# STUDYING GLUON PROPERTIES EXPERIMENTALLY \*

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We argue in this paper the following. (i) A large part of what is observed in high-energy hadron reactions may be rather directly interpretable in terms of gluon interactions. Since gluons do not interact directly with leptons and photons this could be a valuable way to study them experimentally. Earlier work in this direction is briefly summarized. We suggest how several quantities can be reinterpreted in terms of gluon interactions; the rise in  $\sigma_{tot}$ , and the more rapid increase of multiplicity with energy at very high energies, are particularly fruitful to examine. The possibility of interpreting inclusive polarization data in terms of gluon spin properties is considered. Most importantly, we propose that if certain of our predictions on central region particle ratios are correct, then the gluon distribution as a function of x may be measurable at energies in the ISABELLE range. (ii) The structure of gluon jets in mass, multiplicity and momentum is discussed; we suggest that gluon jets will be quite different from quark jets, with more of the energy of the gluon jet going into mass, so hard gluon jets may not exist.

### 1. Introduction

It is believed today that quantum chromodynamics (QCD) [1] is a good candidate for a theory of strong interactions. QCD is a theory of colored quarks interacting with colored gluons. The major new features of the theory, and the ones which are crucial for the theory to have desirable properties such as asymptotic freedom and confinement, depend on the nature of the gluons and their interactions.

The gluons and their properties may be extremely difficult to study. They are thought not to interact with lepton or photon beams, so these probes will not help directly. Although an elegant theoretical edifice has been erected around QCD, additional experimental tests and experimental stimulation are desirable.

It is the purpose of this paper to argue that, under certain conditions, a large part of the data from ordinary hadron high-energy interactions may be interpretable in terms of gluon interactions, and may provide extensive experimental tests of QCD.

In section 2 we will review briefly other recent work that touches on this subject.

Section 3 summarizes the standard views on the relevant properties of gluons, and discusses some more recent and speculative aspects of gluon interactions which

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will be used in the following. In particular, we argue that hard gluon jets may not occur; instead, a finite fraction of the energy may go into particle masses and increased multiplicity.

In section 4 we argue that almost every standard observable in high-energy hadron physics can be reinterpreted in terms of gluon interactions. Particularly fruit-ful quantities to examine are the rise in the total cross section, the growth of the multiplicity at very high energies, central region clusters and especially their mass dependence, and particle ratios for  $x < \frac{1}{2}$ . Inclusive polarization, double pomeron exchange, and other quantities are also considered. Most important, if the predictions for particle ratios substantiate the view that the gluon interactions are dominant, then it is possible that the gluon distribution function, or at least its value at intermediate x, can be measured.

Our predictions are summarized in table 1.

We should remark on our viewpoint and on the spirit with which we expect our analysis to be taken. QCD is not an easy theory to work with. When color separation occurs in an interaction but colorless hadrons emerge, no one knows how to take account of the associated effects. To put it more directly, so far work in QCD mainly addresses properties of quarks and gluons. How these fundamental objects transform into hadronic matter, as seen experimentally, is still a subject of speculation. Although it will be difficult, one should judge the validity of the framework, namely QCD, separately from the attendant speculation. On the other hand, normally the development of a theory benefits considerably from contact with experiment. We have tried to push as hard as we can to relate gluon interactions to experimental quantities, using analogies and speculations and models. We view this work as having value in several ways. (i) By raising questions on these subjects we may stimulate useful theoretical work on QCD. (ii) Some of what we say may be essentially correct, so the relevance of QCD and gluons to experiment will be increased. (iii) Any solutions of QCD (or any equivalent theory) will be able to be tested against experiment by calculating their predictions for the quantities we have discussed.

Of course, some of the quantities we discuss already have interpretations in other languages or models such as the pomeron in reggeon calculus, multiperipheral models, general Mueller-Regge arguments, etc. There is no conflict. It can be fruitful to approach the same questions from the new point of view. For some aspects it will help to give a better picture of how gluons interact but explain no new physics, while for some others (such as the energy dependence of the multiplicity or inclusive polarizations) new explanations or predictions may arise as well.

## 2. Other work

In this section we review other recent work which we have been able to find, to indicate the present status of attempts to interpret hadron interactions in terms of quark and gluon interactions. Several authors have tried to develop a pomeron theory in terms of gluon exchanges. We are aware of three independent analyses. Low [2] has constructed a pomeron model, using MIT bag model arguments, based on single-gluon exchange for the dominant production amplitude. The elastic amplitude is then two-gluon exchange. An important result of his analysis is a calculation of the absolute size of the total cross section in terms of the basic quark-gluon coupling, and a mass characteristic of confinement size which is taken from the form factor. For a typical values of the coupling he indeed obtains  $\sigma_T \simeq 40$  mb. Further, because of the vector nature of the gluons the bare pomeron must have a Regge intercept  $\alpha_P(0) = 1$ . Nussinov's approach [3] is similar to that of Low, with emphasis on the quantum number structure of the pomeron, and the role of color, rather than on constructing a definite model.

Ochme and Heckathorn have argued [4] that in QCD the gluon-gluon scattering amplitude has a singularity at J = 1 at t = 0; the existence of the singularity follows from short-distance behavior and renormalization group arguments. This singularity couples to hadrons and can be interpreted as the pomeron. To have the pomeron at J = 1 requires the translation due to the gluon spin. Their pomeron is gauge-invariant and only appears in positive-signature amplitudes. We interpret this work as providing strong formal support for the more phenomenological analyses of Low and Nussinov, and what appears in the following.

Van Hove and Pokorski have suggested [5] that it is possible to interpret hadronhadron interactions at high energies in the following picture. The valence quarks go right on through in the collision, while the gluons interact strongly and are stripped from the quarks. The quarks emerge to give leading particle and fragmentation effects, while the gluons give rise to the central region clusters. They discuss the cluster properties in some detail and show that at present the data is consistent with interpreting clusters as arising from independent emission of neutral objects, with a flat rapidity distribution and small  $p_{\rm T}$ . All the cluster properties are consistent with interpreting them as arising from gluons. We suscribe to this view, and we will explore possible further implications below, particularly concerning cluster mass and multiplicity distributions.

