

INCLUSIVE AND SEMI-INCLUSIVE PION PRODUCTION IN pp COLLISIONS AT 102 AND 400 GeV/c *

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We present a comparative study of inclusive and semi-inclusive pion production in pp collisions at 102 and 400 GeV/c. In particular, we examine the correlation between transverse and longitudinal momentum variables and the energy dependence of invariant cross sections.

1. Introduction

In this paper we present a comparative study of inclusive and semi-inclusive pion production in proton–proton collisions at 102 and 400 GeV/c, and specifically investigate the validity of various scaling hypotheses which have been proposed for particle production in high-energy collisions †. The data are from an analysis of inelastic events (~ 3000 events at 102 GeV/c and ~ 2200 events at 400 GeV/c) obtained in exposures of the FNAL 30-inch bubble chamber to proton beams at the highest available accelerator momenta †.

A difficulty in the analysis of bubble chamber data at high energies is the lack of particle identification for tracks of momenta in excess of 1.2 GeV/c; below this momentum ionization estimates in the bubble chamber can be used to separate protons from pions. To correct for proton contamination of the π^+ data above 1.2 GeV/c lab momentum, we have assumed that the observed cross sections for inclusive and semi-inclusive proton production, at momenta below 1.2 GeV/c, extrapolate smoothly to higher momenta. The inclusive proton spectrum has been shown previously [3] to be almost independent of the scaling variable $x = 2p_q^*/\sqrt{s}$ for x

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† For a summary of the general features of multiparticle production at high energies, see the reviews given in ref. [1].

† Preliminary results from the 102 GeV/c exposure may be found in ref. [2].

values in the range $0.8 \geq |x| \geq 0.5$ (p_q^* is the c.m. longitudinal momentum and \sqrt{s} is the c.m. energy in the collision). In calculating corrections to the π^+ distributions we have assumed that the inclusive cross section for protons in the region $|x| < 0.5$ remains unchanged from that observed for $0.5 < |x| < 0.8$. Corrections for proton contamination of the π^+ spectra were calculated using a Monte-Carlo program, and the results are displayed along with the *uncorrected* π^+ spectra whenever these corrections are significant.

In the text we refer to the positive spectra as π^+ and negative spectra as π^- . However, we estimate [#] that on the average there exists approximately 1% e^- , 7% K^- and 2% \bar{p} contamination in the displayed negative spectra, and approximately 1% e^+ and 10% K^+ contamination in the positive spectra (in addition to the calculated proton contamination).

2. Inclusive pion production

The inclusive scaling hypotheses of Wilson [5], of Feynman [6] and of Benecke et al. [7] predict that the invariant cross section for single-particle production at fixed values of transverse and longitudinal momentum in the target or projectile rest frame, approaches an energy-independent limit as the colliding energy increases. Expressing the invariant cross section in terms of rapidity in the target frame, $y_{\text{lab}} = \frac{1}{2} \ln[(E + p_q)/(E - p_q)]$, and the transverse momentum p_T of the produced particle, the essence of these scaling hypotheses can be written as follows:

$$\frac{E d\sigma}{d^3p} = f(y_{\text{lab}}, p_T, s) \xrightarrow{s \rightarrow \infty} f(y_{\text{lab}}, p_T), \quad (1)$$

where the region $p_q^* \sim 0$ (i.e., $y^* \sim 0$) is explicitly excluded from the predictions. More recently, Mueller [8] was able to show that inclusive scaling follows from a generalized optical theorem combined with a Regge picture of particle production in which the invariant cross section is expected to develop a ‘‘plateau’’ in the central region of rapidity space as s becomes very large:

$$f(y^*, p_T) \xrightarrow{s \rightarrow \infty} f(p_T).$$

Using the Mueller–Regge picture, the scaling hypothesis can therefore be extended into the previously excluded region of $p_q^* \approx 0$.

We begin our study by displaying in fig. 1 the inclusive production spectra for π^+ and π^- at 102 and 400 GeV/c. The data at the two beam momenta are presented

[#] Material in ref. [1] has been used to estimate \bar{p} and K^\pm background. Kaon contamination was also estimated from data on K_S^0 production (see ref. [4]). Electron contamination stems from the Dalitz decay mode of π^0 mesons.

