ELECTRON-POSITRON ANNIHILATION: STATISTICAL AND THERMODYNAMIC CONSIDERATIONS[☆]

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Lightcone considerations suggest that, in e^+e^- annihilation, the hadronic matter is initially created within a spatial volume in the c.m. frame of the order of E^{-3}, where $E/2$ is the colliding beam energy. Its implications, within the statistical thermodynamic descriptions of Fermi and Landau, are conjectured. A distinctive result is that the multiplicity for e^+-e^- annihilation is a constant.

Recent preliminary experimental indications from SPEAR [1] for e^+-e^- annihilation, taken together with the earlier results from CEA [2], are extremely interesting in that parton scaling ideas [3], at least in their usual simple versions, appear to be inapplicable. It seems that we are back to the beginning of a long search for a viable theoretical description of the annihilation process.

We will in this note make some general remarks which suggest that statistical and thermodynamic considerations are both natural and appealing for discussing e^+-e^- annihilation. A specific realization of such an approach would be, for example, an adaptation of the hydrodynamical model proposed by Landau [4] for hadron-hadron scattering, such as those discussed recently by several authors [5]. While the details of this type of model certainly involve a number of simplifying idealizations, we take the view that the overall picture may still be a compelling way to envision the creation and subsequent "particlization" of hadronic matter in the e^+-e^- annihilation process. The emphasis here will be confined to the more general aspects of this picture, which seem to have the potential for providing an adequate description of the gross features of the experimental data. Our picture differs from those of ref. [5] in one essential aspect. This difference, which we discuss below, will have important consequences.

The e^+-e^- annihilation cross section, in the one-photon exchange approximation, is related to the absorptive part of the vacuum polarization by a time-like photon,

$$\pi_{\mu\nu}(q) = \int d^4x \exp(iqx) \langle 0| j_\mu(x) j_\nu(0) |0\rangle, \qquad (1)$$

where q is the momentum of the photon. The product of currents in eq. (1) can be replaced by a commutator because of the positivity of the energy spectrum of physical states, i.e.,

$$\pi_{\mu\nu}(q) = \int d^4x \exp(iqx) \langle 0| [j_\mu(x), j_\nu(0)] \, 0\rangle. \qquad (2)$$

Micro-causality condition,

$$[j_\mu(x), j_\nu(0)] = 0, \text{ for space-like } x^2, \qquad (3)$$

restricts the support of the integral to the time-like region,

$$x^2 \geqslant 0. \qquad (4)$$

In the c.m. frame, where $q^0 = E$ and $\boldsymbol{q} = 0$, the phase in eq. (2) is simply Et. The important region of con-

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tribution is

$$t \lesssim E^{-1}. \tag{5}$$

Combining eqs. (4) and (5) then gives

$$|x| \lesssim E^{-1}. \tag{6}$$

This result means that the coherent vacuum polarization occurs predominantly in a spatial volume $V_0 \sim E^{-3}$. It would thus seem reasonable to suppose that *in e^+-e^- annihilation the hadronic matter is initially created in a volume of that order* [‡1]

$$V_0 \sim E^{-3}. \tag{7}$$

As a consequence of this observation, we list in the following our conjectures, in sequence of increasing additional assumptions:

(I) A large amount of energy, E, is converted into hadronic matter, in a very small volume. According to eq. (7), the initial energy density is

$$\epsilon_0 \propto E^4, \tag{8}$$

which can be made extremely high by increasing the incident energy of the colliding beams. At sufficiently high energy, the dimension of the initial volume V_0 is much smaller than the characteristic length of strong interaction forces. Under such circumstances, the hadronic system must undergo strong self-interaction and adjustments, immediately after its formation. We believe that for such a system a statistical and thermodynamic description is both natural and appealing. This is to be contrasted with the parton viewpoint [3], according to which the partons are essentially free after their creation. We note here that the preliminary SPEAR data [1] provide encouraging indications that a statistical and thermodynamic description is in the right direction. Further tests bearing on such considerations will be of interest.

(II) We follow Fermi [7] and Landau [4] to assume that the hadronic system can be treated as an ideal fluid in thermal equilibrium, immediately after its formation and before its evolution to eventual "particlization". The initial temperature T_0 is related to the energy density through the Stefan-Boltzmann law,

$$\epsilon_0 \propto T_0^4, \tag{9}$$

and, in view of eq. (8),

$$T_0 \propto E. \tag{10}$$

Thus, at high energy, the hadronic system immediately after its creation is extremely dense and hot. Its entropy is

$$S_0 \propto V_0 T_0^3 \approx \text{constant}, \tag{11}$$

according to eqs. (7) and (10).

(III) This superheated and highly dense matter will expand before it breaks up into actual physical particles. The appropriate variables for describing the collective behavior are conceivably the thermodynamic field quantities such as energy density, entropy density, local temperature, etc. The proposal by Landau [4] is that the expansion is an adiabatic one, governed by the equations for a relativistic ideal fluid. Irrespective of the expansion dynamics, however, as soon as the adiabatic assumption is made, the total entropy of the system is conserved. The average multiplicity, which is proportional to the entropy, is then

$$N \propto S = S_0 \approx \text{constant}, \tag{12}$$

according to eq. (11). This is different from the prediction

$$N \propto E^{3/4}, \tag{13}$$

obtained by previous authors [5] who *assumed* a constant V_0, an assumption which seems to us to be arbitrary and without justification.

