

## USE OF WEIGHTED LEAST SQUARES IN RELATING COMPONENT OUTAGE RATES TO AGE OF VEHICLE†

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**Abstract**—A field survey was conducted to measure the outage rates of various vehicle safety components. The population of vehicles sampled had been subject to a random checklane vehicle inspection program which annually inspected about 6% of the vehicles in the state. A recently developed unified approach to the analysis of multivariate categorical data was utilized to relate the various component outage rates to the age of the vehicle. A method is presented for utilizing commonly available least squares regression programs to accomplish the weighed least squares computations. The results relate component outage rates to the age of the vehicle and have implications for vehicle inspection programs.

### INTRODUCTION

During the summer of 1975, the Highway Safety Research Institute (HSRI) of the University of Michigan in conjunction with the Michigan State Police and the Michigan Office of Highway Safety Planning conducted a field survey as the first part of a project to evaluate the Michigan Checklane Motor Vehicle Inspection Program. The aim of this initial survey was to measure the outage rates for vehicle safety components. The sample of vehicles was obtained from Monroe and Jackson Counties in southeastern Michigan. Sites were chosen and a randomized rotating schedule established to allow for visits to each site at different times of day and days of the week. At each site a systematic sample with a random start was used to select vehicles from the traffic stream for inspection. Further details may be found in Flora, Corn and Copp [1976].

A total of twenty-three components was inspected for each vehicle. These have been grouped and are summarized into five component categories. The proportion of vehicles which passed all components in the inspection is also reported as the "total vehicle", and is the data used for detailed illustration of the procedures and methods. The other components reported here are: brakes, lights, exhaust, tires and steering.

Some explanation of each component is in order. The brakes were tested using a moving stopping test in which a Michigan State Police trooper accelerated the vehicle to twenty miles per hour and then attempted to stop it in a specified lane which was twenty-five feet long and ten feet wide. A comparison of this moving stopping test with a wheel pull inspection is reported in Corn, Landis and Flora [1977]. The vehicle was classified as unsafe if any of the following conditions was observed: (1) the vehicle failed to stop, (2) it pulled to one side, (3) there was a metal-on metal sound from the brakes, or (4) the brake pedal pressure was not within safe bounds. The lights were classified as unsafe if any light component—headlights, direction indicator lights, tail lights, or brake lights—failed to operate, or if the headlight output was too low or seriously out of proper alignment. The exhaust component was classed as

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unsafe if there was excessive smoke (rare) or if it produced excessive noise (indicating possible holes or leakage). Tires were regarded as unsafe if wear bar indicators were visible in two adjacent grooves of the tread, if tread depth was less than two thirty-seconds of one inch, or if there was evidence of tread or side body separation (bulges, breaks, etc.). Unsafe tires for a reason other than insufficient tread were rare. Steering was classed as unsafe if there was excessive play or wear in the linkage, or if excessive force was needed to turn the steering wheel. The total vehicle passed the inspection only if all components were in satisfactory operating condition and if in addition, the vehicle registration, operator's license, and insurance coverages were all in order.

There are two reasons for relating the outage rates for various vehicle safety components to the age of the vehicle. First, this gives useful information about the age at which a vehicle is likely to have a particular component malfunctioning. This could be used to suggest at which ages inspections of vehicles should be done or at which periods specific components should be inspected. Secondly, if comparisons of vehicles in different jurisdictions are to be made to determine the effectiveness of different inspection programs, the fact that the outage rates depend on the age of vehicles implies that age distributions must be similar or an adjustment for the age of the vehicle must be incorporated into the comparisons.

#### METHODOLOGY

The proportion of vehicles with outages of each type of safety component may vary with age and is subject to sampling fluctuations, so it may be advantageous to use a statistical model to investigate the relationship of vehicle age to outage rate. The modelling should result in some smoothing of the data and would generally result in lower standard errors than would be estimated from the individual proportions. Since the variable of interest is a rate—the proportion of vehicles with a particular type of outage—the variance is not constant, but depends on the true outage rate and the number of vehicles observed at that age.

Ordinary linear regression assumes that the errors are independent, have constant variance, zero mean, and follow the normal distribution. In the present situation, the assumption of equal variance is known to be violated, but the other assumptions may be reasonably valid. When the assumption of equal variances is violated, weighted least squares estimation may be used as preferable to ordinary least squares. If the variances are unequal, but known, the weighted least squares procedure produces estimates with the usual optimality properties: minimum variance linear unbiased.

