

SOLAR ENERGY ECONOMICS—THE *A PRIORI* DECISION

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Abstract—The initial decision concerning the economic viability of a solar energy heating system is shown to involve technical, physical, meteorological, geographic, design and cost factors as well as the source of funding and type of arrangements made to finance the system. Four economic/technical models, which include the influence of increasing fuel costs, are presented and compared with other possible kinds of investments to determine the economic viability of the system. It is found that different economic conclusions are both possible and justifiable for investment situations having different constraints.

NOMENCLATURE

A_c ,	collector area [$m_c^2(ft_c^2)$];	F_s ,	fraction of year solar system provides heating [1];
a ,	annual inflation rate [$\$/\$/y$];	F_u ,	utilization factor for solar system [1];
B ,	property equity factor [1];	F_c ,	fraction of clear sky solar radiation incident on collector [1];
b ,	constant, 1.0 [y^{-1}];	F_1 ,	inflation factor, equation (26) [y^{-1}];
C_c ,	annual cost conventional system [$\$/y$];	F_2 ,	inflation factor, equation (30) [1];
$C_{c,mm}$,	annual cost to conventional system for materials in maintenance [$\$/y$];	I ,	cost recovery factor, equation (17), Fig. 1 [$\$/\$/y$];
$C_{c,ML}$,	annual cost to conventional system for labor in maintenance [$\$/y$];	i_d ,	annual discount rate (interest) on mortgage [$\$/\$/y$];
$C_{s,MM}$,	annual cost to solar system for materials in maintenance [$\$/y$];	i ,	annual interest on investment [$\$/\$/y$];
$C_{s,ML}$,	annual cost to solar system for labor in maintenance [$\$/y$];	P_c ,	annual power requirements, conventional system [$J/y(Btu/y)$];
$C_{a,MM}$,	annual cost to auxiliary system for materials in maintenance [$\$/y$];	P_s ,	annual power requirements, solar system [$J/y(Btu/y)$];
$C_{a,ML}$,	annual cost to auxiliary system for labor in maintenance [$\$/y$];	P_a ,	annual power requirements, auxiliary system [$J/y(Btu/y)$];
C_B ,	cost of conventional furnace [$\$/y$];	Q_a ,	annual heating load [$J/y(Btu/y)$];
C_s ,	annual cost of a solar heating system [$\$/y$];	$(Q/A_c)_o$,	total annual clear sky solar flux per unit area of collector [$J/m_c^2-y(Btu/ft_c^2-y)$];
C_a ,	annual cost of auxiliary (conventional) heating system [$\$/y$];	(Q/A_c) ,	total annual clear sky solar flux per unit area of collector [$J/m_c^2-y(Btu/ft_c^2-y)$];
C_o ,	cost of solar system per unit area of collector, equation (16) [$\$/m_c^2(\$/ft_c^2)$];	R_o ,	investment probability (Case I), equation (14), [1];
$C_{s,a}$,	annual cost of solar/auxiliary heating system [$\$/y$];	R_i ,	investment probability (Case II), equation (27), [1];
c ,	constant, 1.0 [y];	R_{ii} ,	investment probability (Case III), equation (31), [1];
C_c^* ,	cost of collector [$\$/m_c^2(\$/ft_c^2)$];	R_{iv} ,	investment probability (Case IV), equation (34), [1];
C_s^* ,	cost of storage per unit volume of storage [$\$/m^3(\$/ft^3)$];	T ,	tax rate [$\$/\$/y$];
C_{Σ}^* ,	cost of equipment and controls for solar system per unit area of collector [$\$/m_c^2(\$/ft_c^2)$];	t ,	time [y];
C_f^* ,	unit cost of fuel [$\$/J(\$/Btu)$];	t^* ,	useful life [y];
C_p^* ,	unit cost of power [$\$/J(\$/Btu)$];	V_s ,	volume of storage unit [$m^3(ft^3)$].
E_o ,	annual useful energy from solar collector per unit area of collector, equation (15) [$J/m_c^2-y(Btu/ft_c^2-y)$];		
\bar{E} ,	mean value of useful energy from solar collector per unit area of collector, equation (23) [$J/m_c^2-y(Btu/ft_c^2-y)$];		
η_o, η ,	annual collector efficiency [1];		
F_a ,	fraction of year auxiliary system provides heating [1];		

Conversion factors (9)

$$(Q_1, Btu/ft_c^2-y) 11\,356.528 = (Q_2, J/m_c^2-y);$$

$$(Q_1, \$/ft_c^2) 10.764 = (Q_2, \$/m_c^2);$$

$$\left(Q_1, \frac{Btu/y}{\$/y}\right) 1055.056 = \left(Q_2, \frac{J/y}{\$/y}\right);$$

$$(Q_1, Btu) 1055.056 = (Q_2, J).$$

INTRODUCTION

AFTER a long period of dormancy solar energy utilization is once again receiving widespread interest and attention. Doubtless this renewed activity by governments, industry, the technical and scientific communities and the general public is principally the result of an increased awareness of the delicate balance presently existing between the world's supply of available fuels and energy and its rate of energy consumption. Because an uninterrupted flow of fuels and energy is essential to maintaining social, political and economic stability in the industrial nations, the matter of energy supply and utilization is now possibly the single most important national policy question facing such countries. The situation for developing nations is probably even more critical since increasing costs for energy will have a retarding influence on economic development and place an increased strain on already burdened economic systems. Perhaps at no other time in the history of the world has the need for genuine international cooperation and understanding been as great as it is now in the matter of energy supply, distribution and utilization.

For the heavily industrialized, energy dependent countries of western Europe, Japan and the United States presently available domestic sources no longer are able to provide sufficient fuel supplies to satisfy current demands nor those for projected national growth. Hence, at least for the near term (10–20 y, perhaps longer), it seems apparent that these nations will be forced to depend on imported energy and/or fuel supplies to support their economies and to maintain their present standards of living. Further, as things now stand, it seems virtually certain that this supply/demand deficit in energy supplies will have to be made up by liquid petroleum in the form of both finished products and crude. This condition is imposed by the approximate 10-y lead time for construction of central power generating facilities, both coal and nuclear. Beyond the near term (perhaps by 2000 AD) it seems possible that with a dedicated development of domestic coal and nuclear (fission) power plants, beginning now, dependence on imported fuels can be reduced if not eliminated, at least for the United States. Should this turn out to be successful, then the petroleum that would otherwise have been consumed by the U.S. and other industrial nations would be available for use in other parts of the world for both energy production and as feed stock for materials processing.

Beyond the matters of energy availability, fuel supply and demand is the concern for environmental protection. Although this problem exists mostly in the industrialized nations and is, to some degree, a consequence of economic and material affluence, it is now being recognized as an important consideration in any industrial/economic development. As a result of environmental concern new constraints are being placed by governments on industrial and plant construction. These inevitably lead to certain tensions between the legitimate requirements of nations for energy and industrial growth and the demands for an environment

which is as sullied as possible. As things now stand the point of equilibrium between the industrial/economic/energy needs of a people and their desires for an acceptably clean environment has yet to be established in any permanent form.