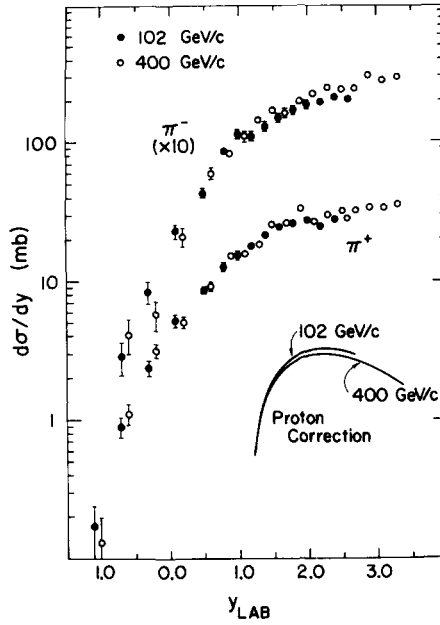


Fig. 1. Single-particle inclusive spectra for π^- and π^+ production. The spectra for positive pions have not been corrected for contamination from kaons, electrons or protons. The estimated corrections for proton contamination of the π^+ data are sketched in the figure (smooth curves).

in terms of a cross section as a function of y_{lab} integrated over p_T and azimuth,

$$\frac{d\sigma}{dy_{\text{lab}}} = \pi \int dp_T^2 E \frac{d\sigma}{d^3p} = \int dp_T^2 \frac{d\sigma}{dy_{\text{lab}} dp_T^2}. \quad (2)$$

We show only the data for production in the backward hemisphere of the center of mass ($y_{\text{lab}} < 2.69$ at 102 GeV/c and $y_{\text{lab}} < 3.37$ at 400 GeV/c), where background and systematic errors are least severe. Also shown are the corrections that must be applied to the π^+ data points to account for proton background. The errors contain statistical as well as systematic uncertainties due to measuring and scanning difficulties. The data are consistent to within $\sim 10\%$ uncertainty with the scaling of π^+ and π^- production in the target fragmentation region for rapidity values of $y_{\text{lab}} \lesssim 1.5$. However, we point out that in a recent detailed study of the energy dependence of the cross section for the fragmentation region ($y_{\text{lab}} < 0$), we have found a significant decrease in pion production from AGS to NAL energies [9] which indicates that scaling in the fragmentation regime may be approached from above. The approach to scaling throughout the fragmentation region remains an interesting question for probing with higher statistics experiments.

In contrast to the result in the fragmentation region, there is a large growth of the pion cross section in the central region ($y_{\text{lab}} \gtrsim 1.5$). In particular, we find that

the cross sections at $y^* = 0$ (i.e., $y_{\text{lab}} = 2.69$ at 102 GeV/c and $y_{\text{lab}} = 3.37$ at 400 GeV/c) have increased by $(45 \pm 10)\%$ for the π^- data and $(30 \pm 10)\%$ for the π^+ data. Application of Mueller–Regge techniques to pion production at $y^* = 0$ suggests a rather slow approach to scaling: $d\sigma(s)/dy|_{y^*=0} = a + bs^{-1/4}$ [8,10]. The 102 and 400 GeV/c data points, corrected for K, e and p backgrounds, yield the following values for these parameters:

	a (mb)	b (mb · GeV ^{1/2})	
π^-	46 ± 6	-100 ± 25	(3)
π^+	44 ± 6	-80 ± 30	

The values of a (~ 45 mb) are about two standard deviations larger than the corresponding values obtained from extrapolations using low-energy experiments [11].

We next investigate the transverse momentum characteristics of π^- production. In fig. 2 the first two moments of the p_T spectra at fixed y_{lab} are shown; the n th moment is given by

$$\langle p_T^n \rangle = \int dp_T^2 p_T^n \frac{E d\sigma}{d^3p} / \int dp_T^2 \frac{E d\sigma}{d^3p}. \quad (4)$$

The integration limits are $0 < p_T < 1.5$ GeV/c. We observe that the moments are s -independent at fixed y_{lab} . Also, the trend of the data with expanding central rapidity region suggests that π^- production has nearly attained a limiting value of $\langle p_T \rangle \sim 0.36$ GeV/c. The second moment would appear to require somewhat higher energies before possibly reaching a saturation value. (The decrease of the moments for $y_{\text{lab}} \lesssim 0$ may be due to kinematic restrictions on p_T for these y_{lab} values.)