In accident data, analyses are often concerned with proportions. In modeling these proportions, the estimation methods of maximum likelihood, minimum chi-squared, or modified-minimum chi-squared are often used. Under the hypothesis that the model is correct, these three methods of estimation are asymptotically equivalent. The method of modified-minimum chi-squared is algebraically equivalent to using weighted least squares [Bhapkar, 1966], and is computationally simplest. Weighted least squares procedures for quite general analyses may be implemented using the recently developed program GENCAT discussed in Landis *et al.* [1976]. However, since this program may be unfamiliar or unavailable to researchers in the accident investigation field, a method for performing the calculations using ordinary regression programs such as those found in common software packages such as SAS, BMD, OSIRIS, or SPSS may be useful.

The method to be described is applicable in the important special case when the variable of interest is a proportion. The proportions are estimated independently in a number of sub-groups, and the aim of the analysis is to study how these proportions are related to variables describing the sub-groups.

To fix the ideas and methods, consider the total vehicle failure rate as an example. The results of the inspection for the specific components are presented later. The data have the form of Table I, which gives the number of vehicles of each age which passed or failed the inspection. The proportion of vehicles failing the inspection is the variable of interest.

Let  $\pi_i$  be the probability that a vehicle of age  $i$  will fail the inspection. Assume that  $n_i$  vehicles of age  $i$  are inspected and  $r_i$  of these fail. (In general the subscript  $i$  could refer to any sub-groups of interest.) Assuming that the vehicles within a group are independent,  $r_i$  will be a

Table 1. Overall results of the vehicle inspection

Year	Age	Pass	Fail	Total
1976	0	27	0	27
1975	1	1151	148	1299
1974	2	1406	484	1890
1973	3	1241	757	1998
1972	4	862	850	1712
1971	5	479	690	1169
1970	6	358	623	981
1969	7	270	753	1023
1968	8	149	599	748
1967	9	86	466	552
1966	10	44	341	385
1965	11	28	242	270
1964	12	18	126	144
1963	13	7	46	53
1962	14	4	38	42
1961	15	0	12	12
1960	16	0	10	10

binomially distributed random variable with parameters  $n_i$  and  $\pi_i$ . One would estimate the  $\pi_i$  by the sample proportions

$$P_i = r_i/n_i.$$

The variance of  $P_i$  would be

$$\sigma_i^2 = \frac{\pi_i(1 - \pi_i)}{n_i},$$

and would be estimated by

$$\hat{\sigma}_i^2 = \frac{P_i(1 - P_i)}{n_i}.$$

Suppose that the relation between the inspection failure rate and the age of the vehicle is to be described by a quadratic model in the age of the vehicle. Then the model may be written

$$P_i = \beta_0 + \beta_1 i + \beta_2 i^2 + \epsilon_i, \tag{1}$$

where  $P_i$  is the observed proportion failing of age  $i$ ,  $\beta_0$  is a constant,  $\beta_1$  is the coefficient of the linear term,  $\beta_2$  is the coefficient of the quadratic term, and  $\epsilon_i$  is the error. The assumption is that

$$\pi_i = \beta_0 + \beta_1 i + \beta_2 i^2,$$

that is, that the true probability of failing inspection is a quadratic function of age. The errors are

$$\begin{aligned} \epsilon_i &= P_i - E[P_i] \\ &= P_i - \pi_i. \end{aligned}$$

The  $\epsilon_i$  thus satisfy

- (1)  $E(\epsilon_i) = 0$ .
- (2)  $\text{Var}(\epsilon_i) = \pi_i(1 - \pi_i)/n_i$ .
- (3) The  $\epsilon_i$  are independent.

(4) The  $\epsilon_i$  are approximately normal for large  $n_i$  (the usual normal approximation to the binomial).

Except for (2) the  $\epsilon_i$  satisfy the usual assumptions for regression. The standard assumption would be

$$(2') \text{Var}(\epsilon_i) = \sigma^2.$$

The method proposed is based on the observation that if weights

$$W_i = [\pi_i(1 - \pi_i)/n_i]^{-1/2},$$

are defined, the weighted errors,  $W_i\epsilon_i$  would have

$$\begin{aligned} \text{Var}(W_i\epsilon_i) &= W_i^2 \text{Var}(\epsilon_i) \\ &= \frac{\pi_i(1 - \pi_i)}{n_i} \cdot \frac{n_i}{\pi_i(1 - \pi_i)} = 1, \end{aligned}$$

and so would have constant variance. Since the  $\pi_i$  are unknown, the  $P_i$  are used as estimates in the computation.

