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ON "THE TANTALIZER" AND "INSTANT INSANITY"

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The object of this note is to clarify the recent history of a frequently published procedure which enables one to solve an intriguing puzzle by means of a graph theoretic technique. Four small cubes of the same size are given, each face having one of four colors. The object is to pile the cubes so that the four different colors are showing on each of the four sides of the pile.

The story started in the 1940's when four Cambridge undergraduates adopted the two pseudonyms of Blanche Descartes and F. de Carteblanche (purportedly Blanche's husband) for the purpose of publishing light and entertaining mathematical notes and poems. The puzzle, attracting considerable attention in Great Britain at that time, was called "The Tantalizer." The more recent "Instant Insanity" puzzle is equivalent, but the four cubes are not colored in the same way.

F. de Carteblanche [1947] published the optimal canonical graph theoretic procedure for solving all such puzzles in the undergraduate mathematics magazine of Cambridge University. Tracing further references in chronological order, Busacker and Saaty [1965, 153] included this solution in their book, as an interesting application of graph theory to puzzles. Brown [1968] worked out an algorithm which did not utilize graph theory explicitly, and Schwartz [1970] improved Brown's approach, also without mentioning graphs. In the meantime, Van Deventer [1970], at a graph theory conference in Kalamazoo, gave an expository talk in which she explained the presentation in Busacker and Saaty [1965]. The most recent contribution to this series by Grecos and Gibberd [1971] is apparently the only independent published rediscovery of the graph theoretic technique of de Carteblanche. In a note added in proof, they acknowledge the priority of Van Deventer [1969] who borrowed it from Busacker and Saaty [1965] who based it on [Carteblanche 1947].

Incidentally, using standard methods of graphical enumeration [Harary and Palmer 1973], it is not difficult to develop a formula for the number of different "Tantalizer" games with four cubes (or for that matter with any number of them). This can be done by associating a graph with each such game, and then counting such graphs. These graphs G have four points labeled according to the four colors, and twelve lines marked with the numbers 1, 2, 3, and 4 showing which of the four colored cubes have corresponding colors on opposite pairs of faces. Thus both loops and multiple lines can occur: a loop

indicates an opposite pair of faces of a cube having the same color while multiple lines show that two cubes have the same pair of colors appearing on opposite faces.

We conclude by describing briefly the method of Leonard Brooks, Cedric Smith, Arthur Stone, and William Tutte [Carteblanche 1947]. Using the graph theoretic terminology of [Harary 1969], let G be the "pseudograph" (in which both loops and multiple lines are admissible) obtained as above from a given set of four colored cubes. One then inspects G to find two line-disjoint 2-factors (spanning regular subgraphs of degree 2), each of which uses lines marked 1, 2, 3, and 4. Any two such 2-factors give all the information required to align the four cubes properly without having to resort to trial and error. If the four cubes have been fiendishly colored so that there do not exist such 2-factors, then the puzzle has no solution.

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