A subject of first priority importance which must always be included in discussions of these kind is the conservation of energy and fuels. Any savings that can be practically realized in the utilization of energy through conservation has the effect, of course, of extending the supply as well as directing valuable resources, such as petroleum, into other uses. Although it may be difficult always to define precisely, the waste of energy by any standard is inconscionable now and always has been.

It is to this situation of energy availability and conservation, economic growth and maintenance and environmental concern that a search is directed for energy sources other than the conventional ones: petroleum, coal, nuclear (fission). Prominent among these alternate energy sources is coal liquefaction and gasification, oil shale, nuclear fusion, geo-thermal and solar, the last three of which are often termed as renewable resources. For the more or less immediate future it is probable that coal and oil shale developments, possibly geo-thermal, will provide the principal contributions to energy supply while nuclear fusion and solar are much longer term sources in terms of significant quantities. It has been variously estimated that the contributions of nuclear fusion and solar will probably not exceed ten percent of the energy requirements in the U.S. until 2000–2010 and beyond even with the rates of technical/economic development anticipated in the preceding years.

Solar energy utilization has much to be said in its favor. It is a source that is clean, silent, abundant, widely available, dependable and essentially inexhaustible. The first two characteristics speak importantly to environmental concern. Solar energy is abundant and widely available in the sense that the quantities of sunlight falling on the surface of the earth are sufficiently large to be made useful in processes in most parts of the earth. In this connection it is perhaps of interest to note that the greatest amount of solar energy falling on the earth's outer atmosphere in any 24-h period is at the south pole on the winter solstice, 21 December. The dependability of solar energy is statistical in that while it is certain that over a given period of time some fraction of the maximum possible solar radiation will reach the surface of the earth neither the magnitude of that fraction nor its statistical confidence limits are presently known to any satisfactory degree over the earth. Solar energy is inexhaustible in terms of the life expectancy of the sun, something like 30–40 billion years. This span of time certainly exceeds that of any human projection for earthly questions of economics and energy.

Solar energy is sometimes said to be "free". This is fallacious since, as is the case for any resource, a considerable investment of capital is necessary to convert the energy from the sun to useful purposes. In fact, it

is probably closer to the truth to say that solar energy is presently the most expensive form of energy utilization. It is, of course, this question that is discussed in this paper. Perhaps the only thing "free" about solar energy is that there are no transportation costs to point of use.

ECONOMICS OF SOLAR ENERGY SYSTEMS

Solar energy conversion systems are capital intensive. That is, a significant initial investment of capital is required to build and install the appropriate equipment and controls to convert the sun's energy to some useful purpose. As in all such systems the decision to proceed with the investment is based on the anticipation of some (generally economic) return during the life of the system which exceeds the total net costs of the system itself over this period. In the case of solar energy thermal conversion systems it is the expected savings in fuel costs over the life of the system compared with the investment costs (including taxes, insurance, maintenance, etc., costs) that provides the economic basis for decision. To be sure, not all legitimate bases for decision are economic. Certainly, reliability of continued supply in the face of instability or expected dislocation of supply of conventional fuels and energy sources is a sound reason to consider the installation of solar conversion equipment. In some instances, such as at remote locations, it may not be possible to use ordinary fuels and a solar system is really the only practical choice. The electric power source by direct photovoltaic conversion on satellites and space vehicles is such an example. Further, the desire to preserve the environment or to reduce the consumption of petroleum, freeing it for other uses, are also reasons for utilizing solar energy. However, the economic questions are subject to a higher degree of quantification and modeling and, because of these, will be the focus of this paper.

Solar energy utilization is generally considered to encompass (a) heating and cooling of buildings and the heating of process water, (b) thermal conversion in Rankine cycle systems, (c) photovoltaic conversion, (d) wind energy, (e) bio-mass conversion and (f) ocean thermal conversion. Although the general approach in this paper would apply to an economic evaluation of all of these systems, the emphasis will be on (a), the heating and cooling of buildings and the heating of process water. This system will help to focus the discussion and it is also that solar energy conversion system which is expected to be the first having wide application. Accordingly, it is much in the eye of the public, government, contractors, and the technical communities.

It is possible to identify two basic modes of economic decision making regarding solar energy supply systems. Although they are not completely unrelated, one logically precedes the other in point of time. The first economic evaluation is a general one which asks if an investment in a solar energy conversion system is economically viable in comparison with other investments which also might be made taking into consider-

ation all relevant physical and economic parameters, including anticipated cost escalation of fuels. The second economic evaluation would follow an affirmative response to the first, namely, if such an investment is economically justified what is the economic optimum form of the system in terms of location, equipment size, type and fuel and labor costs. Such analyses are very complex and usually require detailed (hourly) meteorological and solar data along with performance characteristics of the solar system components. Calculations are normally carried out using a computer. Studies whose principal thrust is mostly of this second kind have been published by Löf and Tybout [1, 2], Buchberg and Roulet [3], Willicutt *et al.* [4], among others. Two recent text and reference books also discuss this type of economic analysis [5, 6].

The emphasis here will be on the first type of economic decision and, accordingly, is called the *à priori* decision. This view examines the present state of solar energy technology in relation to the availability of solar energy at any geographic location, the utilization of the proposed installation, the local economic factors of unit solar system costs and labor costs and compares these with the costs of fuel, investment capital and taxes. Other costs such as insurance and maintenance are implicitly included. Allowance is made for escalation in fuel costs during the life of the solar energy conversion system.

The analysis will apply directly to systems providing for solar heating as these are expected to be practical first. With modification a similar analysis can be formulated for solar heating and cooling systems. Such systems, however, will probably not be found in normal use for many years.

PHYSICAL-ECONOMIC MODEL

To examine the economic viability of a solar energy conversion system in the pattern of the first mode, a model is adopted consisting of two buildings, one which receives its heat and hot water by conventional methods and another which is supplied with heat by a combination of conventional and solar systems. The annual total costs of each system, including those for capital investment, operation, fuel and taxes are compared. Costs for maintenance and insurance are not included explicitly but could be considered by increasing the annual investment charges appropriately by some small amount.

(a) Annual cost of conventional system

The total annual cost of a conventional heating system may be expressed as

$$C_c = C_R(I + T) + Q_a C_F + P_c C_p^* + C_{c.MM} + C_{c.ML} \quad (1)$$

(b) Annual cost of a solar energy supply system

The total annual cost of a solar energy supply system assuming the cost of the capital investment, I , and taxes, T , are the same as for the conventional heating

system is

$$C_s = \left[C_c^* + C_s^* \left(\frac{V_s}{A_c} \right) + C_E^* \right] A_c (I + T) + P_s C_p^* + C_{s,MM} + C_{s,ML} \quad (2)$$

In this result the term C_E^* is the cost of equipment, controls, piping, etc. associated with the solar system expressed on the basis of the area of the solar collector. While it is recognized that such costs are not necessarily directly related to collector area, A_c , their representation as shown enables them to be included in a simple, although somewhat approximate manner. In general they will be relatively small and for any given class of solar systems could be expressed on a unit collector area basis.

