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Abstract:

This paper studies the famous Feldstein and Horioka finding, which is a high correlation between long period averages of savings rates and investment rates across countries. We first confirm the Feldstein-Horioka finding with a more recent data set, and then show that a calibrated complete markets model generates a cross-section savings-investment correlation close to zero. Thus, the cross-section Feldstein-Horioka finding is a puzzle to the complete markets model and further research is needed to account for this puzzle.

We next explore roles of financial frictions in accounting for the cross-section Feldstein-Horioka puzzle. The most popular incomplete markets model, the bond model with natural borrowing constraints, cannot account for the cross-section Feldstein-Horioka puzzle. We then propose the bond model with enforcement constraints in which uncontingent debt contracts are enforced by the threat of permanent exclusion from the markets. This model generates endogenous borrowing constraints, which capture the incentives of countries to repay their debts instead of their abilities to repay under natural borrowing constraints, and accounts for the cross-section Feldstein-Horioka puzzle.

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1. Introduction

The savings-investment puzzle is one of the most important regularities in international finance, which originally identified by Feldstein and Horioka (1980). Feldstein and Horioka find that a cross-country[1] regression of average investment rates on average savings rates results in a regression coefficient close to one. They argue that this regression coefficient should be close to zero in a frictionless open economy world and suggest that this large regression coefficient indicates a high degree of financial frictions in the world economy. Their finding and claims about substantial financial frictions in international financial markets raise a huge debate in the literature. The profession has responded to the Feldstein-Horioka finding from two different approaches.

First, empirical research attempts to refute the finding either by studying different data samples and periods, by adding other variables to the original OLS regression, or by using different estimation methods. However, it turns out that the high correlation between savings and investment is remarkably robust in both OECD and non-OECD countries, though the FH coefficient has tended to decline in recent years.[2] In this paper, we confirm the Feldstein-Horioka finding with a more recent data set with a sample of 57 countries over the period from 1960 to 2000.

Second, theoretical research tries to refute the claims that the regression coefficient on savings is close to zero in a frictionless world, and thus the Feldstein-Horioka finding provides no information about financial frictions. However, most quantitative studies in the literature have focused on a variant of the original finding, that is, the high time-series correlation between savings and investment rates at business cycle frequencies within nations,[3] instead of the cross-section Feldstein-Horioka finding. Baxter and Crucini (1993) and Finn (1990) show that the high time-series correlation between savings and investment rates at business cycle frequencies can arise naturally in a quantitative complete markets model (frictionless capital markets with a complete set of assets). Mendoza (1991) show that the most popular incomplete markets model, the bond model in which

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[1] Data sample: 16 OECD countries (Austria, Austria, Belgium, Canada, Denmark, Finland, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Sweden, United Kingdom, and United States) from 1960 to 1974

[2] Details see Coakley, Kulasi and Smith (1998).

[3] Details see Tesar (1991).

countries can only trade state-uncontingent bonds, can also quantitative produce the high correlation between savings and investment in time series. Thus, the high time-series correlation offers no insight about financial frictions.

Motivated by the time-series research, we first investigate the cross-section regression coefficient on savings using a theoretical model with complete markets. Given our focus, we need to depart from the standard two-country setup and develop a framework with a large number of countries. For the sake of tractability, we use a continuum of small open economies with production. The uncertainty in the model arises from idiosyncratic fluctuations in country-specific total factor productivities (TFP). In our quantitative work, we capture the rich characteristics of (total factor) productivity processes for 57 countries in the sample with a regime switching process.

In contrast to results from time-series studies, the complete markets model generates a regression coefficient on savings close to zero, which confirms the claim made by Feldstein and Horioka. The key to this result is that with complete markets, investment depends on changes in TFP shocks, while savings depends on levels of TFP shocks. Consider an extreme case of a random walk process, in which changes in shocks are independent of levels of shocks. We would expect no correlation between savings and investment rates in cross section. With highly persistent TFP shocks calibrated to the data, the cross-section correlation between savings and investment rates is close to zero.

Thus, we are left with a puzzle: what kinds of financial frictions can account for the Feldstein-Horioka puzzle? We first study the most popular incomplete markets model in the literature: the bond with natural borrowing constraints[4] model. In this model, countries can only trade state-uncontingent bonds instead of a complete set of state-contingent assets. With natural borrowing constraints, countries cannot borrow more than what they can repay without incurring non-positive consumption. The implicit assumption behind the natural borrowing constraints is existence of complete enforcement in the international financial markets: countries repay their debt no matter how low their utilities have to go, so long as it is physically feasible for them to do so.

The bond with natural borrowing constraints model is remarkably unsuccessful in accounting for the cross-section Feldstein-Horioka puzzle. The key reason is that the natu-

[4] As in Aiyagari (1994)

ral borrowing constraints are so loose that the model allows too much borrowing and lending relative to what we observed in the data. Countries are able to shift resources across periods according to their investment needs, though they cannot insure against the idiosyncratic uncertainty within a period. Thus, the bond with natural borrowing constraints model generates a very low FH coefficient relative to the one in the data.

The implicit assumption of complete enforcement in the bond with natural borrowing constraints model is hard to justify given the weak enforcement mechanism on sovereign debts in the international financial markets. Thus, we relax this assumption and consider the bond with enforcement constraints model. In this model, countries are still limited to trading state-uncontingent bonds, but there is only limited enforcement, under which international debt contracts are enforced by the threat of exclusion from international financial markets. That is, at every period, countries cannot borrow more than what they have incentives to repay rather than defaulting and entering financial autarky for any possible realization of shock during the following period.

The bond with enforcement constraints model produces a regression coefficient close to the one observed in the data, and thus can account for the cross-section Feldstein-Horioka puzzle. Compared with the (implicit) infinite default penalty under natural borrowing constraints, the (explicit) default penalty of financial autarky under the enforcement constraints is less severe. As a result, the enforcement constraints generate much tighter borrowing constraints, which endogenously respond to the larger incentives of countries to default under lower default penalty. Quantitatively, the bond with enforcement constraints model generates capital flows, measured by the current account-over-GDP ratios, very close to what we observe in the data. Thus, this model generates a positive regression coefficient close to the data.

This paper is closely related to the literature on sustainability of sovereign debts. One approach of determining the viability of sovereign debt is by looking at the ability of countries to repay, that is, whether countries are able to generate sufficient future trade surplus (or future income) to repay the debt.[5] The bond with natural borrowing constraints model studies the ability of countries to repay by focusing on the solvency of debts. Another approach is to evaluate sustainability of debt by the willingness of coun-

[5] Detailed discussions see Milesi-Ferretti and Razin (1996).

tries to repay. The bond with enforcement constraints model captures the willingness of countries to repay by taking into account the incentives of borrowers. That is, countries have an implicit option to default on their debts and enter financial autarky if they want to. Given this option, at every period, the debt that countries can issue must provide them incentives to repay no matter which shock is realized in the following period. This paper shows that the willingness of countries to repay, instead of the abilities of countries to repay, plays an important role in accounting for international borrowing and lending.

This paper also borrows from a large literature on limited enforcement.[6] The most closely related work is Zhang (1997), which studies the limited enforcement friction with incomplete markets for a pure exchange economy with 2-type agents. The bond with enforcement constraints model studied in this paper builds on Zhang (1997) and extends the analysis to the environment with production economies and a continuum of agents.

The models studied in this paper are rich enough to also look at the time-series implications. In the time-series literature, capital adjustment costs are commonly imposed to match the volatility of investment in the data.[7] Thus, when looking at the time-series correlation, we also impose capital adjustment costs. We find that all the models, with or without financial frictions, generate large time-series correlations between savings and investment rates with almost no change in the cross-section regression results. Hence, this paper confirms the results in the time-series literature that the large time-series correlation offers no insight about financial frictions, and in fact is the result of highly persistent shocks.

The chapter is organized as follows. Section 2 introduces the original Feldstein-Horioka puzzle in cross section. Section 3 describes the theoretical model with complete markets and quantitative results under this benchmark model. Section 4 describes the bond with natural borrowing constraints model. Section 5 presents the bond with enforcement constraints model. Section 6 presents implications on the time-series correlation between savings and investment, and Section 7 concludes.

[6] Kehoe and Levin (1993), Kocherlakota (1996), Alveraz and Jermann (2000), Kehoe and Perri (2002), and etc.

[7] Papers referred to earlier have all imposed capital adjustment costs.

2 The Cross-Section Feldstein-Horioka Puzzle

In Feldstein and Horioka's seminal paper (1980), they estimate the following equation to assess the cross-section long-run relationship between savings rates and investment rates,

$$(I/Y)_i = \gamma_0 + \gamma_1 (S/Y)_i \tag{1}$$

where I is the gross domestic investment (gross capital formation); Y is the gross domestic product (GDP); S is the gross domestic savings[8], defined as GDP minus private and government consumption; and i denotes country indices. For each country i and each year t, Feldstein and Horioka first calculate savings and investment rates $(S/Y)_{it}$ and $(I/Y)_{it}$. Next, they compute period averages of the savings rates and investment rates for each country, denoted by $(S/Y)_i$ and $(I/Y)_i$, and run the cross-section regression of the average investment rates on the average savings rates as specified in Equation (1).

With a sample of 16 OECD countries over the period from 1960 to 1974, they find that the regression coefficient on savings rates is 0.89 with a standard error of 0.07. The regression result shows that in the long run, the average savings rates and the average investment rates are highly, positively correlated in cross section. A large body of literature following this paper shows that this result is robust to all econometric variation examined. A survey article by Coakley, Kulasi and Smith (1998) reviews these findings in details.

Feldstein and Horioka published their original paper in 1980, over 25 years ago. Due to limited data availability at that time, they were only able to consider a small sample of countries. Further, during the past 20 years, the world has witnessed a dramatic increase in international capital flows across countries. To explore whether the Feldstein-Horioka finding is robust to a more recent data set, we run the same regression with a larger sample (57 countries)[9] and a longer time period (1960-2000). We find that the regression coefficient is 0.46 with a standard error of 0.07 (see Figure 1). Thus, we still observe the

[8] Tesar (1991) shows that the result is robust if gross national savings is used instead of gross domestic savings.

[9] See Appendix 1.

positive cross-section correlation between savings rates and investment rates in the long run, though the regression coefficient is much lower than the original estimate made by Feldstein and Horioka. We also experiment with different sub-periods and subgroups of countries to confirm that the finding of the positive regression coefficient is robust.[10] Table 1 shows that the regression coefficient is positive (around 0.50) and significantly different from zero, regardless of countries and periods sampled.