Van Hove and Fiałkowski have extended [6] this picture in yet another direction, to include diffractive production. They argue that diffractive production can be interpreted as the shadow of the gluon scattering. They suggest a nice picture, in which central gluon interactions saturate the unitarity limit, but central pp collisions do not, since zero impact parameter will not be the same for gluons and protons. This allows one to understand the consistency of the data with geometrical scaling, and allows a basic interpretation of many features of pp diffractive interactions. They are able to obtain a reasonable size for the diffraction dissociation cross section.

The above approaches are, we think, in basic agreement, although they are not expressed in similar languages. An alternative, physically different, approach has been pursued by Brodsky and Gunion [7]. Following to some extent Feynman's

notion of wee partons, and assuming these are quarks, they argue for the pomeron as a quark exchange (to drive the inelastic cross section). Although we will not discuss their work further here, they suggest some tests to distinguish their quarkexchange pomeron (QEP) from the gluon-exchange pomeron (GEP) and we comment on one of these. For the QEP the inelastic states are a baryon minus one quark, a  $\overline{3}$  in color SU(3). From the color point of view this is like  $e^+e^- \rightarrow q\overline{q}$ , where the separating colored objects are also triplets, so they predict that pp and  $e^+e^-$  will have the same multiplicity. On the other hand, for the GEP the separating colored objects are octets, which can be thought of as  $3 \otimes \overline{3}$ , or two separating triplets. Thus for the GEP the multiplicity should be roughly twice as large as for  $e^+e^-$ (Brodsky and Gunion give the actual factor as  $\frac{9}{4}$ ).

At present energies the multiplicities in pp and  $e^+e^-$  are about equal, so Brodsky and Gunion argue that the QEP is favored. However, we believe that the theoretical argument should be applied to the asymptotic multiplicity to avoid low-energy effects, i.e., to the coefficient of ln s in the multiplicity. Although that is not directly measurable, it can be obtained by fits or extrapolations. Ref. [8] gives  $1.84 \pm 0.05$  for this coefficient for pp and  $0.75 \pm 0.04$  for  $e^+e^-$  while ref. [9] gives  $1.45 \pm 0.02$  and  $0.67 \pm 0.17$ , respectively \*, both essentially consistent with the Brodsky-Gunion prediction of  $\frac{9}{4}$  for the ratio of pp to ep; the extrapolations [10] give similar results.

Consequently, we lean toward the view that this test favors the GEP over the QEP. On the other hand, since the actual multiplicities are rather universal, and deep inelastic multiplicities do appear to show a  $q^2$  independence, both of which would favor the view of Brodsky and Gunion, at the present time no convincing conclusion can be drawn. A better understanding of the phenomenological implications of the different theoretical models is needed.

Further, we expect that color compensation can be more local for hadrons than in  $e^+e^-$  (see below; this point is also discussed by Sivers [11]), while Brodsky and Gunion assume it is identical for both. While this is the sort of question that will have to be decided by a better grasp of the theory, it does not appear that present knowledge justifies assuming the multiplicity arises by the same mechanism in pp and in  $e^+e^-$ .

The pomeron of Veneziano and Lee [10] can also be thought of in quark-gluon language and gives results qualitatively similar to the gluon pomeron. The origin of multiplicities has been carefully analyzed there and gives an increase of a factor of 2 when diffraction and absorption are included.

One further analysis, by Ochs [12], should be treated here. He has noticed that at the ISR particle ratios such as  $K^+/K^-$ ,  $\pi^+/\pi^-$  are similar at a given x at 0° and at 90°, and are different from unity down to fairly small x (fig. 1 of his paper). Since

<sup>\*</sup> Different values are obtained by different authors because of slightly different choices of functional forms or variables.

hadrons coming only from gluons would give unity for these ratios, the gluons are clearly not dominating at present energies.

This is not inconsistent with our viewpoint, because at present energies the part of  $\sigma_T$  due to gluon interactions is not expected to be large. For example, we can interpret  $\sigma_T$  as having a rising part less than one quarter of  $\sigma_T$  at ISR energies. However, it does point out one of our most clean predictions: these particle ratios must go sharply toward unity in the x region where gluons dominate (presumably  $x \leq \frac{1}{3}$ ) as the energy increases, or our basic approach is simply wrong. We will discuss this in more detail below; present data is consistent with such expectations (see subsection 4.7).

Finally, we just remark here that Heller has made an interesting suggestion [13] relating the observations of inclusive polarization in  $pp \rightarrow \Lambda + x$  to gluon spin properties. In subsect. 4.9 we will extend his considerations and briefly discuss their implications.

In this section we have summarized work by other authors on interpreting much of high-energy hadron-hadron interactions in terms of (quark and) gluon interactions. Next we review and speculate on how gluons are expected to behave.

### 3. Gluon properties

In this section we will summarize the relevant properties that gluons are expected to have, in two parts. The first includes standard, relatively well known ones, and the second set are more speculative, based on recent work [14] by Chang and Yao.

(A) The most important single result is from the momentum sum rule: only about half of the momentum of a proton is carried by constituents which interact with weak and electromagnetic currents (in  $e^-$  and  $\nu$  reactions), and the rest is interpreted as being carried by gluons. Thus we are assured that in all high-energy collisions a sizeable fraction of the hadron consists of gluons which will interact strongly. Recently Politzer has suggested that the momentum fraction carried by gluons is closer to a third [15] than a half; while that would affect the extent to which we can neglect sea effects at any x, it will not modify any qualitative results.