This weighted least squares estimation procedure can be computed directly using GENCAT, or by means of transformations of the data and use of an ordinary least squares regression routine.

A practical problem often arises in applying this procedure to real data, whichever method for computation is used. This problem is illustrated by the data in Table 1. Notice that for age zero, no vehicles failed, while for ages 15 and 16 no vehicles passed. As a result, the  $P_i$  for these cases are zero, one, and one, respectively. The estimated variance would be zero, and the weights could not be calculated—they would be infinite.

This difficulty arises because the weights are estimated using the  $P_i$ , rather than calculated from the  $\pi_i$ . Even with a non-zero  $\pi_i$ , a sampling zero may occur, particularly if  $n_i$  is not too large. For computational purposes a small number—some authors suggest  $\frac{1}{2}$ —is used to replace the zero cells. In the computation here, zero frequencies were replaced by  $\frac{1}{2}$ . For example, with this modification,  $P_1$  becomes 0.0182 (0.5/27.5) rather than zero. With this modification, the weights can be calculated.

Multiply both sides of (1) by  $W_i$  to get

$$W_i P_i = W_i \beta_0 + W_i \beta_1 i + W_i \beta_2 i^2 + W_i \epsilon_i, \quad (2)$$

which can be rewritten as

$$Y_i = \beta_0 X_{0i} + \beta_1 X_{1i} + \beta_2 X_{2i} + \delta_i. \quad (3)$$

In (3) the independent variables,  $X_{0i}$ ,  $X_{1i}$  and  $X_{2i}$  are transformations of the dummy variable  $X_0$  (= 1 for all  $i$ ), the age, and the age squared. The transformed error term  $\delta_i$  should nearly satisfy assumptions (1)–(4) of ordinary regression.

The model (3) can then be fit using ordinary regression on the three variables  $X_0$ ,  $X_1$  and  $X_2$ . Note that no constant term appears, so that the regression is forced through the origin. This elimination of a constant term is an option available in most modern regression programs, often by use of a statement such as `OPTION = MEANZERO` with the regression command.

A sample of the output from such a regression appears as Table 2. Some care in identification of items is required. For comparison, the same analysis was done with GENCAT and the relevant output is summarized in Table 3.

Referring to the output presented in Table 2, the estimated coefficients  $b_0$ ,  $b_1$  and  $b_2$  can be read directly from the column headed "COEFF". The coefficient for variable 9 is the intercept, and those for variables 13 and 14 are the linear and quadratic effects of age, respectively. The error sum of squares (17.037 with 14  $df$ ) is the statistic used to test for lack of fit. This appears in Table 3 as

Table 2. Output from a regression program (transformations used to obtain weighted least squares)

```

COMMAND
TREGRES M=12,9,13,14 DAT=MEANZERO,MATRIX

LEAST SQUARES REGRESSION

ANALYSIS OF VARIANCE OF 12.WTDPROEF  N= 17 OUT OF 17

SOURCE          DF  SUM SQRS  MEAN SQR  F-STAT  SIGNIF
REGRESSION      3   23533.   7844.3    6445.9   .0000
ERROR           14   17.037   1.2170
TOTAL           17   23550.

DAT:  MEANZERO  R-SQR= .9953  SE= 1.1032

VARIABLE        PARTIAL  COEFF      STD ERROR  T-STAT    SIGNIF
9.WT             -.42272   -.17649 -1  .10113 -1 -1.7453   .1028
13.WTDAGE       .99495    .14853  .40036 -2  37.099    .0000
14.WTDAGE2      -.98052   -.58800 -2  .31422 -3 -18.677    .0000

VARIANCE-COVARIANCE MATRIX OF COEFFICIENTS (FIRST 3 VARS)

VARIABLE
9.WT             .10226 -3
13.WTDAGE       -.34532 -4  .16029 -4
14.WTDAGE2      .22949 -5  -.12062 -5  .99113 -7
          9.      13.      14.
          WT      WTDAGE  WTDAGE2
    
```

CHI-SQUARE DUE TO ERROR. Note that the fit is satisfactory in this example ( $P = 0.25$ ).