These results confirm that the Feldstein-Horioka finding is a pervasive regularity in the data. In a world with closed economies, where domestic investment must be fully financed by domestic savings in each period, the cross-section correlation is one. Feldstein and Horioka argued that in a world with complete markets, which is frictionless, the cross-section correlation should be zero. In the following section, we will study the complete markets model to test the above argument in cross section. It turns out that this finding is a puzzle to the complete markets model in cross section and motivates us to explore models with financial frictions.

3 The Complete Markets Model

In this section, we study the cross-section correlation between savings rates and investment rates in the complete markets model, where countries can trade a complete set of Arrow securities in international financial markets without any financial frictions. To study a large number of countries in a tractable fashion, we consider a production economy in a continuum of small open economies framework.

3.1 The Production Technology

There is a homogeneous good which can be either consumed or invested. The production function is the standard Cobb-Douglas given by $AK^{\alpha}L^{1-\alpha}$, where A denotes the productivity shock, K denotes the capital inputs, L denotes the exogenous labor inputs[11] and α is the capital share parameter. At the beginning of each period, each country faces a productivity shock, which has a deterministic trend component and an idiosyncratic

[10] To compare with the Feldstein-Horioka result, we take two sub-periods (1960-1974 and 1974-2000) and two subgroups of countries (16 OECD countries and the rest of the countries).

[11] Here we assume that labor supply is inelastic and abstract from labor and leisure decisions for the sake of simplicity and tractability.

shock component. There is no aggregate uncertainty. Specifically, the productivity shock is given by $A_t = (1+g_a)^t a_t$, where g_a is the deterministic trend, which is common across countries and constant across periods, and a_t is the country-specific idiosyncratic shock, which follows a Markov process with a finite support and a transition matrix Π. We normalize all the relevant variables by the labor supply and the deterministic growth rate $g = (1+g_a)^{1/(1-\alpha)}$.[12] The production function is simplified to $f(k) = ak^{\alpha}$, where small letters denote the variables after normalization.

The history of the idiosyncratic shock is denoted by $a^t = (a_0, a_1, ..., a_t)$. The probability of history a^t is generated by the transition matrix as follows:

$$\pi(a^t) = \pi(a_t \mid a_{t-1})\pi(a_{t-1} \mid a_{t-2})...\pi(a_1 \mid a_0).$$

3.2 The Planner's Problem

Countries are indexed by the initial state $s_0 = (a_0, k_0, b_0)$ where a_0 is the initial idiosyncratic productivity shock, k_0 is the initial capital stock and b_0 is the initial asset holding. In each country, a planner, who makes consumption, capital and savings (borrowing) decisions to maximize the expected discounted lifetime utility,[13] which is given by

$$\sum_{t=0}^{\infty} \sum_{a^t} \beta^t \pi(a^t) u(c(a^t, s_0)), \tag{2}$$

where β denotes the discount factor and c denotes consumption. The period utility function u is assumed to be CES, given by

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma} \tag{3}$$

where σ is the risk aversion parameter.

The planner maximizes welfare subject to resource constraints

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[12] All the $t+1$ variables should be preceded by (1+g). We drop it only for clarity of the model. It is included in all computations.

[13] Focus on the planner's problem in this paper and the decentralization can be constructed accordingly.

$$c(a^t,s_0)+k(a^t,s_0)-(1-\delta)k(a^{t-1},s_0)+\sum_{a_{t+1}|a^t}q(a_{t+1},a_t)b(a_{t+1},a^t,s_0)$$

$$=a_tk(a^{t-1},s_0)^\alpha+b(a_t,a^{t-1},s_0)\,,$$

and no-Ponzi constraints given by

$$b(a_{t+1},a^t,s_0)\geq -B\,,$$

where, δ is the capital depreciation parameter; B is a borrowing limit which is sufficient large such that these constraints never bind in equilibrium; $k(a^t,s_0)$ denotes the capital decision; $b(a_{t+1},a^t,s_0)$ denotes the Arrow security which delivers one unit of goods for each unit of security purchased today if a_{t+1} realizes tomorrow and $q(a_{t+1},a_t)$ denotes the price of such Arrow security.

3.3 The Complete Markets Equilibrium

Complete markets equilibrium is a set of prices $\{q(a_{t+1},a_t)\}$ and allocations $\{c(a^t,s_0),k(a^t,s_0),b(a_{t+1},a^t,s_0)\}$ which satisfy the following conditions:

(1) Allocations solve the planner's problem given prices;

(2) Allocations satisfy the resource clearing conditions given by, for any a^t,

$$\sum_{s_0}\sum_{a^t|a_0}\pi(a^t\mid a_0)(c(a^t,s_0)+k(a^t,s_0)-(1-\delta)k(a^{t-1},s_0)-a_tk(a^{t-1},s_0)^\alpha)=0\,. \qquad (4)$$

The equilibrium can be characterized using nice properties of complete markets without aggregate uncertainty. First, with perfect risk-sharing in complete markets, consumptions are constant over time and across states, i.e., for any a^t,

$$c(a^t,s_0)=C\,. \qquad (5)$$

Second, the Arrow security prices are given by

$$q(a_{t+1},a_t)=\beta\pi(a_{t+1}\mid a_t)\,. \qquad (6)$$

Third, capital flows across countries to equalize the return to capital:

$$k(a^t, s_0) = \left[(1/\beta - 1 + \delta)/(\alpha E(a_{t+1} \mid a^t))\right]^{1/(\alpha-1)}. \tag{7}$$

Finally, the Arrow securities for any history can be solved from the difference between the discounted sum of the consumption path after that history node and the discounted sum of the income flow after that history node.

3.4 Calibration

To quantitatively evaluate the cross-section correlation between savings rates and investment rates, we need to calibrate all the parameters, and more importantly, the world productivity process, to the data. In this section, we describe the calibration in detail.

3.4.1 Calibration of Parameters

The preference and technology parameters are calibrated as follows and are reported in Table 2. The deterministic growth rate g_a is calibrated to match the world average TFP growth rate of 1.3 percent.[14] For the risk aversion parameter σ, we choose the standard literature value 2.0.

All of the other parameters are calibrated to the U.S. data. For the capital depreciation rate δ, we take the ratio of the U.S. average capital consumption allowance to the capital-output ratio, following Stokey and Rebelo (1990). In the data average capital consumption allowance to GDP is 0.107 over the period from 1960 to 2000, and the average capital-GDP ratio is around 1.73.[15] Thus, the capital depreciate rate δ is 6 percent. The discount rate β is calibrated to match the real capital return of 4 percent. The capital share α is calibrated to match the labor compensation share of 0.67. National Accounts Statistics suggest that labor compensation shares of poor countries might be smaller than that in the United Sates. However, the labor compensation in the National Accounts fails to account for self-employed labor. Gollin (2001) showed that for 31 countries, which have enough data to adjust the labor income by including self-employed workers, the labor shares are in the range of 0.6 to 0.85, closer to the labor share in the United States.

[14] Details see the calibration of the world productivity process.
[15] See Christiano (1988).

3.4.2 Calibration of the World Productivity Process

We first compute the productivity processes for the 57 countries in the sample using the standard growth accounting method. Next we analyze the key features of the processes and estimate one stochastic process to capture the key characteristics of the productivity processes observed in the data.

We calibrate the productivity process for each country as follows:

$$\log A_{it} = \log Y_{it} - \alpha \log K_{it} - (1-\alpha) \log L_{it}, \tag{8}$$

where Y_{it} is the real GDP; L_{it} is total employment and K_{it} is the total capital stock constructed from gross investment data.[16] Next we normalize the productivity processes by the average growth rate of 1.3 percent. To make the productivity processes comparable across countries, we use the nominal exchange rates of year 1995 (national currency per US$) to adjust levels of the productivity processes.[17] The normalized productivity processes are presented in Figure 2.

From Figure 2, we observe three key features in the calibrated productivity processes, motivating our choice of the stochastic process. First, the range of the productivity processes is more than 17 times the mean of the standard deviations of each country (see Figure 3). Second, poor countries are more volatile than rich countries on average (see Figure 4). Third, the processes for some countries have different characteristics during different sub-periods (see Figure 5).

The above features of the TFP processes motivate our choice of regime-switching process. There are three regimes.[18] Conditional on being in regime $\Re_t^i = 1,2,3$, we assume that TFP shocks follow an autoregressive process given by

$$a_t^i = \mu_{\Re_t^i}(1-\rho_{\Re_t^i}) + \rho_{\Re_t^i} a_{t-1}^i + \sigma_{\Re_t^i} \varepsilon_t^i, \tag{9}$$

[16] For the detailed data source, refer to appendix 1.

[17] Result is unchanged if using nominal exchange rate in any other year since it is just a scalar and doesn't change the productivity process at all. Since all the real data are in 1995 price, we choose 1995 nominal exchange rate.

[18] The justification for the three-regime specification is as follows. The three-regime specification significantly improves the goodness of fit over the two-regime, while the four-regime specification barely improves the goodness of fit over the three-regime specification.

where a_t^i denotes the idiosyncratic TFP shock of country i at period t, ε_t^i is i.i.d. and drawn from a standard normal distribution $N(0,1)$, and $\Re_t^i$ denotes the regime that country i is in at period t. The probability of switching from one regime to another is given by the transition matrix Π.

We use maximum likelihood method to estimate the unknown parameters: $\Theta = \{\{\mu_{\Re}, \rho_{\Re}, \sigma_{\Re}\}_{\Re=1,2,3}, \Pi\}$. The method we use is an extension of that in Hamilton (1991) for one time series to panel data series. The Expectation-Maximization (EM)[19] principle is used to compute the solutions. The detailed algorithm is described in Appendix 2.

In Table 3, we report the maximum likelihood estimates for the above parameters along with the standard errors. For convenience, the three regimes will be referred to as the low, middle, and high regimes, according to the degree of volatility. From the transition matrix estimated, if at a given time period one country is in a given regime, it will most likely stay in that regime, but there is a positive probability that it may switch to another regime. We use the Tauchen (1986) method to discretize the AR1 process with regime change in the numerical analysis.