The dependence of the π^- invariant cross section on p_T and x is shown in fig. 3.

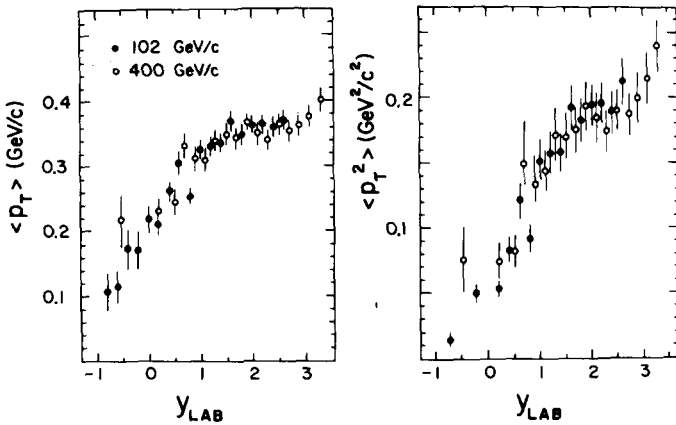


Fig. 2. First two moments (see text) of the π^- transverse momentum spectra as a function of rapidity.

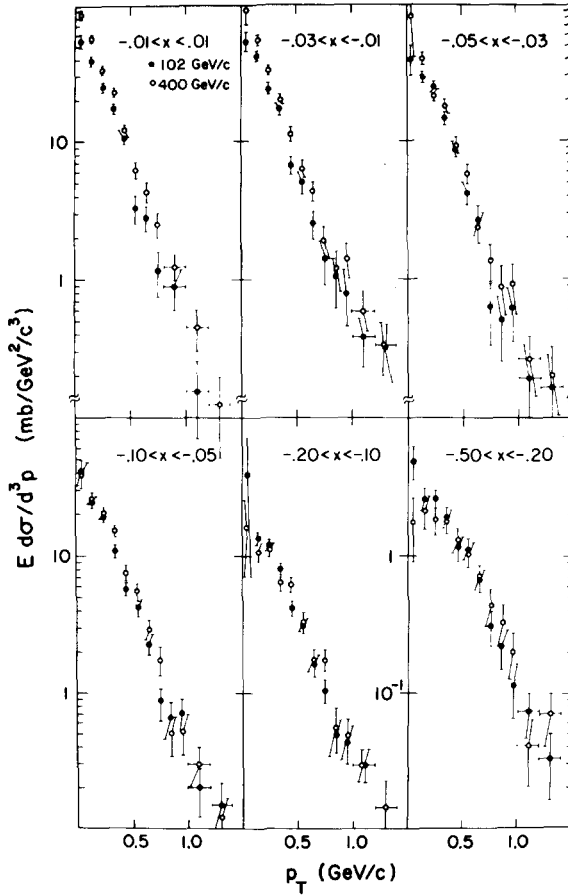


Fig. 3. Single-particle spectra for π^- production. The data have not been corrected for contaminants.

The data are also tabulated in table 1. Similar data have been reported at nearby energies from experiments at FNAL and at the ISR [†]. The scaling violations, noted previously in the rapidity spectra integrated over p_T , are also evident here in the growth of the cross sections for $|x| \leq 0.03$. In this small- x region the cross section for p_T values between 0.2 and 1.5 GeV/c appears to rise uniformly between 102 and 400 GeV/c. Below $p_T \sim 0.2$ GeV/c, the data exhibit a somewhat more rapid growth with increasing s . For the region $|x| > 0.05$ the distribution is only weakly energy dependent over the entire range of p_T values; a possible decrease of the cross section with increasing energy is observed at small p_T in the largest $|x|$ region

[†] For excellent compilations of p_T dependences of cross sections, see ref. [12].

