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TRADE REVERSALS AND GROWTH STABILITY*

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1. Introduction

In this paper we investigate a phenomenon noted in Johnson's (1971) geometrical analysis of trade and growth: that a country could undergo a reversal of its comparative advantage over time as a result of the effect of trade on its growth path.[1] Using a model more general than his, we show that the occurrence of such a reversal requires some instability in the underlying growth process prior to trade.

In Johnson's model, trade reversal is possible only if the investment good is capital-intensive. Uzawa (1963), studying the same two-sector model for a closed economy, showed that this capital-intensity condition could yield unstable steady states. This suggests that there may be a connection between trade reversals and unstable steady states, and that the connection may cast doubt on the likelihood of trade reversals ever occurring.[1a]

Indeed, we will show that, if a closed economy is initially in a globally stable

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[1]The possibility that the direction of trade may change during growth had been noted earlier in the growth-and-trade model of Oniki and Uzawa (1965), and indeed has always been implicit in the factor-proportions explanation of trade. Oniki and Uzawa did not, however, start their economies from positions of autarkic steady state as we do here. Thus the change in trade direction which they obtained may merely reflect a change in comparative advantage which would have occurred, strictly due to growth, even if the economies had remained closed. In fact, since they assumed the investment good to be always labor intensive, trade reversal from initial autarkic steady state would have been impossible in their model. In this paper we limit the term 'trade reversal' to changes in comparative advantage from initial positions of autarkic steady state. Thus a trade reversal as we define it can be attributed only to the opening of trade, and when it occurs, limits the usefulness of the comparative advantage criterion as a determinant of trade.

[1a]Note that this connection between trade reversals and instability is only suggested, and is not proven, by the coincidental appearance of the same capital-intensity condition in the two contexts. Capital-intensity of the investment good is necessary for both phenomena, but is not sufficient.

steady state, then, when it opens to trade at fixed world prices, trade reversals are impossible. Trade reversals are also impossible if the initial steady state is only locally stable, so long as the price change from autarky to free trade is small.

Before proving the foregoing assertions, it might be useful to give a brief, intuitive explanation of the mechanism by which trade reversals may occur.

2. The occurrence of trade reversals

Suppose that an economy, which is initially closed and in steady-state growth, opens to trade with a comparative advantage in the consumption-good industry. By trading, it can obtain the investment good cheaper than it could be produced at home. If savings is a constant fraction of income, then this fact, together with the usual static gain from trade, implies that investment in the country will increase. Since it was initially in steady-state growth, we will now find its capital–labor ratio rising.

Now suppose that the investment good is capital-intensive. Growth of the capital–labor ratio will then cause production to shift toward the investment good. It is true, of course, that demand for the investment good will also rise with the capital–labor ratio, since an increase in the latter also increases both income and savings per capita. But it is quite possible, as illustrated more carefully by Johnson, for the increase in production to be greater than the increase in demand. As this process continues, then the economy can eventually find itself exporting the investment good.

The connection between trade reversals and instability can be illustrated quite simply in this example. The opening of trade causes the capital–labor ratio to grow and the investment good to be first imported, then exported. There must therefore be some point in time, and an associated capital–labor ratio, at which there is no trade in either direction. At that instant the economy must be behaving exactly as though it were closed. But its capital–labor ratio is both above its initial value and increasing. Thus, if we were to close the economy from that point on, it would move away from, rather than toward, the initial steady state of the closed economy. It follows that the initial steady state cannot have been globally stable.

This brief argument captures the essence of the connection between trade reversals and instability. But it lacks both rigor and generality and takes for granted properties of closed and open growth models that may not be altogether obvious. Let us then present our argument more generally and precisely.

3. Proof of assertions

We begin with a general statement of the two-sector growth model for a closed economy,

$$\dot{K} = I(K,p) - \lambda K, \tag{1}$$

$$E(K,p) = 0. \tag{2}$$

Here, K is the economy's capital–labor ratio, I is per capita investment, and E is per capita excess supply of the investment good. I and E are both continuous functions of the capital–labor ratio and of the relative price of the investment good, p. λ is the sum of the rates of population growth and depreciation, both of which are assumed to occur in constant proportion to their respective stocks. The dot over K in eq. (1) indicates its time derivative.

