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ON THE AERODYNAMIC HEATING OF AN OSCILLATING SURFACE

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ABSTRACT

The energy equation for the laminar flow of an incompressible, viscous fluid induced by the axial oscillation of an infinite flat plate is solved by the Laplace transformation technique. Temperature oscillations of the fluid at steady periodic state are analyzed for all Prandtl numbers. It is shown that the oscillation of the plate could change both the fluid temperature and the heat transfer rate appreciably. Results may also be applicable for a compressible viscous fluid.

Introduction

Some earlier works on aerodynamic heating have been summarized in reference 1. Emmons [2] has considered the problem of an insulated flat plate of infinite extent started impulsively from rest in a viscous, incompressible fluid. The same problem has been treated by Bryson [3] for a plate with a surface temperature that varies as a given function of time. Ostrach [4] has analyzed the effect of the surface oscillations on heat transfer by comparing with those for the case of conduction at a stationary surface with the same initial temperature potential.

This paper determines how the axial oscillation of a flat plate can change both the fluid temperature and the heat transfer rate. Analytical results, also applicable for compressible fluids, are obtained through the operation of Laplace transform on the equations governing the laminar transport of heat and momentum in an incompressible viscous fluid.

Analysis

For the laminar flow of an incompressible, viscous fluid induced by the axial (longitudinal) oscillations of an infinite flat plate, the momentum equations, assuming constant physical properties, reduce to

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} \tag{1}$$

where u is the velocity parallel to the plate, t is time, ν is the kinematic viscosity, and y is the coordinate normal to the surface. The associated boundary conditions are

$$u\ (0,t) = U \cos\omega t \tag{2}$$

$$u\ (\infty,0) = 0 \tag{3}$$

where U is the amplitude and ω the frequency of the oscillation.

The appropriate energy equation is

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_p} \left(\frac{\partial u}{\partial y}\right)^2 \tag{4}$$

where T is the temperature, a is the thermal diffusivity, C_p is the specific heat at constant pressure. The associated boundary conditions are

$$T(0,t) = T_s \tag{5}$$

$$T(\infty,t) = T_\infty \tag{6}$$

That is, the oscillating surface is maintained at some uniform temperature T_s. The temperature at infinity may be any finite value.

The problem defined by equations (1), (2) and (3) for the momentum diffusion gives

$$u = U \exp(-\eta) \cos\ (\omega t-\eta) \tag{7}$$

where η is defined as $(\omega/2\nu)^{1/2}y$. Substituting equation (7) into equation (4), the energy equation becomes

$$\frac{\partial\theta}{\partial\tau} = \frac{1}{2\mathrm{Pr}} \frac{\partial^2\theta}{\partial\eta^2} + \exp(-2\eta)[1-\sin 2(\tau-\eta)] \tag{8}$$

in which $\theta = (T-T_s)/(U^2/2C_p)$, $\tau=\omega t$ and Pr is the Prandtl number.

By means of Laplace transformation, the solution of this linear second-order differential equation (3) with the boundary conditions (5) and (6) is obtained. The inverse transformation of the resulting equation yields

$$\frac{\theta(\eta,\tau)}{\mathrm{Pr}/2} = 1 - \mathrm{erf}\ [(\mathrm{Pr}/2)^{1/2}\eta\tau^{1/2}] - \exp(-2\eta) + \frac{1}{2} \exp[2(\tau/\mathrm{Pr}-\eta)].$$
$$\mathrm{erfc}[\tau^{1/2}/(\mathrm{Pr}/2)^{1/2}-(\mathrm{Pr}/2)^{1/2}\eta/\tau] - \frac{1}{2} \exp[2(\tau/\mathrm{Pr}+\eta)].$$
$$\mathrm{erfc}[\tau^{1/2}/(\mathrm{Pr}/2)^{1/2} + (\mathrm{Pr}/2)^{1/2}\eta/\tau] + \frac{1}{2-\mathrm{Pr}} \Big\{ \exp[-(2\mathrm{Pr})^{1/2}\eta].$$

$$\cos\left\{2[\tau-(Pr/2)^{1/2}\eta]\right\} -\exp(-2\eta)\cos[2(\tau-\eta)]$$
$$-\frac{1}{\pi}\int_0^\infty \exp(-\varepsilon\tau)\sin[(2Pr\varepsilon)^{1/2}\eta]\,\frac{\varepsilon d\varepsilon}{\varepsilon^2+4}$$
$$+\frac{1}{\pi}\int_0^\infty \exp(-\varepsilon\tau)\sin[(2Pr\varepsilon)^{1/2}\eta]\,\frac{\varepsilon d\varepsilon}{\varepsilon^2+(4/Pr)^2}\Bigg\} \quad (9)$$

