A STOCHASTIC LEARNING MODEL OF MIGRATION

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It is observed in this paper that although the Harris–Todaro theory is intended to provide insight into rural–urban migration, their formal model describes a theory of equilibrium population densities, and although this can be used for comparative static analyses, it is not immediately applicable to the disequilibrium dynamic process whereby these densities change over time. A very simple stochastic (Markov) choice model found in the psychological adaptive learning literature is applied to the migration problem, and it is found that a dynamic equation similar to that used by Todaro can be obtained explicitly from this model. The possibility that migration rates might accelerate even in the presence of constant urban–rural wage differentials is derived.

1. Introduction

In the growing literature concerning behavioral, adaptive, or otherwise non-traditional theories of consumer and firm behavior, a great deal of stress is placed upon the need for models which are as effective in describing markets which are out of equilibrium as the traditional optimization theory is in describing comparative static equilibria.[1] Although most of the models which are developed in this vein are indeed well-defined under conditions of incomplete market adjustment, examples of the specific contributions which these might make to particular economic problem areas are not plentiful. The purpose of this note is to make one such application, using a simple stochastic learning model to support a theory of rural–urban labor force migration, particularly as it occurs in less-developed countries.

The comparative-static theory of labor-force location is straightforward. In most papers [such as Todaro (1969), Harris–Todaro (1970), Stiglitz (1974)] it is maintained that the urban labor supply in less-developed countries is determined not by the prevailing wage rate alone, but by a composite of the urban wage and the (typically high) urban unemployment rate. Individual workers are assumed

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[1]See, for example, the papers found in Day and Groves (1975), and the references therein.

to compare the income which they can earn from employment in rural agriculture to that which could be received in the city, where the expected urban income is computed from the higher urban wage and various assumptions regarding the probability of finding employment there.

In the cases of the Todaro and Harris–Todaro papers, however, a good deal of stress is placed upon the importance of the dynamic process of migration itself. Todaro, for example, presents as his main hypothesis the relation

$$\frac{\dot{S}(t)}{S(t)} = F\left[\frac{V_u(t) - V_r(t)}{V_r(t)}\right],$$

(1)

where S is the size of the urban labor force and $\dot{S}$ its time derivative, $V_u(t)$ is the discounted present value of expected urban income, and $V_r(t)$ is the discounted present value of expected rural income. The Harris–Todaro paper (1970, p. 126) similarly draws attention to migration as an ongoing phenomenon with the introductory observation that 'migration not only continues to exist, but indeed, appears to be accelerating.'

Despite these suggestions that the central problem of migration is its persistence, possibly reflecting a continuing dynamic adjustment process, even these papers revert to the simple comparative-static model in order to produce their main results. In the Harris–Todaro paper, models are formulated in which the expected urban wage is made equal to the rural wage (that is, in which migration would be zero), and the resulting equilibrium urban unemployment rate is investigated in the light of various tax and minimum wage policies. Even the original Todaro paper concentrated upon a state in which migration is zero except for an 'equilibrium' amount which is calculated by dividing the rate of urban job creation by the employment rate and then subtracting the rate of natural urban population growth.

This reluctance to work with the migration phenomenon itself may be attributable to the fact that dynamic models such as eq. (1), plausible as they certainly are, are not derived from any formal theory, but are simply stated as initial hypotheses. The observation that a rational household must take account of the presence of unemployment in the city, and thus must compare rural income to an expected value for urban income can only be used to derive an equilibrium model for which migration is zero when the returns from the two alternatives are equal. It cannot be used to explain a stable rate of migration when the returns are unequal. Ordinary rates of migration can only occur if many individuals who could migrate do not do so even if the expected urban wage substantially exceeds the rural wage. Of course, a variety of independent variables come immediately to mind which could be used to account for this sluggishness (and many of these play an important role in econometric studies of migration), but none of them are introduced explicitly into the theory. As it stands, eq. (1) states, in effect, that migration decisions are based upon a

rational comparison of income alternatives, but that a large fraction of the population irrationally fails to act on this comparison. That the size of this fraction should vary inversely with the difference between V_u and V_r is plausible, but it is nowhere explained by the theory itself. In short, optimization theory, not being defined for disequilibrium, is simple inadequate to the task.