(c) *Annual cost of the auxiliary heating system associated with the solar system*

Assuming the cost of the auxiliary heating system is essentially the same as that for a conventional heating system† and that the investment costs and taxes also are the same, the annual cost of the auxiliary heating system is

$$C_a = C_B(I + T) + F_a Q_a C_F^* + P_a C_p^* + C_{a,MM} + C_{a,ML} \quad (3)$$

where F_a is the fraction of the annual heating load, Q_a , that would be carried by the auxiliary system in combination with a solar energy system.

(d) *Annual costs of a solar and auxiliary heating system*

The annual cost of a solar and auxiliary system (usually referred to as simply a solar energy heating system) is found by combining equations (2) and (3). Hence

$$\begin{aligned} C_{s,a} &= C_s + C_a \\ &= \left[C_c^* + C_s^* \left(\frac{V_s}{A_c} \right) + C_E^* \right] A_c (I + T) + C_B(I + T) \\ &\quad + F_a Q_a C_F^* + P_s C_p^* + P_a C_p^* + C_{s,MM} + C_{a,MM} \\ &\quad + C_{s,ML} + C_{a,ML} \end{aligned} \quad (4)$$

Now, in seeking an answer to the question of whether or not to consider a solar energy system as opposed to a conventional heating system, these two alternatives can be compared on the basis of their annual costs if an economic decision is to be made. Thus, the difference in the annual costs between these systems may be written from equations (1) and (4) as

$$\Delta C = C_c - C_{s,a} \quad (5)$$

or

$$\begin{aligned} \Delta C &= Q_a(1 - F_a)C_F^* - \left[C_c^* + C_s^* \left(\frac{V_s}{A_c} \right) + C_E^* \right] A_c (I + T) \\ &\quad + (P_c - P_a)C_p^* + (C_{c,MM} - C_{a,MM}) + (C_{c,ML} - C_{a,ML}) \\ &\quad - P_s C_p^* - C_{s,MM} - C_{s,ML} \end{aligned} \quad (6)$$

As an approximation it may be assumed that the differences in the costs for power and materials and labor

for maintenance between the conventional and auxiliary systems are small and can be neglected. Further, in a well designed solar energy system both the power and maintenance costs can be made small. These costs will also be neglected although their effects could be included by increasing the value of $(I + T)$ slightly. With these approximations the annual cost differential between a conventional heating system and a solar/auxiliary heating system becomes

$$\Delta C = Q_a C_F^* (1 - F_a) - \left[C_c^* + C_s^* \left(\frac{V_s}{A_c} \right) + C_E^* \right] A_c (I + T) \quad (7)$$

CRITERIA FOR ECONOMIC VIABILITY OF A SOLAR ENERGY SYSTEM

The result in equation (7) provides the basis for constructing several technical/economic models for evaluating the economic viability of a solar energy heating system. These models are then used to form what is called the *a priori* economic decision. That is, a decision made on economic grounds which determines whether or not a solar energy heating system is economically feasible within a given set of physical, technical and economic constraints. As it turns out, there is not a single economic criterion for this decision as each such judgment depends on a particular basis for an economic determination and there are several possibilities. In the present discussion five cases will be identified and there are doubtless others. The five cases presented here are as follows:

- Case I. Investment consideration on the basis of present values.
- Case II. Investment consideration on the basis of life-cycle values under a mortgage contract with allowance for increasing fuel costs.
- Case III. (a) Investment consideration using income producing capital (non-mortgage contract) with allowance for increasing fuel costs.
(b) Investment consideration using income producing capital (non-mortgage contract) with allowance for property equity and increasing fuel costs.
- Case IV. Redemption of initial investment (pay-off period) with allowance for increasing fuel costs.

These cases all differ in their time periods and the bases for their economic value judgements. Case I is for a period of one-year, the minimum time period considered, and is accordingly identified as "present value". It is possible this minimum period could be extended to two or three years depending on matters of price stability and the willingness of an investor to assume certain risks. The other cases cover a greater period of time and include the very important case of the "life-cycle". Each is formed by integrating in time the "present value" model (case I) under certain assumptions regarding interest rates and anticipated increases in the cost of fuel by both market and inflationary forces. Case IIIb, probably represents the least quantitative model as it includes an allowance for

† This is both a matter of strategy in building design and the market availability of heating units having established ranges of capacity. The cost of such units is usually not large.

financial equity through future sale of property, something which would involve future market factors that may be expected to vary widely and embody subjective considerations as well.

The first term in equation (7) may be written as

$$Q_a C_f (1 - F_a) = Q_a C_f F_s, \quad (8)$$

where F_s is the fraction of the annual building heating load carried by the solar portion of the heating system. Thus, $F_s Q_a$ is the useful annual solar contribution to the total annual heating load of the building and may be written in an equivalent form as

$$F_s Q_a = F_u F_e (Q/A_c)_0 A_c \eta_0 \quad (9)$$

where F_u is the annual utilization factor of the solar energy system to produce useful energy ($0 \leq F_u \leq 1.0$), F_e is the annual average fraction of the maximum solar energy that is incident on the surface of a solar collector having arbitrary orientation at any geographic location ($0 \leq F_e \leq 1.0$), $(Q/A_c)_0$ is the annual average clear sky (maximum) solar heat flux incident on the collector surface for any geographic location, and η_0 is the annual average thermal efficiency of the solar collector ($0 \leq \eta_0 \leq 1.0$). Recognizing the significance of $F_s Q_a$ as expressed above the general criterion for ΔC , equation (7), written on the basis of a unit area of collector becomes

$$\frac{\Delta C}{A_c} = F_u F_e (Q/A_c)_0 \eta_0 C_{f_0} - \left[C_c^* + C_s^* \left(\frac{V_s}{A_c} \right) + C_E^* \right] (I + T), \quad (10)$$

where C_f^* is written as C_{f_0} , the present cost of fuel in order to make it consistent with the other factors in the equation. In this form, the differential cost criterion, $\Delta C/A_c$, embodies the several factors that govern the economics of a solar energy heating system in a fairly simple way: operation (F_u), natural, geographic, meteorological [F_e and $(Q/A_c)_0$], design (η_0 and V_s/A_c) and economic, including the costs of material, labor, production, transportation, local installation, fuel (C_c^* , C_s^* , C_E^* , C_{f_0}) and the costs of ownership (T) and capital (I). As mentioned earlier the costs of insurance and maintenance can be included by increasing the factor $(I + T)$ by some small amount to be established by specific, local considerations.