Another important property of gluons is that they are completely flavor-neutral. They do not distinguish charge, isospin, strangeness, etc. When gluons materialize into colorless mesons, apart from corrections for masses, the population of different mesons should be independent of all quantum numbers.

Since gluons do not recognize the existence of leptons, they can only be studied indirectly at machines with lepton beams, either through decays of charmonium or additional heavy quarkonium states [16], or through gluon bremsstrahlung from a struck quark [17].

Before we make any comments on the distribution properties of quarks and gluons, we should state what is our understanding of such distributions. A hadron always undergoes quantum fluctuations. Thus, gluons are constantly bremsstrahlunged off and pairs are recurrently created and absorbed. When we measure the constituents of this hadron with a probe of certain resolution in time, we should expect to find two distributions. Those partons whose life times are much longer than our resolution time should appear as equilibrium distributions, whereas the others, the wee partons, whose scales are comparable to the scale of the probe may have complicated fluctuations.

When we say the parton distribution is such and such, we are referring to the long time component, as seen by a probe at a certain energy.

In view of the above remarks, we further comment on the gluon distribution. When a quark is struck by an *external* projectile, it produces gluons, which eventually become observed hadrons. These gluons, besides being mostly soft because of the vector nature of the interaction and asymptotic freedom, should not be included in the distribution function, since they do not compose the original hadron. The reader should bear this in mind to avoid misinterpretation of what we have to say about the long time component of the gluon distribution.

The distribution of gluons in x (the fraction of the proton momentum that they carry) should be intermediate between that of the  $q\bar{q}$  sea (which is concentrated at very small x) and that of the valence quarks (which extends to x = 1). This is because the gluons will couple strongly to the  $q\bar{q}$  sea, but many of them will originate by bremsstrahlung from the valence quarks and carry a significant fraction of the valence quark's momentum. Two recent phenomenological analyses of the gluon distribution in x, G(x), are given in ref. [18].

It is important to emphasize that the gluons are not the same as the  $q\bar{q}$  sea, although clearly a sharp distinction between them is not possible. For example, the sea interacts with lepton probes while the gluons do not. At any instant, of order half of the proton momentum is carried by states which are neither valence nor sea quarks; the sea quarks carry only a few percent of the momentum. Indeed, since the gluons carry an order of magnitude more momentum than the sea, while the x distributions are not usually thought to be too different (the gluons having a larger tail at larger x), probably at every x the sea can be neglected for our present purpose of studying strong interactions and hadron production. We are assuming, of course, that production of final state hadrons can be interpreted as coming from gluons; this seems to be consistent with their role in strong interactions.

(B) Now we turn to even more speculative properties of gluons and gluon interactions. Much of the point of view we advocate here is based on results obtained in ref. [14]. While their results do not directly apply to QCD gluons, the models they study do incorporate both gluon splitting and asymptotic freedom [19], which might be expected to be the main determinants of gluon behavior for our purposes.

For trying to decide how gluons will behave, probably the most important gluon property is the existence of strong three-gluon and four-gluon self-couplings. Bebause of these, one gluon rapidly multiplies into a cascade of several gluons. The probability of obtaining more gluons is enhanced when some have appeared; further, because of asymptotic freedom the coupling is expected to be stronger for softer gluons. Theoretically this "gluon splitting" may be fundamental for confinement.

Phenomenologically, gluon splitting suggests that gluon "jets" may be much different from quark jets. While the quark jets are expected to be hard, with much of the momentum of the struck quark in the jet momentum, when a hard gluon is emitted there will be a cascade into a relatively large number of softer gluons, which eventually convert into hadrons. Consequently, we expect much more of the energy of a hard gluon to be converted into hadron mass than would happen for a quark jet.

This implies two important results.

(i) The multiplicity associated with gluons will grow with energy and get to be considerably higher than for quarks. We will discuss this in subsect. 4.6 below.

(ii) There may not be any hard "gluon jets" analogous to quark jets. Instead, gluon jets will be characterized by high multiplicity swarms of hadrons that are totally flavor-neutral.

Other workers [17] who have studied gluon properties and discussed how to look for them have instead assumed that gluon jets would be similar to quark jets, although no compelling reasons have been given. An experimental choice between these approaches would be of great help in getting further insight into the theory. Note that for us the two results of high multiplicity and no hard gluon jets go together.

We should comment on the behavior of quark jets that is normally expected. They remain hard for two reasons. The first is that quark masses keep intermediate states away from strong infrared singularities, and at the same time cause the running coupling constant to have an upper bound, with a scale set by the quark mass; this does not allow a quark to have much gluon bremsstrahlung compared to a gluon. The second is that the most probable dissociation for a gluon of mass  $\sqrt{s}$  is [14] into two gluons each of mass  $\frac{1}{2}\sqrt{s}$ , so the amount of energy taken away from a hard gluon after *n* splittings is proportional to (see fig. 1)

$$\alpha(\sqrt{s}) \alpha^2(\frac{1}{2}\sqrt{s}) \dots \alpha^{2^n}((\frac{1}{2})^n \sqrt{s}),$$

while the energy taken away from a quark in the corresponding configuration is proportional to

$$\alpha(\sqrt{s}) \alpha(\frac{1}{2}\sqrt{s}) \dots \alpha(\sqrt{s}/2n)$$
.

The latter product \* terminates with  $\alpha(m_q)$ , while the former only terminates with a confinement mechanism for zero-mass gluons. The former product is much larger than the latter one, and correspondingly the gluons lose much more energy to soft gluons than do the quarks.

<sup>\*</sup> There is some question whether instead it should be  $(\alpha(m_q))^n$  in the spirit of the free quark approximation, but this does not change our argument. The most probable dissociation pattern was proved for scalar gluons; for vector gluons the needed *non-perturbative* result is technically more difficult to obtain and is not at present known.