Some additional calculations are needed to recover the standard errors of the coefficients from the regression output. The entries in the column headed "STD ERROR" must be divided by the entry labeled "SE" in Table 2 to obtain the "standard deviations of the estimated model parameters" presented in Table 3. For example, the standard error of the linear coefficient is  $(0.0040036/1.1032) = 0.003629$ . Similarly, the variance covariance matrix of coefficients in the regression output must be divided by the error mean square to obtain the variance covariance matrix of GENCAT. Thus the estimated variance of the constant term,  $b_0$  is  $(0.00010226/1.217) = 0.0000840$ . Finally, the chi-squared statistics for testing whether each coefficient is equal to zero can be obtained from the regression output by multiplying each entry in the column headed "T-STAT" by the entry for "SE" and then squaring the results. The  $\chi^2$  for testing  $\beta_2 = 0$  is  $[(-18.667) \times 1.1032]^2 = 424.1$  with 1 *df*. Note that there is some round off error caused by working with only five significant digits.

The reason for these differences is that the regression program assumes that the errors all have variance  $\sigma^2$ , while after the transformations (i.e. using weighted least squares)  $\sigma^2$  should be equal to one. The regression program estimates  $\sigma^2$  by the mean square for error. If the model fits,  $\sigma^2 = 1$ . Thus in the output from the regression program the covariance matrix of the estimated parameters has a factor of  $\hat{\sigma}^2$ , which should be one if the model fits.

The estimate of  $\sigma^2$  is the error sum of squares divided by its degrees of freedom. In the GENCAT output this is denoted as the  $\chi^2$  for error. If this chi-squared statistic is equal to its degrees of freedom, the estimate of  $\sigma^2$  in the regression program is one. Otherwise, even if the lack of fit test is non-significant, the estimate of  $\sigma^2$  will differ from one. Typically, it will be slightly larger than one, although occasionally values less than one will be observed.

Table 3. Summarized output from the GENCAT program

---

TOTAL CHI-SQUARE = 23550.0006  
 DF = 17  
 P = 0.0

ESTIMATED MODEL PARAMETERS:  
 -0.17649E-01    0.14852E+00    -0.58800E-02

VARIANCE-COVARIANCE MATRIX:  
 0.84333E-04    -0.28376E-04    0.18807E-05  
 -0.28376E-04    0.13171E-04    -0.99117E-06  
 0.18807E-05    -0.99117E-06    0.81445E-07

STANDARD DEVIATIONS OF THE ESTIMATED MODEL PARAMETERS:  
 0.02869E-02    0.36292E-02    0.00538E-03

CHI-SQUARE DUE TO ERROR = 17.0347    DF = 14    P = 0.2503

(Test for all three parameters)  
 CHI-SQUARE = 23532.9653    DF = 3    P = 0.0

(Testing linear and quadratic)  
 CHI-SQUARE = 5769.1845    DF = 2    P = 0.0

(Intercept)  
 CHI-SQUARE = 3.7068    DF = 1    P = 0.0542

(Linear term)  
 CHI-SQUARE = 1679.9749    DF = 1    P = 0.0

(Quadratic term)  
 CHI-SQUARE = 524.2312    DF = 1    P = 0.0

---

The sum of squares for regression is the statistic used to test the hypothesis that  $\beta_0 = \beta_1 = \beta_2 = 0$ , i.e. that all coefficients are simultaneously zero. This is produced in GENCAT by using a  $3 \times 3$  identity matrix as the contrast matrix, and the resulting statistic has 3 degrees of freedom. In this context, a measure of the variability explained could be calculated as

$$\frac{\chi^2(\text{regression})}{\chi^2(\text{regression}) + \chi^2(\text{error})} = \frac{23532.97}{23550.00} = 0.9993$$

however, the total in the denominator represents the chi-squared statistic for testing the hypothesis that all the proportions are equal to zero. A more appropriate measure of the variability explained by the model is available from the GENCAT program by using the contrast matrix

$$C = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

which gives the chi-square for testing  $\beta_1 = \beta_2 = 0$ . Then the measure

$$\frac{\chi^2(\beta_1, \beta_2)}{\chi^2(\beta_1, \beta_2) + \chi^2(\text{error})} = \frac{5769.1845}{5769.1845 + 17.0347} = 0.9971$$

may be used. This does not seem to be generally available from the regression programs.