2. Learning

It is proposed in this paper that we investigate the mechanism of migration in terms of the dynamic learning paradigm which psychologists have long employed as a foundation stone for their models of human behavior. Models derived from this theory have the advantage of being dynamic by nature and thus of requiring no adaptation to disequilibrium circumstances. Moreover, they enjoy enough experimental support already to give us confidence that they do in fact reflect our best available understanding of behavior change. For our purposes here, we shall apply one of the simplest of these models: that due to Bush and Mosteller as described in Cross (1973). The approach implied in these theories is entirely behaviorist in that 'decisions' of individuals are treated as random (Markov) variables whose likelihoods are dependent upon each individual's own previous experience and not upon any kind of explicit expected income or utility calculations. The variables are therefore similar in spirit to those found in sample survey studies of migration,[2] except that we will not use migration probabilities as the dependent variables but focus instead upon a worker's choice of location: the individual worker, i, will be found in the urban sector during period t with a probability P_t^i and in the rural sector with a probability $1 - P_t^i$. The value of P_t^i is determined from experience, and there is no presumption that the worker 'knows' anything about the market beforehand.

In this regard, learning models address the problem of uncertainty (in this case the uncertainty of employment in the city) in quite an unconventional way. Traditional maximization approaches require that uncertainty be handled with statistical estimation and search procedures, many of which employ quite sophisticated techniques.[3] In some cases, these even demand some prior information as to the probability distributions which characterize a market. Whereas optimization models therefore require greater and greater sophistication on the part of individual decision-makers as the importance of uncertainty grows, this learning model will make essentially the same assumptions under both uncertainty and certainty. The mathematics found in the two alternative approaches are often similar, but the interpretations are wholly different.

Since our purpose is to focus upon the economic factors which may influence

migration, we will concentrate here upon the economic experience of the worker. A more general model could make similar use of variables reflecting such factors as age, education, family size, or population density in order to derive a more complete view of the migration process. For the sake of a simple model, we will characterize the objective situation as follows: There is one major urban center to which migration is possible. If worker i locates himself in this city, he will find 'modern sector' employment which pays a high wage, W_u, with a probability q_i. If he fails to find such a job there still may exist various forms of marginal employment which will pay a very low wage, W_0. If he does not live in the city, rural employment guarantees an income of W_r.[4] In general, we have $W_u > W_r > W_0$. The likelihood q_i should depend upon quite a number of variables such as the age of the worker, the length of his stay in the city, and, most important, whether or not he had a modern sector job in the previous period $t-1$. In the face of the potential complexity in determining q_i, however, it is common to assume (following Harris–Todaro) that jobs are distributed randomly, and since we are only concerned in this note with demonstrating the usefulness of a learning model, we will preserve this assumption. This makes q_i equal to the employment rate itself, E/S, where E is the number of modern sector jobs.

3. Migration

Beginning with the likelihood P_t^i, we use the actual experience of the individual during t to modify this likelihood to a new value P_{t+1}^i. Naturally, both P_t^i and P_{t+1}^i must be bounded between 0 and 1, and we expect P_{t+1}^i to vary with the degree of success which is encountered at the location which is chosen. The well-known Bush–Mosteller linear learning model provides the simplest function which meets these conditions. For example, if i lives in the city and finds a job, then we write

$$P_{t+1}^i = P_t^i + \alpha_i(W_u)(1 - P_t^i), \tag{2a}$$

where the function $\alpha_i(W)$ describes the rate of learning as a function of the reward (wage) magnitude. We emphasize that 'learning' here is not to be interpreted in the sense of 'finding out,' and that the worker is not being described as someone attempting to estimate q_i. Instead, $\alpha_i(W)$ simply reflects the empirical observation that actions which are met with success tend to be repeated. $\alpha_i(W)$ has the general properties

$$0 < \alpha_i(W) < 1, \qquad \alpha_i'(W) > 0, \qquad \alpha_i''(W) < 0,$$

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[4]We could, with no loss, distinguish a wage for the rural employment from a still lower rural unemployment wage. In keeping with the models already cited, however, we presume that all rural workers are equally employed.