3.5 Results

In this section we simulate the complete markets model and report the results. In the empirical work, we analyze annual data on 57 countries, which include a mix of developed and developing countries, over 41 periods from 1960 to 2000. To be consistent with the data sample of 57 countries over 41 periods, we perform the regression exercise on model simulations of the same number of countries and of the same time periods as in the data. More specifically, we simulate 57 time series of the TFP processes of 1000 periods in each simulation. For each county, given the initial TFP shock a_0, we start with $b_0 = 0$[20] and we use the optimal k_0 for a_0 specified in (7). We then compute all the variables according to the decision rules. After a long period, the world economy is in invari-

[19] Dempster, Laird, and Rubin (1977)

[20] The invariant distribution over debts in the complete markets model is not unique and depends on the initial distribution over debts. We are interested in the invariant distribution of debts generated from an initial distribution of a continuum of closed economies.

ant distribution. Given the shock processes and capital stock and bonds holding at the beginning of the last 41 periods, we compute the equilibrium solutions and the average savings rates and the average investment rates for 57 series. We then run the same regression as we did for the data to compute the coefficient on savings, referred as the *FH coefficient*. We repeat the above simulation procedure 1000 times. In Table 4, we report the mean statistics across 1000 simulations.

The mean of the FH coefficients over 1000 simulations is 0.001 and the standard deviation of mean is 0.02. Thus, the cross-section correlation between savings and investment rates is quantitatively zero in the complete markets model. To understand the result, let's first look at the investment and savings decisions in complete markets. Investment depends on the change of shocks: if the shock today is higher than the shock yesterday, countries will increase investment to build up the higher capital stock desired, while if the shock today is lower than the shock yesterday, countries will decrease the capital stock, meaning that investment level will be low. On the other hand, savings depends on the level of shocks. When the level of shock is low, savings is low and may even be negative. The converse is also true.

For the sake of the intuition, assume that all economies in the world start with the same initial condition, and thus have the same consumption path. Under the invariant distribution, countries with shocks above the mean level will have high average savings rates, and countries with shocks below the mean level will have low average savings rates. However, independent of whether a country is rich or poor, if it has a higher shock in the period following the current period, it will invest more and vice versa. Consider the extreme case of random walk, in which the level of the shock is independent of the change in the shock. We would expect zero correlation in cross section. In the data, the shocks are very persistent (close to random walk) and this is why savings rates and investment rates have zero cross-section correlation.[21]

This result is in stark contrast with the findings in the existing studies on the high time-series correlation between savings and investment rates. In those studies, the positive time-series correlation can naturally arise in a quantitative complete markets model

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[21] If the shock process becomes less persistent than the one we estimate here, the cross-section regression coefficient turns negative instead of positive because of the stronger mean reversion.

as endogenous responses of savings and investment to persistent TFP shocks. However, this chapter shows that the complete markets model does generate a cross-section regression coefficient close to zero. Thus, the Feldstein-Horioka finding is a puzzle to models in the absence of financial frictions.

Having confirmed the Feldstein-Horioka claim that average investment rates and average savings rates should display no correlation across countries in a frictionless world, we are left with a puzzle: What kinds of financial frictions can account for the Feldstein-Horioka puzzle? In Section 4, we will address the above question.

4 The Bond with Natural Borrowing Constraints Model

To account for the cross-section Feldstein-Horioka puzzle, we deviate from the complete markets model to explore incomplete markets models. In this section, we will study the most extensive-used incomplete markets model in the literature, the bond model. The model environment is the same as the complete markets model, except that assets traded are restricted to state-uncontingent bonds instead of a full set of state-contingent Arrow securities.

4.1 The Model

In a world with a continuum of small open economies without aggregate uncertainty, interest rates R are constant over time. Given the interest rate R, the planner in each country s_0 chooses sequences of consumption $\{c(a^t, s_0)\}$, bonds $\{b(a^t, s_0)\}$ and capital stocks $\{k(a^t, s_0)\}$ to maximize the welfare given by (2), subject to resource constraints given by,

$$c(a^t, s_0) + k(a^t, s_0) - (1-\delta)k(a^{t-1}, s_0) + b(a^t, s_0) = a_t k(a^{t-1}, s_0)^{\alpha} + Rb(a^{t-1}, s_0), \quad (10)$$

and natural borrowing constraints given by

$$b(a^t, s_0) \geq \hat{B}(a_t, k(a^t, s_0)). \quad (11)$$

The natural borrowing constraints $\hat{B}(a_t, k(a^t, s_0))$ specify how much a country with the TFP shock a_t and the capital decision $k(a^t, s_0)$ can borrow such that, no matter which shock is realized in the next period, this country can repay its debt without incurring non-positive consumption.

The idea behind the natural borrowing constraint is to capture abilities of countries to repay as a criterion for sustainability of sovereign debts, that is, the viability of sovereign debts is determined by whether countries are able to generate sufficient future trade surplus (or future income) to repay the debt. The natural borrowing constraints implicitly assume complete enforcement in international borrowing and lending, that is, countries will repay their debts no matter how low their utilities have to go as long as it is physically feasible for them to do so.

An equilibrium in the bond with natural borrowing constraints model consists of the world interest rate and a sequence of allocations such that, given the interest rate, the allocations solve the planner's problem in each country and the world resource clearing constraints as in (4) are satisfied.

4.2 Results

To quantitatively study the FH coefficient generated from the bond with natural borrowing constraints model, we calibrate parameters and the TFP process in the model using the same method used in Section 3. We focus on the stationary equilibrium, i.e., density functions over the states are the same over time. The stationary equilibrium can be solved from the following recursive dynamic problem using recursive methods.

$$V(a,k,b) = \max \left\{ \frac{c^{1-\sigma}-1}{1-\sigma} + \beta \sum\nolimits_{a'|a} \pi(a' \mid a) V(a',k',b') \right\},$$

$$\text{s.t. } c + k' - (1-\delta)k + b' \le ak^{\alpha} + Rb\,,$$

$$b' \ge \hat{B}(a,k')\,.$$

To compute the stationary equilibrium, we first guess a world interest rate R. Next, we compute the decision rules for each state (a,k,b) and calibrate the invariant distribution $\lambda^{*}(a,k,b)$[22] using the iterative method together with decision rules. Finally, we compute whether the excess demand of bonds clears under the invariant distribution. If not, we update the interest rate until the bond markets are cleared.

After solving the equilibrium in the bond with natural borrowing constraints model, we simulate the model from the invariant distribution. We first simulate the model by drawing 57 time series of the same length as the data and compute all the relevant variables according to the decision rules. Next, we compute the average savings and investment rates for 57 series and run the Feldstein-Horioka regression. We repeat the above simulation procedure 1000 times. In Table 5, we report average statistics over 1000 simulations. The mean of the regression coefficients is 0.06 with the standard deviation of mean 0.02. Thus, the bond with natural borrowing constraints model generates an FH coefficient much lower than the one in the data, which is 0.46.

This model fails to produce the observed large FH coefficient because the natural borrowing constraints are quite loose and countries are able to shift huge resources across periods. As shown in Figure 6, under natural borrowing constraints, countries are allowed, on average, to borrow up to 7-8 times their current income. In equilibrium, this model generates a current-account-over GDP ratio of about 42 percent, in contrast to only 6 percent in the data. Thus, if sustainability of debts is modeled by abilities of countries to repay, the model generates too much international borrowing and lending relative to what is observed in the data and cannot account for the cross-section Feldstein-Horioka puzzle. In the next section, we will consider sustainability of debts by incentives of countries to repay in accounting for the cross-section Feldstein-Horioka puzzle.

5. The Bond with Enforcement Constraints Model

In this section, we study the bond with enforcement constraints model. In this model, countries are still limited to trading uncontingent bonds. However, there is only limited enforcement in international borrowing and lending. Contracts are enforced by the threat of exclusion from markets. That is, countries have the option to repudiate their debt and the punishment they get for doing so is to be excluded from international financial mar-

[22] Details see Appendix 3.

kets forever. In contrast to natural borrowing constraints, enforcement constraints aim to capture incentives of countries to repay rather than their abilities to repay. The amount that a country can borrow must be such that it has incentives to repay rather than defaulting and entering financial autarky for any possible realization of shock during the following period. In equilibrium, countries do not default, because the amount they can borrow endogenously fluctuates as their state fluctuates, so that repaying the debt is always preferred to defaulting on it. This model is closely related to and builds on Zhang (1997), which studies limited enforcement under incomplete markets in a pure exchange economy with 2-type agents.

5.1 The Model

The model environment is the same as the bond with natural borrowing constraints model. Given the world interest rate R, the planner chooses consumption, capital and bonds holdings $x \equiv \{c(a^t, s_0), k(a^t, s_0), b(a^t, s_0)\}$ to maximize the welfare specified as (2), subject to resource constraints specified as (10) and enforcement constraints given by

$$U(s_0, a^{t+1}, x) \geq V^{AUT}(a_{t+1}, k(a^t, s_0)), \text{ for any } a^{t+1} \text{ following } a^t .$$

The left hand side of the inequality denotes the continuation utility under allocation x,

$$U(s_0, a^{t+1}, x) \equiv \sum_{\tau \geq t+1} \sum_{a^\tau | a^t} \beta^{\tau-(t+1)} \pi(a^\tau \mid a^t) u(c(a^\tau, s_0)) .$$

The right hand side of the inequality denotes the autarky utility given by

$$V^{AUT}(a_{t+1}, k(a^t, s_0)) \equiv \max \sum_{\tau \geq t+1} \sum_{a^\tau | a^t} \beta^{\tau-(t+1)} \pi(a^\tau \mid a^t) u(c(a^\tau, s_0)) ,$$

subject to the resource constraints in the closed economy,

$$c(a^\tau, s_0) + k(a^\tau, s_0) - (1-\delta) k(a^{\tau-1}, s_0) \leq a_\tau k(a^{\tau-1}, s_0)^\alpha .$$

The enforcement constraints require that at any history node, the continuation utility must be greater than or equal to the autarky utility from that node on for any following state.

Formally, the planner's problem, labeled as the original problem, is given by

The original problem:

$$\max_{x} \quad \sum_{t=0}^{\infty} \sum_{a^t} \beta^t \pi(a^t) u(c(a^t, s_0))$$

s.t. $c(a^t, s_0) + k(a^t, s_0) - (1-\delta)k(a^{t-1}, s_0) + b(a^t, s_0) = a_t k(a^{t-1}, s_0)^{\alpha} + Rb(a^{t-1}, s_0)$, (12)

$U(s_0, a^{t+1}, x) \geq V^{AUT}(a_{t+1}, k(a^t, s_0))$, $\forall a^{t+1} \mid a^t$, (13)

$b(a^t, s_0) \geq D$. (14)

The constraint (14) is the short-sale constraint, where the debt limit D can be set low enough such that no countries will bind at this constraint with the presence of the enforcement constraints. This constraint is a necessary condition to rule out the Ponzi scheme, which does not violate the enforcement constraints.