Eq. (1), then, states that the change in the capital–labor ratio equals per capita investment minus the decrease in the capital–labor ratio that would occur automatically due to depreciation and population growth. Per capita investment depends upon the historically determined capital–labor ratio and on the relative price of the investment good. Eq. (2) determines that price as that needed to equate supply and demand for the investment good.

The same model can be reinterpreted to describe growth of an open economy, by simply taking price as exogenous and dropping eq. (2). The $E(.)$ function then serves to determine per capita exports of the investment good.

The per capita investment and excess supply functions, $I(.)$ and $E(.)$, could both be derived from production functions for the two sectors, a behavioral savings assumption, and the assumptions of perfect competition. To do so, however, would limit the generality of our results, since the behavioral and technological assumptions would have to be made explicit.[2]

It turns out that the only assumptions we really need for our results are the following. First, we assume that per capita excess supply of the investment good is an increasing function of the relative price of the investment good. That is, for any K and any p_1 and p_2,

(A1) $\qquad E(K,p_1) < E(K,p_2),\quad$ if and only if $p_1 < p_2$.

Second, we assume that investment per unit of capital is a decreasing function of the capital–labor ratio. Thus, for any p and any K_1 and K_2,

(A2) $\qquad I(K_1,p)/K_1 \geqq I(K_2,p)/K_2,\quad$ if $K_1 < K_2$.

These assumptions may be justified in two different ways. First, we note that both (A1) and (A2) are satisfied in the growth model that Johnson considered. His is the standard model of a two-sector economy with linearly homogeneous production functions, in which investment equals savings and savings is a constant fraction of national income. This model has been analyzed in a variety of ways by a variety of authors, to which the reader may refer for derivations of (A1)

[2]As long as the assumptions that we make about the $I(.)$ and $E(.)$ functions themselves are consistent with standard behavioral and technological assumptions, we therefore gain both generality and simplicity by using them directly

and (A2).[3] Alternatively we may note that both (A1) and (A2) are themselves stability conditions. Assumption (A1) states that excess supply varies directly with price, and this is the standard necessary condition for stability of short-run market equilibrium. Assumption (A2), on the other hand, is a necessary condition for stability of steady-state growth for an open economy which is small enough to face a fixed world terms of trade.[4] Thus, the result we derive will be valid not only for the model analyzed by Johnson, but also for a larger class of growth models which possess these stability properties.[5]

We wish to show that if an ecomony opens to trade from an autarkic steady state, and if its initial direction of trade becomes reversed when it reaches its new steady state with trade, then the initial autarkic steady state cannot have been stable. To prove this, we essentially argue backwards, showing that the process just described cannot be reversed. That is, we show that if the economy reverts to autarky once it is in the free-trade steady state, then it will *not* return to the initial autarkic steady state. Once this is proved, it follows that the initial autarkic steady state was not globally stable.

Thus suppose that the economy begins in a free-trade steady state, with the price of the investment good being $\bar{p}$. Since it is in steady state growth, its capital–labor ratio is stationary at some K_0, which, from eq. (1), must satisfy:

$$I(K_0,\bar{p})/K_0 = \lambda. \tag{3}$$

We assume that the country actually does trade in this steady state; that is,

$$E(K_0,\bar{p}) \neq 0. \tag{3a}$$

Now suppose that we close the economy. In order to clear the market, price will have to change to some p_0, such that:

$$E(K_0,p_0) = 0. \tag{4}$$

K_0, with its corresponding p_0, is the initial condition for the subsequent growth of the closed economy, which will be governed by the dynamic system (1) and (2). That system will generate a continuous time path of the capital–labor ratio, $K(t)$, as well as a continuous time path for the price, $p(t)$.[6]

