ION ACOUSTIC PARAMETRIC DECAY INSTABILITY IN A PLASMA WITH $\beta > 1$

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It is found that in a plasma with $\beta > 1$ a finite amplitude ion acoustic wave is unstable and decays into two shear Alfvén waves propagating in opposite directions. The threshold and growth rate of the instability are determined.

Resonant and non-resonant coherent wave interactions in a plasma have been studied extensively recently. In particular, the parametric decay and the purely growing (or modified decay) instabilities of an Alfvén wave has been studied by Lashmore-Davies and Ong [1]. Furthermore the short time behavior of a non-resonant modulational instability of a finite amplitude Alfvén wave was discussed by Lashmore-Davies [2], while the long time behavior has been investigated by Ionson and Ong [3]. In each of these cases the plasma was assumed to be of the low β type, i.e., $\beta \equiv p/(B^2/4\pi) \ll 1$. In a low β plasma there is only one possible decay instability in regard to an Alfvén wave, namely the decay of a finite amplitude Alfvén wave into a sound wave and another sideband Alfvén wave. On the other hand if $\beta > 1$ the only possible parametric decay instability is that of a finite amplitude sound wave into two Alfvén waves. Physically, this is the result of a reversal between branches of the dispersion curve for the sound mode and Alfvén mode in a plasma with $\beta > 1$. This reversal clearly indicates that the sound wave branch is no longer the lowest frequency normal mode of the high pressure plasma.

The dispersion relation for a sound wave is given by

$$\omega_0^2 = k_0^2 c_s^2/(1 + k_0^2\lambda_{De}^2) ,$$

and for the Alfvén wave

$$\omega = kc_A ,$$

where $c_A = B_0/(4\pi\rho_0)^{1/2}$, $c_s = (k_B T_e/m_i)^{1/2}$, $\lambda_{De}^2 = k_B T_e/8\pi n_0 e^2$ with n_0 the number density of the ambient plasma and $\boldsymbol{B}_0$ the external magnetic field. Moreover we assume $T_e \gg T_i = 0$, and $k_0^2\lambda_{De}^2 \ll 1$.

For a plasma with $\beta \gg 1$, the matching conditions for resonant three wave interactions (conservation relations)

$$\omega_0 = \omega_1 + \omega_2 , \quad \boldsymbol{k}_0 = \boldsymbol{k}_1 + \boldsymbol{k}_2 ,$$

are satisfied by

$$\omega_1 = \tfrac{1}{2}\omega_0[1 + (c_A/c_s)] ; \quad \omega_2 = \tfrac{1}{2}\omega_0[1 - (c_A/c_s)] , \tag{1}$$

$$k_1 = \tfrac{1}{2}k_0[1 + (c_s/c_A)] ; \quad k_2 = \tfrac{1}{2}k_0[1 - (c_s/c_A)] < 0 . \tag{2}$$

Since $\beta \gg 1$ implies $c_s \gg c_A$, it may be seen that ω_1/k_1 and ω_2/k_2 are, to a good approximation, equal to c_A and $-c_A$ respectively.

In order to analyze the decay instability we use the equations of magnetohydrodynamics with an external magnetic field in the x-direction. Thus we let

(a) $\boldsymbol{B}_0 = B_0\hat{x}$, (b) $\boldsymbol{b} = b\hat{y}$,

(c) $\boldsymbol{u} = u\hat{x}$, (d) $\boldsymbol{v} = v\hat{y}$,

and consider plane waves propagating in the x-direction. Note that for Alfvén waves propagating along $\boldsymbol{B}_0$ the two possible linear polarizations are independent, so that the analysis can be applied to either polarization. We then have the following system of one-dimensional MHD equations for a plasma warm with respect to the electrons but cold for the ions:

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0 , \tag{3}$$

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + c_s^2 \frac{\partial \rho}{\partial x} + \frac{\partial}{\partial x}(b^2/8\pi) = 0 , \tag{4}$$

$$\rho \frac{\partial v}{\partial t} + \rho u \frac{\partial v}{\partial x} - (B_0/4\pi) \frac{\partial b}{\partial x} = 0 , \tag{5}$$

$$\frac{\partial b}{\partial t} + \frac{\partial}{\partial x}(ub) - B_0 \frac{\partial v}{\partial x} - \eta \frac{\partial^2 b}{\partial x^2} , \tag{6}$$

where η is the electrical resistivity. Keeping only the lowest order nonlinearity, eqs. (5) and (6) yield

$$\rho_0\left(\frac{\partial^2 b}{\partial t^2} - c_A^2 \frac{\partial^2 b}{\partial x^2} - \eta \frac{\partial}{\partial t}\frac{\partial^2 b}{\partial x^2}\right) = -c_A^2 \frac{\partial}{\partial x}\left(\rho \frac{\partial b}{\partial x}\right)$$
$$- c_s \frac{\partial}{\partial x}\left(\rho \frac{\partial b}{\partial t}\right) - c_s \frac{\partial^2}{\partial t \partial x}(\rho b) . \tag{7}$$

We now let the sound wave be the pump wave of the form

$$\rho = \rho_1 \exp(i\psi_0) + \rho_1^* \exp(-i\psi_0)$$

with $\psi_0 = k_0 x - \omega_0 t$, and look for Alfvén wave solutions of the form

$$b = b_1 \exp(-i\psi_1) + b_2 \exp(i\psi_2) ,$$

where $\psi_1 = k_1 x - \omega_1 t$ and $\psi_2 = k_2 x - \omega_2 t$. As we are considering an "absolute" instability, we let k_0, k_1, and k_2 be all real and ω_1, ω_2 be complex.

We substitute these relations into eq. (7) and obtain a system of homogeneous algebraic equations for the Alfvén wave amplitudes b_1 and b_2. The vanishing of the coefficient determinant yields the dispersion relation. We can now investigate the parametric decay instability in a manner similar to that for a low β plasma. In particular the equation for the imaginary part of ω_1 is

$$\omega_{1i}^4 + \omega_{1i}\eta(k_1^2 + k_2^2)$$
$$+ \omega_{1i}[\eta^2 k_1^2 k_2^2 + 4\omega_{1r}\omega_{2r} - 4c_s^2 k_1 k_2 (\rho_1/\rho_0)^2]$$
$$+ \omega_{1i}[2\eta\omega_{1r}\omega_{2r}(k_1^2 + k_2^2)]$$
$$+ [\eta^2 k_1^2 k_2^2 \omega_{1r}\omega_{2r} - c_A^4 k_1^2 k_2^2 (\rho_1/\rho_0)^2] = 0 , \tag{8}$$

where we have assumed that $\omega_{1i} = \omega_{2i}$, and (ω_{1r}, k) and (ω_{2r}, k) satisfy the matching requirements (1) and (2) earlier. The threshold of the decay instability is found to be

$$|\rho_1/\rho_0|^2 = \omega_0^2\eta^2(1 - (1/\beta))/4c_A^4 .$$

Further the growth rate of the instability may be found by solving for ω_{1i} in eq. (8). For the case when $\eta = 0$ we find that

$$\omega_{1i} \simeq \tfrac{1}{4}\omega_0(1 - (1/\beta))^{1/2} |\rho_1/\rho_0|$$

when

$$|\rho_1/\rho_0| \ll (1/\beta)^{1/2} ,$$

and

$$\omega_{1i} \simeq \tfrac{1}{4}\omega_0(1 - (1/\beta))^{1/2}/\beta^{1/2} ,$$

when

$$|\rho_1/\rho_0| \gg (1/\beta)^{1/2} .$$

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References

[1] C.N. Lashmore-Davies and R.S.B. Ong, Phys. Rev. Letters 32 (1974) 1172.
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[3] J.A. Ionson and R.S.B. Ong, Univ. of Michigan Technical Report 013746-1-T (1975).