The debt limit D can be explicitly specified in the spirit of the natural debt limit. If a country were binding at the debt limit, to ensure the non-negativity of consumption, the debt limit would have to satisfy the following constraints:

$$b(a^t, s_0) = b(a^{t-1}, s_0) \geq -(a_t k(a^{t-1}, s_0)^{\alpha} + (1-\delta)k(a^{t-1}, s_0) - k(a^t, s_0))/(R-1).$$

In order to set D loose enough, we choose the minimum of the right hand side of inequality, that is,

$$D = -(a_{\max} k_{\max}^{\alpha} + (1-\delta)k_{\max} - k_{\min})/(R-1) \tag{15}$$

When the debt level approaches this limit, the utility level approaches negative infinity since the consumption approaches to zero. In the original problem, for any feasible allocation, the short sale constraint will never bind with the presence of the enforcement constraints, which require that the discounted utility along any feasible allocation at any date be no less than the autarky utility (which is finite). This property will be used in the later proof. From here on, we drop the country index s_0 for simplicity of notations.

The world resource clearing constraints are given by (4). Similarly, we can define the *equilibrium of the bond with enforcement constraints model*. The enforcement constraints make this model very hard to compute. Without any transformation, this model has no obvious recursive structure because future decisions regarding consumption enter the current enforcement constraint. This makes the standard dynamic programming approach inapplicable. In the next subsection, we propose a tractable recursive technique to compute the bond with enforcement constraints model. The key idea is to replace the enforcement constraints with endogenous borrowing constraints.

From the debt-constrained literature, we know that the nature of the enforcement constraints is to define an endogenous debt limit based on the risk of the default. This has been illustrated by Alvarez and Jermann (2002) and Zhang (1997). Alvarez and Jermann (2002) study the enforcement constraints in a pure exchange economy with complete markets. Zhang (1997) studies the enforcement constraints also in a pure exchange economy but with incomplete markets. In a pure exchange economy, the autarky utility depends only on the exogenous shocks and so the feasible set is convex, even in the presence of enforcement constraints. Thus, first order conditions are necessary and sufficient for the optimality. In a production economy, the autarky utility at each node is a function of both the realized productivity shock and the amount of the capital stock that a country possesses. Thus, the feasible set is not convex in the presence of enforcement constraints. We need to prove the validity of the approach and to compute the solutions without invoking first order conditions, which are conveniently used in Alvarez and Jermann's and Zhang's works.

5.2 The Computational Method

In this subsection we first construct a transformed problem, in which an endogenous debt limit function replaces the enforcement constraints in the original problem. The endogenous debt limit function is carefully designed such that any solution to the transformed problem is also a solution to the original problem, and vice versa. The transformed problem has an obvious recursive structure and is much easier to compute. Thus, we will solve the transformed problem instead. Here we focus on the discrete grids of the capital stock.

Construct the transformed problem

The debt limit function $B: A \times K \to \mathbb{R}$, where A and K denote the finite set of shock and capital stock, specifies the amount that a country can borrow if the productivity shock today is $a \in A$ and the capital choice is $k' \in K$. The key to constructing the transformed problem is to find an endogenous debt limit function $B^*(a, k')$ such that any solution to the transformed problem is a solution to the original problem, and vice versa. Obvious such debt limit functions are not unique.[23] Here we provide a computationally convenient way to construct a candidate of such debt limit functions. Moreover, a useful property of the debt limit function we constructed is that it prevents default by prohibiting agents from issuing more debt than they are willing to repay, while allowing as much borrowing as possible.

The debt limit function B^* is constructed from the following three steps. First, given any debt limit function B, we define *the W problem with B* as

$$W(a_0, k_0, b_0;\, B) = \max_{c(a^t), k(a^t), b(a^t)} \sum_{t=0}^{\infty} \sum_{a^t} \beta^t \pi(a^t) u(c(a^t)) \tag{16}$$

$$\text{s.t.} \quad c(a^t) + k(a^t) - (1-\delta)k(a^{t-1}) + b(a^t) \le a_t k(a^{t-1})^{\alpha} + Rb(a^{t-1})$$

$$b(a^t) \le B(a_t, k(a^t)),$$

where, the function $W: A \times K \times F \times F^{\#A \times \#K} \to \mathbb{R}$, with $F = [D, 0]$ and D defined as (15), denotes the optimal welfare given the debt limit function $B \in F^{\#A \times \#K}$ and the initial state $(a_0, k_0, b_0) \in A \times K \times F$. Second, we define the operator $T: F^{\#A \times \#K} \to F^{\#A \times \#K}$ on the debt limit function as below:

$$TB(a, k') \equiv \max_{a' \mid \pi(a' \mid a) > 0} \left\{ \tilde{b}(a') : W(a', k', \tilde{b}(a');\, B) = V^{AUT}(a', k') \right\} \quad \text{for all } (a, k'). \tag{17}$$

[23] Any debt limit function under which the feasible allocation set of the transformed problem is a subset of the feasible set of the original problem but includes the optimal solutions of the original problem is a candidate for the transformed problem.

Third, we construct the endogenous debt limit $B^* = \lim_{n\to\infty} T^n B_0$ with $B_0(a,k') = D$ for all (a,k'). We next discuss the validity of each step in details.

We know that the W problem with the debt limit B is well defined and has a recursive structure given by

$$W(a,k,b;B) = \max_{c,b',k'} u(c) + \beta \sum_{a'|a} \pi(a' \mid a) W(a',k',b';B) \tag{18}$$

$$\text{s.t.} \quad c + k' + b' \le ak^{\alpha} + (1-\delta)k + Rb$$

$$b' \ge B(a,k').$$

For convenience of reference, we call W the market utility, which is the maximum utility that a country with state (a,k,b) can obtain from participating the credit market with debt limits B. The above recursive problem has the following useful properties, which help demonstrate validity of the operator T on debt limit functions.

Lemma 1. For fixed (a,k,B), $W(a,k,b;B)$ is continuous in b; for fixed (a,k,b), $W(a,k,b;B)$ is continuous in B.

Lemma 2. For fixed (a,k,B), $W(a,k,b;B)$ is strictly increasing in b.

Lemma 3. Suppose that $\lim_{b\to-\infty} W(a,k,b;B) = -\infty$ and $\lim_{b\to\infty} W(a,k,b;B) = \infty$. There exists a unique $\tilde{b}$ such that $W(a,k,\tilde{b};B) = V^{AUT}(a,k)$ for fixed (a,k,B).

Lemma 1 follows directly from the Maximum Theorem and Lemma 2 follows from the envelope theorem and the strict concavity of the utility function. These two lemmas establish two important properties of the market utility, i.e., continuity with respect to debts and debt limits and monotonicity on debt levels. The above two properties and the Intermediate Value theorem give rise to Lemma 3, which shows that there exists one unique level of debt (cutoff debt) to make countries indifferent between the market utility and the autarky utility.

From the above three lemmas, one can easily show that the operator T in (17) is well defined. However, for TB to be a debt limit function, we still need to show that it is non-positive for any (a,k'). For any future realization of shock, consider the market utility with (a',k',b') where b' equals the debt level zero. Clearly the market utility must be as least as high as the autarky utility because staying in the markets allows countries to smooth future consumption. Thus, the cutoff debt level must be less than or equal to zero.

Lemma 4. If $B(a,k') \le 0$ for all (a,k'), $TB(a,k') \le 0$ for all (a,k').

For a country with the shock a and the capital decision k', the debt limit function $TB(a,k')$ specifies the maximum amount debt that a country is willing to repay rather than defaulting independent of realized states next period. From the definition of TB, the following corollary follows immediately.

Corollary. $b' \ge TB(a,k')$ if and only if $W(a',k',b';\ B) \ge V^{AUT}(a',k')$ for all a' following a.

In the lemma below, we show the monotonicity property of the debt limit functions operator. The proof is delegated to Appendix 4 and the basic arguments are as follows. Suppose that the debt limit function B_1 is tighter than the debt limit function B_2 everywhere. Obviously the market utility under B_1 is lower or equal to the one under B_2 everywhere. Since the autarky utility is independent of debt limits, the amount of debt, with which countries are indifferent between the market utility and the autarky utility under B_2, makes countries default under B_1. The cutoff debt level under B_1 must be tighter than the cutoff level under B_2 everywhere. This property will be used to prove existence of the limit function in step 3.

Lemma 5. For any two debt limit functions B_1 and B_2, if $B_1(a,k') \ge B_2(a,k')$ for all (a,k'), then $TB_1(a,k') \ge TB_2(a,k')$ for all (a,k').

When starting with the debt limit function $B_0(a,k') = D$ for all (a,k'), we know that $B_1(a,k') = TB_0(a,k') \ge D = B_0(a,k')$ for any (a,k') by the discussion above (the short sale constraint is not binding in the presence of enforcement constraints). Using the monotonicity property of the operator T, we have

$$B_0(a,k') \le TB_0(a,k') \le T^2 B_0(a,k') \le .... \le T^n B_0(a,k') \le ..., \text{ for all } (a,k').$$

Fixing any (a,k'), we know that any monotone sequence $\{B_0(a,k'),\ TB_0(a,k'),\ T^2 B_0(a,k'),\ ....\}$ in the compact set $[D,0]$ converges to a limit in the compact set, which is denoted by $B^*(a,k')$, i.e.,

$$B^*(a,k') = \lim_{n\to\infty} T^n B_0(a,k') \quad \text{for all } (a,k'). \tag{19}$$

Now we can define the transformed problem using the debt limit function B^*.

The transformed problem:

$$W(a_0,k_0,b_0;\ B^*) = \max_{c(a^t),k(a^t),b(a^t)} \sum_{t=0}^{\infty} \sum_{a^t} \beta^t \pi(a^t) u(c(a^t)) \tag{20}$$

$$\text{s.t.} \quad c(a^t) + k(a^t) - (1-\delta)k(a^{t-1}) + b(a^t) \le a_t k(a^{t-1})^{\alpha} + Rb(a^{t-1})$$

$$b(a^t) \le B^*(a_t, k(a^t)).$$

In the following theorem, we prove the key result, which shows that any allocation that is optimal in the transformed problem is also optimal in the original problem, and vice versa.

Theorem 6. Any allocation is optimal in the transformed problem defined above, where $B_0(a,k') = D$ for all (a,k') with D defined as in (15), T is defined as (17), and B^* is defined as (19), if and only if it is optimal in the original problem.