With this view, a criterion for economic viability for a proposed solar energy heating system can be developed. This criterion can be stated as follows: "Whenever the differential cost between a conventional heating system and one consisting of a solar/auxiliary system is equal or greater than zero, the solar/auxiliary system is economically justified." In terms of equation (10) this criterion is expressed as

$$\frac{\Delta C}{A_c} \geq 0 \quad (11)$$

or,

$$F_u F_e (Q/A_c)_0 C_{f_0} \eta_0 - \left[C_c^* + C_s^* \left(\frac{V_s}{A_c} \right) + C_E^* \right] (I + T) \geq 0. \quad (12)$$

In another somewhat more useful form, this criterion may be written as

$$R_0 \equiv \frac{F_u F_e (Q/A_c)_0 C_{f_0} \eta_0}{\left[C_c^* + C_s^* \left(\frac{V_s}{A_c} \right) + C_E^* \right] (I + T)} \geq 1.0. \quad (13)$$

Thus, whenever the combination of the various technical, physical and economic factors that are expressed in this equation are such as to cause R_0 to be equal to or greater than 1.0, a solar heating system is economically viable.

ECONOMIC MODELS

Case I—Investment considerations on the basis of present values

The criterion for R_0 in equation (13) is actually the model for this case as it was developed from the analysis for an annual period. This is called here the case for "present values" and can probably be extended to greater periods of time (2–3 y) at the discretion of each potential investor on his judgement of the expected variability of economic cost factors. The factor R_0 , which can be called the investment probability, can be written as

$$R_0 \equiv \frac{(E_0/C_0)}{(I + T)/C_{f_0}}, \quad (14)$$

where

$$E_0 = F_u F_e (Q/A_c)_0 \eta_0, \quad [\text{J/m}^2 \cdot \text{y}, (\text{Btu}/\text{ft}^2 \cdot \text{y})] \quad (15)^*$$

$$C_0 = C_c^* + C_s^* (V_s/A_c) + C_E^*, \quad [\$/\text{m}^2, (\$/\text{ft}^2)] \quad (16)^*$$

The factors E_0 and C_0 separate the governing parameters into relatively easily identifiable groups involving (E_0) operation, physical circumstances and design and (C_0) total installed cost of the solar equipment and controls per unit area of collector. The term $(I + T)/C_{f_0}$ is the ratio of cost of capital and ownership to the present cost of fuel, having the units of J·y·\$ (Btu·y·\$).

The quantity I is generally known as the "cost recovery factor" [6] and is a constant annual cost to the borrower per dollar of initial investment on a mortgage contract apportioned to amortize the borrowed capital during the period of the contract. The magnitude of I depends on the period of time over which the contract (mortgage) is to be amortized and the discount rate (interest) charged by the lender. I is made up of both interest and repayment of the borrowed capital and may be written as

$$I = \frac{i_d(1 + ci_d)^{bt}}{(1 + ci_d)^{bt} - 1}, \quad \frac{\$ \text{COST}/\text{y}}{\$ \text{INVESTMENT}}. \quad (17)$$

The constants b and c have values of unity and are introduced in equation (17) to preserve dimensional homogeneity. I is shown in Fig. 1 for the ranges of mortgage period and interest rates common in the United States.

The investment probability R_0 for this case is given in Fig. 2 as a function of $(I + T)/C_{f_0}$ and (E_0/C_0) . The range of $(I + T)/C_{f_0}$ is chosen to include values of this

*Conversion factors are provided in the nomenclature.

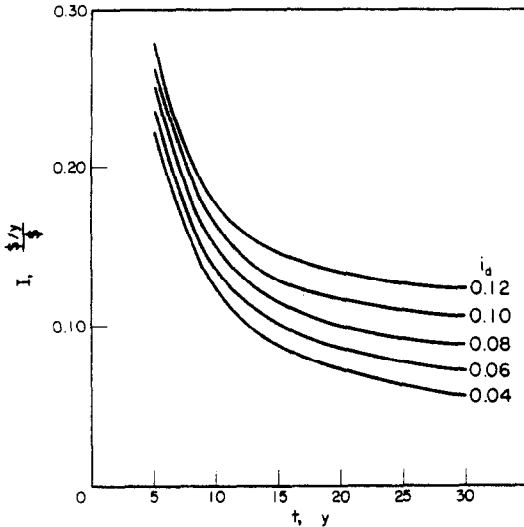


FIG. 1. Cost recovery factor, I.

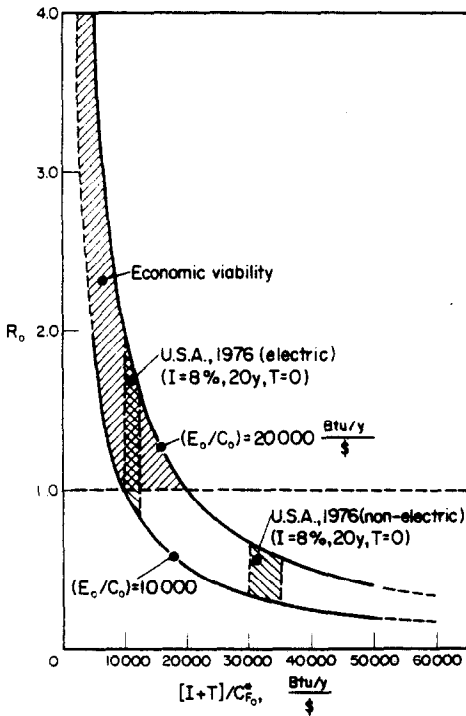


FIG. 2. Investment probability, R_0 (Case I).

parameter as presently exist in the U.S. as well as to encompass possible values that may exist in the future as investment costs, taxes and fuel costs may change. The limits selected for (E_0/C_0) , 10,000–20,000 Btu/y- $\$$ (10.551×10^6 – 21.102×10^6 J/y- $\$$), represent roughly the upper and lower bounds that could be expected at the present time in the U.S.† Of course, the limits shown on Fig. 2 for (E_0/C_0) will correspond to other combinations of the quantities which determine both E_0 and C_0 .

†For these calculations C_0 was taken as $\$25/\text{ft}^2$ ($\$269.10/\text{m}^2$), a cost which is probably a high-mean for installed equipment in the U.S. (1976) and η_0 was set at 0.50, a reasonable estimate for solar and meteorological conditions prevailing over most of the U.S. and available from current solar collector designs.

When R_0 exceeds unity a solar energy heating system can be judged as economically viable under “present” conditions of geographic location, selected design and economic costs. When R_0 is less than unity, the opposite is true. As shown in Fig. 2, the approximate ranges of the controlling parameters for mid-1976 in the U.S. indicate that at this time solar energy is not an economically attractive investment for building heating in competition with non-electrical resistance heating units. These results suggest that in this case the annual system costs currently are 2 to 3 times the annual value of the fuel to be saved, a poor investment proposition. In the case of electrical heating, solar energy heating is definitely economically more attractive, particularly in the southwestern regions where (E_0/C_0) is high, as indicated in Fig. 2. However, it is clear what factors are necessary to change in order for the economic viability of solar heating systems to improve. These factors deserve some discussion at this point as they will also appear in the other economic models.

