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ψ Phenomenology and the Nature of Quark Confinement*

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The nonrelativistic Schrödinger equation is used with confinement potentials that are either fractional power laws or logarithmic functions of the radial coordinate to investigate the spectrum of states, leptonic decay widths, and radiative decays of the ψ family of resonances. The spectrum of states and the leptonic decay widths are in good agreement with the data for the entire class of potentials considered here. The radiative decays are still somewhat large for the standard model but could be brought into agreement with the data when threshold and relativistic effects are taken into account.

1. INTRODUCTION

During the last decade an exciting picture of elementary particle interactions has emerged in which the four or more basic quark constituents interact via a non-Abelian gauge theory. In the standard theory the quarks are fractionally charged. In order to reconcile both fermion properties for the quarks and spatially symmetric wave-functions for the nucleons the quarks are postulated to carry an additional quantum number, color. Each quark type comes in three colors. All observed particles are color singlets. Color is an exactly conserved symmetry mediated by massless vector gluons. However, quarks and gluons are permanently confined. The confinement potential between quarks is usually abstracted to be linear from lattice gauge theories. It is important to remember, however, that at this time the nature of the confinement potential and even the idea of confinement itself are hypotheses, not results, of the theory.

The discovery [1] nearly 2 years ago of narrow, heavy resonances at the Stanford Linear Accelerator Center (SLAC) and at Brookhaven National Laboratory and the subsequent work on the spectroscopy of the new particle family at SLAC and elsewhere has caused great excitement because it is viewed as experimental verification of the above theoretical picture. The first attempt at dealing with the data used a Coulomb potential between quarks [2]. This would be expected up to logarithmic corrections for acymptotic freedom if the short distance behavior of the interaction dominated the above phenomena. Quantitatively the Coulomb potential alone did

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not work well. This indicated that one was sampling primarily the long distance, confinement behavior between quarks. Both linear confining potential [3, 4], or linear potential with a small short distance Coulomb part, and harmonic oscillator [5] potentials have been used as candidates for the confinement potential. Qualitatively the picture works amazingly well. Again qualitatively, however, there are two major discrepancies. First as discussed in [6] the leptonic decay widths of the ψ family particles decrease rapidly with increasing energy [7]. This means that the wave-function squared at the origin, $|\psi(0)|^2$, is depleted rapidly for higher s-wave excitations. The nonrelativistic linear potential model predicts that $|\psi(0)|^2$ should be constant. The harmonic oscillator potential [5] requires elaborate s–d mixing to prevent $|\psi(0)|^2$ from increasing rapidly for the observed states but still has difficulty by predicting a large width for the unobserved partner to the narrow s–d mixed state at 4.4 GeV. Second, the radiative decays from $\psi'(3.7)$ to the p states near 3.5 GeV were an order of magnitude smaller than predicted by theory. A more sophisticated treatment [4] using a linear potential and taking into account the effect on the bound states of the opening of a threshold for charmed meson production reduced the radiative decays by a factor of 3, and brought them to within a factor of 2 of the experimental values. This treatment however, actually increased the leptonic width for the ψ' from the naive theoretical value of 3.4 to 3.6–4.1 keV rather than reducing it to its experimental value.

We suggest the possiblity that the rapid decrease of the leptonic widths for the ψ family of resonances indicates a confining potential which is weaker than a linear potential. In this paper we will investigate within the context of nonrelativistic potential models confinement potentials which are either a fractional power of the radial coordinate r or behave as logatihmic functions of r with emphasis on the ψ family spectrum, leptonic decay widths, and radiative decays of the ψ' to the p state near 3.5 GeV and of the p states to the ground state $\psi/J(3.1)$. In Section 2 the general procedure for this investigation is discussed. In Section 3 the results for various potentials are tabulated and discussed.

2. Procedure

The following potentials are used to analyze the ψ family data.

(i) $V = Ar^{0.1} + B$;

(ii) $V = A \ln r + B$;

(iii) $V = A \ln (1 + r) + B$;

(iv) $V = A[\ln(1 + r)]^{1/2} + B$;

(v) $A[\ln(1 + 0.2r^2)]^{1/2} + B$.

