

## INTERFEROMETRIC GRATINGS WITH PHASE CORRECTION

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A technique for producing diffractive optical elements uses holographic methods to incorporate into the recorded fringe pattern a correction factor for the phase defect of the substrate.

Interferometrically-produced optical elements, using the techniques that have been extensively developed or refined in connection with holography, show considerable promise. We describe a technique which enables high quality optical elements to be produced on a low quality substrate.

We show (fig. 1) a method for producing a diffraction grating with substrate correction. A photosensitive material is coated onto a non optically flat substrate, and for illustration, we show a thickness variation on the surface opposite the emulsion. Let a collimated coherent beam impinge from the left at normal incidence. This beam then acquires a phase defect  $\exp [i\phi(x,y)]$ . We can write this beam as  $u_1 = a_1 \times \exp [i\phi(x,y)]$ . The beam emerges from the plate, strikes a mirror, and is reflected back, traversing the plate a second time. The reflected wave can be written

$u_2 = \sigma a_1 \exp (i\phi)$ , where  $\sigma$  represents the attenuation of the beam after its first incidence on the plate; we assume that  $\exp (i\phi)$  varies sufficiently slowly that in traveling the distance  $2d$  from plate to mirror and back again the phase error retains its original form.

A reference beam  $u_0$  is introduced obliquely, as shown. The plate thus records the intensity distribution

$$I = |u_0 + u|^2, \quad (1)$$

where  $u = u_1 + u_2$ . The recorded signal is a hologram of the defect of its own substrate.

The hologram is at once both a transmission and a reflection hologram, because of the two object beams impinging from opposite directions. If we wish to use the hologram as a transmission grating, we prefer to form interference only between  $u_0$  and  $u_2$ ;  $u_1$  should not interfere with the other beams. This may readily be accomplished in several ways. Recalling that the longitudinal coherence function of a gas laser is periodic and becomes zero for certain values, we need only adjust the distance  $d$  between hologram and mirror so that the interference between  $u_1$  and  $u_2$  is a null at the recording position. We then adjust the path length of the reference beam  $u_0$  so that interference occurs with  $u_2$ , but not with  $u_1$ . The beam  $u_1$  therefore produces only a uniform background, which is insignificant provided the reference beam is several times stronger than the object beam, which is the usual case in holography. The recorded signal is thus

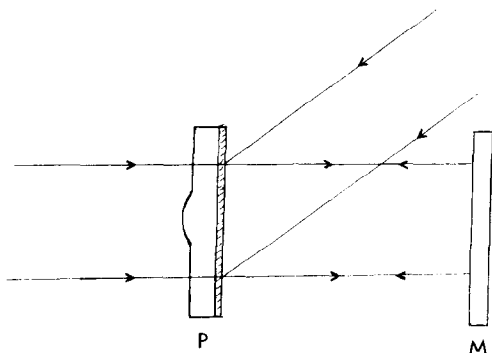


Fig. 1. Method for forming grating. P, photographic plate, with protuberance on left face, emulsion on right face; M, mirror.

$$|u_0+u_2|^2+|u_1|^2 = |u_0|^2+|u_1|^2(1+\sigma^2) + u_0^*u_2+u_0u_2^* \quad (2)$$

In using the grating, let a collimated coherent beam  $u_c$  impinge on the surface from the right at normal incidence. Upon emerging, this beam also acquires the phase defect  $\exp(i\phi)$ . The two first order diffracted beams also have the phase defect that was holographically recorded. Thus, the true image term, which forms one first order diffraction, has the form

$$u_{d+} = u_c u_0^* u_2 = a_c a_0 \sigma a_1 \exp(i2\phi), \quad (3)$$

where the  $a$ 's are the amplitudes of the correspondingly subscripted beams  $u$ . The conjugate beam has the form

$$u_{d-} = u_c u_0 u_2^* = a_c a_0 \sigma a_1 \quad (4)$$

since the phase defect on  $u_2^*$  has the form  $\exp(-i\phi)$ , which cancels the phase defect that  $u_c$  acquires by its passage through the plate. The conjugate wave is thus free from aberration.

In using the grating, the incident beam should for best results impinge at an angle roughly approximating that of  $u_1$ , otherwise the compensation is imperfect. Other problems arise when the grating is used with a polychromatic source.

If the optical element thus formed is a volume hologram, the making and readout steps should be such that when the readout wave impinge at the proper angle for complete cancellation of the phase defect, the Bragg condition for the conjugate wave is satisfied. There are two ways to read out the conjugate image of a volume hologram [1]. First, the readout wave can duplicate the reference wave  $u_0$ , but travel in the opposite direction; the diffracted wave then follows the object wave, but again, travels in the opposite direction. Second, the readout wave can duplicate the object wave, whereupon the diffracted wave follows the reference wave. These two solutions can be written, respectively, as

$$\theta_c = \theta_0 + \pi, \quad \theta_{d-} = \theta_1 + \pi, \quad (5a)$$

$$\theta_c = \theta_1, \quad \theta_{d-} = \theta_0, \quad (5b)$$

where the  $\theta$ 's are the angles of the correspondingly subscripted light beams.

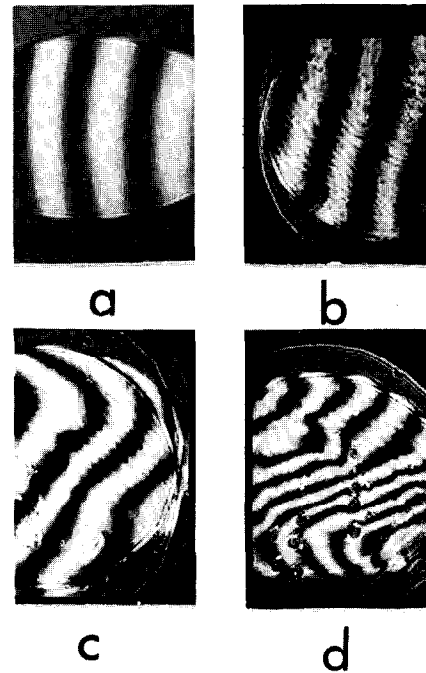


Fig. 2. Experimental results. (a) incident wave; (b) corrected wave, first order; (c) uncorrected wave, zeroth order; (d) twice-aberrated wave, other first order.

Evidently, the readout process we have described is case b. Case a is valid only for narrow-band object signals [1]. This is certainly the case here, where the beam  $u_2$  is of zero spatial frequency bandwidth except for the phase error term, which we assume is of low spatial frequency.

The effectiveness of the technique was demonstrated experimentally (fig. 2). We constructed a phase-error plate by exposing a high resolution photographic plate to a gaussian light distribution, then bleaching the plate to form a variable-index structure. This plate was then attached to an unexposed plate, upon which interferometrically-formed fringes were recorded. The plate was developed, bleaching to improve diffraction efficiency, then was placed back in the original setup, and the phase error plate was reattached. The reason for making a special phase error plate rather than relying upon the imperfection of the recording plate was merely to produce a phase error per our own specification and large enough to easily demonstrate the principle. In practice, of course, we wish to correct the phase er-

rors of the recording plate.

The holographically-corrected grating was illuminated and three diffracted orders were formed, zero order and two first orders. A prism with a slight wedge was used as an interferometer in each of the four beams, the incident and the three diffracted. The zero order shows the phase error existing on the plate. One first

order shows the phase error doubled, the other shows it corrected.

#### Reference

- [1] E.N. Leith, A. Kozma, J. Upatnieks, J. Marks and N. Massey, *Appl. Opt.* 5 (1966) 1303.