That the multiplicity in e^+-e^- annihilation should be a constant seems somewhat surprising. It is encouraging that the preliminary data from SPEAR [1] does give a charge multiplicity [‡2] consistent with a constant value for the c.m. energy from E = 3 GeV to 5 GeV. It is of great interest to see whether such a behavior persists when colliding beam energy is extended to higher values.

[‡1] We have wondered why the standard lightcone argument, which is presented here in full for completeness, has not been made use of to infer the result $V_0 \sim E^{-3}$ and its consequences. J.M. Wang and one of us (HTN) tried some time ago, but we were unable to derive any useful result. We have searched the literature. The only trace we can find bearing on this point is the passing remark made by Bjorken at the 1973 Bonn Conference: "Why not $V \sim Q^{-3}$ leading to $\bar{n}$ = constant?" See ref. [6].

[‡2] There is some confusion as to what the neutral multiplicity is. It is no doubt one of the important experimental questions to be cleared up.

A related consequence of a constant multiplicity is the prediction that the average momentum of the produced hadrons increases linearly with E, i.e.,

$$\langle p \rangle \propto E. \tag{14}$$

(IV) If we further assume that the emission of physical particles takes place locally when the expanding ideal fluid reaches some critical temperature[‡3], T_c, and that the emitted particles (say, pions) obey a Bose-Einstein distribution in the local fluid frame, the momentum spectrum of the secondaries is of the general form ($\omega = \sqrt{\boldsymbol{p}^2 + m^2}$).

$$\omega \frac{dN}{d^3p} = C \int [\exp(\bar{\omega}/kT_c) - 1]^{-1} \times D(\beta, E)\, d(\cos\theta)\, d\beta, \tag{15}$$

where $\bar{\omega}$ is the energy of the particle in the local fluid frame, which is moving with respect to the c.m. frame with velocity β

$$\bar{\omega} = (1 - \beta^2)^{-1/2}(\omega - \beta p \cos\theta), \tag{16}$$

θ being the angle between $\boldsymbol{p}$ and the (radial) direction of the fluid motion. $D(\beta, E)$ is related to the velocity distribution of the fluid during "condensation".

It is clear from eq. (15) that the single-particle momentum spectrum is not of a simple Bose-Einstein form unless there is no fluid expansion, which is ruled out in our case. The details depend on $D(\beta, E)$, which can be obtained by numerically solving the hydrodynamic equations, with specific assumptions about the equation of state and boundary conditions. We shall present our numerical investigations and other details elsewhere. Here, it suffices to mention that since we have $T_0 \propto E$, the ratio T_0/T_c is very large at high energies and the fluid takes a "long" time to cool from T_0 and T_c. The velocity distribution of the fluid corresponding to such a "long" expansion is a relatively smooth one. As a consequence, *the single-particle spectrum will numerically resemble a simple Bose-Einstein curve for relatively small momentum p, but it will definitely become broadened as p increases and in an energy dependent way.*

Concluding, we would like to make one remark: If the statistical and thermodynamic description indeed turns out to be satisfactory for electron-positron annihilation into hadrons, this process will not teach us very much about the fundamental properties of the hadronic constituents at a deeper than macroscopic level.

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[‡3] The experimental fact that the annihilation cross section is a constant at high energies implies that in addition to E their is at least one more important dimensional parameter in the problem. In the thermodynamic approach the critical temperature T_c might be identified with that parameter.

References

[1] B. Richter, invited talk at the APS annual meeting, Chicago, February 1974;
W. Chinowsky, invited talk at the IV Intern. Conf. on Experimental meson spectroscopy, Boston, April 1974.

[2] A. Litke et al., Phys. Rev. Lett. 30 (1973) 1189.

[3] S. Drell, D. Levy and T.M. Yan, Phys. Rev. 187 (1969) 2159;
N. Cabibbo, G. Parisi and M. Testa, Nuovo Cimento Lett. 4 (1970) 35;
R. Feynman, Photon-hadron interactions (W.A. Benjamin, New York, 1972).

[4] L.D. Landau, Izv. Akad Nauk USSR, Ser. Fiz. 17 (1953) 51;
L.D. Landau and S. Belenkij, Usp. Fiz. Nauk 56 (1955) 309.

[5] E.V. Shuryak, Phys. Lett. 34B (1971) 509;
P. Carruthers and Minh Duong-Van, Phys. Lett. 44B (1973) 507;
F. Cooper, G. Frye and E. Schonberg, Phys. Rev. Lett. 32 (1974) 862, and Yeshiva University preprint (1974);
J. Baacke, Phys. Lett. 49 B (1974) 297.

[6] J.D. Bjorken, in Proc. 1973 Intern. Symp. on Electron and photon interactions at high energies, Bonn, Germany, 1973, to be published).

[7] E. Fermi, Prog. Theor. Phys. 5 (1950) 570.