but it is most convenient here to approximate this function with the linear form $\alpha_i' W + \alpha_0$ where α_i' is the slope of the function $\alpha_i(W)$ in the vicinity of W.

Even though $P_{t+1}^i > P_t^i$ in this example, the worker may nevertheless go back to the rural sector at the end of the period – that is, he quits and goes home with probability $1 - P_{t+1}^i$. In this regard we are already departing from the properties of optimization theory. In both the Todaro and Harris–Todaro models, it is assumed that anyone who migrates to the city stays there so long as the expected urban economic opportunities are greater. In fact, however, some out-migration, even among the urban employed, is a common phenomenon in less-developed countries, and a dynamic model ought to reflect that fact.

The value which is taken by P_{t+1}^i depends upon whether the worker lives in the city and whether he gets a job. If the worker fails to find an urban job, then[5]

$$P_{t+1}^i = P_t^i + (\alpha_1' W_0 + \alpha_0)(1 - P_t^i). \tag{2b}$$

If he lives in the rural sector, earning W_r with certainty, a similar formula is applied to $(1 - P_t^i)$, the probability of staying in the rural sector, and this reduces to

$$P_{t+1}^i = P_t^i(1 - \alpha_1' W_r - \alpha_0). \tag{2c}$$

Combining (2a)–(2c) with their associated likelihoods[6] and simplifying, we can obtain an expected value for P_{t+1}^i:

$$E[P_{t+1}^i] = P_t^i + c' P_t^i(1 - P_t^i)(W_i - W_r), \tag{3}$$

where $W_i \equiv q_i W_u + (1 - q_i) W_0$.

If N represents the total population available for urban–rural employment (assumed to be fixed for the purposes of this note), then the expected urban

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population at time t is given by

$$S(t) = \sum_{i=1}^{N} P_t^i,$$

and the expected urban population at time $t+1$ is given by

$$S(t+1) = \sum_{i=1}^{N} E[P_{t+1}^i].$$

Expected migration, $M(t)$, is the difference $S(t+1) - S(t)$ and, using eq. (3), this becomes

$$M(t) = \sum_{i=1}^{N} \alpha_i' P_t^i (1 - P_t^i)(W_i - W_r). \tag{4}$$

Finally, accepting the Harris–Todaro assumption that every city dweller has the same chance of employment, $(q_i = E/S)$, then eq. (4) becomes

$$M(t) = (\bar{W} - W_r) \sum_{i=1}^{N} \alpha_i P_t^i (1 - P_t^i). \tag{5}$$

Since the summation term is positive whatever the values of the individual probabilities, this implies that $M(t) > 0$ whenever the expected urban wage exceeds the rural wage, and this is the main proposition which we wished to obtain.[7]

4. Properties of the migration function

Since the migration model given by eq. (1) was not explicitly derived from any underlying dynamic adjustment theory, there has naturally been considerable debate over its most appropriate form. Todaro, for example, uses the size of the urban labor force, S, as the base from which to measure the rate of migration. Zarembka (1970) has objected to this specification on the grounds that it is the rural population that provides the migrants, and that therefore $(N-S)$ should be used as the base. In fact, eq. (4) does not support either of these positions. In order to get a simple picture of the operation of eq. (4), let us assume

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[7]Incidentally, the term V_r in the denominator of the argument of (1), which is meant to restrict Todaro's function to proportionate wage differences, has its counterpart in eqs. (4) and (5) in the value of α_i' which, as an approximation to the slope of $\alpha_i(W)$, declines as W_r increases.