DISCUSSION

The preceding section described a method to use ordinary least squares regression programs to perform weighted least squares computations for the important special case of data which are proportions. The output from a typical regression program was compared with that from GENCAT to indicate the identifications of the items from the regression output with the test statistics from GENCAT. In a number of instances, notably the variances, standard errors, and test statistics, some modification of the regression output was necessary to obtain the appropriate statistics.

A similar procedure could easily be applied if the data are not proportions, but have unequal variances and zero correlations. In general, if the covariance matrix is  $V$ , there exist orthogonal matrices  $P$  and  $P'$ , such that  $PVP' = I$ , the identity matrix. Then in matrix notation, the model

$$Y = XB + \epsilon,$$

where the errors are assumed to be multivariate normal with zero mean and covariance matrix  $V$ .  $MVN(0, V)$ , may be transformed to

$$PY = PXB + P\epsilon,$$

$$Z = PXB + \delta$$

where the errors  $\delta$  are multivariate normal  $MVN(0, I)$ . However, the determination of the required matrix  $P$  is generally quite tedious and requires a special program unless  $V$  is diagonal. More theoretical discussion of the general transformation approach and weighted least squares can be found in texts such as Draper and Smith [1966] or Neter and Wasserman [1974].

Other factors could be included in the model. Examples might be manufacturer, car size, etc. The inclusion of these would result in a larger number of subpopulations being defined. Some reduction in the number of model years considered would be necessary to avoid excessive numbers of populations with empty cells.

Polynomial models in proportions are clearly not the only models which could relate the outage rate to the age of the vehicle or to combinations of factors. A logistic transformation of the proportions could be considered, or a probit model. While the GENCAT program automatically calculates the linear Taylor series approximation to quite complicated transformations of the proportions, in order to use an ordinary least squares program, the investigator would have to calculate the appropriate estimates of the variances and form the transformation from them. While this can be done, the advantage of an ordinary regression program is quickly lost if much external calculation must be done. However, as long as a linear model in the proportions themselves is desired, the method described here may save effort.

For comparison, Table 4 gives the coefficients of the same model estimated from ordinary least squares regression. In this table, the proportions themselves are regarded as the dependent variables observed. No consideration of the different variances is included. The parameter estimates are not extremely different in this case, but they could be if some points with large variances did not agree well with the fitted quadratic curve. In fact, in using some sort of variable selection procedure to determine the degree of polynomial or the other form of the model, one

Table 4. Results of regression of proportion failing on age

	OLS		WLS		
	Coefficient	Standard error	Coefficient	Standard error	
$b_0$	0.0248	0.03537	-0.0176	0.01011	0.00917
$b_1$	0.1300	0.00958	0.1485	0.00400	0.00363
$b_2$	-0.0045	0.00055	-0.0059	0.00031	0.00029

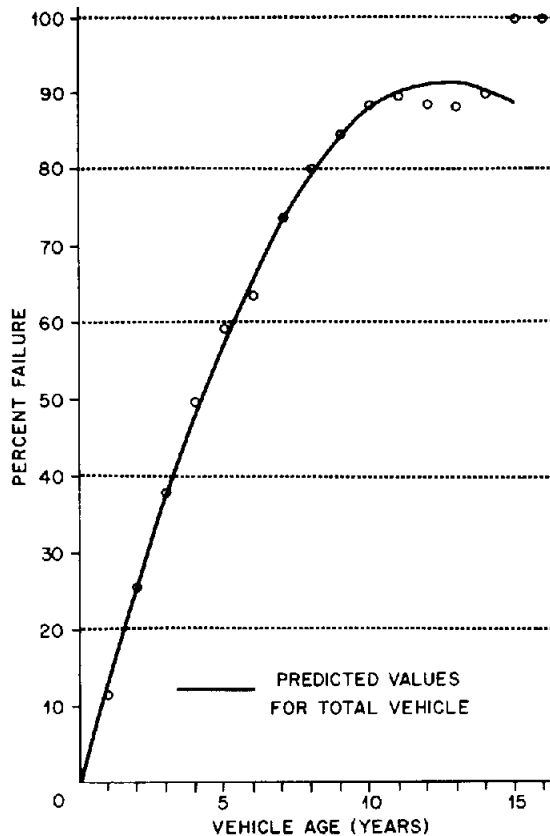


Fig. 1. Total vehicle inspection failure rate by age with quadratic model.

might well select quite different models depending on whether weighted least squares or ordinary least squares was used. Even if a particular model is assumed, the use of the weighted least squares procedure results in reduced estimates of the standard errors of the parameters (and hence of the predicted values).