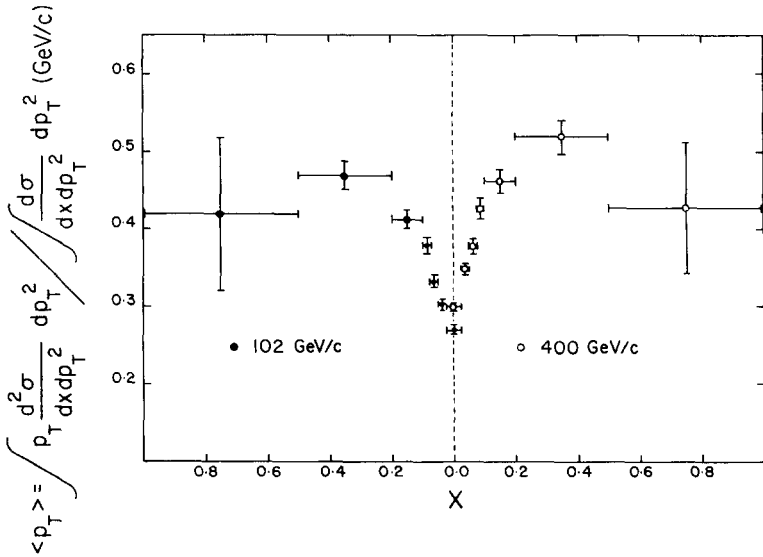


Fig. 4. First moment (see text) of the π^- transverse momentum spectra as a function of the x variable.

shown ($-0.50 < x < -0.20$). As the value of $|x|$ increases, the data appear to exhibit a flattening of the invariant cross section at small p_T (i.e., on the average, higher p_T values appear to be more favored at larger x). This fact is reflected in the "sea gull" effect [13] observed in fig. 4 where the $\langle p_T \rangle$ is shown to increase with $|x|$ for $|x| < 0.5$. (For clarity the 400 GeV/c data have been plotted after reflection about $x = 0$.) A comparison of the behavior of $\langle p_T \rangle$ (figs. 2 and 4) suggests that the invariant cross section is described more simply using the rapidity variable rather than x .

Summarizing our findings for inclusive pion production, we observe single-particle spectra for π^+ and π^- which are consistent with scaling in the fragmentation region, but exhibit sizeable violations of scaling ($\sim 30\%$) in the central region. The $\langle p_T \rangle$ for π^- seems to saturate near $\langle p_T \rangle \sim 0.36$ GeV/c for production in the central region of rapidity. In the small- x region, particularly for $|x| < 0.03$, scaling violations for π^- production are observed throughout our p_T range (0–1.5 GeV/c) with the violations being most pronounced for the rather low p_T data of $p_T \lesssim 0.2$ GeV/c[‡]

3. Semi-inclusive pion production

The asymptotic scaling of inclusive production has many interesting consequences. In particular, Koba, Nielsen and Olesen (KNO) have shown that the assumption

[‡] These results are consistent with the latest precision measurements of inclusive pion production at the ISR reported by the British-Scandinavian Collaboration [14].

of inclusive scaling leads to a scaling of the multiplicity distribution of the kind [15]

$$\sigma_n \xrightarrow{s \rightarrow \infty} \frac{1}{\langle n \rangle} \psi \left(\frac{n}{\langle n \rangle} \right). \quad (5)$$

Here n refers to the inelastic charged-particle multiplicity and ψ is the scaling function. Recent tests of this hypothesis (KNO-1) have shown [16] that the multiplicity data at FNAL energies are in good agreement with its predictions; although this may seem puzzling when one considers the observed scaling violations noted in the previous section, it has been pointed out by Weingarten that multiplicity scaling does not require that Feynman scaling hold to high accuracy [17]. Extending their work, KNO have proposed semi-inclusive single-particle densities should also scale in a similar manner (KNO-2) [18]

$$\rho_n(y, p_T, s) = \frac{1}{\sigma_n} E \frac{d\sigma_n}{d^3p} \xrightarrow{s \rightarrow \infty} \phi \left(y, p_T, \frac{n}{\langle n \rangle} \right). \quad (6)$$