The inclusion of a small Coulomb part attributable to asymptotic freedom is also considered for cases (ii) and (iii). The eigenvalue spectrum and wavefunctions are determined by straightforward numerical integration of the Schrödinger equation for each potential. The constant A in each of the above potentials is adjusted to fit the energy difference between $\psi/J(3.1)$ and $\psi'(3.7)$ and the constant B is adjusted to fix the absolute magnitude of the ground state at 3.1 GeV. The rest of the S-wave spectrum and all the p- and d-wave states are predictions of the model. The leptonic decay widths for each of the resonances are calculated using the Van Royan and Weisskopf formula [8]

$$\Gamma(V \to e^+e^-) = 16\pi\alpha^2 \left[\sum a_i \frac{e_i}{e}\right]^2 |\psi_V(0)|^2 \frac{1}{m_V{}^2} \tag{1}$$

where a_i and e_i are the Clebsch–Gordan coefficient and charge of the ith constituent quark of the resonance V, α is the fine structure constant, e is the electron charge, $\psi_V(0)$ is the wavefunction at the origin for the meson, and m_V is the meson mass. The sum over i includes only quark flavors. Color has already been summed. The mass of the final state leptons is neglected. The value of the bound state wavefunction at the origin is evaluated using

$$|\psi_V(0)|^2 = (1/4\pi)[u'(0)]^2 \tag{2}$$

where prime denotes differentiation and $u(r)$ is the solution to the radial Schrödinger equation

$$[-(1/2m)(d^2/dr^2) + V(r)]\, u(r) = Eu(r). \tag{3}$$

m is the reduced mass of the quark–antiquark system in Eq. (3). Given that the complete radial wavefunction $R = u(r)/r$ it is well known that

$$\frac{1}{2m}[u'(0)]^2 = \int_0^\infty R^2 \frac{dV}{dr} r^2\, dr = \int_0^\infty u^2(r) \frac{dV}{dr}\, dr. \tag{4}$$

The value of $u'(0)$ was calculated both by using the knowledge of the wavefunction with the definition of the derivative and by numerical integration of Eq. (4). In all cases the results agreed very well which served as an internal check on the consistency of the numerical results. Using the quark charge possibilities $+\frac{2}{3}$ or $-\frac{1}{3}$ and Eq. (1) the reduced mass of the quark–antiquark system was varied in the computer calculation in order to fix the absolute magnitude of the leptonic decay width for $\psi/J(3.1)$. All of the rest of the leptonic decay widths are then predictions of the model.

The radiative decays are then calculated in the usual way using

$$\Gamma(\psi' \to {}^3P_J + \gamma) = \frac{4}{3}\alpha\omega^3 \sum_i \left[a_i \frac{e_i}{e}\right]^2 \frac{2J+1}{9} \int_0^\infty R_{nl} R_{n'l'} r^3\, dr \tag{5}$$

where ω is the energy of the emitted γ ray, J is the total angular momentum of the final P state, and R_{nl}, $R_{n'l'}$ are the radial wavefunctions for the ψ', ${}^3P_J$ states, respectively. The other symbols are defined as in Eq. (1). The radial integral in Eq. (5) is calculated numerically and thus must be an underestimate due to the truncation of

TABLE I

Energy Spectrum of States (GeV)[a]