Although the solution given by equation (9) was developed for an incompressible viscous fluid with constant properties, it is equally applicable for a compressible viscous fluid as discussed in references 3 and 4.

After a sufficient time has elapsed to allow the distribution of temperature to become purely periodic, and the influence of the initial distribution has passed away, $y/2(at)^{1/2}$ which is Y/2 will be very small and equation (9) reduces to

$$\frac{\theta(\eta,\tau)}{Pr/2} = 1 -\exp(-2\eta) + \frac{\exp[-(2Pr)^{1/2}\eta]}{(2-Pr)}\Bigg\{ 1 + \exp\left\{ -2[2-(2Pr)^{1/2}]\eta\right\}$$
$$-2\exp\left\{-[2-(2Pr)^{1/2}]\eta\right\}\cos\left\{[2-(2Pr)^{1/2}]\eta\right\}\Bigg\}^{1/2}\cdot$$
$$\cos\left\{2\tau-\tan^{-1}\left\{\frac{\sin[(2Pr)^{1/2}\eta] -\exp[-[2-(2Pr)^{1/2}]\eta]\sin 2\eta}{\cos[(2Pr)^{1/2}\eta] -\exp[-[2-(2Pr)^{1/2}]\eta]\cos 2\eta}\right\}\right\} \quad (10)$$

The first two terms on the RHS of equation (10) express the mean value of the temperature oscillations. The amplitude and phase lag of the temperature oscillation may be obtained from equation (10) as follows:

Amplitude of the temperature oscillation

$$= \frac{\exp[-(2Pr)^{1/2}\eta]}{2-Pr}\Bigg\{ 1 + \exp\left\{-2[2-(2Pr)^{1/2}]\eta\right\}$$
$$-2\exp\left\{-[2-(2Pr)^{1/2}]\eta\right\}\cos\left\{[2-(2Pr)^{1/2}]\eta\right\}\Bigg\}^{1/2} \quad (11)$$

Phase lag of the temperature oscillation

$$= \tan^{-1}\left\{\frac{\sin[(2Pr)^{1/2}\eta]-\exp\left\{-[2-(2Pr)^{1/2}]\eta\right\}\sin 2\eta}{\cos[(2Pr)^{1/2}\eta]-\exp\left\{-[2-(2Pr)^{1/2}]\eta\right\}\cos 2\eta}\right\} \quad (12)$$

Results

Equation (10) illustrates that the frequency of the temperature oscillation is twice that of the plate oscillation. The mean values of the temperature in dimensionless form as shown in Fig. 1 increases exponentially to unity in the direction normal to the oscillating plate. For fluids having small Prandtl numbers, these mean values of the temperature approach unity at a distance closer to the plate. The amplitude of the temperature oscillation expressed by equation (11) and graphically illustrated in Fig. 2 is a function of Pr and has a maximum at a certain distance from the surface of the plate. As the Prandtl number increases, the maximum value of the amplitude decreases and its location is shifted away from the plate. The temperature oscillation has its maximum at a physical time when the cosine term in equation (10) becomes unity.

The temperature oscillation expressed by equation (10) is graphically illustrated in Figs. 3 and 4 for a fluid having Pr=2. Figure 3 shows that at any instant the fluid temperature takes its mean value periodically with respect to dimensionless distance Y.

This phenomenon may be revealed by the investigation of equation (12). The investigation of equation (12) also shows that at any instant the fluid temperature oscillates with increasing period in Y for a fluid having Pr less than 2, and with decreasing period in Y for a fluid having Pr less than 2.