temporarily that all individuals are identical. This would make $P_t^i = S(t)/N$ for all i. Now eq. (5) becomes

$$M(t) = (\bar{W} - W_r)\alpha' \left(1 - \frac{S(t)}{N}\right) S(t),$$ (5')

or

$$\frac{M(t)}{N - S(t)} = \frac{S(t)}{N}\alpha'(\bar{W} - W_r),$$

This formulation differs from that of both Todaro and Zarembka in that the learning process applies to the entire population. In this model, $M(t)$ is the net summation of rural–urban migration and urban–rural migration, whereas Zarembka and Todaro only considered the effects of wage differentials on the rural population, presuming that those who move to the city never go back 'home.'

The migration process described by eq. (4) will eventually lead to a stable population distribution. Net migration will reach zero if $W_i = W_r$ for all i and the comparative static properties of an economy in such an equilibrium can be evaluated as usual.[8] Even in this equilibrium, of course, many individual workers are changing location: it is only the net flow of migrants which is zero.

Finally, the functional form of eq. (4) has an important dynamic implication. For any constant value of $W - W_r$, the character of the migration function is that of a logistic curve (indeed, the logistic is often characterized as 'the learning curve'). For small values of P_t^i (equivalent to a predominantly rural population), $M(t)$ is correspondingly small. $M(t)$ is larger for larger P_t^is, reaching a maximum value when the population is approximately equally distributed between rural and urban components (that is, $P^i(1 - P^i)$ is maximized when $P^i = 0.5$). $M(t)$ falls again if the population shifts still further. Since most less-developed countries are predominantly rural, we would conclude that they are still in the rising phase of this process, and that if the values of $W_i - W_r$ are maintained at present levels, migration will not only continue, but will accelerate.

Although the quotation from Harris–Todaro given earlier conveys a recognition that an acceleration in migration may in fact be taking place, their use of an equilibrium model diverts attention away from this condition and suggests only that migration is a continuing response to disequilibrium. The view obtained from eq. (4) is much more pessimistic. Present investment and wage policies in less-developed countries are often designed to maintain a $W - W_r$

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[8]It may be worth adding that in the determination of W_i, one should take account of the fact that many urban workers are already more or less established in their jobs, so that, for them, q_i is large, or even equal to 1. The unemployment rate among the rest of the urban population may be much higher (and W_i correspondingly lower) than gross employment/population figures would suggest. W_0 is an equally important influence. The availability of marginal employment opportunities can raise W_i considerably by making the state of the urban unemployed bearable.

differential in the face of migration. Emphasis is put upon the expansion of urban job opportunities as a means of holding down unemployment despite constant or even rising urban wage rates. According to the dynamic implications of our learning model, such attempts to accommodate development policies to current levels of migration are entirely hopeless; maintenance of the $W - W$ differential in the face of an accelerating tide of rural–urban migrants will become a practical impossibility, and inevitably, W will, one way or another, be driven down to W_r.

5. Information

Although it is not entirely in keeping with the strict behaviorism embodied in the learning model we have used, many readers of this paper have inquired about the possibility that some workers are influenced by the successes and failures of others. If a rural worker learns (in the sense of 'finding out') that an acquaintance has found high-paying urban employment, then he, too, may be more inclined to relocate in the city. In our notation, P_t^i may be increased to some larger value $(P_t^i)'$, where the extent of the increase is a function of the wage differential $W_u - W_r$, which has been encountered by the acquaintance. Similarly, information that the acquaintance is unemployed would discourage relocation in proportion to the wage differential $W_r - W_0$.

If experience is the great teacher, and information of this sort is impeded by poor information channels and widely dispersed rural populations, then this new learning is insignificant, and we can use the original model as described by eq. (4). If this is not the case, then we must introduce information flows into the original model by replacing P_t^i in eq. (3) by $(P_t^i)'$ from this second learning process. Such combined models in fact are fairly easy to construct, and these prove to have qualitative properties which differ very little from eq. (4).

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