Case (i)	$V = 6.55\, r^{0.1} - 4.24$,		$q = \frac{2}{3}$	$m = 0.60$			
s	3.10	3.68	4.03	4.27	4.47		
p	3.52	3.91	4.18	4.39	4.58		
d	3.80	4.10	4.32	4.51			
	$V = 6.85\, r^{0.1} - 4.24$,		$q = -\frac{1}{3}$,	$m = 1.46$			
s	3.10	3.69	4.03	4.27	4.47		
p	3.52	3.92	4.19	4.39	4.56		
d	3.80	4.10	4.32	4.50	4.65		
Case (ii)	$V = 0.73 \ln r + 2.26$,		$q = \frac{2}{3}$,	$m = 0.55$			
s	3.10	3.69	4.01	4.24	4.43		
p	3.54	3.91	4.16	4.35			
d	3.81	4.08	4.29	4.47			
	$V = 0.73 \ln r + 2.60$,		$q = -\frac{1}{3}$	$m = 1.40$			
s	3.10	3.69	4.01	4.24	4.41		
p	3.54	3.91	4.16	3.54	4.49		
d	3.81	4.08	4.28	4.44	4.57		
Case (iii)	$V = 1.0 \ln(1 + r) + 1.71$		$q = \frac{2}{3}$	$m = 0.70$			
s	3.1	3.68	4.04	4.30	4.51	4.69	
p	3.50	3.91	4.20	4.43	4.61		
d	3.79	4.10	4.35	4.54	4.72		
	$V = 1.16 \ln(1 + r) + 1.86$,		$q = -\frac{1}{3}$,	$m = 1.80$			
s	3.10	3.69	4.06	4.34	4.56	4.75	4.91
p	3.49	3.92	4.23	4.47	4.67	4.84	5.00
d	3.78	4.12	4.38	4.60	4.77	4.93	5.07
Case (iv)	$V = 2.32[\ln(1 + r)]^{1/2} + 0.36$,		$q = \frac{2}{3}$,	$m = 0.60$			
s	3.10	3.69	4.00	4.21	4.39		
p	3.54	3.90	4.14	4.32			
d	3.81	4.07	4.26	4.43			
	$V = 2.35[\ln(1 + r)]^{1/2} + 0.66$		$q = -\frac{1}{3}$	$m = 1.50$			
s	3.10	3.69	4.02	4.24	4.40	4.53	4.64
p	3.53	3.91	4.16	4.34	4.48	4.60	
d	3.81	4.08	4.28	4.44	4.56	4.67	

Table continued

TABLE I (*Continued*)

Case (v)	$V = 1.24[\ln(1 + 0.2r^2)]^{1/2} + 1.86,\quad q = \frac{2}{3},\quad m = 0.70$							
s	3.10	3.69	4.02	4.23	4.39			
p	3.52	3.91	4.15	4.33	4.48			
d	3.80	4.08	4.27	4.42	4.58			
	$V = 1.38[\ln(1 + 0.2r^2] + 2.07\quad q = -\frac{1}{3}\quad m = 1.80$							
s	3.10	3.69	4.05	4.30	4.48	4.63	4.75	4.85
p	3.49	3.92	4.20	4.41	4.57	4.71	4.85	4.90
d	3.78	4.10	4.34	4.51	4.65	4.77	4.87	4.96
Case (ii)′	$V = 0.685 \ln r - 0.1/r + 2.31\quad q = \frac{2}{3}\quad m = 0.50$							
s	3.10	3.68	3.99	4.21	4.41	4.65		
p	3.53	3.89	4.13	4.33	4.54			
d	3.80	4.06	4.26	4.45				
	$V = 0.685 \ln r - 0.1/r + 2.65\quad q = -\frac{1}{3}, m = 1.20$							
s	3.10	3.70	4.02	4.23	4.40	4.53	4.65	
q	3.56	3.92	4.16	4.34	4.48	4.61		
d	3.83	4.09	4.29	4.44	4.56			
Case (ii)″	$V = 0.645 \ln r - 0.2/r + 2.40,\quad q = \frac{2}{3}\quad m = 0.50$							
s	3.10	3.68	3.98	4.19	4.39			
p	3.54	3.89	4.12	4.31				
d	3.80	4.05	4.24	4.44				
	$V = 0.62 \ln r - 0.2/r + 2.72,\quad q = -\frac{1}{3}\quad m = 1.10$							
s	3.10	3.68	3.99	4.19	4.34	4.47		
p	3.56	3.90	4.13	4.29	4.43			
d	3.82	4.06	4.24	4.38				
Case (iii)′	$V = 0.93 \ln(1 + r) - 0.1/r + 1.80\quad q = \frac{2}{3}\quad m = 0.60$							
s	3.10	3.69	4.03	4.29	4.49			
p	3.51	3.91	4.19	4.41	4.60			
d	3.79	4.10	4.33	4.52				
	$V = 1.04 \ln(1 + r) - 0.1/r + 2.00,\quad q = -\frac{1}{3}\quad m = 1.50$							
s	3.10	3.69	4.05	4.32	4.53	4.70	4.85	4.98
p	3.51	3.93	4.22	4.55	4.63	4.79	4.92	5.05
d	3.80	4.12	4.36	4.56	4.73	4.87	5.09	