Here we only describe the strategy of the proof for the "only if " part and the detailed proof refers to Appendix 4. Suppose that an allocation $x^T = \{c^T(a^t), k^T(a^t), b^T(a^t)\}$ is optimal in the transformed problem and we want to show that x^T is also optimal in the original problem. It is easy to show that x^T is feasible in the original problem. The key is to prove the optimality of x^T in the original problem, which is achieved by contradiction. Suppose that an allocation $x^O = \{c^O(a^t), k^O(a^t), b^O(a^t)\}$, which is feasible in the original problem, dominates x^T. If x^O is also feasible in the transformed problem, we arrive at a contradiction to the optimality of x^T in the transformed problem. The feasibility of x^O is obtained recursively. Clearly x^O is feasible in the W problem with $B_0 = D$ everywhere. This indicates that x^O is also feasible in the W problem with $B_1 = TB_0$ because x^O satisfies the enforcement constraints in the original problem. Repeating the same arguments, we can show that x^O is feasible in the W problem with any $T^n B_0$, and hence x^O is feasible in the transformed problem which is the W problem with the limit of sequence $\{T^n B_0\}_{n=1}^{\infty}$, B^*.

The technique to construct the transformed problem also manifests the computational approach to solve the transformed problem. We start with the debt limit function $B_0(a,k') = -D$, and solve the transformed problem given the debt limit function. Then we update the debt limit function according to the operator T. We repeat until the debt limit function converges. The detailed algorithm is described in Appendix 5.

The endogenous debt limit function is plotted in Figure 7. The amount that countries can borrow is strictly increasing in the capital stock. For low levels of capital, countries are poor and the benefit of default (having debt written off) is large relative to the cost of default (losing international risk-sharing). When countries get richer, the benefit of default relative to cost of default goes down, so countries have less incentive to default. Thus, rich countries are allowed to borrow more than poor countries. In Figure 8, we present the endogenous debt limit over output ratios as a function of the capital stock. The debt limit over output ratio is not strictly increasing in capital stocks due to the concavity of the production function. On average, the debt limits are quite tight, about 20 percent of output.

5.3 Results

In this subsection, we simulate the bond with enforcement constraints model and report the simulation results. The simulation method is the same as the bond with natural borrowing constraints model. The simulation results are reported in Table 6. The bond with enforcement constraints model generates a positive FH coefficient, 0.51, which is significantly different from zero and close to the FH coefficient in the data, 0.46. Thus the bond with enforcement constraints model can account for the cross-section Feldstein-Horioka puzzle.

The intuition of the result is as follows. Together with state-uncontingent bonds, enforcement constraints generate state-uncontingent debt limits. The state-uncontingent nature of debt limits means that countries can only borrow up to the amount at which, under any realization of uncertainty in the next period, countries still prefer to repay. Thus, the bond with enforcement constraints model restricts the amount of international borrowing and lending and generates a current-account-over-GDP ratio of about 6 percent, similar to what is observed in the data.[24] With such tight endogenous borrowing constraints, this model generates the positive FH coefficient close to the data.

When compared with the natural borrowing constraints in the previous section, the endogenous borrowing constraints are much tighter, which is shown in Figure 9. This comes from the difference of enforcement mechanism of debt contracts. Under the natural borrowing constraints, there is complete enforcement and the implicit default penalty is infinitely costly. However, under the enforcement constraints, there is limited enforcement and the explicit default penalty is the autarky utility, which is less costly. Hence, under enforcement constraints, countries have more incentives to default when there are lower penalties, and they are not allowed to borrow as much as when there are fewer incentives to default resulted from higher penalties. The implications on the size of international borrowing and lending also help us understand and evaluate two distinctive approaches to sustainability of sovereign debts. Clearly the willingness of countries to repay matches international capital flows better than the abilities of countries to repay.

In the bond with enforcement constraints model, there are two types of frictions: limited span of assets and limited enforcement of debt contracts. These two frictions are both important for accounting for the cross-section Feldstein-Horioka puzzle and either friction alone does not work. We have shown that limited span of assets alone, as in the bond

[24] See Table 7.

with natural borrowing constraints model[25], allows substantial borrowing and lending among countries, and thus drives down the cross-country correlation between average savings and investment rates. We have also shown in the previous version of this paper that limited enforcement of debt contracts alone still allows considerable borrowing and lending with volatile productivity process and fails to account for the cross-section Feldstein-Horioka puzzle.[26]

Thus, it is interaction between the two frictions above that help us generate capital flows close to the data and account for the cross-section Feldstein-Horioka puzzle. The key is that when combined with uncontingent bonds, enforcement constraints generate state-uncontingent debt limits, which guarantee that no matter which state is realized, countries have incentives to repay. With a volatile productivity process, the constraints become more binding rather than looser in the case of limited enforcement friction alone.

6 Time-series Correlation between Savings and Investment

The models studied in this paper are rich enough to look at both the time-series and the cross-section implications. One natural question is what are implications on time-series correlation between savings and investment rates within countries of the models studied here. In this section, we will address this question.

It is well known that, in standard international business cycle models, capital flows instantly to countries with higher rates of return. This feature exaggerates the volatility of investment relative to the data, and thus underestimates the correlation between investment and savings. To overcome these issues, usually capital adjustment costs are introduced. Thus, when looking at the time-series correlation, we also impose capital adjustment costs in the complete markets model and the bond with natural borrowing constraints model. Models with enforcement constraints do not need capital adjustment costs because enforcement constraints in nature hinder capital flows.

Following the literature, we specify the capital adjustment cost as

[25] In the bond with natural borrowing constraints model, the percentage of countries binding at the borrowing constraints is close to zero, 0.01. Thus, we can think of this model capturing the friction of limited span of assets.

[26] In Zhang (2005), we show that the complete with enforcement constraints model, with the limited enforcement friction alone, cannot account for the cross-section Feldstein-Horioka puzzle either.

$$\phi(K_{t+1}, K_t) = \chi\left((K_{t+1} - K_t) / K_t\right)^2 K_t / 2,$$

where χ denotes the adjustment parameter and is calibrated to match the volatility of investment in the data. Thus, the law of capital accumulation is give by

$$I_t = K_{t+1} - (1-\delta)K_t + \phi(K_{t+1}, K_t).$$

We find that all the models, with or without financial frictions, generate the positive time-series correlation between savings and investment rates with almost no change in the cross-section regression coefficient, as shown in Table 8. Hence, this paper confirms the results in the time-series literature that the positive time-series correlation offers no insight about financial frictions, and in fact is the result of highly persistent shocks.

7 Conclusion

In this paper, we take advantage of one of the strong regularities in the data – the positive cross-section correlation between average savings and investment rates – to investigate frictions in international financial markets. The paper makes several contributions. Firstly, we find that both the friction on the span of assets and the friction on debt contract enforcement are important for understanding savings and investment behaviors across countries. We also find that, closely related to the literature on sustainability of sovereign debts, the willingness of countries to repay captures the observed international capital flows better than the ability of countries to repay. Secondly, we have developed a tractable model of a world economy with a large number of countries that is a useful alternative to the standard two-country models, especially for issues involving cross sections.[27] Thirdly, at a technical level we have developed a tractable method to compute the bond with enforcement constraints model in the production economy with a continuum of countries.

[27] Contemporaneously Castro (2004a and 2004b) also developed the same framework to study the cross-country income distribution and development regularities related to capital accumulation.

Appendix 1: Data Description

Country sample

There are a total of 68 countries with all relevant data series available for the whole period (1960-2000). We delete 11 countries, which have RGDP per capita in 1995 less than $1050 (about 4 percent of the US RGDP per capita in 1995). These countries are Burundi, the Dominican Republic, Ghana, Kenya, Lesotho, Malawi, Malta, Niger, Rwanda, Togo, and Zambia.

The 57 countries remaining in the sample are Austria, Australia, Belgium, Cameroon, Canada, Chile, Columbia, Costa Rica, Côte d'lvoire, Cyprus, Denmark, Egypt, El Salvador, Fiji, Finland, France, Germany, Greece, Guatemala, Honduras, Israel, India, Iran, Ireland, Iceland, Italy, Jamaica, Japan, Korea, Malaysia, Mexico, Morocco, the Netherlands, New Zealand, Norway, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, the Philippines, Portugal, Senegal, Singapore, South Africa, Spain, Sri Lanka, Sweden, Switzerland, Thailand, Trinidad & Tobago, Tunisia, the United Kingdom, the United States, Uruguay, Venezuela, and Zimbabwe.

Data source

All of the following series are from the World Development Indicators 2004 for 1960-2000: nominal GDP, nominal private consumption, nominal gross capital formation, real GDP, and real gross capital formation. Total employment data are from three different sources. For 24 OECD countries, employment data are from the OECD databases. For the following 19 countries, we obtain employment data from national statistics: Cameroon, Chile, Columbia, Costa Rica, Cyprus, Egypt, El Salvador, Fiji, Israel, India, Jamaica, Malaysia, Pakistan, the Philippines, Singapore, Thailand, Trinidad & Tobago, Uruguay, and Venezuela. For the remaining 14 countries, employment data are supplemented by the Penn World Table[29] using the following formula:

$$\text{Employment} = \frac{\text{real GDP per capita} * \text{population}}{\text{real GDP per worker}} \,.$$

[29] The missing data for the first two groups are also backed up and interpolated using the Penn World Table data.

System of National Accounts

Our result for 16 OECD countries in 1960-1974 is 0.68, which is different from 0.89 in the original Feldstein-Horioka study due to changes of systems of national accounts (SNA). In the data source used by Feldstein and Horioka, National Accounts of OECD countries (1974), the 1953 SNA and the 1968 SNA are used for 16 OECD countries. In our data source, the World Development Indicators (2004), most countries use the 1993 SNA. The adoption of the 1993 SNA involved a lot of changes, some of which were simply reclassifications of items between various components of GDP, but others involve adding new transactions or suppressing old ones. Among all the components of GDP, the largest overall revisions have affected gross fixed capital formation. The 1993 SNA has broadened the concept of investment to include several types of expenditure that were not formerly considered to be capital spending such as spending on computer software and expenditures on mineral exploration, entertainment, artistic works, etc. The above changes led to an upward revision of gross capital formation and also led to a decrease of the regression coefficient of investment rates on savings rates.

Appendix 2: Estimation of the World Productivity Process

In this appendix, we describe an EM algorithm used to obtain the maximum likelihood estimates of parameters in the regime-switching process specified in (9). The log-likelihood function is given by

$$L(\Psi;\Theta) = \sum_{i=1}^{N} \log(f(\Psi^i;\Theta)),$$

where $\Psi^i = \{a_T^i, a_{T-1}^i, ..., a_1^i\}$ is a vector containing all the observations on country *i's* TFP; $\Theta = \{\{\mu_{\Re}, \rho_{\Re}, \sigma_{\Re}\}_{\Re=1,2,3}, \Pi\}$ denotes the parameters to be estimated; $\Re$ denotes regime; N denotes number of countries; T denotes number of periods; and

$$f(\Psi^i;\Theta) = \sum_{\Re^i} f(a_T^i \mid \Re_T^i, a_{T-1}^i;\Theta)....f(a_2^i \mid \Re_2^i, a_1^i;\Theta)\, p(\Re_T^i \mid \Re_{T-1}^i)...p(\Re_2^i \mid \Re_1^i)\, p(\Re_1^i).$$

Due to the nonlinearity of the maximum likelihood function, we cannot solve the parameters analytically. We use an EM algorithm, which is described below, to iteratively solve the maximum likelihood estimates. Start with an initial guess of the parameters Θ_{n-1}.