First, it should be observed that an important economic parameter is $(I+T)/C_0^*$. This is the ratio of the sum of the rates of investment costs and taxes to the cost of fuel prevailing under the “present” circumstances. Hence, it is not only fuel costs that determine the economic attractiveness of solar energy systems, as is often supposed, but rather the ratio of the costs of investment plus costs of ownership to fuel costs that represent a significant economic parameter. What coupling may exist between $(I+T)$ and C_0^* is difficult to know since a great many influences are involved but in any kind of a free market it seems likely that as one increases so will the other and vice-versa. In any event, an increase in the cost of fuel alone does have a very pronounced effect on making solar energy a better investment, as is clear by Fig. 2. In all probability an increase in fuel costs by at least a factor of 2, is not entirely unlikely in the near term. According to data supplied by the U.S. Department of Labor, the costs of petroleum based fuels has increased in the U.S. at rates varying between 17–65% per year, depending on the type of fuel, in the 2-year period September 1973–September 1975. The costs of electrical energy has seen similar increases as well although at generally the lower rates. Every indication is for a continuing increase in fuel and energy costs in the foreseeable future with increases estimated at 200–400% for gas, oil and electricity by the year 2000 [7,8]. Doubtless these increases alone will do much to propel solar energy systems to economic viability, perhaps even as early as 1980–85.

The investment costs I are dependent on the prevailing economic conditions, money supply and cost of capital. At any given time one can find some variation within a community but probably not much. The taxes T on the value of the solar energy system will probably not be greater than the prevailing tax rates and the local formula for assessing the value of the system. In some states of the U.S. the state legislatures have passed laws or are considering laws that will exempt solar energy systems from state sales tax and permitting

local real estate assessing bodies to waive property improvement assessment on solar systems.

The second significant parameter that influences R_0 is the ratio (E_0/C_0). This ratio involves operational considerations (F_u), geographical locations and meteorological effects [F_e and $(Q/A_c)_0$], design (η_0) and the economic material and labor costs for manufacture, transportation and installation of a solar energy system (C_0). The latter costs are also geographically dependent owing to the dependence of shipping costs on distance and the variability of labor cost between different sections of a country. In the U.S. for mid-1976 C_0 is probably in the range of \$15/ft² to \$25/ft² (\$161.48/m² to \$269.10/m²) for the 1 to 2 glass cover flat plate collector with adequate storage and auxiliary equipment and controls. The factor E_0 is not particularly subject to effective change as it involves such quantities as F_e and $(Q/A_c)_0$ that are dependent on location and weather. Also, for a given installation the utilization factor, F_u , is probably reasonably fixed. Hence, the only really variable quantity in E_0 is the thermal efficiency of the solar collector, η_0 . This quantity, however, is closely related to C_0^* and may be expected to vary with it more or less directly. In fact, it can be shown that an increase in R_0 with increase in η_0 , that is, that

$$\frac{dR_0}{d\eta_0} \geq 0 \quad (18)$$

only when,

$$\frac{dC_0}{C_0} \leq \frac{d\eta_0}{\eta_0}, \quad (dC_0 \equiv dC_0^*) \quad (19)$$

Accordingly, little economic advantage seems possible if E_0 is increased by using a collector of increased thermal efficiency if the cost of the collector is increased roughly a corresponding amount. The same is true by selecting less efficient and less costly collectors. Hence, the collector efficiency η_0 is an important but apparently, not a primary factor in establishing the economic viability of solar energy systems.

Considering all the quantities that influence the investment probability R_0 , and recognizing an inevitable increase in C_0^* , it seems that C_0 is the most important one over which there is some control. This quantity is the cost of an installed solar energy system per unit area of collector. Within this cost the manufacturing cost of the collector itself (\$7.00–\$15.00/ft², U.S.A., 1976) and its cost of installation (about equal to C_0^*) are the principal cost factors. Accordingly, if economic viability of solar energy systems is to be realized the most promising factor to reduce is C_0^* and the corresponding costs of labor at installation. What can be done about the latter is problematical since at present fairly high cost mechanical trades are required. One approach would be to simplify installation procedures by design to the point where low cost labor would qualify for the work. Further, if the general labor situation in any locality would allow for competitive bidding for solar energy installations some reduction in installation costs would doubtless result. However,

probably the single most important component in C_0^* on which significant improvement can reasonably be expected is the cost of manufacturing of a solar collector. In many ways, it seems apparent that a growth of solar energy systems really hinges on the matter of cost reduction of collectors through innovative design and mass production techniques and manufacturing methods. It is impossible to say what this will actually involve but the history of cost reduction certainly gives a basis for encouragement, whether it be automobiles or solid-state devices.

Case II—Investment consideration on the basis of life-cycle values under a mortgage contract with allowance for increasing fuel costs

Perhaps the most realistic of all the economic models discussed here is this one as it deals with integrated costs and savings over an extended period of time. This period is called the life-cycle of the solar equipment and also is usually set equal to the period of amortization of the original borrowed capital. The assumption is that this period is selected such that the equipment will have essentially expired and have no value when the amortization is realized. In addition, allowance will be made for escalation in fuel costs during the active life of the equipment, and mortgage. Thus, since solar energy systems are capital intensive this model allows for their economic evaluation on the basis of future fuel savings over an extended period of time, something not considered in the "present value" model in Case I.

The formulation for the "instantaneous" (annual) values that determine economic viability, inequality (12), may be rearranged slightly and integrated in time, as follows

$$\int_0^t F_u F_e (Q/A_c) \eta C_0^* dt \geq \int_0^t \left[C_0^* + C_0^* \left(\frac{V_s}{A_c} \right) + C_0^* \right] (I+T) dt \quad (20)$$

If the fuel costs increase with an annual rate a its time dependency can be expressed as

$$C_0^* = C_0^* (1+ca)^{bt} \quad (21)$$

Hence, since $(I+T)$ may be considered constant and C_0 is the initial cost, the inequality becomes

$$C_0^* \int_0^t F_u F_e (Q/A_c) \eta (1+ca)^{bt} dt \geq C_0 (I+T)t \quad (22)$$

Now, if \bar{E} is defined as the mean effective annual value of the given quantities over the time period t , it may be written as

$$\bar{E} = [F_u F_e (Q/A_c) \eta]_m \quad (23)$$

and, the inequality becomes on integration,

$$\bar{E} C_0^* \left[\frac{(1+ca)^{bt} - 1}{b \log_e(1+ca)} \right] \geq C_0 (I+T)t \quad (24)$$

† Any other time dependency may be used, including one that is non-uniform. It only should be capable of integration by equation (20).