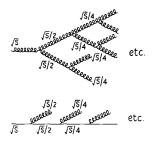


Fig. 1. Graphs to illustrate the estimate of the energy carried away from a hard gluon and a hard quark. The energies assigned to each line represent the most probable dissociation configuration for gluons, whereas for the quark case they are chosen to compare with the gluon case even though it is an improbable dissociation configuration, quarks preferring to dissociate into soft gluons.

#### Table 1

The predictions we give are summarized here, in two categories, (A) those for which no further input is needed, and (B) those for which additional information such as the gluon distribution function or QCD solutions is needed to make a precise prediction. Each prediction is treated in an appropriate section of the text.

#### A. No further input needed for predictions

- (i) Gluon "jets" are not hard (compared to quark jets).
- (ii) Gluon "jets" have high multiplicity (compared to quark jets).
- (iii) The multiplicity in hadron collisions goes as  $\langle n_{\pi} \rangle = cs^{\gamma}$  at very high energies.
- (iv) Particle ratios  $\pi^+/\pi^-$ ,  $K^+/K^-$ ,  $\overline{p}/p$  in the region  $0 \le x \le x_0$  approach unity as s increases, where  $x_0$  is about  $\frac{1}{3}$ .
- (v) If (iv) is correct, the size of the gluon distribution function in x can be determined near  $x_0$ .
- (vi) For inclusive polarizations,  $P(pp \rightarrow pX)/P(pp \rightarrow \Lambda X) \simeq 0.4$ ,  $P(pp \rightarrow nX)/P(pp \rightarrow \Lambda X) \simeq 0.55$ , in a certain kinematic region.
- (vii) Central region clusters must have properties consistent with originating from flavorneutral gluons, and the number of clusters must grow with s.
- (viii) Factorization is increasingly violated as s increases.
- (ix) The pomeron in double pomeron exchange in an SU(3) singlet.

#### B. Additional QCD input needed

- (i) The coefficient and power (c and  $\gamma$  in (iii) above) of s for the multiplicity are calculable in QCD.
- (ii) The rise in  $\sigma_T$  is due to gluon interactions, and writing  $\Delta \sigma_T = A(\ln s)^K$ , A and K are simply related to QCD quantities.
- (iii) The x and  $p_{\perp}$  distributions for inclusive polarizations can be calculated in certain kinematical regions.
- (iv) The mass distribution, multiplicity, and number of central region clusters as functions of energy can be calculated.

The absence of hard gluon jets may not make it harder to observe gluon bremsstrahlung. In  $e^+e^- \rightarrow q\bar{q}g$ , rather than three jets [17] one would observe q and  $\bar{q}$  jets that are not back-to-back with the missing momentum taken by a flavor-neutral hadron swarm. In  $\gamma^* + q \rightarrow q + g$ , [17] even more dramatically, only a single quark jet would appear, opposite a flavor-neutral hadron swarm.

Another important area for theory and experiment is the form of the gluon-gluon scattering amplitude. Theoretical work [20] on solving QCD will find a gg excitation spectrum, perhaps with an effective threshold, perhaps with resonances ("glue balls"), a certain strength determined by the effective coupling, and some characteristic high-energy fall-off. We will see below that several kinds of observables in hadron reactions, perhaps even the total cross section, may give experimental evidence about the gluon-gluon scattering amplitude. Predictions based on the above arguments are summarized in table 1.

#### 4. Reinterpretations and predictions

Now we turn to discuss specific hadronic experimental quantities and how we might use them to learn about gluon properties and interactions. This list of possibilities includes the total cross section, the real part of the forward amplitude, the elastic differential cross section, central region clusters and their properties, multiplicities and particularly the growth in multiplicity at very high energies, particle ratios, double pomeron exchange, and inclusive polarizations. Our most important result is probably the possibility that the behavior of particle ratios will permit a measurement of the gluon distribution function. The predictions are summarized in table 1.

### 4.1. The total cross section

Let us adopt the point of view [5] described in sect. 2 that in a high-energy collision the valence quarks are forward scattered and give rise to fragmentation region products, while the gluons interact. If the kinematical conditions are right, several pairs of gluons could be excited. Suppose the gluon-gluon cross section is large in the kinematical region

 $s_{gg} = s_0$ ,

where we guess  $s_0 \approx 4 \text{ GeV}^2$ . Assume the average momentum carried by a gluon is  $\bar{x} = 0.15$  (e.g., with a distribution  $xG \sim (1-x)^5$ ) and for simplicity let all gluons have the average  $\bar{x}$ .

Then for protons of momentum p the total energy squared is  $s = 4p^2$ , and the gluon pair subenergy is

$$s_{gg} = (\bar{x}p + \bar{x}p)^2 = \bar{x}^2 s$$

Presumably the strong gluon-gluon interaction will cause the total cross section to rise; for our guesses it should begin to show this effect at

$$s \simeq s_{gg}/\bar{x}^2 = 180 \text{ GeV}^2$$
,

or at  $p_{\rm L} \simeq 90$  GeV, a reasonable estimate of where the rise actually begins.

This estimate is very crude, but illustrates the basic idea: perhaps the rise in  $\sigma_T$  can be interpreted as due to gluon interactions. Of course, we cannot prove this, but we can show that if it were correct it would be very fruitful, and that it is consistent with several other aspects of hadron interactions.

We also cannot show that pairwise gluon interactions will dominate, although it is reasonable that they should be important. However, if gluons interact strongly, when one hadron emits a pair they will self-interact, perhaps effectively forming a massive object, giving a shorter range and less important effect. Consequently, our approximation might be a good one.

A better calculation would begin by writing

$$\sigma_{\rm T}^{\rm G} = \int \mathrm{d}x_1 \mathrm{d}x_2 G(x_1) G(x_2) \sigma_{\rm gg}(s_{\rm gg}) , \qquad (1)$$

where  $\sigma_T^G$  is the part of  $\sigma_T$  due to pairwise gluon interactions,  $s_{gg} = x_1 x_2 s$ ,  $G(x_1)$  is the gluon distribution function giving the probability of finding a gluon with momentum fraction  $x_1$  (and similarly for  $x_2$ ) and  $\sigma_{gg}$  is the gluon-gluon cross section. If we can determine the shape of G(x) from particle ratio data (see below) and its normalization from the momentum sum rule, this would already give a useful constraint on  $\sigma_{gg}$ .