One other feature of the alternative methods of doing the calculations may be noted. Typical regression programs provide a standard set of output with one or two optional sets of output available. In using GENCAT, all of the tests of significance of parameters must be obtained by using contrast matrices. Thus, the user must specify and input all of the contrast matrices needed to obtain tests of the parameters, which are usually desired. This adds some inconvenience to the use of GENCAT. On the other hand, it also provides more flexibility to the user in the contrasts that can be tested. The tradeoff is one of convenience for generality.

#### APPLICATIONS

In the example already discussed, the relationship of the total vehicle inspection failure rate to the age of the vehicle is developed. The actual data together with the quadratic model fit to the data are presented in Fig. 1. Such information would be necessary in order to compare the overall inspection rates among two or more jurisdictions since the age distribution might well differ.

A similar approach was used to develop models relating the outage rate for specific components to the age of the vehicle. The results of the weighted least squares analyses for the several components are summarized in Table 5. The data together with the best fitting smooth curve are plotted in Figs. 2-6 for the components tires, steering, lights, brakes, and exhaust.

The failure rate for tires—generally insufficient tread—follows approximately a quadratic function with age, rising to a maximum of about 30% of the vehicles in this set. There appears to be some indication of a systematic departure from the model, perhaps related to replacement



Table 5. Relation of component outage rates to age

Component	WLS Model	Coefficients	Std. Errors	Chi-square	Goodness of Fit
Tires	quadratic	$b_0 = -0.0374$	0.00388	146.91	$R^2 = 95.5\%$
		$b_1 = 0.0425$	0.00248	295.72	Error $\chi^2=18.27$
		$b_2 = 0.0013$	0.00026	25.92	13 df p=.145
Steering	quadratic (squared term only)	$b_2 = 0.00017$	0.00005	12.52	$R^2 = 63.9\%$ Error $\chi^2=8.41$ 12 df p=0.08
Lights	Cubic (no squared term)	$b_0 = -0.0224$	0.00978	5.26	$R^2 = 92.2$
		$b_1 = 0.0822$	0.00664	153.34	Error $\chi^2=21.93$
		$b_3 = 0.00011$	0.00005	4.43	14 df p=0.08
Exhaust	No model fit adequately				Lack of fit linear 74.46 quadratic 63.39 cubic 24.74
Brakes	linear	$b_0 = -0.0285$	0.00212	181.13	$R^2 = 98.25$
		$b_1 = 0.0283$	0.00079	1295.04	Error $\chi^2=23.03$ 13 df p=0.04

time, but there was insufficient evidence to document such a trend. The systematic pattern may be noted in that the observed failure rate drops below the predicted at age 5 and 6, then again at ages 9 and 10, and at 14 and 15. This might be related to a general 4-5 yr tire replacement cycle, rather than a careful continual monitoring by owners with replacement when wear indicators begin to show.

The rate of steering defects in the population was quite small, with a maximum of less than 4%. Nevertheless, an increasing trend was noted with approximately a quadratic shape. A relatively large amount of variation in the rates is noted. This is a consequence of the fact that typically only very few cars with steering problems were found for any given age.

Lights were the most frequent components causing a failure of the inspection. The

