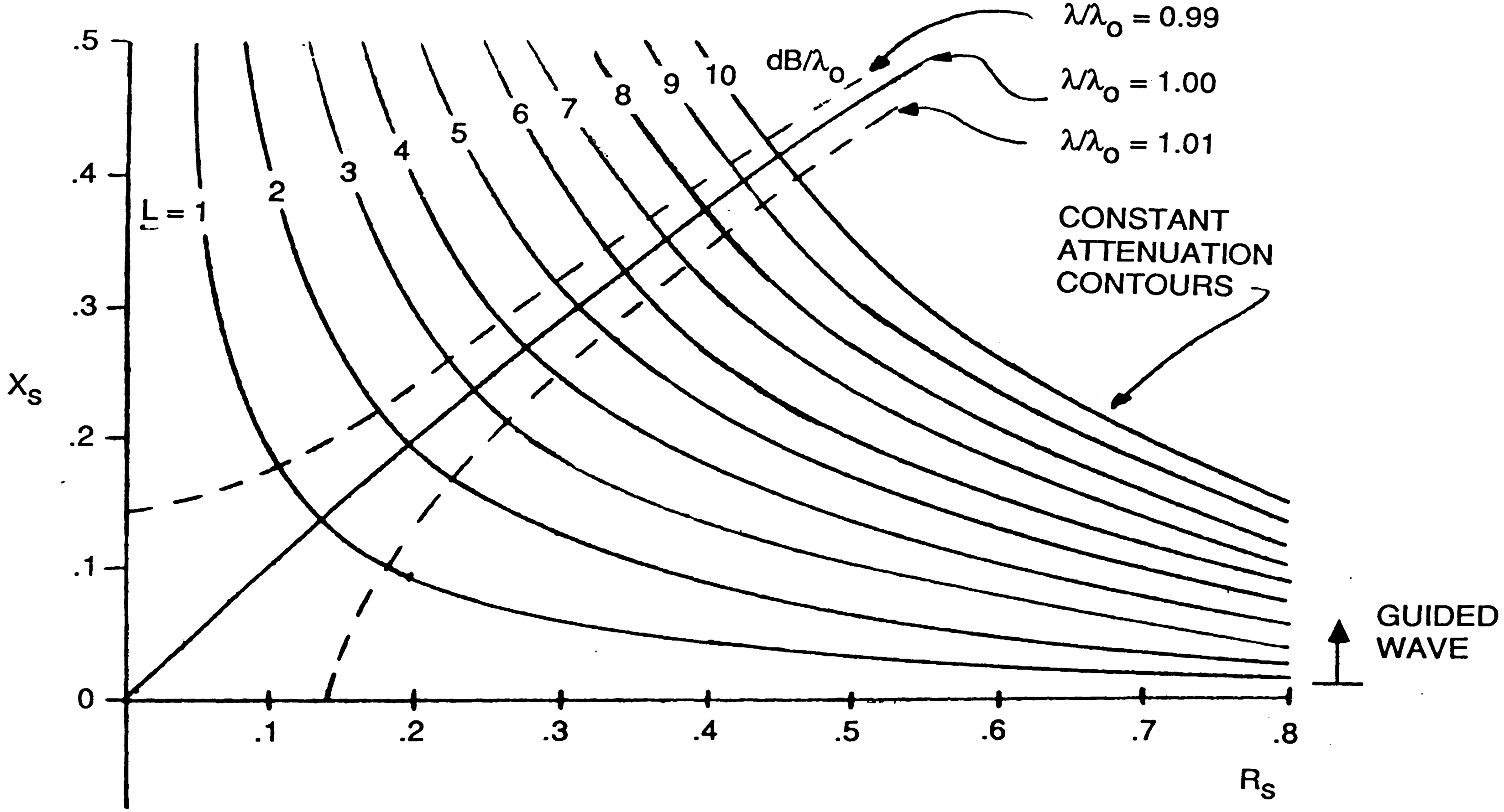

Figure 6. Contours of constant surface wave loss and velocity for an impedance plane.