In particular, the way in which  $\sigma_{gg}$  changes with  $s_{gg}$  will be reflected in the s dependence of  $\sigma_{T}$ . Any theory or solution of QCD can test its distribution function and cross section here. The presence of a threshold or resonances in gluon-gluon scattering, the size of  $\sigma_{gg}$ , and the high-energy behavior of  $\sigma_{gg}$  will all have an effect. One could even imagine relatively local variations in the energy dependence of  $\sigma_{T}$ .

There are of course too many unknowns to determine anything uniquely at present. But knowledge from other observables reflects back on these same quantities (see below) and theoretical knowledge will increasingly constrain them too.

If G(x) behaves (as expected) as 1/x, and if  $\sigma_{gg}$  has a threshold behavior  $s_{gg}^{1+\epsilon}$  with  $\epsilon \ge 0$  (again, as expected), then for  $\sigma_{gg} \rightarrow \text{constant}$  as  $s_{gg} \rightarrow \infty$  the asymptotic rise of  $\sigma_T$  is like  $\ln^2 s$ , while if  $\sigma_{gg}$  falls as a power of  $s_{gg}$  the asymptotic rise of  $\sigma_T$  is  $\ln s$ . Thus the QCD behavior can be tested.

The threshold behavior of the rise may be relevant too. Simple arguments suggest  $\sigma_{gg} \sim s_{gg} f(s_{gg})$  where f is a slowly varying function. This would give a rise going like  $s^{1\pm \epsilon}$  with  $\epsilon$  in principle calculable, though we cannot presently determine if the experimental behavior of approximately  $s^{1.1}$  would come out. Also if the rise is due to gluon interactions it would have to be the same for meson-baryon and baryon-baryon interactions, while if it were due to flavor-carrying constituents it should differ for different reactions since the appropriate distributions would have differ-

ent powers. While this cannot yet be tested clearly because there is no unique way to separate the reggeon contributions, at higher energies it will be a useful check.

In this section we have speculated that the physical origin of the rise in  $\sigma_T$  is the onset of gluon-gluon interactions. Perhaps by adopting this viewpoint it will be possible to say why and in precisely what way  $\sigma_T$  rises, and to test QCD calculations as well.

### 4.2. Factorization

We are not aware of any reason why factorization should be an exact result; in fact, it appears factorization should not hold. Consider  $\pi\pi$ ,  $\pi$ N, NN cross sections. While it is reasonable that the valence quarks give 4/6/9 ratios for these, the gluon cloud around any color singlet hadron would be the same size if it were determined by the total color charge as one might assume. Then the gluon contributions would give equal cross sections for all of these. Similar kinds of remarks will apply for NN  $\rightarrow$  NN, NN + NN\*, NN  $\rightarrow$  N\*N\*. Consequently, it will be the deviations from factorization that are of interest, and the factorization properties of parts of the cross sections. For example, perhaps the leading particle cross sections due to valence quarks for  $\pi\pi$ ,  $\pi$ N, NN will remain in the 4/6/9 ratios while the rising parts of each will originate from gluons and all be equal asymptotically.

It will not be easy to uncover such effects. For example, the above cross sections have quite different reggeon parts which must be removed (NN is exotic and the others are not). Also, it is hard to find factorization violations at all [21]. Consider the scattering of hard spheres, which doesn't factorize. The factorization condition reads  $(4r_1^2) (4r_2^2) = (r_1 + r_2)^4$ . For  $r_1/r_2 = \frac{2}{3}$ , this gives 576 = 625 which would be hard to distinguish experimentally from agreement.

### 4.3. Real part of the forward amplitude

Whenever the imaginary part of the forward amplitude (i.e.,  $\sigma_T$ ) has energy dependence, analyticity requires an associated real part, which is essentially given by  $\frac{1}{2}\pi d\sigma_T/d \ln s$  at high energies. If structure in the gluon-gluon scattering amplitude gives structure in  $\sigma_T$ , there will also be structure in Re M(0). It is possible that Re M(0) is more sensitive to some kinds of effects. However, even estimating such an effect requires going beyond the simple incoherence approximation of eq. (1) to calculate an amplitude rather than a cross section, and we do not know how to do that.

### 4.4. Differential cross section

There is little that can be said here at present. In quark-gluon language the properties of  $d\sigma/dt$  reflect the basic confinement mechanism, how a hadron is held together in a structure of a certain size and quantum numbers. As theory progresses

and more insight is obtained into working with models of a hadron,  $d\sigma/dt$  may become a useful constraint.

### 4.5. Central region clusters

As discussed in sect. 1, Van Hove and Pokorski have suggested that inelastic, central region, hadrons arise from gluons. They review the experimental evidence and propose that the clusters, in terms of which the data are interpreted, originate as gluons.

The observation of Ochs mentioned in sect. 1, that particle ratios are not unity even at small x, imply that the Van Hove-Pokorski suggestion cannot be the whole story at present energies. Nevertheless, we consider it a useful hypothesis and one which we expect will become more correct at higher energies. If it is basically correct, studying the clusters may lead to information about gluon properties.

We envisage a mechanism along the following lines. Upon interaction, in the infinite-momentum frame, non-wee gluons will be shaken off a hadron. These will undergo splitting into a number of softer gluons which will convert into hadrons when they reach some minimum momentum and a sufficiently strong coupling. More than one cluster could arise from a single hard gluon. It is possible, in principle, to calculate the number of clusters, the cluster multiplicity, and the cluster mass distribution (threshold behavior, peak, resonances, high-mass fall-off) in QCD; such calculations have been done [14] in simpler theories which include the main features of gluon splitting and asymptotic freedom. Further complications will occur due to scattering of hard gluons. Although the situation will be very complicated, it seems fair to be optimistic that the main qualitative features of QCD in this area will be testable by studying cluster properties.