The semi-inclusive densities integrated over p_T for π^- and π^+ production at 102 and 400 GeV/c are shown in figs. 5 and 6. As for the inclusive spectra, we show only the data from the backward hemisphere of the center of mass (for economy

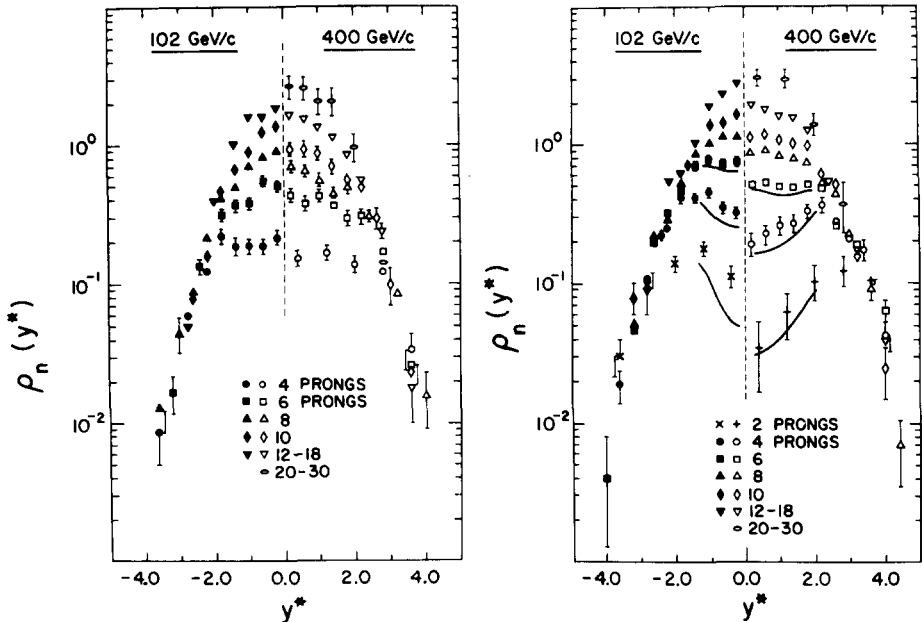


Fig. 5. Semi-inclusive single-particle densities for π^- and π^+ production, in (a) and (b), respectively. The data at 400 GeV/c are shown reflected about $y^* = 0$. The curves shown in (b) for the low multiplicity results ($n = 2, 4, 6$) represent estimates of the π^+ data corrected for proton background. The higher multiplicity data have not been corrected for any contaminants.

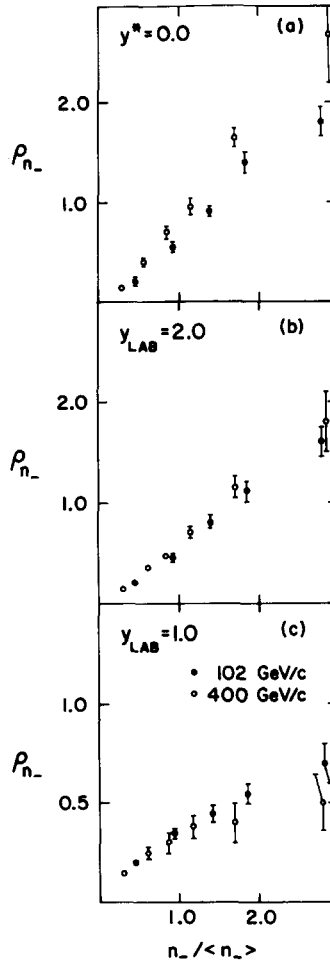


Fig. 6. Semi-inclusive single-particle densities as a function of the KNO scaling variable $n_- / \langle n_- \rangle$.

of presentation the 400 GeV/c data have been plotted after reflection about $y^* = 0$) The curves shown under the low-multiplicity results in fig. 6 ($n = 2,4,6$) represent π^+ data corrected for proton contamination (corrections for the higher multiplicities have not been calculated). For the highest multiplicities the cross sections have been summed over the indicated range of n .