[a] q and $2m$ are the charge and the mass of the quark with the new quantum number, respectively.

TABLE II

Leptonic Decay Widths (keV)

	E	Γ_{ee}	E	Γ_{ee}
(i)	$V=6.55r^{0.1}-4.24, q=\frac{2}{3}, m=0.60$		$V=6.85r^{0.1}-4.24, q=-\frac{1}{3}, m=1.46$	
	3.10	5.02	3.10	4.76
	3.68	1.99	3.69	1.88
	4.03	1.18	4.03	1.12
	4.27	0.84	4.27	0.79
	4.47	0.67	4.47	0.60
(ii)	$V=0.73 \ln r+2.26, q=\frac{2}{3}, m=0.55$		$V=0.73 \ln r+2.60, q=-\frac{1}{3}, m=1.40$	
	3.10	4.90	3.10	4.98
	3.69	1.76	3.69	1.79
	4.01	1.02	4.01	1.03
	4.24	0.71	4.24	0.71
	4.43	0.60	4.41	0.54
(iii)	$V=1.0 \ln(1+r)+1.71, q=\frac{2}{3}, m=0.70$		$V=1.16 \ln(1+r)+1.86, q=-\frac{1}{3}, m=1.80$	
	3.10	4.98	3.10	4.78
	3.68	2.24	3.69	2.28
	4.04	1.39	4.06	1.47
	4.30	0.99	4.34	1.06
	4.51	0.76	4.56	0.82
(iv)	$V=2.32[\ln(1+r)]^{1/2}+0.36, q=\frac{2}{3}, m=0.60$		$V=2.35[\ln(1+r)]^{1/2}+0.66, q=-\frac{1}{3}, m=1.50$	
	3.10	5.26	3.10	4.90
	3.69	1.88	3.69	1.84
	4.00	1.06	4.02	1.05
	4.21	0.73	4.24	0.71
	4.39	0.60	4.40	0.53
(v)	$V=1.24[\ln(1+0.2r^2)]^{1/2}+1.86, q=\frac{2}{3}, m=0.70$		$V=1.38[\ln(1+0.2r^2)]^{1/2}+2.07, q=-\frac{1}{3}, m=1.80$	
	3.10	5.08	3.10	4.36
	3.69	2.05	3.69	2.14
	4.02	1.15	4.05	1.29
	4.23	0.77	4.30	0.88
	4.39	0.60	4.48	0.65
(ii′)	$V=0.685 \ln r-0.1/r+2.31, q=\frac{2}{3}, m=0.50$		$V=0.685 \ln r-0.1/r+2.65, q=-\frac{1}{3}, m=1.20$	
	3.10	4.63	3.1	4.73
	3.68	1.60	3.7	1.58
	3.99	0.92	4.02	0.89
	4.21	0.65	4.23	0.61
			4.40	0.45

Table continued

TABLE II (*Continued*)

(ii″) $V=0.645\ln r-0.2/r+2.40$, $q=\frac{2}{3}$, $m=0.50$		$V=0.62\ln r-0.2/r+2.72$, $q=-\frac{1}{3}$, $m=1.10$	
3.10	5.11	3.10	4.70
3.68	1.69	3.68	1.49
3.98	0.95	3.99	0.82
4.19	0.67	4.19	0.55
		4.34	0.41
(iii′) $V=0.93\ln(1+r)-0.1/r+1.80$, $q=\frac{2}{3}$, $m=0.60$		$V=1.04\ln(1+r)-0.1/r+2.00$, $q=-\frac{1}{3}$, $m=1.50$	
3.10	4.57	3.10	4.61
3.69	1.93	3.69	2.00
4.03	1.18	4.05	1.24
4.29	0.83	4.32	0.93
4.49	0.66	4.53	0.67