As said earlier, the results of ref. [14] may not be directly applicable to QCD. This is aggravated by the observation that the concept of a virtual gluon decaying into an arbitrary number of soft gluons is not a gauge-independent one. Nevertheless, let us assume that this virtual gluon can be anchored onto some physical source and that in some unspecified gauge \* what we have learned can provide a basis for discussion.

Then, write s for the invariant mass squared of a cluster arising from the splitting of a hard gluon, and define  $s_{max}$  to be a typical mass separating hard gluons from soft gluons. The existence of  $s_{max}$  is intuitively acceptable. It has to do with the transition of a weak coupling theory (ultraviolet) to a strong coupling theory (infrared). At the same time, we are aware that ultimately gluons have to materialize into pions and other hadronic matter once a sufficiently long wave-length limit is reached. Thus  $\sqrt{s_{max}}$  must be several times a typical hadronic value, say 2-5 GeV. The mass

<sup>\*</sup> If the (unjustified) leading ln sum is used, and if the production agent is a color singlet source, then the gauge choice is the light-cone gauge [22].

distribution \* for a cluster then comes out to be [14]

$$F(s) \simeq (\sqrt{s})^{\gamma} e^{\beta \sqrt{s}}$$

where  $\gamma$  depends on how many gluons each parent decays into. We expect  $\gamma \simeq -\frac{1}{2}$ , since this obtains if the gluons interact *via* either three-point or four-point vertices.  $\beta$  is a function of the running gauge coupling, the form of which we do not quite know, because of our lack of understanding of how fundamental fields materialize into hadrons. If we neglect this aspect then we expect

$$\beta \simeq c \ln \alpha_{\rm s}$$
  $s < s_{\rm max}$ ,  
 $\simeq \exp\left(-\frac{c'}{\alpha_{\rm s}}\right)$   $s > s_{\rm max}$ ,

where

$$\alpha_{\rm s} \simeq \frac{\alpha_0}{1 + \frac{25}{12\pi} \alpha_0 \ln \frac{s}{s_0}}$$

is the running gauge coupling constant, and c and c' are calculable numerical constants. In fact c = 1, if gluon splitting (i.e., no hadronization) is the only physical mechanism. This has the important consequence that all the hard gluon energy goes into the masses of the soft gluons. Hence

$$\begin{split} F(s) &\sim (\sqrt{s})^{\gamma} e^{\beta \sqrt{s}} \qquad s < s_{\max} , \\ &\sim (\sqrt{s})^{\gamma} \qquad s > s_{\max} , \end{split}$$

where the second line follows from the dependence of  $\beta$  on  $\alpha_s$  and from  $\alpha_s \sim 1/\ln s$ .

While these are not exceedingly precise predictions, they do indicate how the cluster properties may be related to fundamental quantities in QCD. Perhaps an interplay of theory and experiment over a period of time will lead to useful results.

It is also possible in principle to estimate the number of hard gluons emitted in a collision (which is presumably uniquely related to the number of clusters) and how this quantity should grow with energy.

## 4.6. Multiplicity at high energies

One of the main implications of the importance of gluon splitting is that a hard gluon has a high probability of cascading into a number of soft gluons. These, being confined, will have to convert into hadrons. A large part of the energy of a hard

<sup>\*</sup> With our assumptions, *F(s)* is proportional to the total hadronic cross section for two gluon annihilation *via* a single s-channel gluon.

gluon will end up in hadron masses, so the multiplicity of hadrons will be large.

If a non-vanishing fraction of the gluon energy always ends up in masses, then the multiplicity will grow as a power of the energy rather than as  $\ln E$ . Possibly this is the physics underlying the increasingly rapid growth \* of multiplicity at cosmic ray energies.

If at high energies a significant part of the cross section is due to gluon cascades or jets, we can proceed as follows to estimate how the associated multiplicity would behave.

The physical reason the multiplicity grows as a power can be seen as follows. The mass distribution function we wrote down earlier is a generating function for multiparticles and therefore useful information such as average multiplicity can in principle be extracted. However, in view of our present limited understanding of how quarks and gluons convert into hadrons, it is fruitful at this juncture to relate quantities to some phenomenological parameters. These parameters should be calculable and allow contact to be made with fundamental inputs, such as the gauge coupling constant.

Let us accept the premise that once a gluon is formed, it will continue to split in such a way that a finite fraction of the original energy goes into making mass for the daughters. (This fraction would have been unity, had there been no hadronization.)

Let r be the average number of daughters into which each parent can split, and let  $\epsilon$  be the ratio of each daughter's mass to her parent's. From energy conservation

$$\epsilon \leq \frac{1}{r}.$$

Let  $\sqrt{s}$  be the mass of a cluster (in the sense of the "mass" of a virtual photon), then after successive splittings each daughter has

$$m_0 \equiv \sqrt{s} ,$$
  

$$m_1 = \epsilon m_0 ,$$
  

$$m_2 = \epsilon m_1 ,$$
  

$$\vdots$$
  

$$m_i = \epsilon m_{i-1} ,$$
 etc

This splitting terminates after n steps if  $m_n$  commeasures a hadronic mass, say  $m_n$ , i.e.,

...

$$m_n = \epsilon^n \sqrt{s} \simeq m_\pi$$

\* See, for example, the review by Gaisser [23]. Gaisser remarks that one possibility for interpreting extensive air shower data (although not a preferred one) is "transfer of much of the energy to production of many relatively slow particles". or

$$n = \ln(\sqrt{s/m_{\pi}})/\ln(1/\epsilon).$$
<sup>(2)</sup>

Presumably after the *n*th step there are only pions left, so the multiplicity is

$$n_{\pi} \simeq c(r)^{n} \simeq c(r)^{\ln(\sqrt{s/m_{\pi}})/\ln(1/\epsilon)} = c(\sqrt{s/m_{\pi}})^{\ln r/\ln(1/\epsilon)} , \qquad (3)$$

which shows that the multiplicity has power dependence on the cluster energy  $\sqrt{s}$ . A constant c is included to recognize that the hadronic mass scale  $m_{\pi}$  we picked is not unique and also that more than one soft gluon is involved in making a pion.