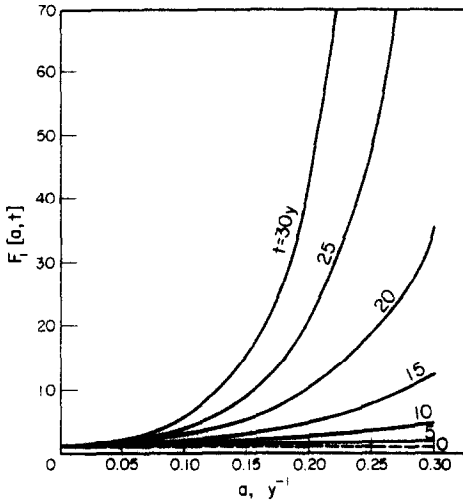


FIG. 3. The inflation function, $F_1(a, t)$.

or,

$$\frac{(\bar{E}/C_0)}{(I+T)/C\%_0} \left[\frac{(1+ca)^{bt} - 1}{bt \log_e(1+ca)} \right] \geq 1.0 \quad (25)$$

and, defining an inflation function, given in Fig. 3, as

$$F_1(a, t) \equiv \frac{(1+ca)^{bt} - 1}{bt \log_e(1+ca)} \quad (26)$$

The inequality (23) is now written as

$$R_i \equiv \frac{(\bar{E}/C_0)F_1(a, t)}{(I+T)/C\%_0} \geq 1.0. \quad (27)$$

In this result R_i is the investment probability for this case. It is a function of the additional variables of time period (life-cycle) and inflation rate on fuel costs. R_i must exceed unity in order for an investment in a solar energy heating system to be economically viable considering life-cycle costing and an increase in fuel costs during the life of the equipment and mortgage.

The investment probability, R_i , equation (27), for this case of economic evaluation is shown in Fig. 4

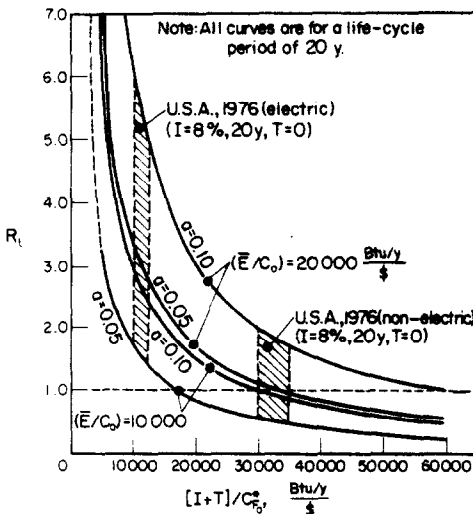


FIG. 4. Investment probability, R_i (for a period of 20 y) (Case II).

for a life-cycle period of 20 y. The values of (\bar{E}/C_0) of 10 000–20 000 Btu.y-\$ (10.551×10^6 – 21.102×10^6 J/y-\$) are roughly the limits that may be expected within the continental U.S. in 1976 and represent that combination of availability of solar energy, solar system performance and system costs currently existing in this region. Two rates of fuel cost escalation for the 20-y period are also shown, namely, 0.05 and 0.10 \$/y. What the actual rate of inflation will be is, of course, a matter of some speculation. The model adopted here, equation (21), is patterned after that for growth of invested capital, compounded annually. Doubtless, during a period as long as 20-y some variability in inflation rates must be anticipated. However, in view of the uncertainty in the future value of this rate and the fact that some average rate could probably represent the inflationary effects of any time period with reasonable satisfaction (assuming no financial collapse), the model adopted will suffice. For the past two years the annual rates of increase in the cost of petroleum products, natural gas and electricity in the U.S. have been greater than 15% and as high as 65%. While these rates may be high and may not be sustained over longer periods there is certainly no evidence in the world's energy and fuel markets to suggest a stabilization of prices. Accordingly, the annual inflation rates of 0.05 and 0.10 for a 20-y life-cycle period may prove to be reasonable.

Returning to Fig. 4, it is clear that in the sunshine abundant regions of the U.S. [$(\bar{E}/C_0) = 20000$] or, for those circumstances in which present solar system costs could be cut in half, a solar energy heating system is economically viable today in competition with non-electric fuels, for a 20-y investment, even with an annual inflation rate of 5%. Should the annual inflation rate be greater then the economic attractiveness of such an investment is enhanced. When the economic comparison is made with electrical resistance heating a solar energy system is already a much better economic alternative, something which has been noted by Löf and Tybout [1, 2], as is evident in Fig. 4. Further, any combination of reduced investment costs and taxes with a corresponding increase in fuel cost, $C\%_0$, also creates a favorable economic condition for investment in solar energy systems. The award of a "grant" from some agency for a solar system puts the grantee in the position of having zero investment cost (probably with tax relief, too) and, accordingly, enjoying the status of the limiting case (Figs. 2 and 4) of $(I+T) = 0$, the most favorable economic condition. This, of course, does not alter the total economic picture since such an investment is simply paid for by other parties.

Case III—(a) Investment consideration using income producing capital (non-mortgage contract) with allowance for increasing fuel costs but no property equity ($B = 0$). (b) Investment consideration using income producing capital (non-mortgage contract) with allowance for a maximum property equity ($B = 1.0$) and increasing fuel costs

This case corresponds to a situation in which an individual or a corporation considers using its own capital, rather than borrowed capital, for investment in a solar energy heating system. Two limiting sub-cases of this can be identified as those in which allowance for equity (through resale) of the solar system is considered at a maximum (case b) and minimum (case a) value in the economic evaluation. These limiting cases are identified by the value of the parameter B , which, in fact, describes an entire family of possible cases.

The economic model for these cases is formulated using the inequality (12) in a slightly revised form. The basis for this model is that economic viability of a solar energy heating system is realized when the value of fuel savings plus any property equity that may accrue for the system over a period of time is equal to or greater than the increase in value that would have been obtained had the original capital been used in an interest bearing investment. Hence, economic viability exists over period t when,

$$\int_0^t F_s F_e (Q/A_c) \eta C \# dt + C_0 \left[1 - \frac{t}{r^*} \right] B \geq C_0 (1 + ci)^{bt} - C_0 \tag{28}$$

where $C_0 = C_c^* + C_s^* (V_s/A_c) + C \#$, [$\$/m^2 (\$/ft^2)$], r^* = expected useful life of the solar system [y], B = property equity factor (case a, $B = 0$, case b, $B = 1.0$) and i = expected interest rate on invested capital [$\$/\$/y$]. As will be noted the equity of the property is expressed simply as some factor B times a linear decrease in time of the initial value of the property C_0 . When the period t equals the useful life r^* , the property is assumed to have zero value. Initially, its resale value is $C_0 B$, where B is probably less than unity. Allowance for increasing fuel costs is introduced by equation (21). Hence, the inequality (28) becomes

$$\frac{\left(\frac{E}{C_0} \right) C \#_0 \left[\frac{(1+ca)^{bt} - 1}{b \log_e(1+ca)} \right] + B \left[1 - \frac{t}{r^*} \right]}{(1+ci)^{bt} - 1} \geq 1.0 \tag{29}$$

and, defining another inflation function as

$$F_2(a, t) = \frac{(1+ca)^{bt} - 1}{b \log_e(1+ca)} = F_1(a, t) \cdot t. \tag{30}$$

The investment probability R_i for this case becomes

$$R_i \equiv \frac{\left(\frac{E}{C_0} \right) C \#_0 F_2(a, t) + B \left[1 - \frac{t}{r^*} \right]}{(1+ci)^{bt} - 1} \geq 1.0. \tag{31}$$

As in the previous cases economic viability in this mode of investment consideration exists only when R_i is equal to or greater than 1.0, or the inequality in (31) holds.