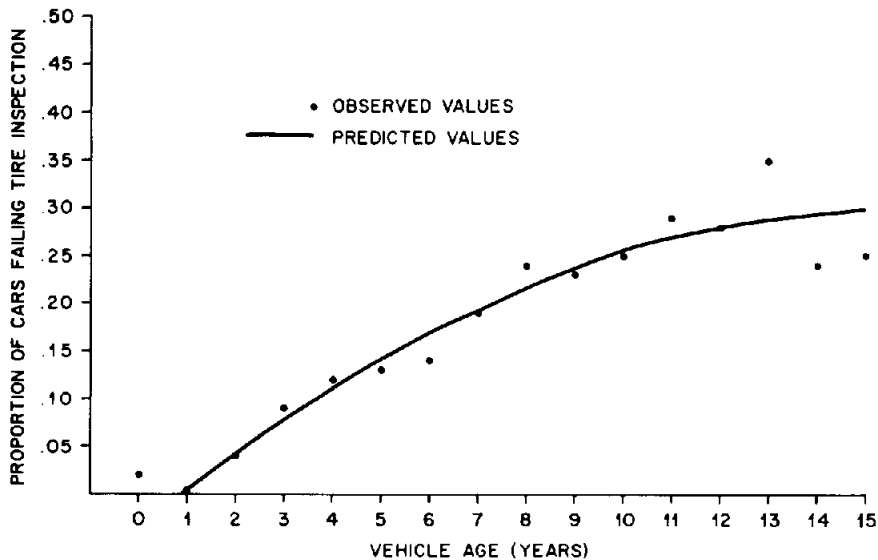


Fig. 2. Proportion of cars failing tire inspection by age with quadratic model.

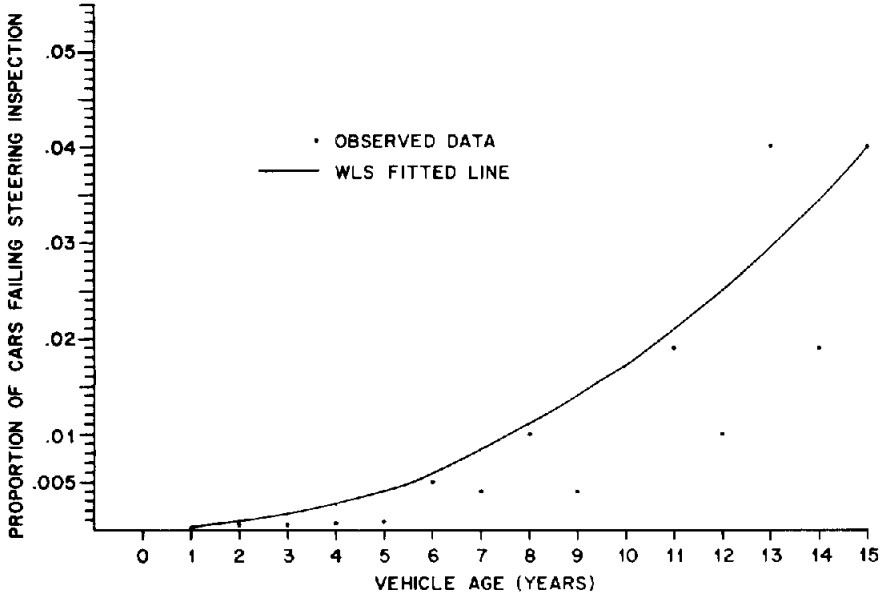


Fig. 3. Proportion of cars failing steering inspection by age with quadratic model.

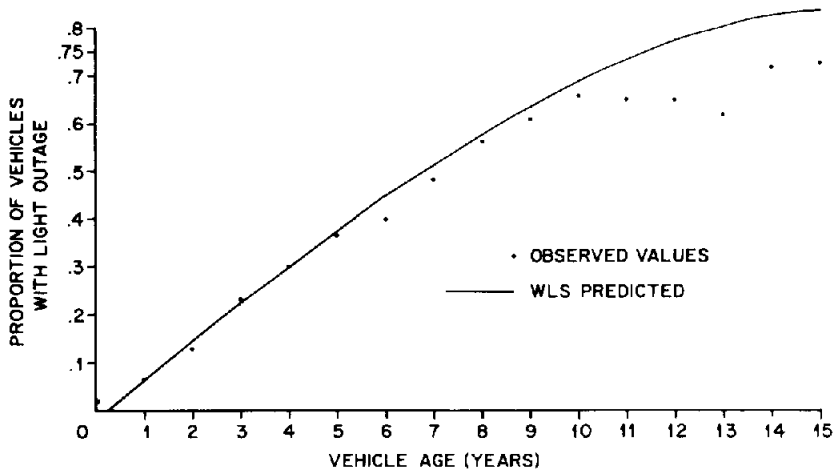


Fig. 4. Proportion of cars with light outages by age with cubic model.