In QCD  $r \simeq 2$  or 3. We can experimentally gain some insight in r and  $\epsilon$  by looking at the maximum momentum allowable for a pion, given a cluster mass. Define v as the velocity of the daughters in the rest system of their parent after the *i*th split. Then energy conservation gives

$$\frac{rm_{i+1}}{\sqrt{1-v^2}}=m_i,$$

and using  $m_{i+1} = \epsilon m_i$ , we have

$$\frac{r\epsilon}{\sqrt{1-v^2}} = 1$$

or

$$v = \sqrt{1 - r^2 \epsilon^2}, \qquad (4)$$

which shows that v is independent of *i*. Clearly, a pion will have the maximum momentum if all the velocities of its ancestors are aligned. It can be shown that in this case

$$v_{\pi}^{\max} = \frac{(1+v)^{n} - (1-v)^{n}}{(1+v)^{n} + (1-v)^{n}},$$

$$P_{\pi}^{\max} = \frac{m_{\pi}v_{\pi}^{\max}}{\sqrt{1-(v_{\pi}^{\max})^{2}}} = m_{\pi}\frac{(1+v)^{n} - (1-v)^{n}}{2(1-v^{2})^{n/2}}.$$
(5)

Reasonable guesses might be c = 1, r = 2,  $\epsilon = \frac{1}{4}$ ,  $\sqrt{s} = 2$  GeV. These give  $P_{\pi}^{\max} = 0.9$  GeV,  $\langle n_{\pi} \rangle = 3.78$ , and a multiplicity growth of  $s^{1/4}$ . Eqs. (2)–(5) should provide useful information on r and  $\epsilon$ , which hopefully may be compared with values for r and  $\epsilon$  deduced from QCD.

The main difference from models which give a power of  $\ln s$  for the multiplicity growth comes from the basic property of what fraction of the initial mass goes into particle masses. If the ratio of the daughter masses  $m_{\rm K}$  to the original mass  $m_0 = \sqrt{s}$  stays non-zero as  $m_0 \rightarrow \infty$ , then the multiplicity grows as a power. If  $m_{\rm K}/m_0$  vanishes as  $m_0 \rightarrow \infty$ , then the multiplicity grows as a power of  $\ln s$ . For the multiplicipheral model, for example,  $m_{\rm K} \sim \sqrt{m_0}$  and  $\langle n_{\pi} \rangle \sim \ln s$ . This is familiar from the kinemati-

cal constraint on the subenergies that  $s_{12}s_{23}/s$  is finite in the multi-Regge limit with momentum transfers cut off, so that each subenergy will typically grow as a fractional power of s.

### 4.7. Particle ratios and the gluon distribution function

When gluons turn into hadrons we expect complete independence of flavor quantum numbers. In practice there will be corrections due to mass breaking, both in phase space and in effective coupling. (It is amusing to note that

$$\alpha_{\rm s}(m_{\rm K}^2)/\alpha_{\rm s}(m_{\pi}^2) \simeq 1/\left\{1 + \frac{25}{12\pi}\alpha_{\rm s}(m_{\pi}^2)\ln(m_{\rm K}^2/m_{\pi}^2)\right\} \simeq \frac{1}{25}$$

This would give about the correct  $K/\pi$  suppression, and similarly,

$$\alpha_{\rm s}(m_{\overline{n}}^2)/\alpha_{\rm s}(m_{\pi}^2) \simeq 1/40 ,$$

so if hadron masses set the scale here, perhaps it will become possible to make such corrections.)

More important, particle ratios such as  $\pi^{-}/\pi^{+}$ ,  $K^{-}/K^{+}$ ,  $\overline{p}/p$  will not need mass corrections. These should be unity at any x where gluons dominate. As discussed above, Ochs has observed that except at small x these ratios are not unity at present energies. Thus our strongest prediction is that as energy increases, particle ratios  $(\pi^{-}/\pi^{+}, K^{-}/K^{+}, \overline{p}/p)$  should approach one in any region of x where gluons dominate. As mentioned in the discussion of  $\sigma_{T}$ , at ISR energies the gluon contribution  $\leq_{1}^{1}$  of the total so it is reasonable that the particle ratios are not one. But by high ISABELLE energies there should be a rapid approach of the particle ratios to one in the region  $0 \leq x \leq \frac{1}{3}$ .

Present data go in this direction at  $p_{\perp} = 0.4$  (where the data [24] is most favorable to our interpretation. For  $\sqrt{s} \ge 23$  GeV and  $x \le 0.1$  the  $\pi^+/\pi^-$  ratio is consistent with unity. For K<sup>+</sup>/K<sup>-</sup> the ratio decreases from about 4 at x = 0.35 to be consistent with 1 at  $x \simeq 0.1$ . For p/ $\bar{p}$  the ratio is not 1 at any observed s, x but it appears to extrapolate to 1 at x = 0 at  $\sqrt{s} \ge 23$  GeV. The energy dependence [25] at x = 0 is consistent with out expectations also. For  $\pi^+/\pi^-$  at  $\sqrt{s} = 6.84$  GeV one has  $\pi^+/\pi^- \simeq 1.5$ , while for  $\sqrt{s} \ge 19.6$  GeV  $\pi^+/\pi^- = 1$ . At  $\sqrt{s} = 23$  GeV K<sup>+</sup>/K<sup>-</sup>  $\simeq 1.3$ , while for  $\sqrt{s} = 63$  GeV, K<sup>+</sup>/K<sup>-</sup>  $\approx 1$ . For p/ $\bar{p}$  even at  $\sqrt{s} = 63$  GeV the ratio is 1.6.