R_i is given in Figs. 5 and 6 for two annual rates of inflation, 0.10 and 0.15 and for the two limiting values of the property equity factor B . It is this factor that determines the two cases (a) and (b). For case (a), $B = 0.0$ and for case (b), $B = 1.0$. In each of the figures the rate of expected interest i is 8%, a value close to

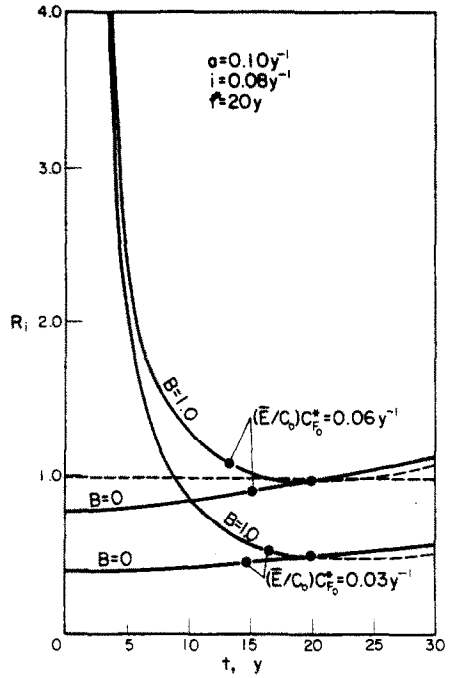


FIG. 5. Investment probability, R_i (Case III, $a = 0.10 \$/\$/y$).

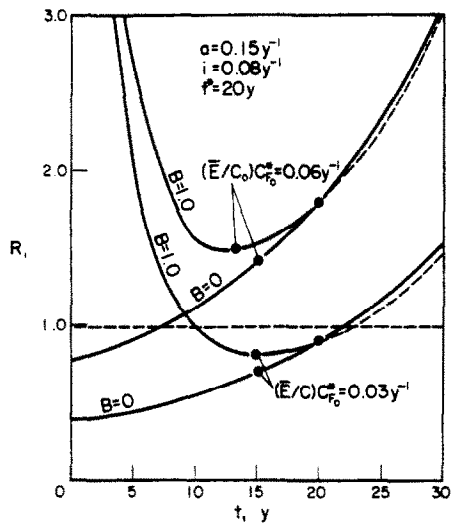


FIG. 6. Investment probability, R_i (Case III, $a = 0.15 \$/\$/y$).

that fairly easily obtained currently in many common investments in the U.S.

These results produce two interesting effects. The first is the strong influence that fuel escalation costs have on the economics of this type of an investment and the second is the effect of including equity in the solar system. Although it is to be expected, an increase in the inflation rate from 0.10 to 0.15 results in a viable economic investment after about 7 y for regions having the larger value of $(E/C_0)C \#_0$ (southwest U.S., for example) and after about 22 y for those having the smaller value (generally the eastern half of the U.S.), even for no consideration of equity in the system, Fig. 6. Lower rates of inflation cause these periods of

economic viability to be extended, as is shown by Fig. 5, to about 20 y for the larger value of $(\bar{E}/C_0)C\#_0$. There is no reasonable economic investment for circumstances corresponding to the lower value of $(\bar{E}/C_0)C\#_0$ for a fuel escalation rate of 0.10 y^{-1} .

When the effect of property equity ($B > 0$) is considered the economic picture changes significantly. For both rates of fuel cost increase the influence of maximum equity ($B = 1.0$) is to create a very favorable economic condition during the initial periods of investment (up to 15 y, approximately). It is during this period that a combination of fuel savings and property equity are considerably greater than income that would have been realized on an 8% investment, compounded annually. A point in time is reached at which these two effects balance, which is shown in Figs. 5 and 6 by a minimum in the curves. This minimum is at a value of R_t less than one for all conditions except those corresponding to the higher inflation rates and higher values of $(\bar{E}/C_0)C\#_0$. Once this minimum is reached the economic attractiveness of the investment improves with time, probably because of increasing fuel cost savings compared with the potential return on the interest bearing investment. The two cases, $B = 0$ and $B = 1.0$, merge when the period of investment equals the useful life of the system. Beyond this time there is a small negative difference between the two cases (dotted curves) indicating a cost penalty in disposing of equipment whose useful life has been exceeded. It is possible its "junk" value would tend to reduce this difference but such an effect is too speculative to consider at this time.

Case IV — Redemption of initial investment (pay-off period) with allowance for increasing fuel costs

The final case discussed here is one that is often used by those considering the purchase of tools, machinery or physical systems capable of producing income or reducing operational costs. This involves an evaluation of the period to pay-off the amount of an initial investment using income or savings from the purchased systems. Although it is the simplest of the cases presented here it probably is the least sophisticated since it ignores the costs of interest and service on borrowed capital or the income that could be produced through other kinds of investments using personal or corporate capital. In all probability this view of an investment masks the true economic optimum use of capital. Using the same assumptions as in the previous cases, ignoring any possible equity in property† and allowing for escalation in fuel costs, the criterion for this case is

$$\bar{E}C\#_0 \int_0^t (1+ca)^{bt} dt \geq C_0. \quad (32)$$

Defining the investment probability as R_t for this case,

the inequality (32) becomes

$$R_t \equiv \left(\frac{\bar{E}}{C_0} \right) C\#_0 \left[\frac{(1+ca)^{bt} - 1}{b \log_e(1+ca)} \right] \geq 1.0 \quad (33)$$

or,

$$R_t \equiv \left(\frac{\bar{E}}{C_0} \right) C\#_0 F_2(a, t) \geq 1.0. \quad (34)$$

This investment probability R_t is given in Fig. 7 as a function of time and several annual inflation rates for values of $(\bar{E}/C_0)C\#_0$ of 0.03 and 0.06 y^{-1} . An economically viable investment for this case exists when R_t is equal to or greater than 1.0. The time corresponding to the "pay-off" period is that time for which R_t is equal to unity. The influence of even a mild inflation very significantly affects the "pay-off" period, as can be seen. For an increase in inflation from zero to 5%, the pay-off period is reduced from about 33 y to approximately 20 y for a circumstance in which $(\bar{E}/C_0)C\#_0$ is 0.03 y^{-1} . Further, for the same rate of inflation the "pay-off" period is less for conditions corresponding to the larger $(\bar{E}/C_0)C\#_0$, as would be expected.

