LETTER TO THE EDITOR


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The work of nailing seems to be bound up principally in "indentation" of the wood and friction work at the tip and along the nail shank. The fracture work of splitting the wood is very small. For example, the fracture toughness (R) of many timbers is perhaps 0.2 < R < 1 lb-in./in². Penetration to a distance x, with a split of width 2c alongside the nail of diameter D, generates a new crack area of some 2c x. If x = 3 in. and 2c = 4D (± on high side), we have for a ¼ in. dia. nail,

fracture work = (0.2 ~ 1) 4 (¼) 3 = (0.6 ~ 3) lb-in.

Fig. 4(a) in Salem et al. gives, for a 3-in. penetration, total work of at least

( ~ 190 x 3) = 570 lb-in.

with a reduced shank nail. With a full diameter nail, the work is greater. Thus fracture work is trivial. However, the mechanics of cracking seem to play a part in determining the sideways pressure (p) and particularly the extent of circumferential contact between the nail and wood (i.e. 4θ in the notation of Salem et al.), where the contact stresses may be relieved by splitting alongside the nail.

Let us model the process of frictionless nailing in terms of a deep elliptical part-through crack, as shown in Fig. 1, which grows because of the strip of pressure p contained by 2θ on both faces. The penetration x is the semi-major axis of the ellipse. Following Newman, the remote boundary stresses σ that will cause propagation of a semi-elliptical surface crack (Fig. 2(a)) are given by

\[ \sigma = \frac{K_I}{\sqrt{ax}} \frac{1}{M_d(Q)} \]

where \( K_I \) is the stress intensity factor, \( M_d \) an "elastic magnification factor on stress intensity" and Q is an elastic shape factor for an elliptical crack. \( M_d/Q \) is given graphically in ref. (1) for various (x/t) and (x/c) combinations. For x/c > 1, which is the range of interest for nailing, \( M_d/Q \) is approximately independent of (x/t), i.e. of the penetration relative to the block thickness. Calculations show that for x/c > 1, σ decreases slightly as x increases (cf. \( M_d/Q \) decreases as x/c increases), but it is permissible to take an average value for σ which is independent of x/t, and depends only on \( K_I \) and c.

Fig. 1. Frictionless nailing modelled as propagation of a "part-through" semi-elliptical crack of semi-major axis x (the nail penetration) and minor axis c (the length of split accompanying nailing).

Fig. 2. (a) The part-through crack with remotely applied stresses. (b) The part-through crack with stresses applied on the crack faces.
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\[ \sigma = \frac{K_f}{(2\pi) \sin^{-1}(b/c) \sqrt{(\pi c)}} \]  

where \( \sigma \) acts over a strip of width \( 2b \) symmetrically within the crack length \( 2c \), with the crack going right through the block. Note that when \( b = c \),

\[ \sigma = K_{II}/\sqrt{(\pi c)} \]

which is the standard solution for remote stresses in a through cracked plate. Assuming that equations (1) and (3) may be thought of as a corresponding pair, let us heuristically modify equation (1) to determine the part-through equivalent of equation (2), i.e.

\[ p = \frac{K_f}{(2\pi) \sin^{-1}(b/c) \sqrt{(\pi c)}} \times (\frac{M_f}{\sqrt{Q}}) \]

Then, \( p/K_f \) may be worked out for various values of \( 2c/D \) (assuming straight sided crack openings tangential to the nail, Fig. 1, to determine the associated \( \theta_0 \) and \( b \)). Such results are used later in this note.

Returning to “real” nailing, which involves considerable work of indentation and work of friction, penetration by the nip is similar to cone indentation, where the pressure resisting indentation \( (p') \) is given by \( p'(1 + \mu \cot \alpha) \), where \( p' \) is pressure in the absence of friction. This latter pressure must be some 2-8 \( \sigma_y \), where \( \sigma_y \) is some average yield strength of the wood beneath the indentation, which fudges anisotropic effects. Because of compressibility effects in timber, the conversion factor may be too large. The nip angle \( (2\alpha) \) is about 60°, and the coefficient of friction between wood and steel is perhaps 0.2-5. Thus

\[ p' \approx 2.8 \sigma_y (1 + 0.2 \cot 30°) \approx 3.8 \sigma_y \]

Consequently, the force \( F_r \) resisting nip indentation is \( p'(\pi D^2/4) \). Note that this is constant, and independent of depth of penetration (because successive indentations are supposedly geometrically similar). Displacements \( x_\alpha \) (Fig. 2 in Salem et al.) are elastic or compressibility effects presumably. Fig. 5 in Salem et al. gives \( p' = K_\alpha/(\pi D^2/4) = 3100 \text{ lb/in}^2 \) (along the grain) and \( 4500 \text{ lb/in}^2 \) (radially normal to the grain). Thus \( \sigma_y \) is some 820 \text{ lb/in}^2 (beneath a nail driven in along the grain) and 1200 \text{ lb/in}^2 (beneath a nail driven in radially normal to the grain), noting that these values may really be greater if \( p' < 2.8 \sigma_y \).