TABLE III

Radiative Decay Widths (keV)

	V (potential)	q	m	$\Gamma(\psi'(3.7)\to{}^3P_1(3.5)+\gamma)$	$\psi({}^3P_1(3.5)\to\psi(3.1)+\gamma)$
(i)	$6.55r^{0.1}-4.24$	$\frac{2}{3}$	0.60	75.4	506
	$6.85r^{0.1}-4.24$	$-\frac{1}{3}$	1.46	7.9	52
(ii)	$0.73\ln r+2.26$	$\frac{2}{3}$	0.55	84.9	530
	$0.73\ln r+2.60$	$-\frac{1}{3}$	1.40	8.7	52
(iii)	$1.0\ln(1+r)+1.71$	$\frac{2}{3}$	0.70	60.3	471
	$1.16\ln(1+r)+1.86$	$-\frac{1}{3}$	1.80	5.6	47
(iv)	$2.32[\ln(1+r)]^{1/2}+0.36$	$\frac{2}{3}$	0.60	80.6	489
	$2.35[\ln(1+r)]^{1/2}+0.66$	$-\frac{1}{3}$	1.50	7.8	50
(v)	$1.24[\ln(1+0.2r^2)]^{1/2}+1.86$	$\frac{2}{3}$	0.70	61.3	457
	$1.38[\ln(1+0.2r^2)]^{1/2}+2.07$	$-\frac{1}{3}$	1.80	5.4	48
(ii′)	$0.685\ln r-0.1/r+2.31$	$\frac{2}{3}$	0.50	101.9	571
	$0.685\ln r-0.1/r+2.65$	$-\frac{1}{3}$	1.20	10.5	57
(ii″)	$0.645\ln r-0.2/r+2.40$	$\frac{2}{3}$	0.50	106	555
	$0.62\ln r-0.2/r+2.72$	$-\frac{1}{3}$	1.10	12	60
(iii′)	$0.93\ln(1+r)-0.1/r+1.80$	$\frac{2}{3}$	0.60	73.7	529
	$1.04\ln(1+r)-0.1/r+2.00$	$-\frac{1}{3}$	1.50	7.2	53

the upper limit of integration. The numerical integration was carried out far enough, however, so that we feel the error due to this truncation is small. The decay rate for $^3P_J \to \psi/J(3.1) + \gamma$ is given by

$$\Gamma(^3P_J \to \psi/J(3.1) + \gamma) = \frac{4}{9}\, \alpha\omega^3 \sum_i \left[a_i \frac{e_i}{e}\right]^2 \int_0^\infty R_{nl} R_{n'l'} r^3\, dr \tag{6}$$

where all the symbols are defined as in Eq. (5). All parameters in the model are fixed by the energies of $\psi/J(3.1)$ and $\psi'(3.7)$ and the leptonic width of $\psi/J(3.1)$ so that all the radiative decays are true predictions of the model.