First, update the conditional probabilities of regimes in each period for each country using Bayesian rule. Next, given the conditional probabilities, compute Θ_n with the maximum log-likelihood method. Iterate the above procedures until Θ converges.

Appendix 3: Algorithm for the Bond with Natural Borrowing Constraints Model

Given any interest rate R, the recursive problem is well defined and we can solve for decision rules $c(a,k,b)$, $k'(a,k,b)$ and $b'(a,k,b)$. The decision rules together with the transition matrix of the productivity shocks define a Markov transition function over the states $Q:(A\times B\times K)\times\mathcal{P}(A)\times\mathcal{B}(K)\times\mathcal{B}(B)\rightarrow[0,1]$ as follows:

$$Q(a,k,b,A,K,B)=\sum_{a'\in A}\begin{cases}\pi(a'\mid a) & \text{if } k'(a,k,b)\in K \text{ and } b'(a,k,b)\in B\\ 0 & \text{otherwise}\end{cases}, \qquad (21)$$

where, $\mathcal{B}(K)$, $\mathcal{B}(B)$ and $\mathcal{P}(A)$ be the Borel set of K, the Borel set of B and the power set of A.

Given the above transition function, we can define the operator T on the space of probability measures $\Lambda(A\times K\times B,\ \mathcal{P}(A)\times\mathcal{B}(K)\times\mathcal{B}(B))$ as the following:

$$(T\lambda)(A,K,B)=\sum_{a\in A}\sum_{k\in K}\sum_{b\in B}Q(a,k,b,A,K,B)\lambda(a,k,b) \qquad (22)$$

for all $(A\times K\times B)\in\mathcal{P}(A)\times\mathcal{B}(K)\times\mathcal{B}(B)$. Note that the operator T maps the set of measurable function Λ into itself. An invariant distribution λ^* over the state space is defined as the fixed point of T.

We start by guessing an interest rate $R<1/\beta$ and solve the recursive problem for the decision rules c,k',b' to construct the transition function Q as Equation (21). Next, compute the invariant distribution λ^* by iterating from an initial guess according to (22) until it converges. We then check if the excess demand of the bonds holdings is cleared. If not, increase the interest rate R if $\overline{b}<0$ and decrease R if $\overline{b}>0$. Repeat the above procedures until the excess demand is sufficiently close to zero.

Appendix 4: Proofs

Proof of Lemma 5: By the definition of the operator T, we have

$$TB_1(a,k') = \max_{a'|\pi(a'|a)>0} \{\tilde{b}(a';B_1):\ W(a',k',\tilde{b}(a';B_1);B_1) = V^{AUT}(a',k')\},$$

$$TB_2(a,k') = \max_{a'|\pi(a'|a)>0} \{\tilde{b}(a';B_2):\ W(a',k',\tilde{b}(a';B_2);B_2) = V^{AUT}(a',k')\}.$$

By $B_1(a,k') \geq B_2(a,k')$ for all (a,k'), we have $W(a,k,b;\ \underline{B}_1) \leq W(a,k,b;\ \underline{B}_2)$ for any (a,k,b) since the feasible set under B_1 is a subset of the feasible set under B_2. This means that

$\tilde{b}(a';B_1) \geq \tilde{b}(a';B_2)$ for all a' following a and for all (a,k').

Thus $TB_1(a,k') \geq TB_2(a,k')$ for all (a,k'). *Q.E.D.*

Proof of Theorem 6: "Only if" part of the theorem:

First, we need to show that $x^T = \{c^T(a^t), k^T(a^t), b^T(a^t)\}$ is feasible in the original problem. The resource constraints are obviously satisfied. The short sale constraints are satisfied by monotonicity of the operator T. We need to show that the enforcement constraints are satisfied. From $B^*(a,k') \geq B_n(a,k') = T^n B_0(a,k')$ for any (a,k') and all n, we have

$b^T(a^t) \geq B_n(a_t, k^T(a^t))$ for all n.

By the corollary, we have

$W(a_{t+1}, k^T(a^t), b^T(a^t);\ \underline{B}_{n-1}) \geq V^{AUT}(a_{t+1}, k^T(a^t))$ for any a_{t+1} following a^t for any n.

From the continuity of W on B for fixed (a,k,b), we have

$W(a_{t+1}, k^T(a^t), b^T(a^t);\ B^*) \geq V^{AUT}(a_{t+1}, k^T(a^t))$ for any a_{t+1} following a^t.

Furthermore by the optimality of x^T in the transformed problem, we have

$$U(a^{t+1}, x^T) = W(a_{t+1}, k^T(a^t), b^T(a^t); B^*) \text{ for any } a_{t+1} \text{ following } a^t.$$

From the above two inequalities, we have

$$U(a^{t+1}, x^T) \geq V^{AUT}(a_{t+1}, k^T(a^t)) \text{ for any } a_{t+1} \text{ following } a^t.$$

Thus the enforcement constraints are satisfied at each history node.

Second, we need to show that x^T is optimal in the original problem by contradiction. Assume there is another allocation x^O which is feasible under the original problem and delivers higher welfare than x^T, i.e.,

$$\sum_{t=0}^{\infty} \sum_{a^t} \beta^t \pi(a^t) u(c^O(a^t)) > \sum_{t=0}^{\infty} \sum_{a^t} \beta^t \pi(a^t) u(c^T(a^t)). \tag{23}$$

We establish the contradiction by showing that x^O is feasible in the transformed problem. Obviously the resource constraints in the transformed problem are satisfied. We need to show $b^O(a^t) \geq B^*(a_t, k^O(a^t))$, which is proven by induction.

Clearly, we have $b^O(a^t) \geq B_0(a_t, k^O(a^t))$, where $B_0(a, k') = D$ for any (a, k'), by construction of D. Thus, x^O is feasible under the problem with the debt limit B_0, and we have

$$W(a_{t+1}, k^O(a^t), b^O(a^t); B_0) \geq U(a^{t+1}, x^O) \text{ for any } a_{t+1} \text{ following } a^t.$$

By the feasibility of x^O in the original problem, we also have

$$U(a^{t+1}, x^O) \geq V^{AUT}(a_{t+1}, k^O(a^t)) \text{ for any } a_{t+1} \text{ following } a^t.$$

From the above two inequalities, we conclude that

$$W(a_{t+1}, k^O(a^t), b^O(a^t); B_0) \geq V^{AUT}(a_{t+1}, k^O(a^t)) \text{ for any } a_{t+1} \text{ following } a^t.$$

Thus, $b^O(a^t) \geq TB_0(a_t, k^O(a^t))$ from the corollary.

Repeating the above arguments, we have $b^O(a^t) \geq T^n B_0(a_t, k^O(a^t))$ for all n. Thus, $b^O(a^t) \geq B^*(a_t, k^O(a^t))$ for any a^t, with $B^*(a_t, k^O(a^t)) = \lim_{n\to\infty} T^n B_0(a_t, k^O(a^t))$. So the allocation x^O is feasible in the transformed problem.

From the optimality of x^T and feasibility of x^O in the transformed problem, we have

$$\sum_{t=0}^{\infty}\sum_{a^t} \beta^t \pi(a^t) u(c^T(a^t)) > \sum_{t=0}^{\infty}\sum_{a^t} \beta^t \pi(a^t) u(c^O(a^t)),$$

which contradicts (23). Thus, the allocation x^T is optimal in the original problem.

The "if" part of the theorem can be proved using the similar arguments. *Q.E.D.*

Appendix 5: Algorithm for the Bond with Enforcement Constraints Model

We compute solutions to the bond with enforcement constraints model by computing the transformed problem with endogenous borrowing constraints. We start with an initial guess of the world interest rate $R < 1/\beta$. Next, we guess the initial debt limit function B_0 (start with very loose debt limits) and iterate over the debt limit functions as following. Given any debt limit function $B_j(a, k')$, we solve the value function $W(a, k, b\,; B_j)$ and policy functions $\{c(a,k,b), k'(a,k,b), b'(a,k,b)\}$. The new debt limit function is given by

$$B_{j+1}(a, k') = \max_{a'|\pi(a'|a)>0} \{\hat{b} : W(a', k', \hat{b}; B_j) = V^{AUT}(a', k')\}.$$

We repeat the above procedures until the debt limit function converges. We compute the invariant distribution over the states and compute the excess demand in the bond markets. If the bond markets clear, we are done. Otherwise we update the interest rate R and repeat the above procedures until the markets clear.

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Table 1: The Cross-Section Regression Coefficients

	1960-2000	1960-74	1974-2000
Full Sample (57 countries)	**0.46** (0.07)	0.51 (0.07)	0.46 (0.07)
16 OECD	0.61 (0.13)	0.68[28] (0.11)	0.49 (0.14)

Note: numbers in the bracket are standard errors of the coefficient above.

Table 2: Summary of the Parameters

Average TFP growth rate g_a	0.013
Capital share in production α	0.33
Capital depreciation rate δ	0.06
Risk aversion parameter σ	2.00
Discount rate β	0.96

Table 3: Maximum Likelihood Estimation Results

Regime		Low	Middle	High
σ		0.017 (0.0000)	0.027 (0.0001)	0.058 (0.0006)
ρ		0.993 (0.002)	0.995 (0.004)	0.987 (0.008)
μ		3.60 (0.13)	6.96 (0.56)	1.52 (0.05)
P	Low	0.95 (0.04)	0.01 (0.02)	0.04 (0.02)
	Middle	0.04 (0.03)	0.90 (0.05)	0.06 (0.03)
	High	0.07 (0.03)	0.05 (0.03)	0.88 (0.05)

Note: numbers in the bracket are standard errors of the coefficient above.

[28] See Appendix 2.1.3.