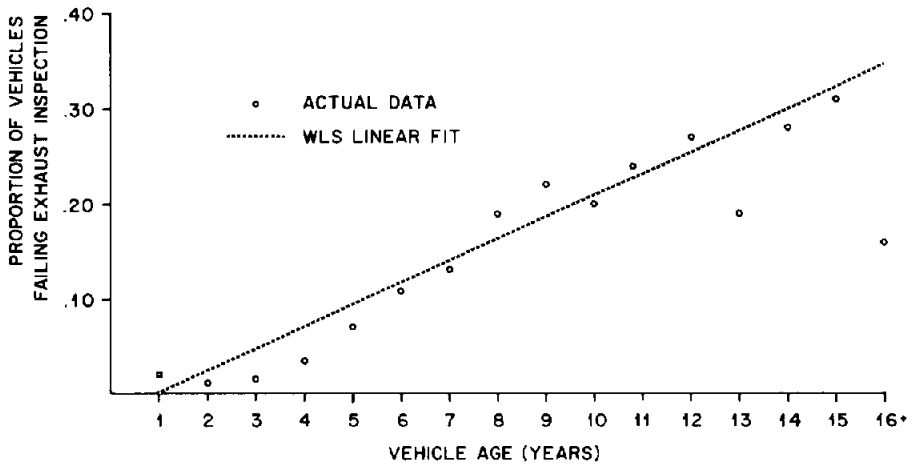


Fig. 5. Proportion of cars failing exhaust inspection by age with linear model.

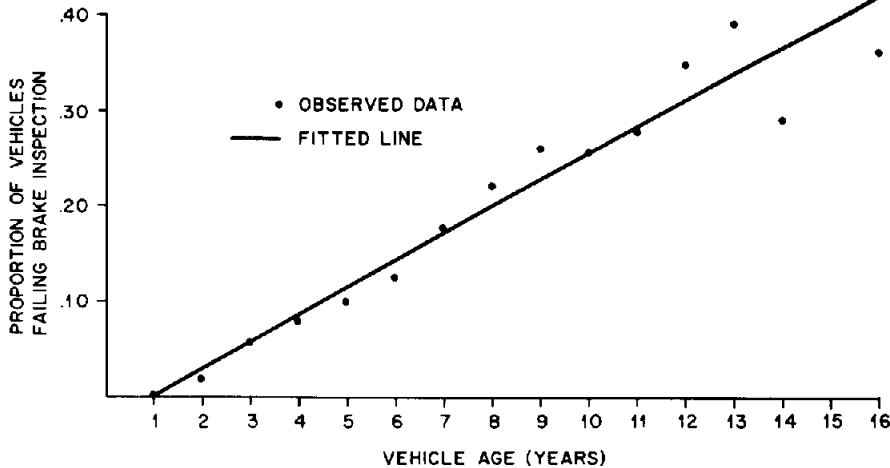


Fig. 6. Proportion of cars failing moving stopping test by age with linear model.

relationship of outage rate for lights to age of the vehicle is seen in Fig. 4. The actual model which best fits is a cubic with no quadratic component. However, a quadratic or even a linear could also be used reasonably well.

The exhaust system exhibits a strong dependence on age, but no simple polynomial model (up to cubic) seems to fit adequately. A linear trend through ages 1–10 yr seems reasonable, but the large fluctuations or outlying points at age 12 and 15 appear to make it very difficult to fit these data.

The condition of the brakes as measured by the moving stopping test shows a strong linear dependence on age as is apparent in Fig. 6. There is a marginal fit of the linear term to the data. Basically there seems to be somewhat too much variation at the older ages which has led to some indication of lack of fit. Neither the quadratic nor cubic terms improved the fit significantly, so the general conclusion is that the proportion of cars failing the moving stopping test depends on the age of the car in approximately linear fashion, but that for the older ages, there is more variation in the failure rates than would be expected.

#### SUMMARY

The proportion of vehicles failing a mechanical inspection is a function of age of the vehicle, with older vehicles exhibiting a higher failure rate. The relationship of the failure rate to the age is, in general, increasing with age, but different shapes and rates of increase with age are noted for different components. Since different populations of cars typically have different age distributions, comparison of inspection failure rates, from state to state, for example, may be misleading. In addition, overall vehicle failure rates may differ because of inclusion of different components in the inspection. Use of a model to describe the relation of a specific component's failure rate to the age of the vehicle may be accomplished using weighted least squares. The necessary calculations may be performed by standard regression programs with slight manipulations of the data and with careful interpretation of the output.

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