3. Results

The results for the potentials listed in Section 2 are given in Tables I, II, and III. In Table I the s-, p-, and d-wave spectrum for each potential is listed. The spectrum is itself not very sensitive to the type of confinement potential provided that it belongs to this general class of weak confinement potentials. The s-wave spectrum is very encouraging since in addition to the $\psi/J(3.1)$ and $\psi'(3.7)$, two states generally fall in the experimentally congested 3.9- to 4.3-GeV region and there is a candidate for the 4.4-GeV resonance. The lowest p-wave states for these classes of potentials lie in the 3.51–3.56-GeV range, slightly higher than the 3.5-GeV experimental value. Potential (iii) is somewhat better than the others in this respect. Table II lists the leptonic widths for the s-wave states for each potential. In each case the leptonic width drops very rapidly with increasing energy. Again potential (iii) agrees the best with the experimental data. The leptonic width for $\psi'(3.7)$ tends to be a little low using the other potentials. This is not, however, sufficient to rule out these potential forms as possible confinement potentials since threshold corrections could bring them into better agreement with the data. Table II only serves to indicate that this general type of potential is very promising. Figure 1 depicts the energy dependence of $\Gamma(\psi \to l^+l^-)\, M_\psi^2$ for the potentials considered in this text, while Fig. 2 shows the slow variation of the same quantity for linear plus various small Coulomb potentials. In Table III the radiative decay widths of $\psi'(3.7) \to {}^3P_1(3.5) + \gamma$ and ${}^3P_1(3.5) \to \psi/J(3.1) + \gamma$ are listed for all cases. The other radiative decays are easily calculated from these using Eqs. (5) and (6). We find that the radiative decays for the standard model with quark charge $+\frac{2}{3}$ and effective quark mass between 1 and 1.4 GeV are still a factor 3 or more too large. This is not, however, incompatible with experiment when threshold and recoil effects are considered. The effect of opening charmed meson production channels reduced some of the radiative decays by as much as a factor of 3 in [4]. Also recent work [9] on relativistic recoil corrections indicates that these effects reduce the radiative width by a factor of 2–3. Thus the corrected results should agree well with the experimental data. The alternative model in which the constituent quark has charge $q = -\frac{1}{3}$ and mass about 3 GeV gives uncorrected widths that are already too small and so is incompatible with the present data. This latter model is also unlikely since the recently discovered charmed mesons [10] in e^+e^- annihilation favor $q = +\frac{2}{3}$ for the new qurk charge. It is interesting to note that the effect of introducing a small Coulomb potential in addition to the confinement potential as in cases (ii) and (iii) has the effect of reducing the effective quark mass required to fit the data.

In conclusion the weak confinement potentials studied in this paper appear very promising. It is easy to fit the spectrum of observed states with two s-wave states

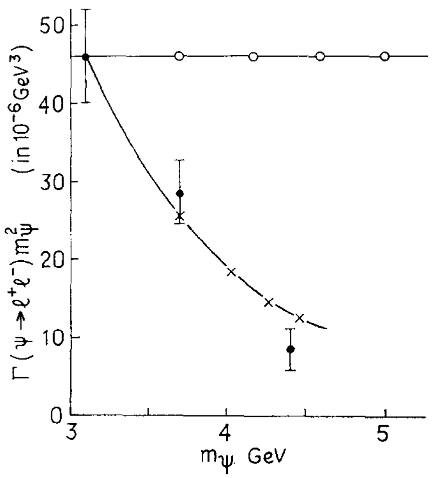

FIGURE 1(i)

FIGS. 1(i)–(v). Energy dependence of $|\psi_S(0)|^2$ for a class of potentials considered in the text. The horizontal line is for a linear potential.

		V	q	m
(i)		$6.55r^{0.1} - 4.24$	$\frac{2}{3}$	0.60
		$6.85r^{0.1} - 4.24$	$-\frac{1}{3}$	1.46
(ii)		$0.73 \ln r + 2.26$	$\frac{2}{3}$	0.55
		$0.73 \ln r + 2.60$	$-\frac{1}{3}$	1.40
(ii′)	□	$0.685 \ln r - 0.1/r + 2.31$	$\frac{2}{3}$	0.50
	×	$0.685 \ln r - 0.1/r + 2.65$	$-\frac{1}{3}$	1.20
(ii″)	□	$0.645 \ln r - 0.2/r + 2.40$	$\frac{2}{3}$	0.50
	×	$0.62 \ln r - 0.2/r + 2.72$	$-\frac{1}{3}$	1.10
(iii)	□	$1.0 \ln(1 + r) + 1.71$	$\frac{2}{3}$	0.70
	×	$1.16 \ln(1 + r) + 1.81$	$-\frac{2}{3}$	1.80
(iii′)	□	$0.93 \ln(1 + r) - 0.1/r + 1.80$	$\frac{2}{3}$	0.60
	×	$1.04 \ln(1 + r) - 0.1/r + 2.00$	$-\frac{1}{3}$	1.50
(iv)	□	$2.32[\ln(1 + r)]^{1/2} + 0.36$	$\frac{2}{3}$	0.60
	×	$2.35[\ln(1 + r)]^{1/2} + 0.66$	$-\frac{1}{3}$	1.50
(v)	□	$1.24[\ln(1 + 0.2r^2)]^{1/2} + 1.86$	$\frac{2}{3}$	0.70
	×	$1.38[\ln(1 + 0.2r^2)]^{1/2} + 2.07$	$-\frac{1}{3}$	1.80