With a full shank nail, the friction force from the sides of the nail upon further indentation is

\[ \mu[p(4\theta_0/2\pi) \pi D] \times \]

(neglecting height of nip). Values for \( p \) and \( \theta_0 \) depend upon the assumed geometry of splitting alongside the nail, i.e. \( 2c/D \). However, \( p \approx 1.8 \sigma_y \) from considerations of indentation of rigid cylinders against incompressible flat plates, noting again that compressibility in wood may reduce the constraint. Here \( \sigma_y \) is the mean indentation yield or crushing stress perpendicular to the direction of nail travel. The estimates determined earlier for \( \sigma_y \) are in the direction of nail travel, so that to determine the limiting sideways pressure \( p > 1.8 \sigma_y \) the previous \( \sigma_y \) values should be swapped it seems, i.e.

\[ p > 1.8 \times (1200) = 2200 \text{ lb/in}^2 \] (alongside a nail driven in along the grain)

\[ p > 1.8 \times (820) = 1500 \text{ lb/in}^2 \] (alongside a nail driven in radially normal to the grain).

Again, the real values of \( \sigma_y \) would vary with grain direction in the plane of penetration.

The contact angle associated with this sideways pressure, and also the extent of sideways splitting alongside the nail \( (2c) \), may be determined from the fracture analysis discussed earlier. The critical stress intensity factor for many woods at incipient splitting is some \( (200-400) \text{ lb/in}^2 \)7 precise values varying with orientation, temperature and moisture content.

Thus, if \( p > 2200 \text{ lb/in}^2 \), \( p/K_f > 11 \) or 5-5, and fracture calculations show that \( 2c/D \) then is 2-5 or 1-7 (i.e. the wood splits \((2.5 - 1) D/2 \approx 0.7D \) on either side of the nail, or for \( K_f = 400 \text{ lb/in}^2 \) in about 0-3D, and also that \( (4\theta_0/2\pi) = 0.25 \) or 0-4, respectively). Consequently,

\[ \mu[p(4\theta_0/2\pi) \pi D] = 82 \text{ or 131 lb/in} \]

(nail driven along the grain), for a 0-236-in. nail assuming \( \mu = 0.2 \). Similarly, for \( p > 1500 \text{ lb/in}^2 \) (nail driven radially normal to the grain) \( p/K_f > 7.5 \) or 3-8, and \( 2c/D = 2 \) or 1-4 with \( (4\theta_0/2\pi) = 0.33 \) or 0.55. Thus,

\[ \mu[p(4\theta_0/2\pi) \pi D] = 74 \text{ or 122 lb/in.} \]

Now these values represent the slopes in the force—penetration diagrams in Salem et al., which we have measured from their Fig. 4(a) as 89 lb/in. (radially normal penetration) and 84 lb/in. (along the grain); elsewhere in the paper a figure of 140 lb/in is quoted. The agreement with our calculated values is remarkably reasonable. The calculated values show the “wrong trend”, i.e. the radially normal slope is predicted to be less than that along the grain slope. Had we not inverted the sideways \( \sigma_y \) values, the trend would have had the correct sense (cf. we have no information for penetration tangentially normal to the grain, the results of which would influence anisotropic calculations).

We also note that the slope of the force—penetration diagram varies directly with \( D \). This trend is seen in Fig. 4(b) of Salem et al., where nails varying in diameter from 0.236 to 0.1155 in. were driven along the grain (diameters given in the legend of the figure in terms of length). Now if the slope is 84 lb/in for \( D = 0.236 \), we would expect a slope of \( (0.155/0.236) 84 = 55 \text{ lb/in.} \) for the 0.155-in. nail driven along the grain. The experimental value is rather smaller (~ 40 lb/in), which perhaps is to be expected because the
spacing of the wood grain is likely to be important, and smaller diameter nails fit more easily in between the spacing of the grain.

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REFERENCES