Mean temperature gradient over a period of $2\pi/\omega$ may be obtained by integrating the equation resulting from differentiating equation (10) with respect to y with respect to time from any instant t_o to $t_o + 2\pi/\omega$ and dividing the result by $2\pi/\omega$

$$\frac{\partial\theta}{\partial\eta} = \Pr\ \exp(-2\eta) \tag{13}$$

The heat flux to the surface over a period of $2\pi/\omega$ is

$$\bar{q}/A = -\frac{\mu U^2}{2}\left(\frac{\omega}{2\nu}\right)^{1/2} \tag{14}$$

The amount of heat transfer expressed by equation (14) is entirely due to the aerodynamic heating and is also equal to the work done per cycle in oscillating the surface.

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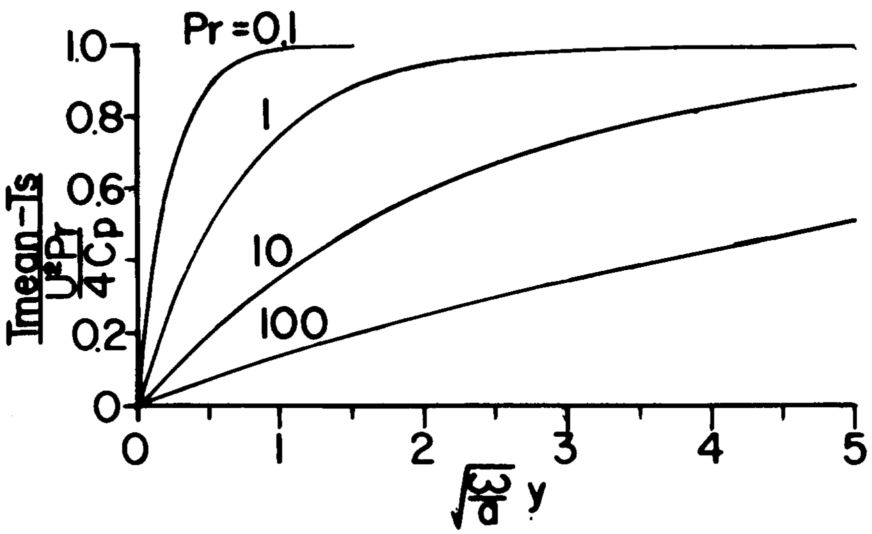