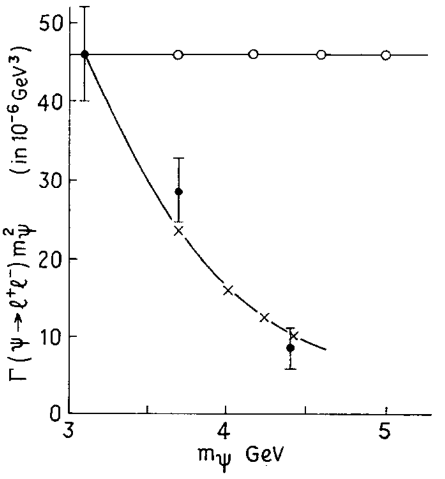

FIGURE 1(ii)

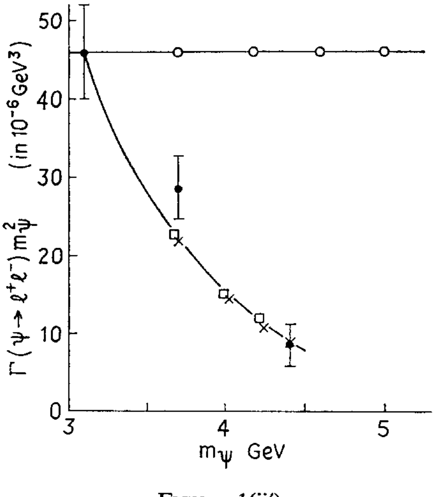

FIGURE 1(ii′)

falling in the 3.9- to 4.3-GeV region and p-wave states falling near 3.5 GeV. The leptonic widths predicted by these potentials fall rapidly with increasing energy in agreement with the experimental data. The radiative decays are still too large for the standard model with quark charge $+\frac{2}{3}$ but should come into good agreement with the data when threshold effects and relativistic recoil corrections are taken into

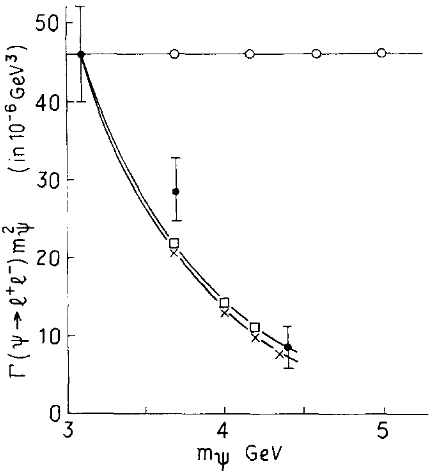

FIGURE 1(ii″)

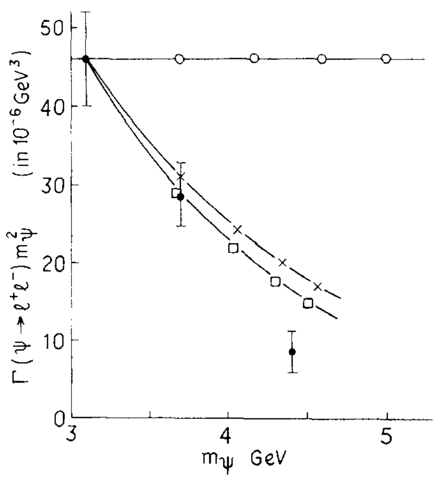

FIGURE 1(iii)

account. A model with larger quark mass and charge $-\frac{1}{3}$ does not fit the present radiative decay data. The experimental data and the simple nonrelativistic approach used here are not sensitive enough to determine the precise radial dependence of the confinement potential. They do seem to indicate, however, that a low power law or logarithmic form is favored over previous treatments. It would be interesting to see