Lemma: If p_0 is greater (less) than $\bar{p}$, then $p(t)$ must be greater (less) than or equal to $\bar{p}$ for all t. That is:

$$(p_0-\bar{p})[p(t)-\bar{p}] \geqq 0, \quad \text{for all } t \geqq 0.$$

[3]In Johnson's article, both of our assumptions may be observed in his figures 4 and 5. Assumption (A1) follows from the movement in opposite directions of the production point and the income expansion line, OI_s, when price changes. (A2) follows from the relative slopes of his investment requirements curve, PQ, and his income expansion path, OI_s (since 'required' investment is proportional to the capital stock). (A1) is also explicitly derived in Oniki and Uzawa (1965, pp. 24–25), and both (A1) and (A2) are derived in Vanek (1971, pp. 378–384).

[4]See Vanek (1971). Note that it is yet a third stability property that we intend to relate to trade reversals, which is stability of steady-state growth for a closed economy.

[5]We discuss several such extensions in our concluding section 4.

[6]Since we have assumed that (2) can be solved for p as a continuous function of K.

Since a comparison of the autarkic market-clearing price with the world price, $\bar{p}$, indicates comparative advantage and the direction of potential trade, this Lemma says that comparative advantage will not reverse during autarkic growth. It follows that if there exists an autarkic steady state with opposite comparative advantage, that steady state will not be reached by autarkic growth starting from K_0 and therefore is not globally stable.

Proof of the Lemma: Suppose the contrary. That is, suppose that at some time $t_1 > 0$,

$$(p_0-\bar{p})[p(t_1)-\bar{p}] < 0. \tag{5}$$

Then, by continuity, there must be a time t_2, $0 < t_2 < t_1$, at which

$$p(t_2) = \bar{p}. \tag{6}$$

Since the economy is closed,

$$E[K(t_2), \bar{p}] = 0, \tag{7}$$

and it follows by comparison with (3a) that

$$K(t_2) \neq K_0. \tag{8}$$

There are then two cases to consider. First, suppose that $K(t_2) > K_0$. Then by (A2) and (3),

$$I[K(t_2), \bar{p}]/K(t_2) \leqq I[K_0, \bar{p}]/K_0 = \lambda, \tag{9}$$

so that, from (1),

$$\dot{K}(t_2) \leqq 0. \tag{10}$$

But $K(t_2) > K_0$ also implies that $\dot{K}$ must have been positive at some time prior to t_2. Thus from continuity again, there exists a time t_3, $0 < t_3 < t_2$, at which

$$\dot{K}(t_3) = 0. \tag{11}$$

Now in the dynamic system, (1) and (2), both $\dot{K}$ and p are uniquely determined by K alone, so that once K stops moving, the entire system must remain stationary forever afterward. That is, (11) implies that $K(t) = K(t_3)$ for all $t > t_3$. But this means, since $t_1 > t_2 \geqq t_3$, that $p(t_1) = \bar{p}$, contradicting (5).

The same contradiction follows if $K(t_2) < K_0$, as may be seen by simply reversing the inequalities (9) and (10). Thus the Lemma is established.

In our application of the Lemma, we compared the autarky price with $\bar{p}$ to determine comparative advantage. Since $\bar{p}$ is the free-trade steady-state price, this procedure is valid only if the country in question is too small to affect the world price. Thus, we have established the connection between trade reversals and autarkic growth stability only for the case of a *small* economy.

In fact, however, the connection also holds for a two-country world in which both countries affect the prices. Assumption (A1) implies that when the two countries change from free trade to autarky, their prices must move in opposite directions. Thus, the price must fall below $\bar{p}$ in one country and rise above $\bar{p}$ in the other. From the Lemma, the initial qualitative relationship between the two countries' prices will continue throughout autarkic growth. Comparative advantage cannot reverse, therefore, during the approach to autarkic steady state. It follows that if trade between two countries reverses during the approach to free-trade steady state, at least one of the countries must have started from an autarkic steady state that was not globally stable.