Table 4: The Complete Markets Model

Mean of γ_1	0.001
Mean of standard deviation of γ_1	0.020
Standard deviation of mean	0.021
Standard deviation of standard deviation	0.003

Table 5: The Bond with Natural Borrowing Constraints Model

Mean of γ_1	0.060
Mean of standard deviation of γ_1	0.027
Standard deviation of mean	0.029
Standard deviation of standard deviation	0.004

Table 6: The Bond with Enforcement Constraints Model

Mean of γ_1	0.051
Mean of standard deviation of γ_1	0.047
Standard deviation of mean	0.104
Standard deviation of standard deviation	0.009

Table 7: Current Account over GDP Ratio

Data	**0.06**
Complete Markets	0.41
Bond with Natural Borrowing Constraints	0.42
Bond with Enforcement Constraints	0.06

Note:
(1) The current account over GDP ratio is defined as ratio of the absolute value of current account to GDP.

(2) Statistics in the data and in the models are taken average over time and across countries in the sample.

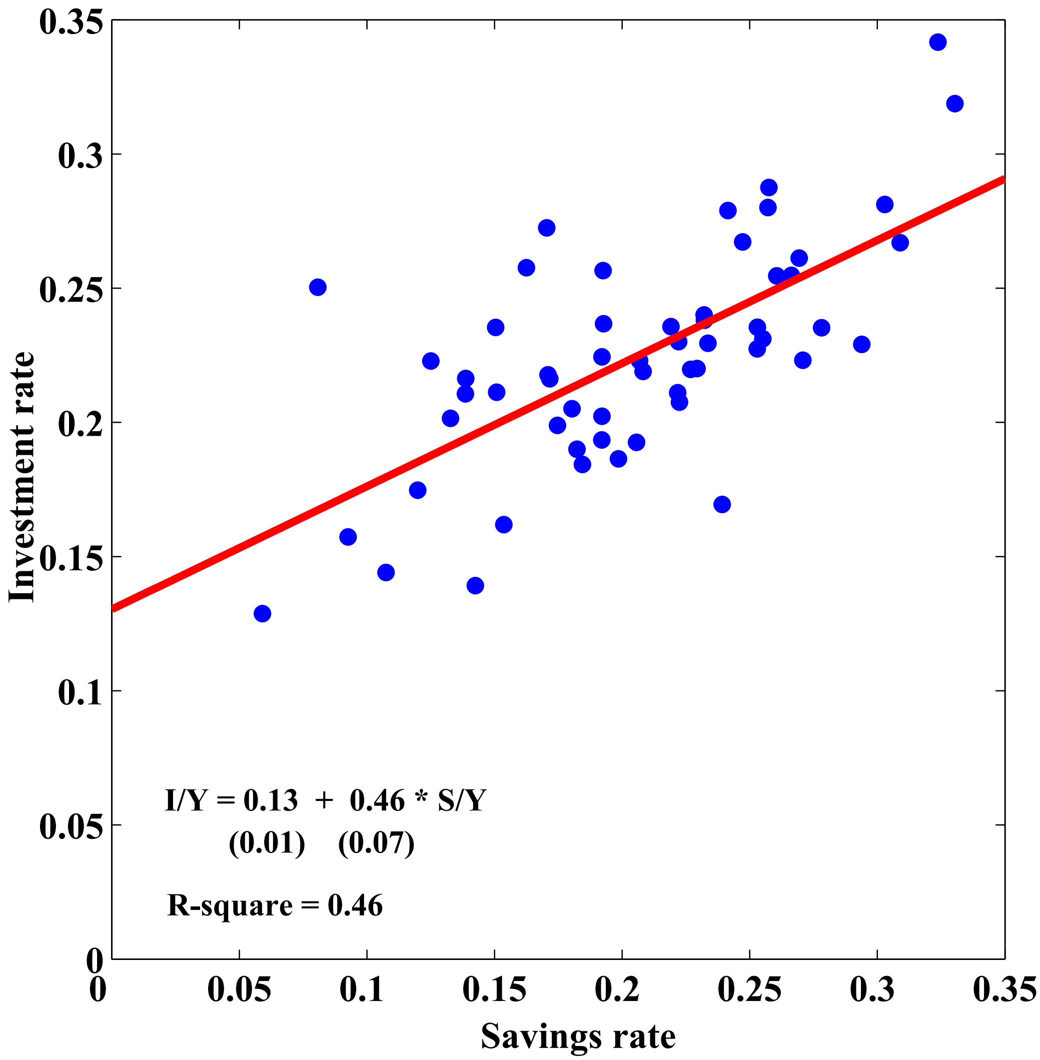
Table 8: The Time-Series Correlation

Data	**0.68**
The complete markets model	0.58
The bond with natural borrowing constraints model	0.65
The bond with enforcement constraints model	0.68

Note:

All statistics are calculated by averaging the time-series correlation between savings and investment rates across countries.

Figure 1: The Cross-Section Feldstein-Horioka Regression



Note:

The regression sample: 57 countries over the period from 1960 to 2000.

Figure 2: The TFP Processes

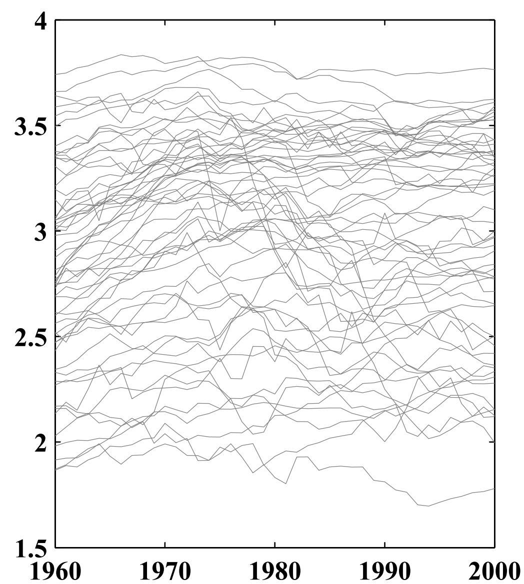


Figure 3: Feature 1

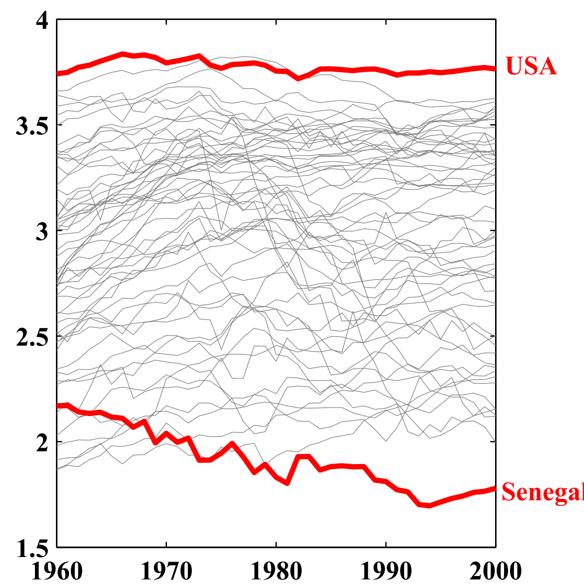


Figure 4: Feature 2

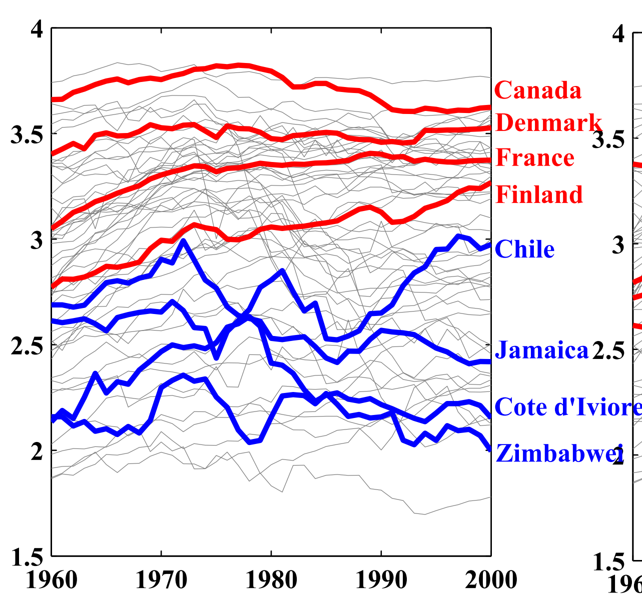
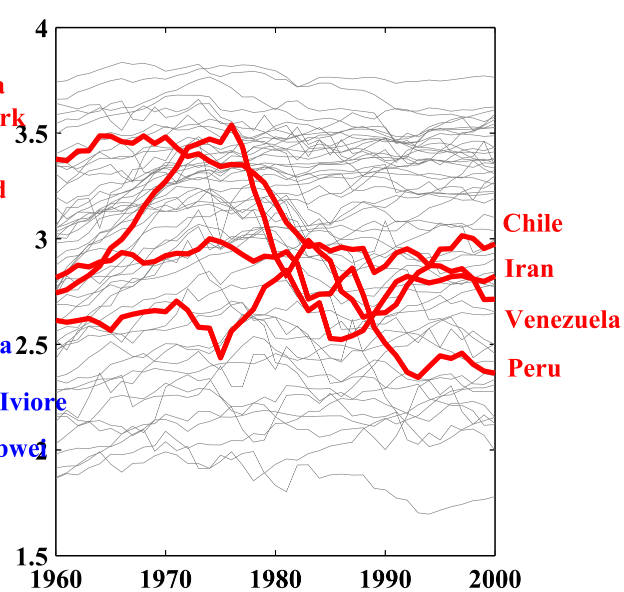
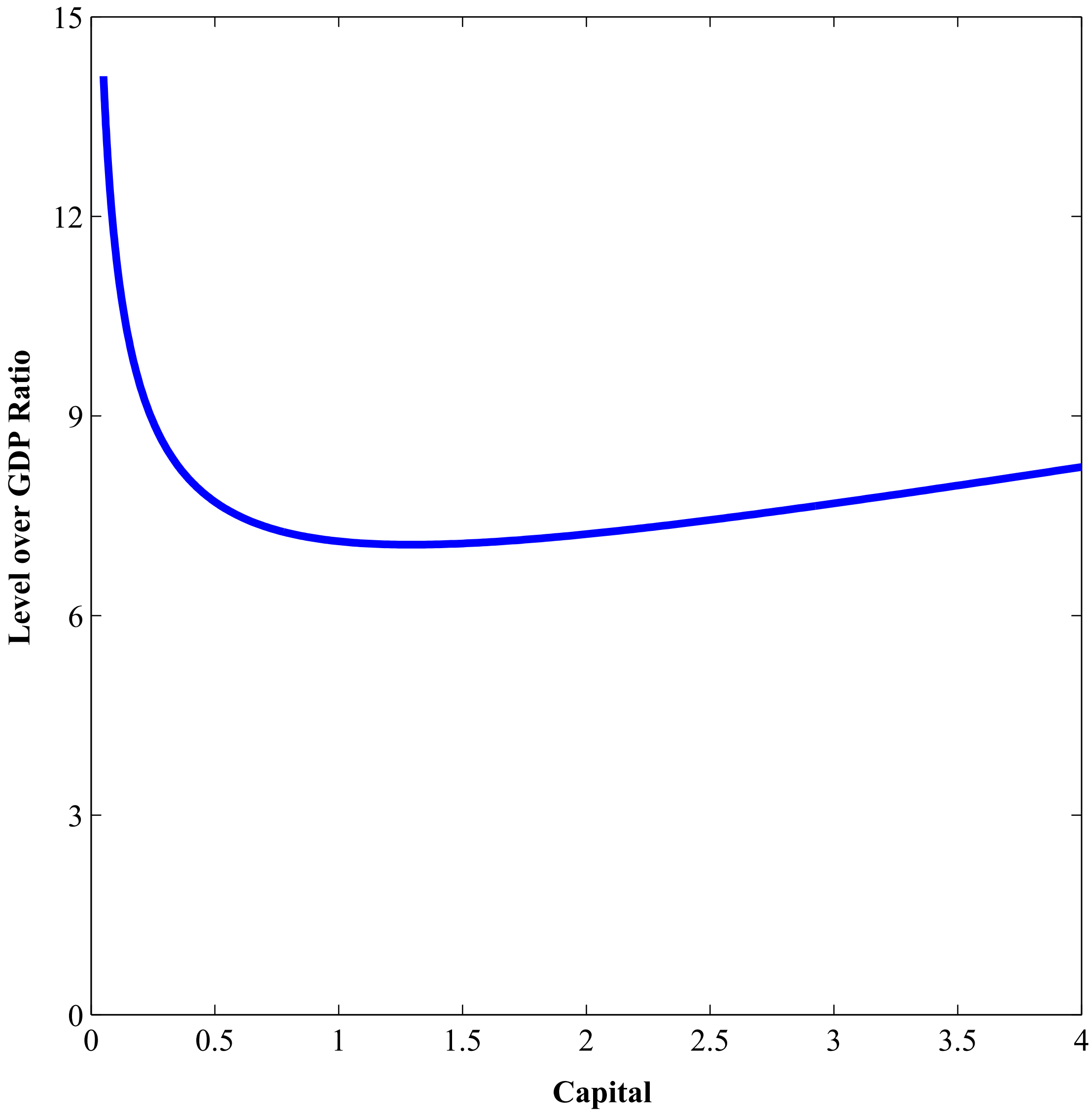