If this approach to unity is observed to happen there is a considerable bonus, namely it automatically provides information about the shape of the gluon distribution function. Normally the gluon distribution function is not directly measurable since it cannot be probed with lepton or photon beams. It is deduced from the energy-momentum sum rule by assuming that whatever momentum is not carried by quarks at a given x is carried by gluons. The sum rule provides only one moment however, and there is considerable freedom in how the momentum is distributed between sea, valence quarks and gluons for  $x < \frac{1}{3}$ .

1

Suppose the particle ratios at high energies go to unity in a certain x region, and then deviate increasingly from unity as x increases. For  $x \ge 0.15$  we know from lepton production that the  $q\bar{q}$  sea is negligible, and we know approximately the valence quark distribution. Should the particle ratios go to unity in the region  $x \ge 0.15$ , we will know that the gluon distribution dominates the valence quarks, and where the particle ratios rise from unity, the gluon and valence-quark distributions are comparable. Depending on the form the particle ratio data takes, it may be possible to determine several parameters in a functional form chosen for the gluon distribution. In particular, it should be possible to find out if the gluon distribution extends to larger x than is commonly thought (e.g., to  $x \le 0.5$ ).

## 4.8. Double pomeron exchange

Another way to get information on gluon interactions and on the gluon interpretation of the pomeron is to study the double pomeron exchange process, where the final protons are detected at  $x \approx 1$  and a cluster of particles is produced near zero rapidity, separated by several units of rapidity from the end hadrons. It is crucial to check [26] that the distribution in each rapidity separation is pomeron-like so reggeon "background" is not included. The gluon interpretation of the pomeron clearly implies that such a cross section must be non-zero. The mass and angular distribution properties of the cluster will reflect on the gluon interactions. Finally, the cluster will have to be completely flavor neutral, since the pomeron is usually expected to have an SU(3) non-singlet part this provides in principle an important test. However, the correction needed for particle masses may keep this from being useful at the present time. Because of the requirement that the cluster be a color singlet, we do not have a way of estimating the cross section size or detailed properties of the cluster, but increasing understanding of QCD will allow someone to do so.

### 4.9. Inclusive polarization

Heller [13] has suggested the model shown in fig. 1 as a way to generate large inclusive polarization. All conventional models predicted quite small polarizations, while what is observed [27] is large, of order 25%, for the  $\Lambda$  polarization perpendicular to the  $\Lambda$ p plane. If such a mechanism can be shown to dominate in certain kinematical regions it would give useful checks on the gluon spin, the effective quark-gluon coupling, and effective quark and gluon masses in interactions.

The spin of the  $\Lambda$  will be the spin of the s-quark in a constituent quark approximation, since the (ud) pair are an isosinglet and a spin singlet. We have calculated the expected polarization using just the darker lines of fig. 1 to see if we can find some criteria to test whether this mechanism is reasonable.

We find that polarization is indeed generated, and the effect could be large. How large depends on making a model for the unknown initial quark-quark scattering,

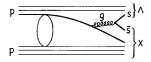


Fig. 2. This model is used in subsect. 4.9. to calculate the inclusive polarization.

which must give a complex, spin-dependent scattering. The only clear prediction we can make is that the result must have as a factor the mass of the polarized quark since for zero-mass quarks their spin is in the direction of motion and cannot give polarization perpendicular to the scattering plane. This allows the prediction that the polarization in inclusive nucleon production and  $\Lambda$  production are in the ratio of the non-strange and strange constituent quark masses,  $m_u/m_s \simeq 0.62$  using the results of ref. [28]. Heller gives a correction factor of  $\frac{17}{19}$  for neutron and  $\frac{2}{3}$  for proton, due to the extent to which one quark carries the spin of the hadron in an SU(6) wave function. Combining these, we predict that

$$\frac{P(pp \to pX)}{P(pp \to \Lambda X)} \approx 0.42 ,$$
$$\frac{P(pp \to nX)}{P(pp \to \Lambda X)} \approx 0.55 .$$

We plan to construct a more specific model for the entire process and see if the magnitude and x and  $p_{\perp}$  dependences of these polarizations can help rule out or confirm the relevance of the basic mechanism of fig. 1.

Also interesting is the possibility of inclusive polarization measurements in leptoproduction reactions, with polarized beams. In the appropriate x regions the same gluon mechanism could dominate and allow detailed study of quark-gluon couplings.

# 5. Conclusions

We have argued for the point of view that much may be learned about gluon interactions from hadron data. While this is admittedly optimistic, some of the arguments are strong enough to take seriously, and the increased knowledge available if our approach were correct seems more than sufficient to justify careful consideration of the ideas.

We have suggested that the physical origin of the rise of  $\sigma_T$  is gluon interactions, and that detailed properties of the rise can be related to basic aspects of QCD. Similar results hold for central region clusters. We believe that QCD predicts that particle multiplicities grow as a power of s at large s rather than as ln s; this may explain effects in cosmic ray data. We predict an important feature of particle ratios at higher energies will be rapid approach to unity over the region in x where the gluon distribution function dominates; if this is observed it may allow the experimental determination of the gluon distribution functions. Since gluons do not interact directly with leptons or photons this may be the most direct way to measure the distribution function.

We also suggest that gluon jets will not be hard like quark jets, but will convert most of their energy into hadron masses, giving high-multiplicity, flavor-neutral, slowly moving groups of hadrons. Our predictions are summarized in table 1.

Even if not all of our speculations are borne out, we hope that the possibility of additional experimental tests of QCD is sufficient justification for pursuing these questions.

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