We note that while the semi-inclusive densities at $y^* = 0$ often differ by orders of magnitude, at large $|y^*|$ the density appears to be only weakly dependent on the charge multiplicity n . Although the individual spectra in fig. 5 do not show evidence for the presence of a central plateau (save for perhaps the $n = 4 \pi^-$ data), there is apparently a general flattening of the π^- data as the beam momentum in-

Table 2
Mean transverse momenta for π^- mesons

Incident momentum (GeV/c)	$\langle p_T \rangle$ (GeV/c) ^{a)}				
	Prong multiplicity				
	4-6	8-10	12-18	20-30	all
102	0.351 ± 0.007	0.345 ± 0.007	0.314 ± 0.015		0.341 ± 0.005
400	0.365 ± 0.010	0.357 ± 0.007	0.345 ± 0.006	0.357 ± 0.020	0.352 ± 0.005

a) These results have not been corrected for K^- , e^- or \bar{p} contaminants.

creases from 102 to 400 GeV/c. The data for π^+ production, particularly in the low multiplicities, exhibit enhancements at large $|y^*|$ which are somewhat more pronounced in the corrected spectra (curves in fig. 5b). These peaks can be attributed to the fragmentation of incident protons which produce an excess of positive pions at large $|y^*|$.

Using the π^- data shown in fig. 5a, we have selected three representative rapidity values, $y^* = 0$, $y_{lab} = 1$ and $y_{lab} = 2$, at which to test the validity of the KNO-2 hypothesis. For π^- production, the hypothesis can be written as follows:

$$\rho_{n_-}(y) = \phi \left(y, \frac{n_-}{\langle n_- \rangle} \right), \quad (7)$$

where $n_- = \frac{1}{2}(n - 2)$ is the number of negative particles in each event, ρ_{n_-} is the particle density function integrated over p_T , and ϕ is the postulated scaling function. The data in fig. 6a clearly indicate that no single function of $n_-/\langle n_- \rangle$ describes the energy dependence of ρ_{n_-} at $y^* = 0$. However, the data in figs. 6b,c, for lab rapidities $y_{lab} = 1$ and $y_{lab} = 2$, agree more closely with a unique $n_-/\langle n_- \rangle$ dependence. Observations similar to these have been noted in a recent study wherein semi-inclusive π^- production at AGS energies was compared with the FNAL data at 205 GeV/c [†].

Thus, we find that in the fragmentation region of rapidity ($y_{lab} \lesssim 1.5$), where the data are in reasonable agreement with the inclusive scaling hypothesis, the KNO-2 semi-inclusive scaling hypothesis also seems to be valid. Conversely, in the central rapidity region ($y^* \sim 0$) where the inclusive scaling hypothesis is violated, KNO-2 is also violated [‡].

[†] It is worthwhile to point out that in a more detailed investigation of KNO-2 scaling for momenta between 6 and 400 GeV/c we have found a small but significant systematic disagreement between the data and the scaling prediction in the target fragmentation region (Schindler et al. [19]).

[‡] This result is not very surprising because $d\sigma/dy$ is related to the ρ_n functions and to the KNO-1 scaling function ψ via $d\sigma/dy = \int_0^\infty dz \rho_2(y,z) \psi(z)$, where $z = n_-/\langle n_- \rangle$; and since $\psi(z)$ is relatively energy independent and $d\sigma/dy$ does not scale, particularly in the central

As a final semi-inclusive characteristic, we display in table 2 the $\langle p_T \rangle$ values, averaged over rapidity, in the inclusive and semi-inclusive π^- data samples. The slight increase in $\langle p_T \rangle$ between the two energies is presumably due to additional contributions from central production ($\langle p_T \rangle \sim 0.36$ GeV/c) in the 400 GeV/c data (c.f. fig. 2). Within our experimental errors, $\langle p_T \rangle$ displays little or no multiplicity dependence at both energies*.

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region, it follows that the ρ_n functions will exhibit a variation with incident momentum, especially in the regime where $d\sigma/dy$ does not scale.

* See, however, the data presented in ref. [20].