Figure 5: Feature 3



Note: TFP processes graphed here are logged and normalized.

Figure 6: Natural Borrowing Constraints



Note: GDP is output with mean productivity shock at any given capital stock.

Figure 7: Endogenous Borrowing Constraints, Level

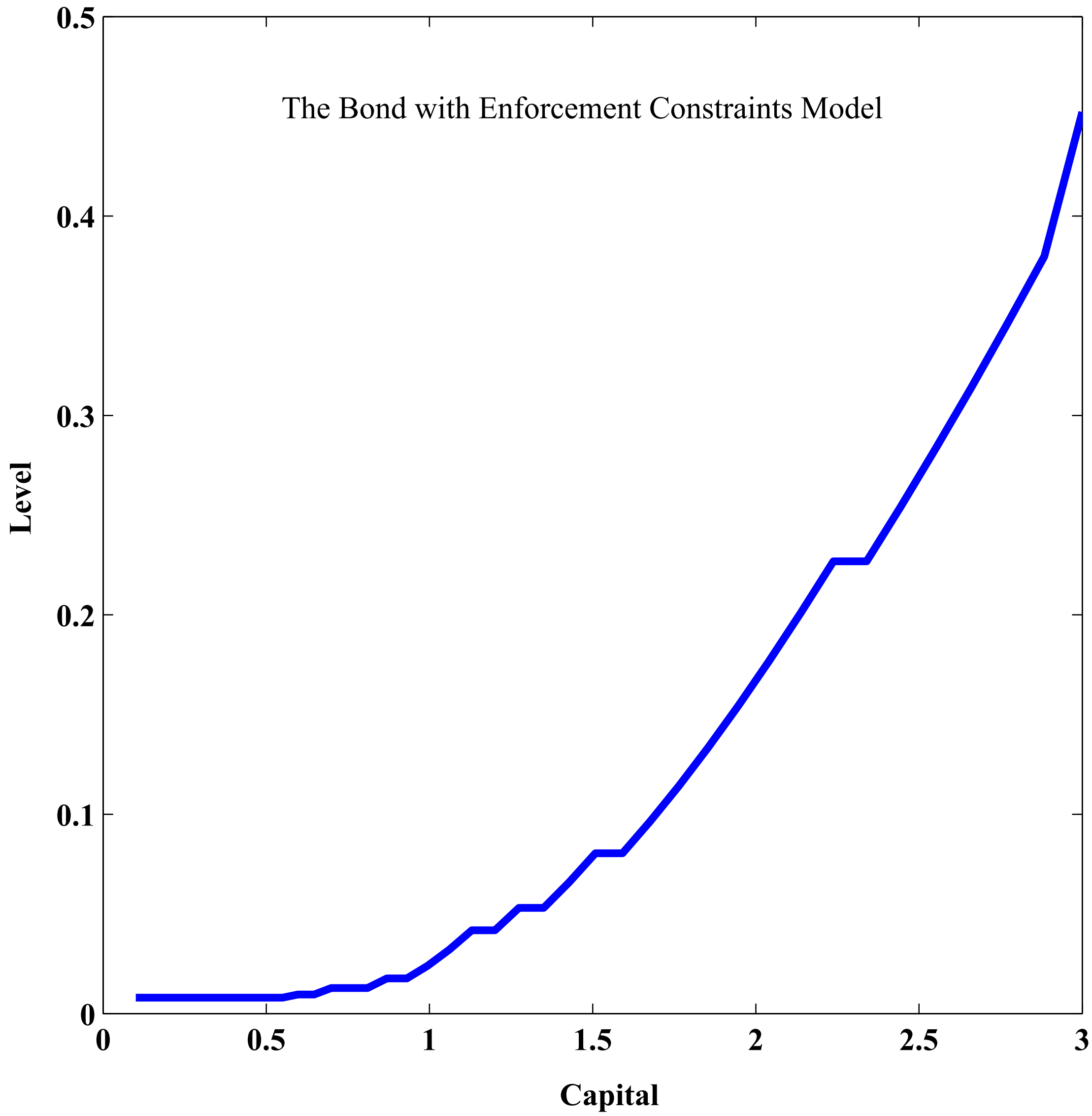
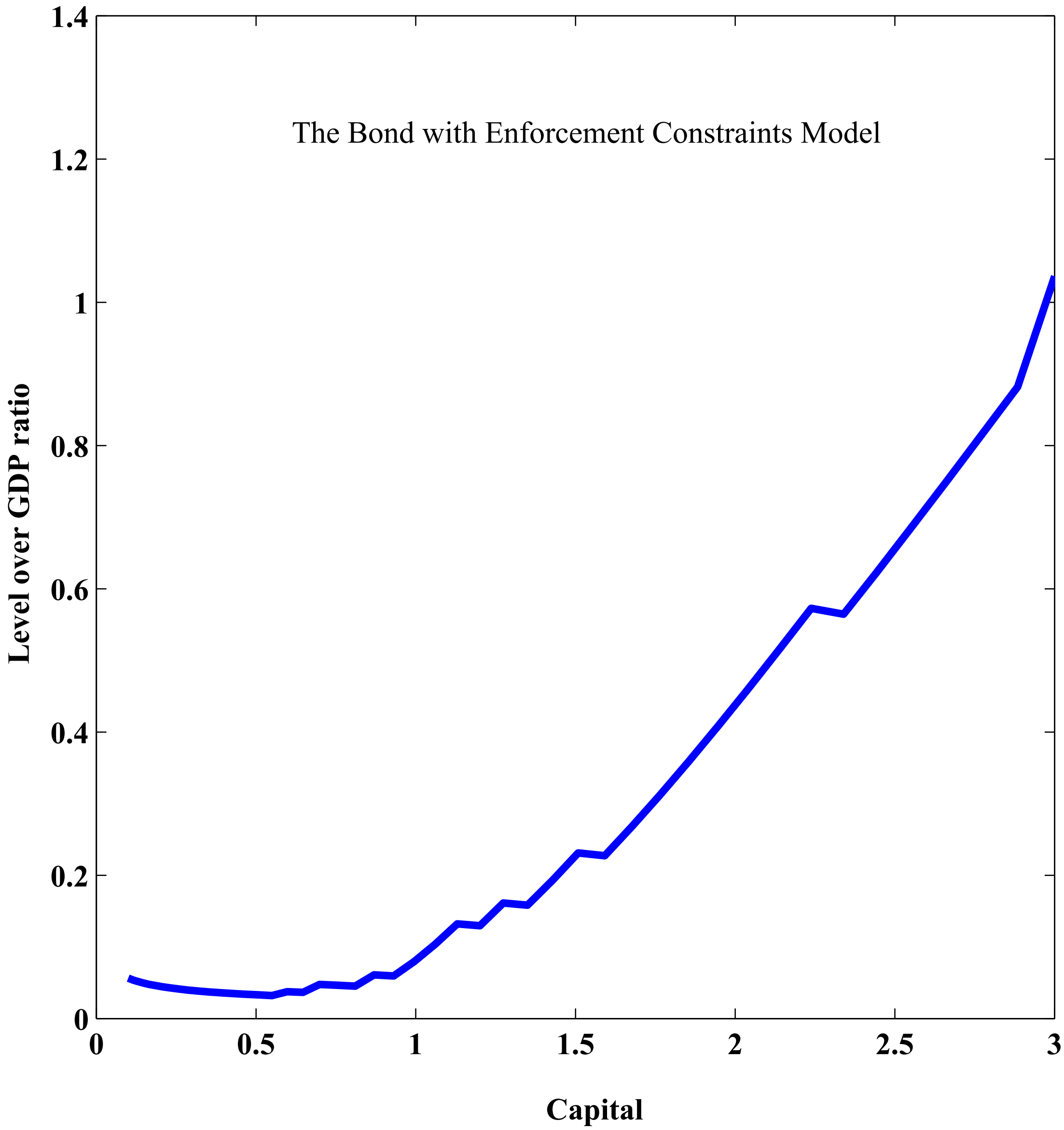


Figure 8: Endogenous Borrowing Constraints, Ratio



Note: GDP is output with mean productivity shock at any given capital stock.

Figure 9: Endogenous and Natural Borrowing Constraints