FIG. 1
Mean values of temperature oscillation

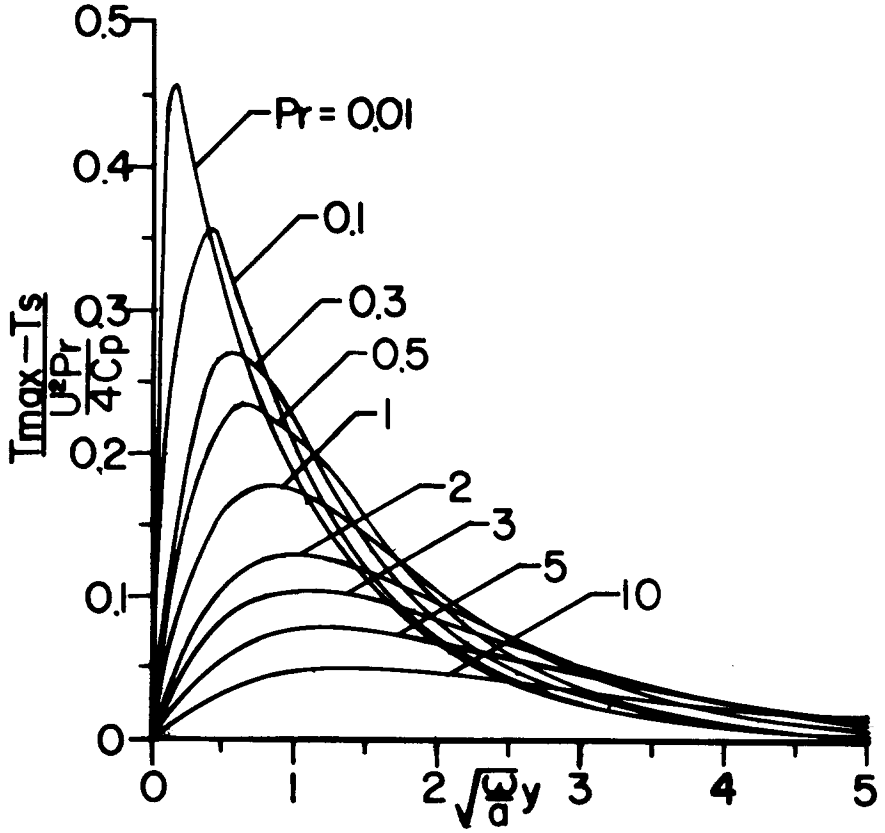

FIG. 2
Amplitude of temperature oscillation

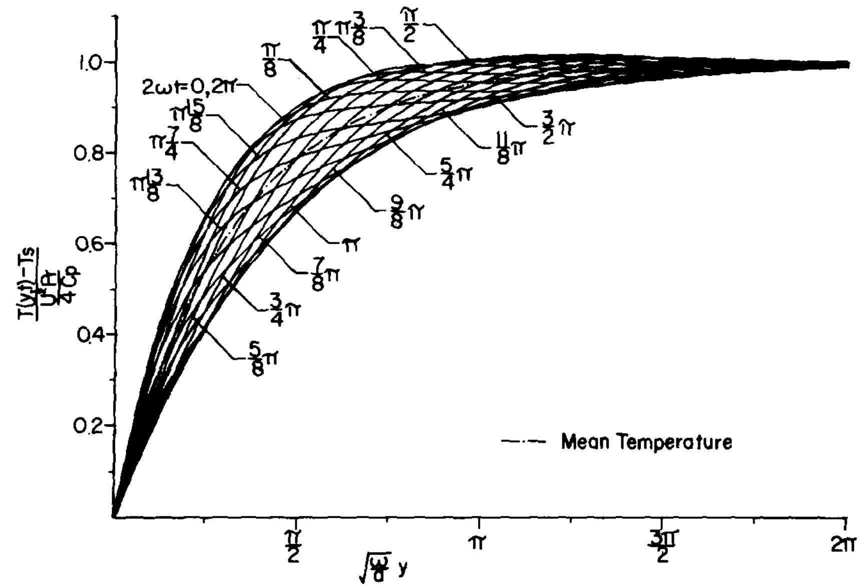

FIG. 3
Temperature oscillation of fluid with Pr=2

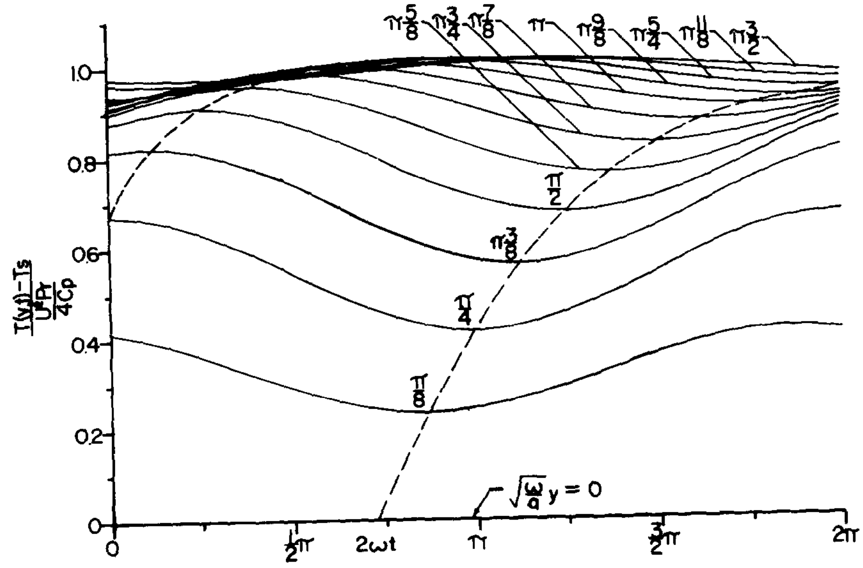

FIG. 4
Temperature oscillation of fluid with Pr=2