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Letter Section

On the use of the volumetric thermal expansion coefficient in models of ocean floor topography

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ABSTRACT

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The use of the volumetric thermal expansion coefficient, instead of the linear coefficient, in successful models of ocean floor topography implies that the elastic rigidity of the lithosphere relaxes, enabling isostasy to be achieved. However, the presence of a thin elastic lid in the lithosphere, inferred from gravity investigations, implies some rigidity at the top of the lithospheric column and suggests that the volumetric thermal expansion coefficient derived from rheologically uniform models of the topography is about 15% too small.

In their excellent book *Plate Tectonics*, Le Pichon et al. (1976) review the basic theory that relates the evolution of ocean floor topography to the thermal evolution of the oceanic lithosphere. As they note, many authors have explained the systematic increase in ocean depth away from the ridge crest by thermal contraction of the lithosphere as it moves away from the spreading axis. The theory that relates the topography to the thermal history is an isostatic theory, with mass balance between lithospheric columns achieved through a temperature dependent density, introduced via the volumetric thermal expansion coefficient $\rho(T) = \rho_0(1+\alpha_v T)$. However, Le Pichon et al. also note (p. 162) that the isostatic theory assumes that all the expansion of the lithosphere upon heating will be in the vertical direction and that lateral expansion is insignificant. They then point out the dilemma that expansion in only one direction should properly be characterized by the linear thermal expansion coefficient α_l, while it is the volumetric thermal expansion coefficient α_v, some three times greater than α_l, that successfully predicts the evolution of oceanic topography.

Approaching the problem of the topography not isostatically but directly through thermoelasticity theory seemingly confirms the dilemma, but in the end resolves it. Let us consider the semi-infinite region $z \geqslant 0$ with a temperature distribution $T(z)$ in the depth range $0 \leqslant z \leqslant h$ and zero below. Because the temperature distribution is one-dimensional (vertical), the displacements likewise will be, and all derivatives with respect to horizontal coordinates will vanish. The relevant thermo-elastic equations (see, e.g., Boley and Wiener, 1960) yield:

$$(\lambda+2\mu)\frac{\partial^2 w}{\partial z^2} - (3\lambda+2\mu)\alpha_l \frac{\partial T}{\partial z} = 0$$

where w is the vertical displacement, α_l the linear thermal expansion coefficient, and λ and μ are the Lame elastic constant and rigidity, respectively. The boundary conditions are that the surface $z = 0$ be stress free and that $w = 0$ at $z = h$. The appropriate solution is:

$$w(z) = -\alpha_l \frac{3\lambda+2\mu}{\lambda+2\mu} \int_z^h T(z)\mathrm{d}z$$

The presence of α_l, the linear thermal expansion coefficient, appears to confirm the dilemma of Le Pichon et al. However, the resolution of the dilemma resides in the coefficient of the integral.

As previously noted, all the displacement is vertical, and recalling that seismological observations indicate that $\lambda \simeq \mu$ for the crust and upper mantle, one obtains the coefficient of the integral as $\sim 5\alpha_l/3$, less than the $3\alpha_l$ ($\simeq \alpha_v$) required by the isostatic theory of the topography. This is because the full expansion of a vertical column proportional to $3\alpha_l$ is resisted by the elastic rigidity. However if the rigidity is relaxed ($\mu \rightarrow 0$) one obtains the coefficient of the integral as $3\alpha_l \simeq \alpha_v$, and the apparent dilemma is resolved.

Under what conditions can one assume the rigidity to vanish? Not under the short term applied stresses of elastic wave propagation, because shear waves do propagate through the oceanic lithosphere and show $\mu \simeq \lambda$. Apparently, under longer term vertical loadings, however, the rigidity effectively relaxes throughout a substantial part of the lithosphere and enables isostasy to be achieved. Analyses of oceanic gravity anomalies and isostasy suggest that the best rheologic model for the oceanic lithosphere is a 20—30 km elastic plate overlying a weak fluid (Watts, 1978). Provided the fluid layer is substantially thicker than the elastic plate, the thermal topography will be controlled principally by the expansion of the fluid, for which the use of the volumetric thermal expansion coefficient is appropriate. Parsons and Sclater (1977) estimate the total thickness of the thermal boundary layer to be 125 km, with an effective volumetric thermal expansion coefficient of $3.2 \cdot 10^{-5}$ °C^{-1}. Of this 125 km layer an upper 20—30 km elastic zone is a moderately small but not negligible fraction. Accordingly, one can attribute the topography principally but not entirely to the expansion of the weak zone beneath the thin elastic lid; a complete analysis should address the expansion within the elastic zone as well. The displacement at the surface of the elastic-fluid composite can be calculated as follows:

$$w(0) = (5\alpha_l/3) \underbrace{\int_0^{h_1} T_0 \, (1 - z/z_2)\, dz}_{\text{elastic}} + 3\alpha_l \underbrace{\int_{h_1}^{h_2} T_0 \, (1 - z/z_2)\, dz}_{\text{fluid}}, \quad h_2 > h_1 \qquad (1)$$

where the integrand is the temperature difference between an initial high temperature T_0 throughout the boundary layer, and an equilibrium temperature distribution of $T_0 z/h_2$ after cooling; h_1 and h_2 are the thicknesses of the elastic lid and full boundary layer, respectively. With an elastic lid of 25 km, for this surface displacement to be equivalent to the expansion of an entirely fluid layer with $\alpha_V = 3.2 \cdot 10^{-5}$ °C^{-1} requires α_l to be $1.27 \cdot 10^{-5}$ °C^{-1}, or equivalently $\alpha_V (\simeq 3\alpha_l)$ to be $3.8 \cdot 10^{-5}$ °C^{-1} throughout the composite boundary layer, because of the constraining effect of the elastic lid. The tabulation by Skinner (1966) of α_V for eight olivine samples at 800°C (approximately the mean temperature of the steady state boundary layer) shows a mean of $3.7 \cdot 10^{-5}$ °C^{-1}, in close although perhaps fortuitous agreement with the requirements of the thermoelastic analysis of the boundary layer with an elastic lid.

In conclusion, because the oceanic lithosphere behaves principally as a weak fluid in response to vertical loads, the thermal expansion topography can be successfully modelled using the volumetric thermal expansion coefficient. However, because the upper few tens of kilometers of the lithosphere behave elastically, the apparent volumetric thermal expansion coefficient estimated from the thermal boundary layer analysis is likely to be some fifteen percent smaller than the intrinsic volumetric thermal expansion coefficient of the material.

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