4. Conclusion

We have shown that a reversal of trade can occur during the approach from an autarkic steady state to a free-trade steady state, only if the autarkic steady state was not globally stable. This result bears obvious resemblance to Samuelson's Correspondence Principle.[7] Comparison of trade patterns in different steady states is essentially a comparative static exercise. Samuelson showed that comparative static ambiguities can often be removed by assuming dynamic stability. In our case, if we assume that no unstable steady states exist for the closed economy, then a reversal of trade during growth of the open economy is ruled out. Alternatively, we can merely assume that the country begins in a stable steady state before trade, whether or not other, unstable, steady states exist. In that case, trade reversal is ruled out so long as the price change from autarky to free trade is sufficiently small.[8]

Note, however, that this analogy with the Correspondence Principle is not perfect. The Principle allows one to rule out certain static changes on the grounds that the new equilibrium, being unstable, cannot be reached. In our case, however, there is nothing to prevent trade reversals from occurring if an economy does manage to start from an unstable autarkic steady state. We have not shown the free-trade growth process to be unstable. On the contrary, our assumption (A2) assures growth stability for the case of a small open economy, and in fact two-country growth can also be stable even if one or both of the countries possess unstable autarkic steady states.[9] Instead what we have shown is that trade reversals imply instability of autarkic steady states and thus that the initial position needed for trade reversal is unlikely.

[7]See Samuelson (1947).

[8]Small enough, that is, to leave the economy in the neighborhood with respect to which the autarkic steady state was locally stable.

[9]The possibility that trade can have a stabilizing effect on growth should not be surprising. Vanek (1971) has shown that the steady state of a small open economy with proportional savings must be unique and stable, even though the same economy if closed may possess unstable steady states. In the case of two countries of comparable size, this possibility is more difficult to demonstrate, but can, I believe, be shown by adapting the tools of Oniki and Uzawa (1965) to the case of a capital-intensive investment good.

We mentioned in discussing assumptions (A1) and (A2) that they were general enough to encompass a larger class of growth models than that analyzed by Johnson. We have already shown our result to hold for the two-country extension of Johnson's model. It remains to note several other modifications of his model for which our assumptions, and therefore our result, remain valid.

An obvious candidate for modification in Johnson's model is the savings assumption. If we replace the assumption that savings are proportional to income with the popular alternative that they depend only on profits,[10] then it can be shown that our assumption (A2) remains valid. Assumption (A1), however, does not. It is well known that the closed two-sector growth model with such 'classical savings' may, if the investment good is capital-intensive, possess unstable short-run market equilibria, thus violating assumption (A1). If, however, we also assume that elasticities of substitution in production are sufficiently large, then as shown by Drandakis (1963), such short-run instability is ruled out. Thus our result is valid for a growth model with classical savings, so long as substitution elasticities are sufficiently large.[11]

Since our result is valid for both proportional and classical savings assumptions, we would expect it also to be valid for a combination of the two. Thus if there are separate savings propensities for wage and profit income, then, again subject to the qualification regarding substitution elasticities, both assumptions (A1) and (A2) remain valid.[12]

Alternatively, we could assume savings and investment both to be functions of a simultaneously determined rate of interest, as well as income and the return to capital. Uzawa (1963) used such an assumption in an extension to his closed growth model, though he assumed the investment good to be labor-intensive. It should be possible to adapt his formulation, taking care to assure short-run market stability, and obtain a more general model which satisfies our assumptions. As long as savings, investment, and production of the investment good can be determined, in per capita terms, as static functions of K and p, or of other variables which depend on K and p, then our eqs. (1) and (2) correctly describe the model.

[10]This assumption was used by Uzawa (1961) in his first treatment of closed two-sector growth, and has also appeared in the growth and trade literature as one of the savings assumptions used by Stiglitz (1970).

[11]Alternatively, we could generalize our conclusion by saying that trade reversal requires *either* instability of the autarkic steady state *or* short-run instability of some autarkic market equilibrium.

[12]This hybrid savings assumption has been criticized by Pasinetti (1962) as requiring individuals with two sources of income to save differently out of each. His alternative of specifying savings propensities for groups within the population is attractive, but clearly leads to a model different from ours. His assumption requires explicit consideration of the separate capital stocks owned by each group, so that a country cannot be described by a single differential equation such as (1).

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