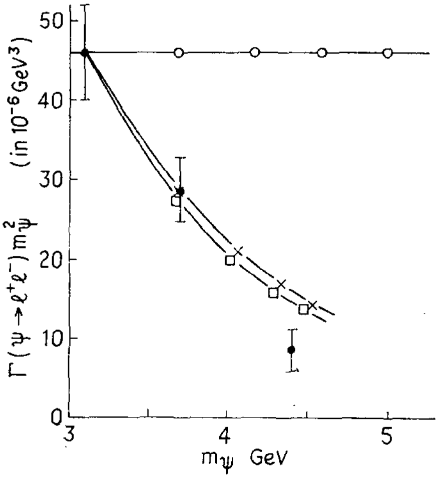

FIGURE 1(iii')

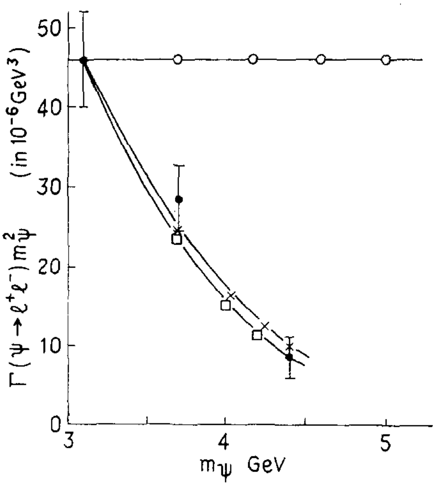

FIGURE 1(iv)

if such a logarithmic form could be derived from non-Abelian gauge theory calculations.

One of us (M. M.) would like to thank J. Sanchez-Guillen for helpful discussions on the effects of relativistic corrections to this approach.

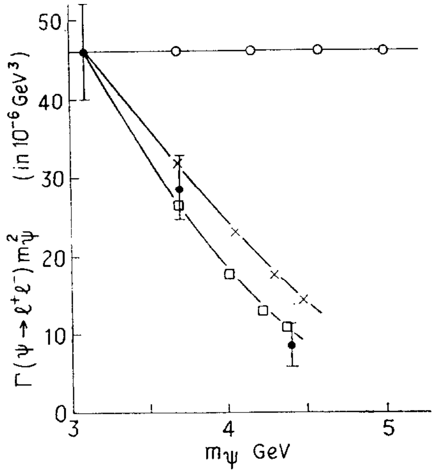

FIGURE 1(v)

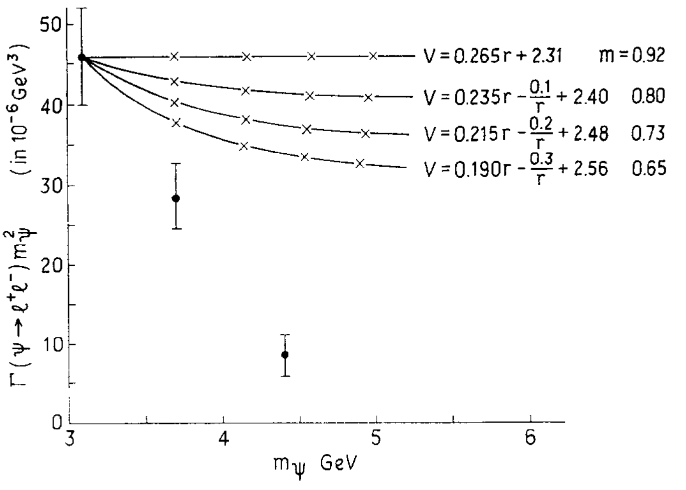

FIG. 2. Energy dependence of $|\psi_S(0)|^2$ for a linear potential and a linear potential plus Coulomb potential. The crosses indicate the energy spectra and the dots with error bars are the experimental points. m stands for the reduced mass of the quark.

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