Some measure of the economic reliability of this case may be obtained by comparing it with some results from Case II, which considers life-cycle cost evaluation, Fig. 4. For an annual rate of inflation of 5%, the "pay-off" period in Fig. 7 (Case IV) is 20 y for $(\bar{E}/C_0)C\#_0$ of 0.03 y^{-1} . In the case of life-cycle analysis (Case II) taking an 8%-20 y mortgage (i.e. 20 y "pay-off" period) the value of R_t for an inflation rate of 5% and (\bar{E}/C_0) of 10000 [roughly the same as $(\bar{E}/C_0)C\#_0$ of 0.03 for $C\#_0$ equal to \$3 per 10^6 Btu] is only about 0.5, indicating an unattractive investment. If a 20-y "pay-off" period is thought to be advantageous in this

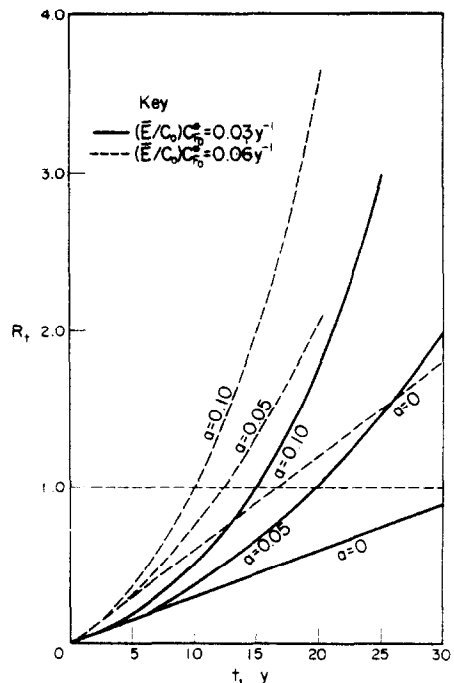


FIG. 7. Investment probability, R_t (Case IV).

†If the "pay-off" period is less than the useful life of the equipment, equity in the system could be considered using the same formulation as in Case III. This would act to make an investment under this case appear more attractive.

circumstance from the criterion of Case IV, there is an evident contradiction when examined under the more realistic conditions of Case II. The same conclusion is found by a comparison of this example under Case IV with that in Case III(a) or III(b) for a 20-y life in a situation where capital could have been invested at 8% interest, compounded. In this instance R_i is only 0.278 indicating that such an income producing investment would have been a more economically viable decision in the first place as opposed to borrowing capital at 8% interest under a 20-y mortgage in installing a solar energy heating system or making a decision to install such equipment on the basis of a seemingly favorable 20-y "pay-off" period. However, this conclusion is certainly not a general one since with other circumstances such as a higher annual rate of inflation, lower collector costs, reduced investment costs, etc., economic viability of solar energy heating systems will be realized.

CONCLUSIONS

The economic evaluation of solar energy supply systems is shown to involve first a determination as to whether such systems represent an economically viable investment. Once this *à priori* decision has been formulated, then a specific design of solar energy system can be selected and optimized. The initial economic decision is found to depend on a number of technical, physical, meteorological, geographic, design and cost factors as well as on the source and type of financing arrangements available. Four economic models are presented that are appropriate to this *à priori* decision. Each is based on the premise that an economically viable system is one that must, at some time, produce financial benefits, primarily in terms of fuel savings, that are equal to or greater than the cost of the investment. These models, which allow for increasing fuel costs with time, include economic/technical evaluation

of solar energy heating systems for investment, (I) at a given time, (II) under a mortgage contract, (III) by use of personal or corporate capital, and (IV) on the basis of a "pay-off" period. It is found that differing economic conclusions are both possible and justifiable for investment situations having different constraints. The principal element in improving the economic feasibility of solar energy systems is shown to be the unit cost of the solar collector, something which seems possible by innovative design and the use of mass production methods and new manufacturing techniques. Expected increases in fuel costs also will cause investment in solar energy systems to become attractive.

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ECONOMIE DE L'ENERGIE SOLAIRE—UN JUGEMENT *A PRIORI*

Résumé—Il ressort que la décision initiale concernant la viabilité économique d'un système de chauffage par l'énergie solaire fait appel à des facteurs techniques, physiques, météorologiques, géographiques, de conception et de coût aussi bien qu'à l'origine des fonds et le type de dispositions assurant le financement du système. Afin de déterminer la viabilité économique du système, quatre modèles technico-économiques qui tiennent compte de l'influence de l'accroissement des coûts du fuel sont présentés et comparés à d'autres types possibles d'investissements. On conclut que différentes solutions économiques sont également possibles et justifiables dans des types d'investissement supposant des contraintes différentes.

DIE WIRTSCHAFTLICHKEIT DER SONNENENERGIENUTZUNG—DIE *A-PRIORI*-ENTSCHEIDUNG

Zusammenfassung—Es wird gezeigt, daß die zunächst zu treffende Entscheidung hinsichtlich der Wirtschaftlichkeit von Sonnenenergie-Heizsystemen sowohl technische, physikalische, meteorologische, geografische Faktoren, Bauart- und Kostenfaktoren als auch Kapitalquelle und Art der Finanzierung einschließt. Es werden vier wirtschaftlich-technische Modelle vorgestellt, die den Einfluß steigender Brennstoffkosten beinhalten. Die Wirtschaftlichkeit des Systems wird durch Vergleich mit anderen möglichen Arten von Anlagen ermittelt. Es zeigt sich, daß für Anlagenbedingungen mit unterschiedlichen Randbedingungen verschiedene wirtschaftliche Schlußfolgerungen sowohl möglich wie gerechtfertigt sind.

ЭКОНОМИЧНОСТЬ ИСПОЛЬЗОВАНИЯ СОЛНЕЧНОЙ ЭНЕРГИИ. АПРИОРНОЕ РЕШЕНИЕ

Аннотация — Показано, что предварительная сушка экономической жизнеспособности солнечной системы нагрева основывается на технических, физических, метеорологических, географических, проектных и материальных факторах, а также включает источник и способы финансирования системы. Предложены четыре техникоэкономических модели, в которых учтено влияние возрастающей стоимости топлива и дано сравнение с другими возможными способами определения экономической жизнеспособности системы. Найдено, что различные экономические выводы о целесообразности системы солнечного нагрева являются возможными и оправданными в условиях ограниченных источников финансирования.